Implementing Monetary Policy Without Reserve Requirements*

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Abstract

We propose a new framework to implement monetary policy in a corridor system, which does not rely on imposing reserve requirements. A main lending facility is introduced, where banks can borrow a limited amount of cash at a rate that we suppose is in the middle of the corridor. There is no maintenance period, so that short term rates are not affected by an end of period effect. We formally show that this framework will reduce the volatility of the overnight rate relative to systems that use reserve averaging. Also, parametrizing the framework, we show that the variability of the short term rate can be practically eliminated without affecting trading in the interbank market.

1 Introduction

Most central banks now implement monetary policy by targeting some level for short term money market rates, normally the overnight rate. It is therefore important for the credibility of monetary policy that money market rates are effectively steered to the target set by the central bank. To adjust the overnight rate and control its variability, central banks use mainly three instruments, with which they steer the marginal value of liquidity. First, using open market operations, central banks steer aggregate liquidity in the system. Second, central banks can offer a lending and a deposit facility, where commercial banks can borrow or deposit reserves at some fixed interest rates. These rates then form a natural corridor for the overnight money market rate, which limits the size of its fluctuations. Generally, central banks use a combination of both tools. For instance, the Federal Reserve conducts reverse repos every day and although it does not offer a deposit facility with a rate different from

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zero, it maintains a lending facility, the discount window. Similarly, the European Central Bank offers a deposit and a lending facility and conducts weekly refinancing operations and occasional fine tuning operations. Finally, central banks can also require commercial banks to hold a fraction of their deposits as reserves. When banks are required to hold an *average* amount of reserves over a certain period, the maintenance period (usually one month), these reserve requirements can be used to buffer against their liquidity shocks. The buffer role of average reserves functions in a simple way. Whenever a bank receives an (unexpected) liquidity shock, it can either adjust its reserves holdings for that day or, if still possible, borrow funds on the interbank market. Adjusting reserves balance for a day is an option as banks need to fulfill requirements *on average*: If a bank taps in its reserves today, then it will have to adjust its reserves by the corresponding amount later in the period. As it also helps to smooth idiosyncratic shocks, the buffering function of reserve requirements is an important tool to maintain a low level of volatility of the short term money market rates. Moreover, reserve requirements is a tool for reducing the number of open market operations, as it is helpful in absorbing fluctuations in aggregate liquidity.

However, reserve averaging creates issues of its own. First, from a practitioner point of view, it is rather complicated for banks to identify their optimal reserve requirements fulfillment path.[1] Second, the buffer function of average reserves is inexistent in the last day of the maintenance period and, as is the case in the euro money market, the variability of rates will increase as the end of the maintenance period approaches.[2] The volatility on the last day of a maintenance period is basically a consequence of banks being eager to borrow (or lend) the required reserves and dodge the penalty rate associated with a recourse to the facilities. To some extent, this volatility spills over to days farther away from the end of the maintenance period. Third, there is evidence that the overnight rate does not satisfy the martingale property according to which the prevailing rate is the one expected at the end of the maintenance period.[3] Perez-Quiros and Rodriguez-Mendizabal (2006) argue that risk-neutral banks back-load their reserve requirements to use the buffer function of required reserves to its full extent, thus putting upward pressure on rates during the last days of the maintenance period. Similarly one could also argue that risk-averse banks front-load their reserve requirements so as to limit the recourse to the borrowing facility on the last

[1] See Cassola (2007) for a recent study on banks required reserves fulfillment path in the euro area.

[2] See for instance Würtz (2003) or Hamilton (1996).

