Euler Equations and Money Market Interest Rates:

A Challenge for Monetary Policy Models

by

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ABSTRACT

Standard macroeconomic models (with or without nominal rigidities) assume that the money market interest rate is equal to the interest rate implied by a consumption Euler equation in which the real (nominal) interest rate is proportional to the expected growth rate of real (nominal) consumption. The empirical literature shows that a monetary tightening slows consumption growth for a few quarters. A decline in expected consumption growth will reduce the real interest rate implied by the Euler equation. This is a problem because the empirical literature shows that money market rates respond in the opposite direction. We calculate real and nominal interest rates first for standard, additively separable preferences with constant relative risk aversion and then for three specifications of preferences with habit persistence. In addition to presenting summary statistics, we plot estimates of the time series of interest rates from 1965 to 1999. We find that money market rates are low when the implied Euler equation rates are high and the spread between them appears to vary systematically with indicators of monetary policy. Most strikingly, for several of the specifications, money market interest rate rose relative to the consumption Euler equation rates during the period of Federal Reserve tightening in 1979-1982. This poses a challenge for models that equate the two.

* The earlier version of this paper was titled, “The Spread Between the CCAPM Interest Rate and the Treasury-bill Rate is Correlated with Monetary Policy.” We have benefitted from discussions with Rochelle Edge, B. Ravikumar, and Michael Woodford. The usual disclaimer applies.

I. Introduction

The analysis of monetary policy has advanced markedly with the development of optimizing models that include nominal rigidities.[1] Goodfriend and King (1997) characterize this literature as one in which "macroeconomics is moving toward a New Neoclassical Synthesis"–henceforth, NNS. An attractive feature of the NNS is that its models, which incorporate explicit optimizing behavior, can yield welfare-based comparisons of alternative monetary rules. Moreover, NNS models are generally more successful than models without nominal rigidities in replicating key features of the response of macroeconomic time series to a number of shocks. Basic NNS models, however, have difficulties in accounting for the apparently persistent effects of monetary shocks on output and inflation, as Chari, Kehoe and McGratten (2000), for example, point out.

Adding habit persistence to NNS models – possibly, in conjunction with real rigidities – seems to hold promise in addressing these persistence problems and other empirical regularities. Fuhrer (2000), for example, shows that adding habit persistence to consumers preferences in an NNS model can generate the "hump shaped," gradual response of real spending and inflation to shocks, including monetary policy shocks, found in unrestricted vector autoregressions.[2]

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[1] For example, Goodfriend and King (1997), Rotemberg and Woodford (1997, 1999), King and Wolman (1999), and Erceg, Henderson, and Levin (2000).

[2] Habit persistence has generally been examined as a solution to the equity premium puzzle (for example, Abel (1990), Constantinides (1990), and Campbell and Cochrane (1999)). More recently, Boldrin, Christiano, and Fisher (2001) introduce habit persistence (along with factor-market rigidities) into a real business cycle model and show that doing so improves its ability to account for the observed persistence in output growth, the co-movement of employment across sectors over the business cycle, and the excess sensitivity of consumption to current income.

The NNS models share another problem, one that results from equating the money market interest rate with the interest rate implied by the consumption Euler equation. This building block is central to the models as it provides the link between monetary policy, typically modeled as interest rate setting by the central bank, and real variables.[3] The empirical literature using vector autoregressions to study the effects of monetary policy shocks finds that an unexpected monetary tightening leads to a rise in real and nominal interest rates and calls this the "liquidity effect." This same literature finds that a monetary tightening reduces real GDP (Christiano, Eichenbaum, and Evans (1999)) and real consumption (Fuhrer (2000)) and their growth rates. In this paper we show that these two results are difficult to reconcile in a model that equates the interest rate implied by a consumption Euler equation with money market interest rates.

We consider standard additively separable CRRA preferences and three models of preferences with habit persistence. The intuition, however, is clearest if the representative consumer has additively separable log utility and consumption is lognormally distributed. The consumption Euler equation implies that the real interest rate is proportional to the expected growth rate of real consumption. The empirical literature shows that a monetary tightening has a small effect on consumption in the first quarter following the tightening. In the following few quarters, the decline in consumption increases so that expected consumption growth declines.[4] A

[3] An earlier literature examined one implication of equating money market interest rates to the rates implied by consumption Euler equations – what Weil (1989) called the "risk-free rate puzzle." Combining consumption growth with the Euler equation of representative consumer with standard, additively separable utility implies a real interest rate that is much greater than observed money market rates. In addition, Rose (1988) and others show that standard consumption Euler equations cannot explain the persistence of real short-term interest rates.

[4] This is the "hump-shaped" response mentioned above.

decline in expected consumption growth will reduce the real interest rate implied by the Euler equation. This poses a puzzle because the empirical literature shows that money market rates respond in the opposite direction.[5]

Although the puzzle we highlight and many of our results are closely related to the liquidity effect literature (e.g. King and Watson (1996)), our focus is broader. When we compute real interest rates from consumption Euler equations, we find that they are <u>negatively</u> correlated with real money market rates. This presents a challenge for models that equate the two.

There are two potential ways of resolving this puzzle. One is to consider models that sever the link between consumption Euler equations and money market interest rates. In this paper we pursue the other possibility, which is to consider alternative preferences.[6] As we note above, adding habit persistence to NNS models allows these models to better replicate the response of real variables to monetary shocks. Here we ask whether adding habit persistence to preferences will resolve the puzzle. We find that it does not. In the consumption Euler equations we examine, whether derived from standard additively separable CRRA preferences or from preferences embodying habit persistence, a decline in expected consumption growth is associated with a decline in real interest rates. And because expected inflation declines as well,

[5] In a simple model with one-period price rigidity, an increase in real interest rates implied by a consumption Euler equation can accompany a monetary contraction because consumption drops for one period, leading to an increase in expected consumption growth. Nominal interest rates can, however, rise or fall, depending impact on expected inflation. See, for example, King (1991) and Ohanian and Stockman (1994). But this prediction about consumption growth is inconsistent with the empirical evidence, which raises further issues we address in this paper.

[6] Changing the supply side of the model leaves the consumption Euler equation unchanged and therefore cannot resolve the puzzle.

nominal interest rates implied by consumption Euler equations fall in response to a monetary tightening.

The plan of the paper is as follows. In section II, we calculate real and nominal interest rates first for standard additively separable preferences with constant relative risk aversion and then for three specifications of preferences with habit persistence. We begin with Fuhrer's (2000) model. We turn to the models proposed by Abel (1999) and Campbell and Cochrane (1999) because they are specified to avoid the excessive interest rate volatility that characterizes other habit models, including Fuhrer's.[7] In addition to presenting summary statistics, we plot estimates of the time series of interest rates from 1965 to 1999. This allows us to examine the implied behavior of interest rates during specific historical episodes. We find a substantial spread between the interest rates implied by the consumption Euler equations and observed money market rates. That the two rates differ is not surprising – indeed it has been examined in detail in the "risk-free rate puzzle" literature. What distinguishes our results from this earlier literature is that we find the spread is not constant – money market rates are low when the implied Euler equation rates are high – and it appears to vary systematically with indicators of monetary policy. Most strikingly, the spread declines noticeably during the period of Federal Reserve tightening in 1979-1982. That is, money market interest rate rose relative to the consumption Euler equation rates. In section III, we present evidence on the sensitivity of the spread between the interest rates computed from each of the models and observed money market interest rates to two commonly used measures of monetary policy shocks. We show that interest rates implied by the models fall in response to a monetary tightening while money market

[7] We considered other habit specifications as well but because they exhibit excessive volatility similar to Fuhrer's model we chose not to present the results.

interest rates rise. This implies a challenge for the NNS monetary models. In section IV we conclude by discussing ways in which the NNS models might be modified to meet this challenge.

II. Computing Real and Nominal Interest Rates

In this section we compute real and nominal interest rates implied by consumption Euler equations for four sets of consumers' preferences and compare them with money market rates. In each model, we assume that a representative agent chooses consumption and holdings of two riskless one-period bonds – one that pays one unit of the consumption good and one that pays one dollar. The consumer is assumed to maximize lifetime utility,

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} E_t u(C_s, Z_s)$$

subject to a sequence of budget constraints, where β is the consumer's discount factor and Z_s is the reference, or habit, level of consumption in period s. The first order conditions imply that the prices of the bonds are,

$$\frac{1}{1+r_t} = \beta \frac{E_t\left(\partial U_t / \partial C_{t+1} \right)}{E_t\left(\partial U_t / \partial C_t \right)} \qquad \text{and} \qquad \frac{1}{1+i_t} = \beta \frac{E_t\left(\partial U_t / \partial C_{t+1} \; P_t / P_{t+1} \right)}{E_t\left(\partial U_t / \partial C_t \right)}$$

where r_t is the real interest rate, i_t is the nominal interest rate, and P_t is the price of one unit of the consumption good. The models we consider differ in their specification of the period utility function and therefore in the implied marginal rates of substitution.

2.1 The Standard Preferences

We begin by assuming the representative agent has the standard, additively separable CRRA preferences (so $Z_s = 0$). The period utility function is,

$$u(C_t) = \frac{1}{1-\alpha} C_t^{1-\alpha},$$

where α is the coefficient of relative risk aversion. The corresponding Euler equation is,

$$(1) \qquad (1+i_t)^{-1} = \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\alpha} \frac{P_t}{P_{t+1}} \right]$$

Next, we follow Fuhrer and assume that the dynamics of consumption can be described by the vector autoregression (written in companion form),

$$(2) \qquad Y_t = A_0 + A_1 Y_{t-1} + v_t$$

and let c_t=log(C_t) be the first element (and π_t=log(P_{t+1}/P_t) be the second element) of the vector Y_t.[8] In addition, we assume that the error term, ν_t, is iid N(0,Σ). Under conditional lognormality the Euler equation implies that nominal interest rates are given by,

[8] We adopt the convention that a lower case letter denotes the log of the corresponding upper case letter except for interest rates. As in Fuhrer (2000), the other variables in the VAR are the log of the Journal of Commerce industrial materials commodity price index, the log of per capita real disposable income, the federal funds rate, and the log of per capita real nonconsumption GDP. In addition we follow Fuhrer by measuring consumption as per capita real expenditures on nondurables and services, measuring inflation as the log change in the price index for personal consumption expenditures, and beginning our estimation of the VAR in 1966:1. Unlike Fuhrer, we do not detrend consumption, income, and GDP. Instead, we include a (segmented) time trend in the VAR.

(3) $$\left(1+i_t\right)^{-1}=\beta\exp\left[-\alpha\left(E_t c_{t+1}-c_t\right)-E_t\pi_{t+1}+\frac{\alpha^2}{2}V_t c_{t+1}+\frac{1}{2}V_t\pi_{t+1}+\alpha\operatorname{cov}_t\left(c_{t+1},\pi_{t+1}\right)\right].$$

The expression for real interest rates is identical to (3) without the terms involving (log) inflation.

Assuming α=2 and β=0.993 and the moments obtained from the VAR (see the appendix), we compute the implied real interest rate shown in Figure 1. The contrast between the behavior of the model-generated real rate and the observed ex-post real rate is striking.[9] Most notably, the model-generated rate falls when the money market rate rises during the Volker disinflation and model-generated real rates are high during the late 1970s and early 1990s when market rates are low. The stark difference in the behavior of the two rates can also be seen in Table 1, which presents summary statistics. The average real rate implied by the consumption Euler equation exceeds the ex-post real money market rate by nearly 450 basis points and the correlation between the two is -0.38.

As we discuss above, the reason that the model interest rate fails to mimic the behavior of money market rates in response to monetary policy shocks is clear from (3). A monetary tightening is followed by a decline in GDP and in consumption for several quarters, so expected consumption growth falls. And from (3) a decline in expected consumption growth will reduce the real interest rate implied by the Euler equation. But the empirical literature shows that money market rates respond in the opposite direction. Changing preference will change the

[9] We have also compared the implied rate to an estimate of ex-ante real interest rates obtained by adjusting nominal rates by the one-quarter ahead forecast of inflation from the VAR, as well as nominal rates and find that all three sets of plots convey the same message.

details of the Euler equation, but the role of expected consumption growth will be an enduring feature.[10]

One reason that the interest rates implied by the consumption Euler equation differ substantially from money market interest rates is that an Euler equation might not describe the consumption choices of all individuals, perhaps due to liquidity constraints. Campbell and Mankiw (1989) find that they are able to fit aggregate consumption data well by assuming one group of individuals consumes all of their disposable income while another group chooses consumption optimally over time (without liquidity constraints). The consumption of the first group represents roughly half of aggregate disposable income. In order to determine if this explains the results in Figure 1 and Table 1, we compute the consumption of individual whose consumption obeys the Euler equation as, $c_t^* = \log(C_t - 0.5\, Yd_t)$, and use this series both in the VAR and to compute the Euler equation interest rates. We find that doing so reduces the correlation coefficient between the real Euler equation rate and the ex-post real money market rate from -0.38 to -0.23, but the behavior of the two rates still differs substantially (as the negative correlation coefficient suggests).

2.2 Fuhrer's Model

Fuhrer (2000) assumes that the representative consumer's period utility function is,

$$u\left(C_t, Z_t\right) = \frac{1}{1-\alpha}\left(\frac{C_t}{Z_t^{\gamma}}\right)^{1-\alpha},$$

where the reference or habit level of consumption evolves as, $Z_t = \rho Z_{t-1} + (1-\rho)C_{t-1}$ and $0 \leq \gamma \leq 1$ is

[10] This is the case, for example the non-expected utility preference examined by Epstein and Zin (1987a, b) and Weil (1989, 1990). See also Kocherlakota (1990, 1996).

a parameter indexing the importance of habit. When he estimates his model, Fuhrer finds that the estimate of ρ is close to zero and insignificant, so that his period utility function is,

$$u\left(C_t, Z_t\right) = \frac{1}{1-\alpha}\left(\frac{C_t}{C_{t-1}^{\gamma}}\right)^{1-\alpha} = \frac{1}{1-\alpha}\left(\frac{C_t}{C_{t-1}} C_{t-1}^{1-\gamma}\right)^{1-\alpha}.$$

Unlike in standard models, utility is not separable over time because the current period's choice of consumption affects next period's utility. Alternatively, current utility depends on both the current level of consumption and on the growth of consumption from last period. As a result, consumers will want to smooth both consumption and its growth rate.