[3] Prati et. al. (2002) find that the euro overnight rate drops towards the end of the maintenance period. Also, Perez-Quiros and Mendizabal (2006) find evidence that the rate increases on the last trading day of the maintenance period.

day of the maintenance period, thus putting downward pressure on rates. Whether there is an upward or a downward pressure on rates, the fact that the overnight rate is not a martingale is problematic, as it is either a symptom that there is some complications in the money market, or that the central bank is unable to achieve the rate it targets.[4] As a matter of facts, it seems that the ECB resorted on several occasions to open market operations on the last day of the maintenance period in order to counter the deleterious effects of reserve averaging. Another drawback of required reserves is that they increase the overall size of a central bank's balance sheet, which is inconsistent with the lean balance sheet principle. As a matter of principle, a central bank should aim to reduce the amount of financial resources it absorbs into its balance sheet. Even though reserve requirements are sometimes fully remunerated they can still be regarded as a consumption of financial resources, tying up collateral of banks that could potentially be used more profitably.[5]

Against this background, we propose an implementation framework which does not use reserve requirements, and therefore eliminates the problems associated with dealing with maintenance periods. The buffer role of reserve requirements is instead replaced by a Main Lending Facility (MLF). In this system, commercial banks hold remunerated current accounts with the central bank. In case their balance is negative, banks borrow overnight at the MLF, at fixed rate and against proper collateral, any amount of reserves up to a limit. In the proposed framework, the central bank still operates the lending and deposit facilities and one may suppose that the MLF rate is the target rate of the central bank. Therefore, if banks need liquidity (have negative balance on their current account) that they cannot obtain on the interbank market, they will automatically be directed to the MLF and then to a residual lending facility. Since the amount borrowed at the MLF is capped, banks will still have recourse to the residual lending facility if they receive a liquidity shock that is larger than the borrowing limit at the MLF. In this framework, the central bank carries out refinancing operations, calibrated with the view to steer the average expected draw from the MLF to be one half of the aggregate limit. In this way, the central bank ensures equal possibility to absorb liquidity draining and liquidity absorbing shocks. The frequency and average volume of these operations need to be made consistent with the MLF limit. We show that the lower the size of the limit, the more precisely the expected draw from the MLF should be calibrated in order to avoid fluctuations in the overnight rate. A more frequent calibration of the draw from the MLF in turn necessitates a larger average volume

[4] On the contrary, Cassola (2007) argues the Euro area overnight rate (the EONIA) satisfies the martingale property.

[5] See Papadia and Würtz (2007) for a detailed analysis of the lean balance sheet principle.

in, and a higher frequency of, refinancing operations. At the extreme the MLF limit could be set sufficiently large to absorb all fluctuations in the temporary component of the liability side of a central bank's balance sheet. To illustrate the effect of introducing the MLF, the two tables below show a simplified version of a central bank balance sheet under the two frameworks. The first table shows the balance sheet with reserve averaging.

Assets		Liabilities	
Refinancing operations (1 week & 3 months)	300	Banknotes	400
Net financial assets	200	Reserve requirements	100
Total assets	500	Total liabilities	500

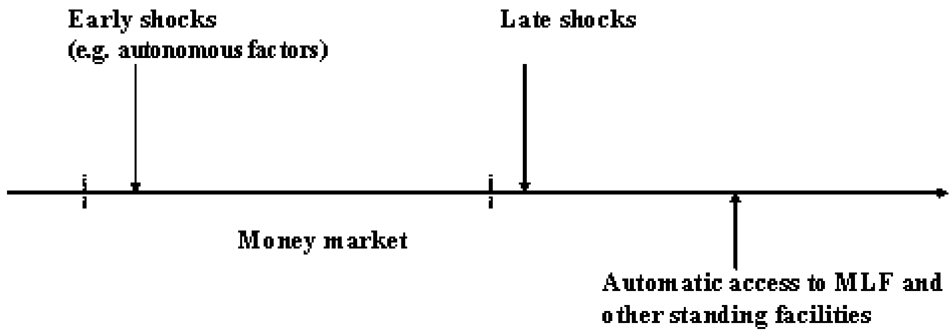
The following table presents the simplified balance sheet of a central bank that would adopt the MLF framework.