Nominal interest rates in Fuhrer's model are then given by,

$$\text{(4)} \qquad \beta\left(1+i_t\right) = \frac{E_t\left[C_{t-1}^{-\gamma(1-\alpha)} C_t^{-\alpha} - \beta\gamma C_{t+1}^{1-\alpha} C_t^{-\gamma(1-\alpha)-1}\right]}{E_t\left[\left(C_t^{-\gamma(1-\alpha)} C_{t+1}^{-\alpha} - \beta\gamma C_{t+2}^{1-\alpha} C_{t+1}^{-\gamma(1-\alpha)-1}\right)\left(P_t / P_{t+1}\right)\right]} \qquad ,$$

and real interest rates are defined similarly. The assumption that consumption (and inflation) are conditionally log normal implies that equation (4) is,

$$\beta\left(1+i_t\right)^{-1} = \frac{\exp\left(a_t\right) - \beta\gamma \exp\left(b_t\right)}{\exp\left(d_t\right) - \beta\gamma \exp\left(e_t\right)},$$

where,

$$a_t = \gamma(\alpha - 1)c_{t-1} - \alpha c_t$$

$$b_t = \left(\gamma(\alpha - 1) - 1\right)c_t + (1-\alpha)E_t c_{t+1} + \frac{(1-\alpha)^2}{2} V_t c_{t+1}$$

$$d_t = \gamma(\alpha - 1)c_t - \alpha E_t c_{t+1} - E_t \pi_{t+1} + \frac{\alpha^2}{2} V_t c_{t+1} + \frac{1}{2} V_t \pi_{t+1} + \alpha \operatorname{cov}_t\left(c_{t+1}, \pi_{t+1}\right)$$

$$e_t = \left(\gamma(\alpha - 1) - 1\right)E_t c_{t+1} + (1-\alpha)E_t c_{t+2} - E_t \pi_{t+1} + \frac{\left(\gamma(\alpha - 1) - 1\right)^2}{2} V_t c_{t+1} + \frac{(1-\alpha)^2}{2} V_t c_{t+2} + \frac{1}{2} V_t \pi_{t+1}$$
$$+(1-\alpha)\left(\gamma(\alpha - 1) - 1\right)\operatorname{cov}_t\left(c_{t+1}, c_{t+2}\right) - (1-\alpha)\left(\operatorname{cov}_t\left(\pi_{t+1}, c_{t+2}\right) + \operatorname{cov}_t\left(\pi_{t+1}, c_{t+1}\right)\right)$$

The corresponding expression for real interest rates excludes the terms involving (log) inflation. Again, using the conditional moments from the VAR, and the parameter values reported by Fuhrer (2000), we can compute the time series of real and nominal interest rates implied by Fuhrer's model.

As can be seen from Figure 2 and Table 1, the time series of real interest rates implied by Fuhrer's model bear little resemblance to observed (ex-post) real money market rates. The average real rate computed from the model is about twice the average computed from the data and the standard deviation of the real rate computed from the model is about 12 time that computed from the data. Even more striking is the range of variation of the real rates computed from the model, with a minimum of less that -60 percent and a maximum of more than 90 percent.[11]

The reason that adding habit persistence raises interest rate variability is that it strengthens the desire for a smooth path of consumption. Greater interest rate movements are therefore needed to induce consumers to overcome their desire for smooth consumption and willingly accept a given volatility of consumption. Jermann (1998) shows that introducing an elastic supply of capital can reduce interest rate variability, but would do so at the expense of inducing a lower volatility of consumption than is observed. Restoring realistic consumption variability by introducing costs of adjusting the capital stock re-introduces excessively volatile interest rates.

Another way of stating this feature of habit models is that interest rates are often quite sensitive to changes in the path of consumption. As a result, there is often a trade-off between realistic interest rate behavior and realistic consumption behavior.

Fuhrer's model of habit is hardly alone – the same problem is shared by nearly all habit models. As we note above, we focus on Fuhrer's model only because it has been successful in other regards.[12] Despite these other successes, our calculations suggest that these models are missing a fundamental feature of money market interest rates. We next turn to two other habit

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[11] We experimented with a smaller VAR to see if doing so would reduce the variability in our estimates of expected one-quarter-ahead consumption growth and therefore reduce the variability of the implied interest rates. We found that doing so had little effect.

[12] An interesting new direction is proposed by Edge (2000), who introduces nonseparability of consumption and real balances in household preferences along with habit persistence in a model with time-to-plan and time-to-build in investment. She is able to generate impulse responses with both a decline in interest rates and a hump-shaped response of consumption to a monetary expansion. Although we haven't fully explored the implications of the nonseparability of consumption and real balances, when we compute interest rates from her log-linearized Euler equation without nonseparability and with actual consumption and price data, we find the excessive interest rate volatility that plagues other habit models.

specifications that have been parameterized to overcome this problem of excessive interest rate volatility.

Table 1
Summary Statistics for Real and Nominal Interest Rates
(percent per annum)

		Rates Computed from Models					
		Conditionally Lognormal Consumption				iid Consumption Growth	
Real Rates	Data	CRRA	Fuhrer	Abel	Campbell - Cochrane	Abel	Campbell-Cochrane
Mean	2.73	7.20	5.72	7.55	2.31	6.06	2.34
Std Deviation	2.42	1.85	29.18	25.47	1.83	2.42	0.00
Minimum	-2.63	1.14	-62.03	-70.93	-3.68	0.34	2.34
Maximum	10.65	10.18	94.29	52.42	5.26	15.75	2.34
Corr(data, model)		-0.38	-0.08	-0.38	-0.38	0.20	0.00
Nominal Rates	Data	CRRA	Fuhrer	Abel	Campbell - Cochrane		
Mean	7.22	11.72	10.22	12.02	6.82		
Std Deviation	3.10	1.83	29.26	24.43	1.78		
Minimum	2.99	7.96	-54.84	-62.15	3.18		
Maximum	17.78	16.03	103.60	55.36	11.07		
Corr(data, model)		-0.04	-0.15	-0.80	-0.05		

2.2 Abel's Model of Catching Up with the Joneses

Like Fuhrer, Abel (1999) specifies the representative agent's period utility function in a way that depends on the ratio of current consumption to a reference, or habit, level of consumption,

$$u(C_t, Z_t) = \frac{1}{1-\alpha}\left(\frac{C_t}{Z_t}\right)^{1-\alpha} .$$

There is, however, an important difference – habit is assumed to be external, rather than internal. That is, the reference level of consumption depends on lagged aggregate consumption, rather than lagged individual consumption. In particular, Abel assumes that $Z_t = \overline{C}^{\gamma}_{t-1}G^{\eta t}$, where $\overline{C}_{t-1}$ is lagged aggregate consumption and G is the unconditional gross growth in reference consumption.

Assuming that habit is external greatly simplifies the representative agent's intertemporal marginal rate of substitution. In Abel's specification, nominal interest rates are then given by,[13]

$$(5) \qquad \left(1+i_t\right)^{-1} = G^{\eta(\alpha-1)}\beta E_t\left[\left(\frac{C_t}{C_{t-1}}\right)^{\gamma(\alpha-1)}\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha}\frac{P_t}{P_{t+1}}\right] \quad .$$

Assuming conditional log normality,

$$(6) \left(1+i_t\right)^{-1} = G^{\eta(\alpha-1)}\beta\exp\left[-\gamma(\alpha-1)c_{t-1} + \left(\alpha+\gamma(\alpha-1)\right)c_t - \alpha E_t c_{t+1} - E_t\pi_{t+1} + \frac{\alpha^2}{2}V_t c_{t+1} + \frac{1}{2}V_t\pi_{t+1} + \alpha\,\mathrm{cov}_t\left(c_{t+1},\pi_{t+1}\right)\right].$$

We compute real and nominal rates using Abel's specification under two sets of assumptions. First, we assume consumption growth is iid lognormal (as in Abel (1999)) and choose parameters using Abel's algorithm, which matches the mean and variance of real money market rates to those computed from a linearized version of his model. Next, we assume consumption and inflation are conditionally lognormal and compute the conditional moments from the vector autoregression (2).

As can be seen in Figure 3A and Table 1, the iid lognormal specification does not suffer from the extreme volatility of the real interest rate found in Fuhrer's specification. The average real rate, about 6 percent at an annual rate, is too high by a factor of more than two, but its

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[13] In equilibrium individual and aggregate consumption growth rates will be equal.

standard deviation is virtually identical to that found in the data.[14] And the wild swings in real rates computed from Fuhrer's specification are notably missing here. The implied model rate differs significantly from the money market rate in much of the sample (particularly around the time of the recessions of 1973:4 - 1975:1 and 1990:3 - 1991:1 as well as during the late 1970s and early-to-mid 1990s), but it does appear to capture the early-1980s disinflation fairly well.