Assets		Liabilities	
Main lending facility	100	Banknotes	400
Refinancing operations (1 week & 3 months)	100	Reserve requirements	0
Net financial assets	200		
Total assets	400	Total liabilities	400

Note that building the buffer function of the MLF on the asset side of the balance sheet, rather than on the liability side as is the case of required reserves, reduces the size of the balances sheet and is therefore consistent with the lean balance sheet principle.

We propose a simple model of the MLF framework, as a first step to analyse how the MLF framework fares relative to a setup with reserve requirements. The result of the stylised analysis is that a reasonable limit on the MLF reduces the variability of the interbank rate significantly, without affecting much interbank market activity. The reason is that, with an aggregate liquidity deficit, the interbank market rate is set at the MLF rate so that banks trade their idiosyncratic shocks away. Also, we show that the MLF just takes over the role of reserves in an average reserve requirement system. In the Appendix, we use the model of the interbank market rate determination with required reserves proposed by Gaspar et. al. (2007) to compare the variability of short term rates, and the activity on the interbank market for both implementation frameworks.

The paper is structured as follows. In Section 2, we present the theoretical environment for the new framework. In Section 3 we solve for an equilibrium and present some basic



results. In Section 4 we study how MLF systems compare with reserve averaging systems. Section 5 extends the basic analysis to consider the effects of transaction costs on trading activity. We consider different ways to set up the MLF limits in Section 6 and we conclude in Section 7.

2 Environment

This environment is partially based on Poole (1968) and Gaspar, Perez-Quiros, Rodriguez-Mendizabal, (GPR, 2007). There are n commercial banks and a central bank. Commercial banks maintain deposits with the central bank, called current accounts, to fulfil payments obligations. We will call 'balances' the amounts on these current accounts. At the start of the day, all current accounts are cleared in the sense that banks have a zero balance on their current account.

A typical day for a commercial bank can be decomposed in three stages. In the **first stage**, banks receive an early liquidity shock caused by autonomous factors and central banks operations. Given balances on their current account, banks can trade balances in the interbank market. In a **second stage**, banks receive a (late) liquidity shock. Given the size of their current account balance after this shock, banks are directed to one of the three standing facilities, the main lending facility (MLF) where they can borrow up to a limit B_i for each bank i at rate i_b, the residual lending facility (RLF) where they can borrow any amount at a rate $i_\ell \geq i_b$ and finally, positive account balances are swept into the deposit facility, where the remuneration rate is i_d. In the **last stage**, and depending on their financial claims contracted in the previous two stages, banks receive interest rates on their current account and pay interest rates from their current account. Banks will seek to maximise(minimise) the interest payments they receive(make) from their operations. We abstract from default issues, so that the central bank does not require any collateral.

Banks are not required to hold reserve, so there is no maintenance period. Instead, banks use all profits accuring in the last stage, so that they start each cycle/day with a zero account balance. As a result, there is no dynamics involved, and we can assume that there is only one day without loss of generality.

Following the early shock bank i starts the day with an amount of balances $s_i \in \mathbb{R}$.[6] The aggregate amount of balances is then $\sum_{i=1}^{n} s_i = S$. We assume that there is a structural liquidity deficit due to, for instance, banknotes in circulation, which is partially offset by the central bank's conduct of liquidity providing operations. Hence, in general S is negative. We do not model central bank's operations here. We describe in details below the functioning of the interbank market and of the three standing facilities.

Interbank market When the interbank market opens, banks' balances are heterogeneous so that they have different expected liquidity needs. As a consequence, they have an incentive to trade in the interbank market if there is a wedge between the lending rate and the deposit rate. When bank i accesses the interbank market, we will denote its trade by y_i. If $y_i < 0$ then bank i lends balances, while if $y_i > 0$, bank i borrows balances. The interbank market rate is set so as to clear the market.

Standing facilities and end of day shocks At the end of the trading session, given banks' initial balances and their trades on the interbank market, bank i has balances $s_i + y_i$. Each bank then receives a late shock $\lambda_i \backsim F(\mu_i, \sigma_i)$ which distribution is common knowledge. This shock can be bank specific, and is not necessarily independent across banks. The bank must have enough balances to cover this liquidity shock. Otherwise, the bank is automatically directed toward the standing facilities provided by the central bank to borrow balances. If a bank ends the day with a positive account balance, it is automatically swept to the deposit facility.