The extreme volatility reappears, however, when we allow expected consumption growth to vary over time. Although the model rates exhibit less high-frequency volatility than those computed from Fuhrer's model, the standard deviation of the model's real interest rate exceeds that observed in the data by a factor of more than ten and range from less than -60 percent to more than 50 percent. In addition, the correlation between the implied Euler equation rates and the money market is strongly negative.[15] Figure 3B shows that the lowest real interest rates implied by the model are found around the time of the Volker disinflation when money market rates were at their peak. The implied real rates are also markedly negative during the 1973:4 - 1975:1 and 1990:3 - 1991:1 recessions.

In part, this reappearance of extreme volatility may be due to the use of parameters calibrated under the assumption of iid consumption growth. In order to reduce the volatility in the implied rates, we searched for parameter values that matched the average interest rate and

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[14] The difference between the average model rate and the average market rate is somewhat surprising because the parameters are chosen to match the two. The difference arises because the size of the linearization error. Average consumption growth and average real interest rates are higher in our sample than in Abel's. When we calibrate the model to our sample, the error from linearization turns out to be higher than that reported in Abel (1999).

[15] The correlation coefficient is virtually identical to that we compute using power utility because γ is much smaller than α so that expected consumption growth dominates lagged consumption growth in the Euler equation.

minimized its standard deviation, without regard to matching the average equity premium. As can be seen in Figure 3C, this alternate set of parameter values results in a considerable reduction in volatility, but the negative correlation between the money market rate and the real rate implied by the consumption Euler equation is apparent.

2.3 Campbell and Cochrane's Model of External Habit

Like Abel, Campbell and Cochrane (1999) assume that habit is external, but unlike Abel and Fuhrer, they assume that period utility depends on the difference between consumption and the reference, or habit, level (rather than the ratio of the two) and rewrite the period utility function in terms of the "surplus consumption ratio," $S_t = (C_t - Z_t)/C_t$. In particular, they assume,

$$u(C_t, Z_t) = \frac{1}{1-\alpha}\left(C_t - Z_t\right)^{1-\alpha} = \frac{1}{1-\alpha}\left(C_t S_t\right)^{1-\alpha}.$$

Nominal interest rates are then,

$$(7)\ \left(1+i_t\right)^{-1} = \beta E_t\left[\left(\frac{C_{t+1}}{C_t}\frac{S_{t+1}}{S_t}\right)^{-\alpha}\frac{P_t}{P_{t+1}}\right].$$

The reference, or habit, level of consumption – and therefore the surplus consumption ratio – adjusts over time as aggregate consumption changes.[16] Campbell and Cochrane assume the log of the surplus consumption ratio evolves as,

$$s_{t+1} = \left(1-\phi\right)\bar{s} + \phi s_t + \lambda\left(s_t\right)\left(E_t c_{t+1} - c_t\right),$$

where ϕ and $\bar{s} = \log \bar{S}$ are parameters. By assuming consumption growth is iid log normal and

[16] As in Abel (1999), in equilibrium with identical consumers, aggregate and individual consumption will be equal.

choosing $\overline{S} = \sigma_c(\alpha/(1-\phi))^{.5}$, $1+\lambda(s_t) = (1/\overline{S})(1-2(s_t-\overline{s}))^{.5}$ Campbell and Cochrane obtain a constant real rate.

$$(1+r_t)^{-1} = \beta \exp\left[-\alpha\left(E_t c_{t+1} - c_t\right) - \alpha(\phi - 1)(s_t - \bar{s}) + \frac{\alpha^2 \sigma_c^2}{2}\left(1 + \lambda(s_t)\right)^2\right] = \beta \exp\left[-\alpha\left(E_t c_{t+1} - c_t\right) + \frac{\alpha}{2}(1-\phi)\right].$$

As the consumption nears the reference, or habit, level, the log of the surplus consumption ratio approaches negative infinity. By assuming that the effect of consumption uncertainty rises as the log surplus consumption ratio falls, Campbell and Cochrane are able to engineer risk premia – and, therefore, a desire for precautionary saving – that rise sharply as consumption approach its habit level. This precautionary savings motive moves in a way that exactly offsets effects of desired intertemporal substitution on real interest rates, leaving real rates constant if consumption growth is iid.

As with the other models, we assume that consumption and inflation are conditionally log normal and compute real and nominal interest rates.[17] The results are summarized in Table 1 and in Figure 4. As can be seen in table 1, when we assume consumption growth and inflation are iid log normal, the real interest rate is constant at an annual rate of 2.34 percent. Allowing expected consumption (and inflation) to vary according to the VAR has almost no impact on the average real and nominal interest rates in the Campbell-Cochrane specification and adds relatively little to the volatility of either rate. In fact, the volatility of both rates is below that observed in the data. As is clear from Table 1, although the Campbell-Cochrane specification succeeds in eliminating the problem of excessive volatility in interest rates, money market rates and the

[17] We use the parameter values chosen by Campbell and Cochrane in our calculations.

implied consumption Euler equation rates are negatively correlated.[18] This negative correlation is readily seen in Figure 4. The model rates and market rates diverge sharply during the Volker disinflation and differ substantially in a number of other periods. For example, the model rates rise in the late 1970s when market rates decline, decline in the early 1980s and, again in the late 1980s, when market rates rise. In the next section, we link these differences to differing responses of model rates and market rates to monetary policy shocks.

III. The Response of Model and Market Interest Rates to Monetary Policy Shocks

In this section we raise additional doubts about the use of Euler equations whether from standard models with power utility or from habit models to obtain short term interest rates in models of monetary policy. The problem we examine – the sensitivity of the spread between interest rates generated from these Euler equations and money market interest rates to monetary policy shocks – is suggested by the figures discussed in Section II. We begin by showing that the spread rises when monetary policy eases and declines when monetary policy tightens. We then show that this reaction of the spread to shocks to monetary policy does not arise simply because market rates respond more than do model rates, but because the two rates move in opposite directions in response to a monetary policy shock.

We begin with regressions of the spread (defined as the model-generated interest rate less the money-market interest rate) on four lags of the spread and (individually) two measures of monetary policy suggested by Christiano, Eichenbaum, and Evans (1999). The first is the federal funds rate and the second is the ratio of nonborrowed reserves plus extended credit to

[18] The correlation coefficient is virtually identical to that obtained with power utility because both Euler equations depend on expected consumption growth plus a constant.

total reserves (which we refer to as the S-ratio, after Strongin (1995)).

Table 2
Response of Interest Rate Spreads to Monetary Policy
(standard errors in parentheses)

Real Rates	Conditionally Lognormal Consumption				iid Consumption Growth	
Monetary Policy Indicator	CRRA	Fuhrer	Abel	Campbell - Cochrane	Abel	Campbell-Cochrane
Federal Funds Rate	-0.563 (0.075)	-1.433 (0.962)	-3.820 (0.761)	-0.563 (0.075)	-0.104 (0.065)	-0.240 (0.042)
S-Ratio	0.086 (0.013)	0.791 (0.216)	0.937 (0.105)	0.086 (0.014)	0.007 (0.016)	0.034 (0.010)
Nominal Rates	Conditionally Lognormal Consumption					
Monetary Policy Indicator	CRRA	Fuhrer	Abel	Campbell - Cochrane		
Federal Funds Rate	-0.439 (0.060)	-1.435 (0.966)	-3.536 (0.738)	-0.447 (0.064)		
S-Ratio	0.068 (0.010)	0.795 (0.217)	0.911 (0.102)	0.068 (0.010)		

Notes: In each regression, the spread (defined as the interest rate computed from each model's Euler equation less the federal funds rate) is regressed on four lags of the spread and one of the two indicators of monetary policy. Only the coefficients and corresponding standard errors for the monetary policy variables are reported.