There are three standing facilities, the main lending facility (MLF), the residual lending facility (RLF) and the deposit facility (DF). Any bank i that ends the day with a negative account balance is first directed to the MLF. The amount borrowed from the MLF is limited and cannot be more than $B_i \geq 0$ for bank i. All banks pay the MLF rate i_b on any amount borrowed there. Once the MLF cap is reached, bank i is automatically directed to the RLF and the central bank charges $i_\ell \geq i_b$ on any amount borrowed there. Finally, positive account balances are remunerated at the DF rate i_d. We will assume that shocks are evenly

[6] s_i also denotes the size of the early liquidity shocks. We therefore do not model explicitly this shock.

distributed across banks, so that a positive shock for bank i corresponds to a negative shock for some bank j and there is no aggregate liquidity creation.

Settlement After the liquidity shock is realised, banks repay loans and redeem deposits. Therefore, the amount of balances of bank i at this stage is

$$s_i' = s_i - \lambda_i - y_i\,(1+i_m) - b_i\,(1+i_b) - \ell_i\,(1+i_\ell) + d_i\,(1+i_d)$$

where b_i is the amount borrowed from the MLF, ℓ_i is the amount borrowed at the RLF and d_i is the amount deposited at the DF. We assume that, at any stage, bank i seeks to maximize s_i'.

3 Equilibrium

We now solve for the equilibrium in the interbank market, i.e. the interbank market clearing interest rate, first finding an expression for banks' payoff as a function of their trading activities on the interbank market.

Liquidity shocks and access to standing facilities We let $V\left(\bar{s}_i, y_i\right)$ be the expected payoff of bank i when it exits the interbank market with balances $\bar{s}_i$, of which y_i has been borrowed from the interbank market.

Each bank then receives a late shock $\lambda_i \backsim F\left(\mu_i, \sigma_i\right)$. If the bank has enough balances to cover this shock ($\bar{s}_i \geq \lambda_i$), then it still has a positive balance on its current account $\bar{s}_i - \lambda_i$, which is remunerated at the deposit facility rate i_d. Otherwise, if $\bar{s}_i < \lambda_i$, bank i's current account is automatically credited with the amount $\lambda_i - \bar{s}_i > 0$, so that it holds a zero current account balance. If $\bar{s}_i + B_i \geq \lambda_i$, bank i's current account is credited with the $b_i = \lambda_i - \bar{s}_i$ and is charged the MLF rate i_b. Bank i repayment to the central bank is then $\left(\lambda_i - \bar{s}_i\right) i_b$ for all λ_i such that $\bar{s}_i < \lambda_i \leq \bar{s}_i + B_i$. Otherwise, if $\lambda_i > \bar{s}_i + B_i$, then bank i's current account is credited with B_i from the MLF, and with the remaining missing amount $\lambda_i - \left(\bar{s}_i + B_i\right)$ from the lending facility. In this case, bank i's repayment from shock λ_i to the central bank is $\left[\lambda_i - \left(\bar{s}_i + B_i\right)\right] i_\ell + B_i i_b = \left(\lambda_i - \bar{s}_i\right) i_\ell + B_i\left(i_b - i_\ell\right)$. Therefore, bank i's automtic recourse to the facilities can be summarised as follows:

$$\begin{aligned}
&\lambda_i \leq \bar{s}_i,\ \text{deposit } \bar{s}_i - \lambda_i \text{ at the DF} \\
&\bar{s}_i < \lambda_i \leq \bar{s}_i + B_i,\ \text{borrow } \lambda_i - \bar{s}_i \text{ from MLF} \\
&\bar{s}_i + B_i < \lambda_i,\ \text{borrow } B_i \text{ from MLF, borrow } \lambda_i - \left(\bar{s}_i + B_i\right) \text{ at RLF}
\end{aligned}$$

Therefore, bank i's expected payoff from borrowing y_i on the interbank market $V(\bar{s}_i, y_i)$ has the following expression

$$
\begin{aligned}
V(\bar{s}_i, y_i) &= -(1+i_m)\, y_i + \int_{-\infty}^{\bar{s}_i} (1+i_d)(\bar{s}_i - \lambda_i)\, f(\lambda)\, d\lambda - \int_{\bar{s}_i}^{\bar{s}_i + B_i} (1+i_b)(\lambda_i - \bar{s}_i)\, f(\lambda)\, d\lambda \\
&\quad - \int_{\bar{s}_i + B_i}^{+\infty} \left[(1+i_b) B_i + (1+i_\ell)(\lambda_i - (\bar{s}_i + B_i))\right] f(\lambda)\, d\lambda.
\end{aligned}
$$

The expected value of leaving the interbank market with balance $\bar{s}_i$, of which y_i has been borrowed on the interbank market is constituted of four parts: first, the cost to repay the loan contracted on the interbank market. Second, the expected gains from depositing reserves at the DF when the liquidity shock is not large. Third, the expected cost of borrowing at the MLF if the liquidity needs are lower than the cap. Finally, the expected cost of borrowing up to the cap at the MLF and even more at the RLF when the liquidity needs are greater than the cap. The bank will seek to borrow liquidity on the interbank market to maximize this expected value.

Interbank Market In the interbank market, a bank solves

$$
Z(s) = \max_y V(s+y, y)
$$

Note that we have omitted the dependency of this value function on the aggregate state S. Implictly, we are therefore assuming that banks are too small to be able to affect the equilibrium rate on their own.[7] The first order condition gives $V_y + V_s = 0$. From the definition of V, we have (see the Appendix for details),

$$
\begin{aligned}
V_y(s_i + y_i, y_i) &= -(1+i_m) \\
V_s(s_i + y_i, y_i) &= (1+i_\ell) - (i_b - i_d) F(s_i + y_i) - (i_\ell - i_b) F(s_i + y_i + B_i) \\
&= (1+i_d) F(s_i + y_i) + (1+i_b)\left[F(s_i + y_i + B_i) - F(s_i + y_i)\right] \\
&\quad + (1+i_\ell)\left[1 - F(s_i + y_i + B_i)\right]
\end{aligned}
$$

In words, the value of an incremental increase in borrowing on the interbank market is the cost to pay back this loans, $-(1+i_m)$. Also, the marginal value of bank's account balance is the expected gain from having a positive account balance at the end of the day. If the shock is not severe, the extra balance can be deposited at the DF and earn interest rate

[7] See Ewerhart et. al. (2006) for a framework where this is not the case.

i_d. If the schock takes some intermediate values (between $s_i + y_i$ and $s_i + y_i + B_i$) then the incremental balance is used to cover the shock, and therefore saves on the borrowing cost at the MLF, $1 + i_b$. Finally, if the shock is severe enough, the incremental balance is used instead of having recourse to the RLF, which saves $1 + i_\ell$. In equilibrium, the marginal benefit of borrowing on the interbank market is the marginal benefit of increasing one's account balance V_s, while the marginal cost is the interbank market rate, $1 + i_m$. Hence, y_i solves

$$i_d F(s_i + y_i) + i_b [F(s_i + y_i + B_i) - F(s_i + y_i)] + i_\ell [1 - F(s_i + y_i + B_i)] = i_m \quad (1)$$

Figure 1 illustrates the demand curve for banks on the interbank market for different level of caps, where borrowing (lending if negative) is on the x-axis. Note that the cap has a non-linear effect on the demand schedule. In particular, as the size of the cap becomes relatively large compared with the uncertainty regarding the size of the liquidity shock, banks' demand schedule become very elastic, up to the point where it reaches its cap. The reason is that when the interbank market rate is higher than the MLF rate, a bank arbitrages both rates, by lending on the interbank market, as it can borrow from the MLF at a cheaper rate. As the borrowing limit increases, the lending activity for all interbank rates above the MLF rate also increases. Importantly, since the demand is very elastic for rates in a neighborhood of the MLF rate, a large variation in the uncertain component of the demand for liquidity can be accomodated by a relatively small change in the interbank market rate. Therefore, the equilibrium rate volatility is decreasing when the limit on the MLF becomes larger.