The regression results are reported in Table 2. The results for real and nominal rates are virtually identical – both show that monetary expansions are associated with a decline in the money market rate relative to the model rate. The estimated coefficients for the federal funds rate are negative for all of the preferences so the measured spread (model rate - market rate) rises when monetary policy eases. When we assume that the data are conditionally lognormal, the coefficients are highly significant for power utility and for two of the three habit models. The coefficient for the third, Fuhrer's model, is significant at only the 14 percent level. Given the extreme volatility of rates computed from this model, it is somewhat surprising that the coefficient is measured as precisely as it is. And when we assume that consumption growth is iid lognormal, the coefficient estimated using the real rate generated from Campbell and Cochrane's model is highly significant while that using the rate generated from Abel's model is significant at only about the 11 percent level.

The results from the regressions using the S-ratio are even stronger. Each of the estimated coefficients is positive, again indicating that a monetary expansion (an increase in the S-ratio) increases the measured spread (reduces the money market rate relative to the model-generated rate). Except for the coefficient estimated using Abel's model with iid lognormal consumption, each of the coefficients is highly significant. These results all suggest that the habit models, like the standard CRRA models, are missing something systematic about the way that monetary policy influences real and nominal interest rates.

Next, we add the spreads (one at a time) and the indicators of monetary policy to the basic vector autoregression and examine the responses of the spreads to innovations to monetary policy. Figure 5 contains the responses of the spread between the model-generated and money market real interest rates to shocks to the S-ratio (the left-hand figures) and the federal funds rate (the right-hand figures). A monetary expansion (a positive shock to the S-ratio or a negative shock to the federal funds rate) results in a rise in the spread – that is, a decline in the market rate relative to the model rate and the response is quite persistent for power utility and for two of the three habit models. The spread computed from Abel's specification rises for six quarters following a shock to the S-ratio and for nine quarters following a shock to the federal funds rate. Both of these are statistically significant for four quarters. The response of the spread computed from Campbell and Cochrane's model is even more persistent and is significant for six quarters following the shock. The response of the spread computed from Fuhrer's model is much less persistent and is statistically significant only upon impact – but again, this is not surprising given the extreme high frequency variability found in the real rates generated from that model.

We also estimate the vector autoregressions including the spread between the nominal interest rates generated by the models and the federal funds rate. The impulse response functions are virtually identical to those in Figure 5, and, in the interest of space, we do not report them. This is, of course, not surprising because the variability of expected inflation is quite small, especially relative to the variation in the model-generated real interest rates.

Finally, we estimate the vector autoregressions including the model-generated real interest rates to determine if the extent to which the decline in the spread is due to the behavior of the model rates. Each of the impulse responses is remarkably similar to those reported in Figure 5. A monetary expansion is followed by an increase in the model-generated real interest rates. This is in marked contrast to the evidence from the empirical literature on monetary policy that real money market rates decline following an easing of monetary policy.

Like the regression evidence, the impulse response functions show that the three habit models, like the standard CRRA model, fail to capture the effect of monetary policy on real and nominal interest rates observed in the data.

IV. Conclusion

The empirical literature shows that monetary policy has a liquidity effect – that is, an unexpected monetary tightening raises real and nominal interest rates. The same literature finds that a monetary tightening reduces both real GDP and consumption and their growth rates for several quarters following the tightening. In this paper we show that these two results cannot be reconciled in models that equate the interest rate implied by a consumption Euler equation with money market interest rates.

This problem arises because a decline in expected consumption growth will reduce the real interest rate implied by the Euler equation. Although adding habit persistence to NNS models has allowed these models to better replicate the response of real variables to monetary shocks, we find that adding habit persistence to preferences does not resolve the problem we have identified. In the consumption Euler equations we consider, whether derived from additively separable CRRA preferences or from preferences embodying habit persistence, a decline in expected consumption growth is associated with a decline in real interest rates. We also find that the difference between the interest rates implied by consumption Euler equations and money market rates appears to vary systematically with indicators of monetary policy and that interest rates implied by the models fall in response to a monetary tightening while money market interest rates rise. It is, of course, possible that some other preferences could resolve the puzzle, but doing so would require that the impact of expected consumption growth be reversed.

Alternatively, a model that drives a wedge between the interest rate implied by a consumption Euler equation and money market rates could resolve the puzzle. Limited participation models and models attributing liquidity services to bonds are two alternatives within the representative agent paradigm.[19] Lucas (1990), Fuerst (1992), and Christiano and Eichenbaum (1992, 1995, 1997) assume that households do not adjust their money holdings immediately following a monetary policy shock. Instead, the impact of a monetary shock fall on financial intermediaries, which, in turn, adjust their lending to firms. As a result, money market interest rates are no longer given by a consumption Euler equation.

Bensal and Coleman (1996) and Canzoneri and Diba (2000) introduce a spread by

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[19]A third alternative is heterogeneous agent models with liquidity constraints (Huggett and Ospina (1999)).

allowing bonds to provide transactions services. In Canzoneri and Diba's model, an expansionary open market operation increases the ratio of money to bonds; this in turn lowers the money market interest rate by changing the marginal transactions services of money and bonds. In practice, it remains to be seen if this prediction is empirically significant.

Appendix

The conditional moments that we use to compute real and nominal interest rates are obtained from the vector autoregression,

$$Y_t = A_0 + A_1 Y_{t-1} + \nu_t \,,$$

where $c_t = \log(C_t)$ and $\pi_t = \log(P_{t+1}/P_t)$ are the first and second elements of the vector Y_t and the error term, ν_t, is iid $N(0,\Sigma)$.

Conditional Expectations. The expectation of one-period-ahead and two-periods-ahead consumption and inflation are just the first and second elements of the vectors,

$E_t Y_{t+1} = A_0 + A_1 Y_t$,

$E_t Y_{t+2} = A_0(I+A_1) + A_1^2 Y_t$.

Conditional Variances and Covariances. The conditional second moments are constant and given by the 1,1 and 2,2, and 1,2 elements of,

$V_t(Y_{t+1}) = \Sigma$,

$V_t(Y_{t+2}) = A_1 \Sigma A_1' + \Sigma$,

$C_t(Y_{t+1}, Y_{t+2}) = \Sigma A_1'$.

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Figure 1: Real Interest Rates
Ex Post and CRRA Euler Equation

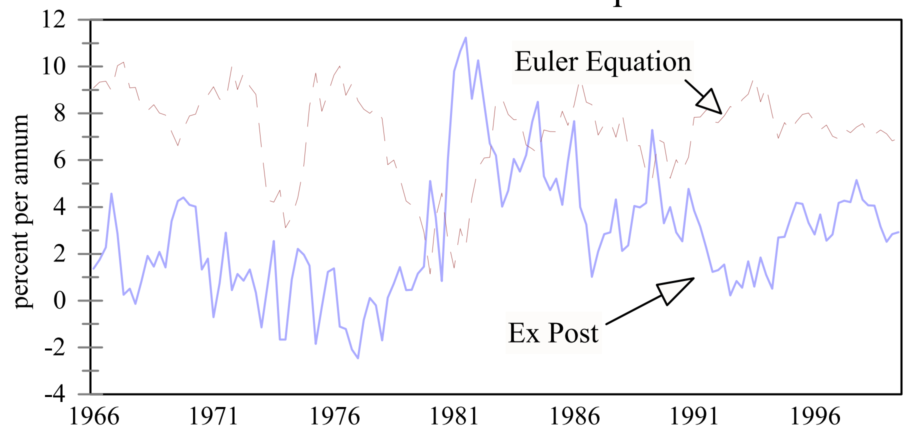


Figure 2: Real Interest Rates
Ex Post and Fuhrer Euler Equation

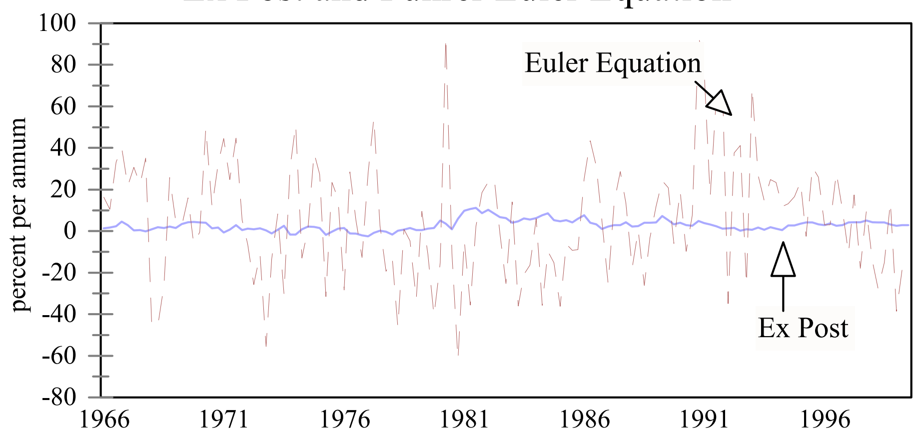


Figure 3A: Real Interest Rates
Ex Post and Abel Model (iid)

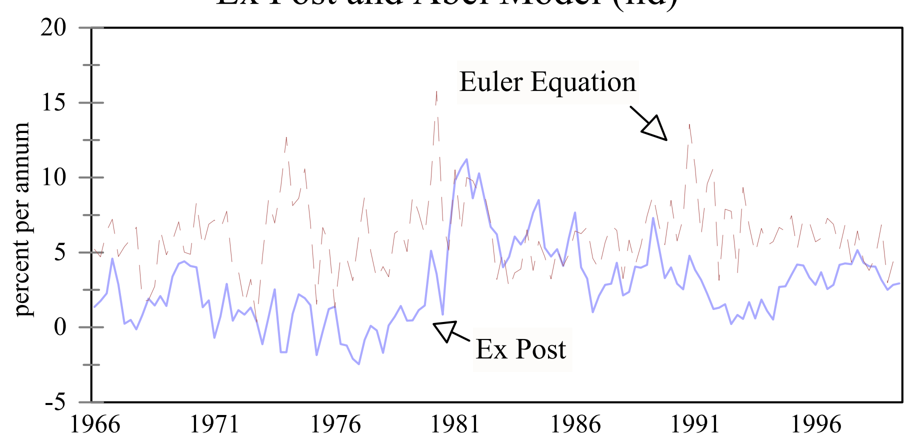


Figure 3B: Real Interest Rates
Ex Post and Abel Model

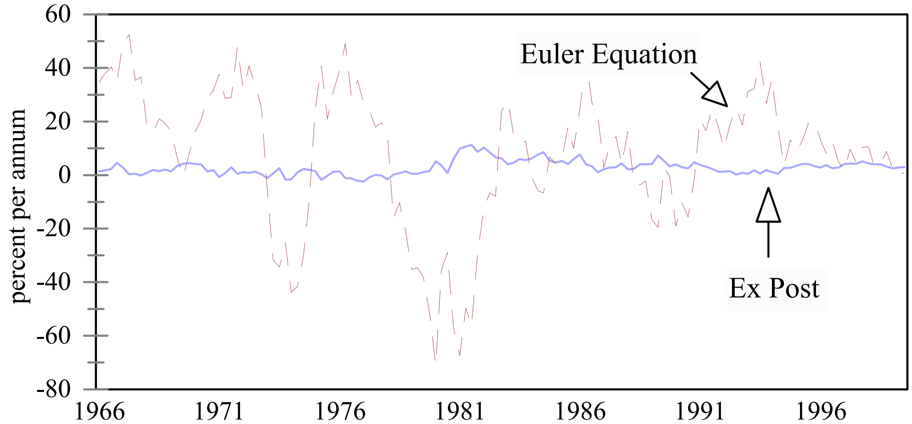


Figure 3C: Real Interest Rates
Ex Post and Abel Model (alternate)

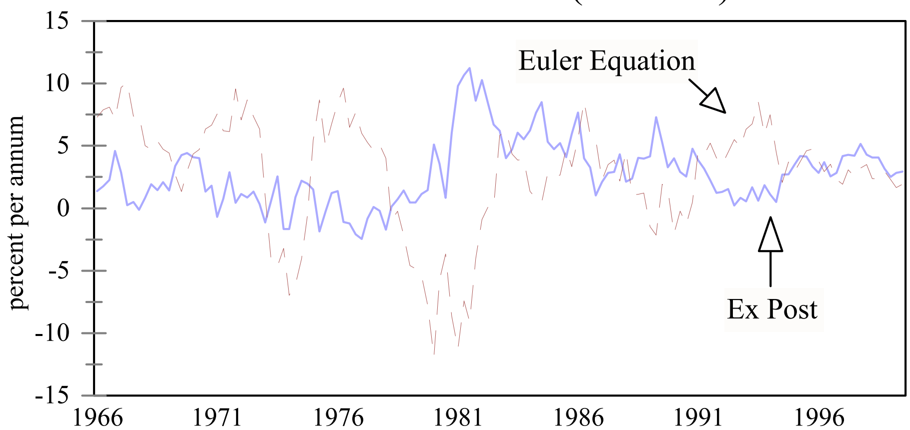


Figure 4: Real Interest Rates
Ex Post and Campbell-Cochrane Model

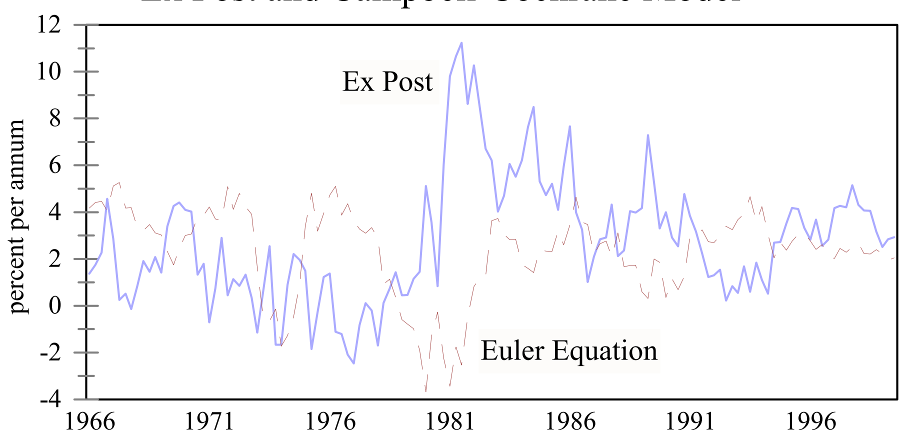
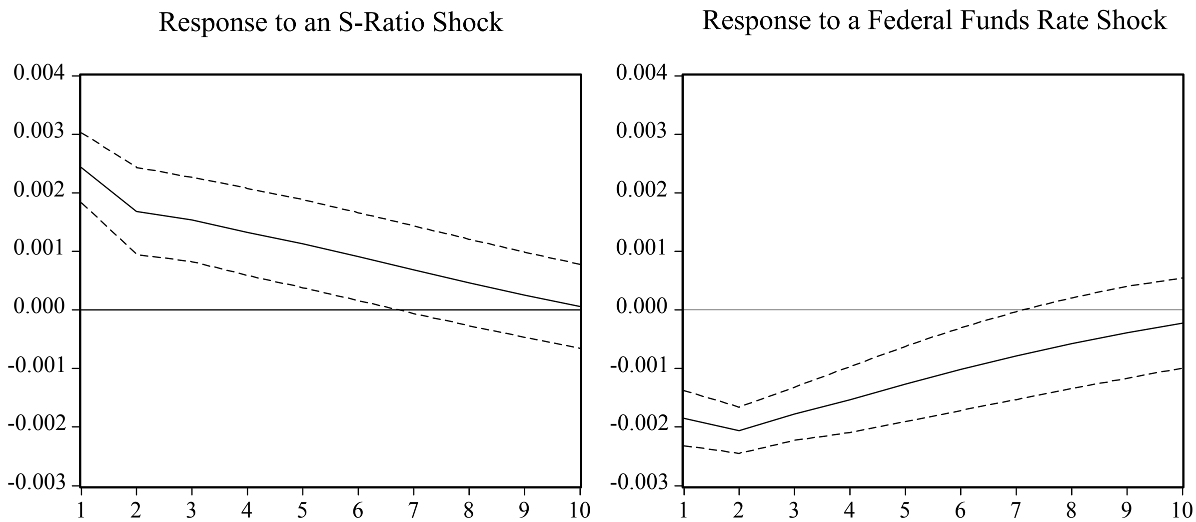
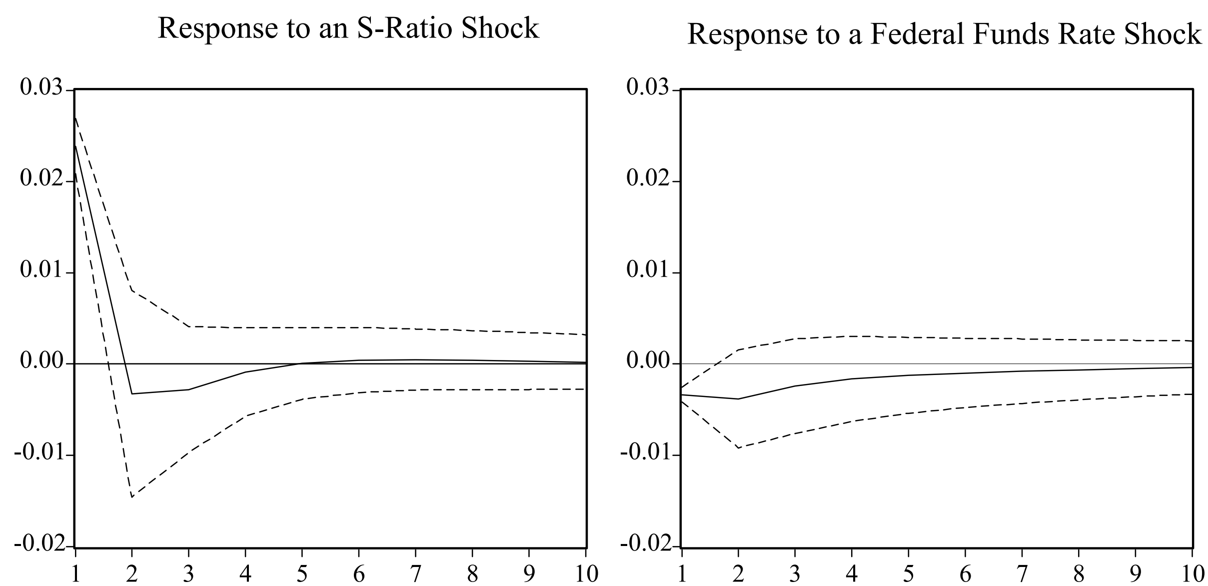


Figure 5: Response of Real Spread to Monetary Policy Shocks

A. Power Utility

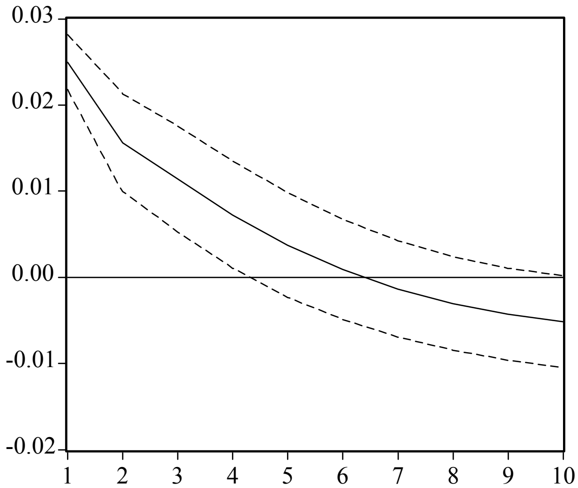


B. Fuhrer's Model

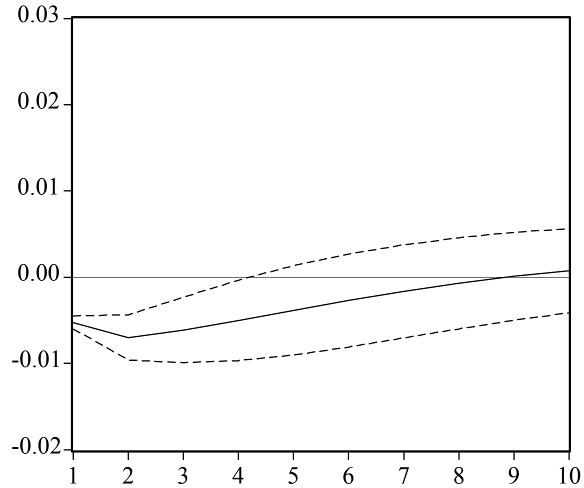


C. Abel's Model

Response to an S-Ratio Shock

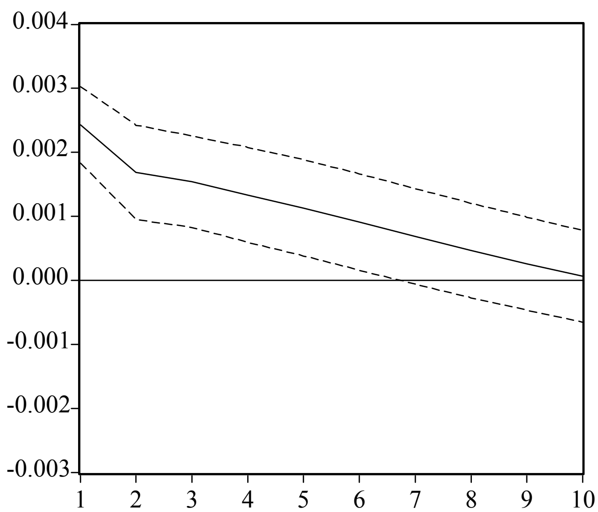


Response to a Federal Funds Rate Shock



D. Campbell and Cochrane's Model

Response to an S-Ratio Shock



Response to a Federal Funds Rate Shock

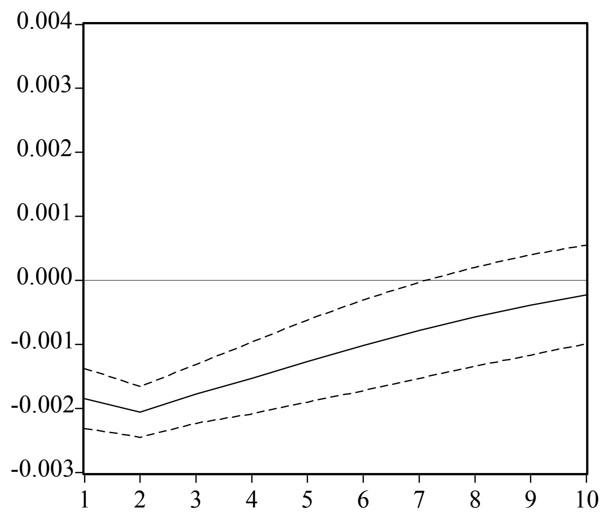
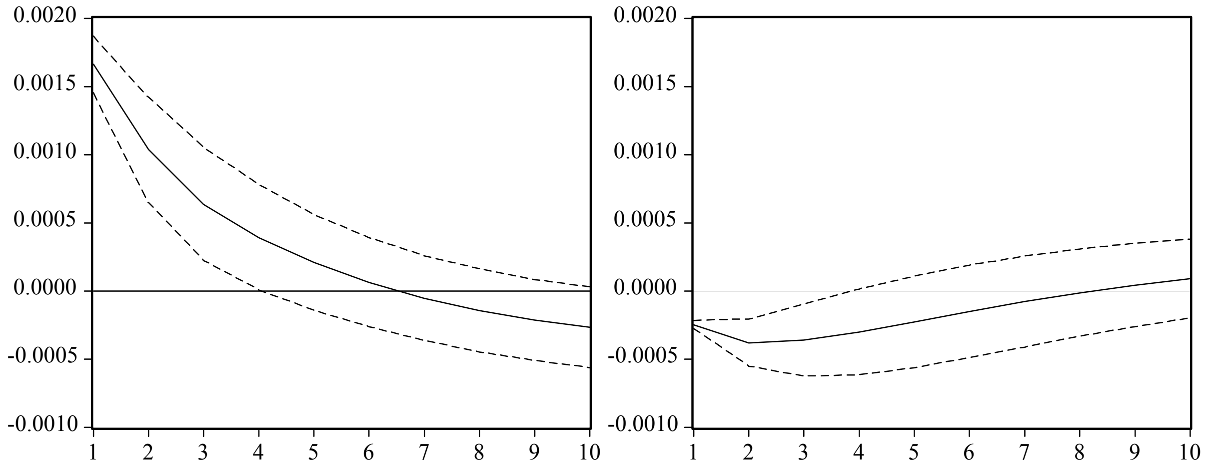


Figure 6. Response of Model Real Rate to Monetary Policy Shocks

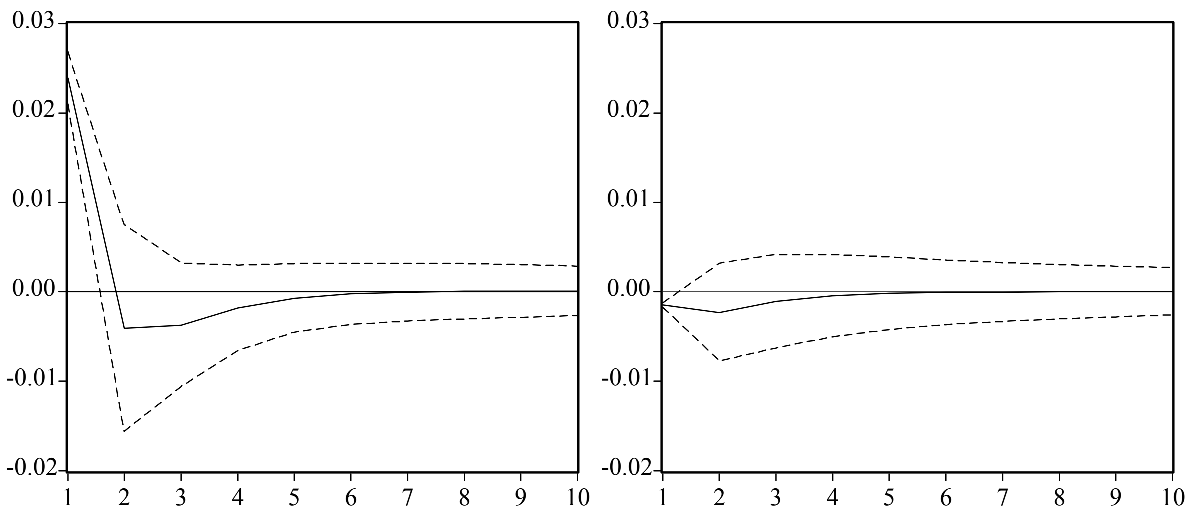
A. Power Utility

Response to an S-Ratio Shock | Response to a Federal Funds Rate Shock



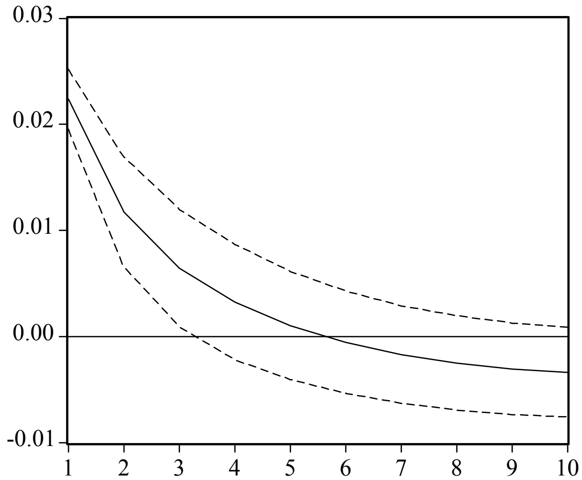
B. Fuhrer's Model

Response to an S-Ratio Shock | Response to a Federal Funds Rate Shock

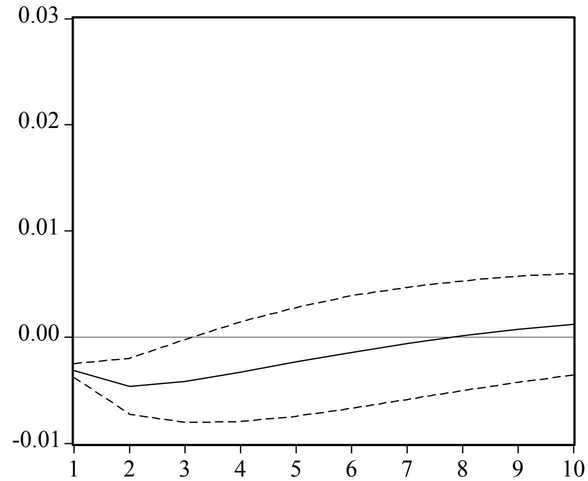


C. Abel's Model

Response to an S-Ratio Shock

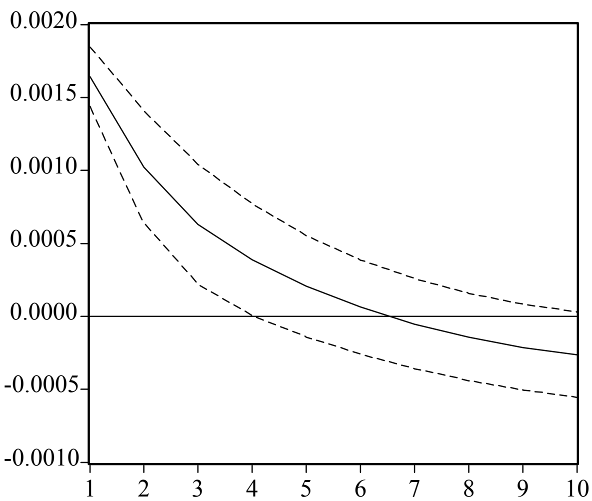


Response to a Federal Funds Rate Shock



D. Campbell and Cochrane's Model

Response to an S-Ratio Shock



Response to a Federal Funds Rate Shock