Note that when banks are perfectly homogeneous, so that $s_i = s_j = s$, and $B_i = B_j = B$ for any $i \neq j$, the only equilibrium is when $y_i = y_j = 0$, so that the equilibrium rate satisfies,

$$i_d F(s) + i_b [F(s + B) - F(s)] + i_\ell [1 - F(s + B)] = i_m.$$

In this case, Figure 1 shows, the interbank market rate will only be in a region near i_b if the structural liquidity deficit - defined as $\sum_{i=1}^{n} s_i/n = s < 0$ - is sufficiently large. Below we show rigorously that this intuition is correct. Finally, Figure 1 also shows that a relatively small cap size will not affect the variability of the interbank market rate much relative to a standard daily reserve requirement system. We can now define an equilibrium for this economy.

Definition 1 *Given policy rates* (i_ℓ, i_b, i_d), *and aggregate balances* S, *an equilibrium is an interbank market rate* i_m *and allocation* $\{y_i\}_{i=1}^{n}$, *such that, given* i_m, y_i *solves (1) for all i and* $\sum_i y_i = 0$.

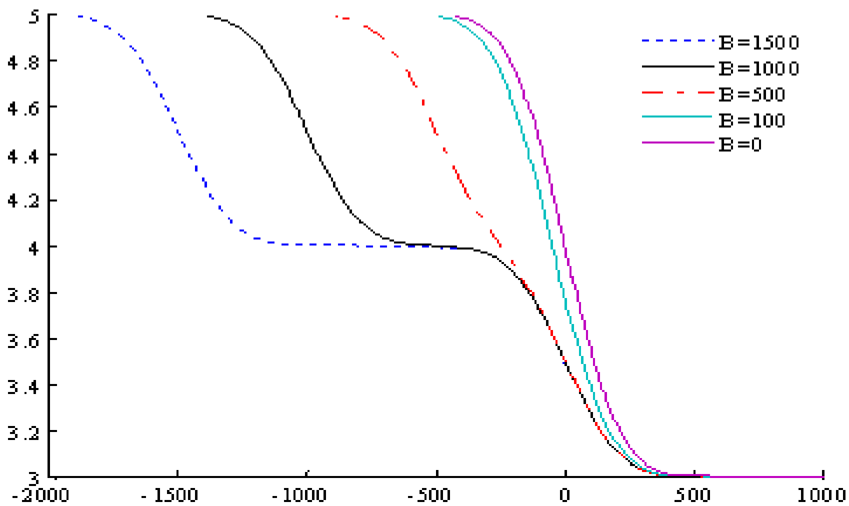


Figure 1: Demand function of a bank on the interbank market, given shocks are distributed according to a $N(0, 166)$, where $i_d = 3$, $i_\ell = 5$ and $i_b = 4$, for different levels of caps.

Proposition 1 *An equilibrium exists and is unique.*

Proof. Let us first define the function $\psi_i(x) = (i_b - i_d) F(x) + (i_\ell - i_b) F(x + B_i)$. Note that ψ_i is a function of B_i and therefore can differ across banks. $\psi_i(x)$ is defined over the interval defining $F(\lambda)$ (which we will take to be $[-\infty, +\infty]$). Hence $\psi_i(-\infty) = 0$ and $\psi_i(\infty) = i_\ell - i_d$. Also, ψ_i is strictly increasing over this interval. Therefore, its inverse $\psi_i^{-1}$is well defined. Then an equilibrium is defined by i_m and y_i such that $\psi_i(s_i + y_i) = i_\ell - i_m$ for all i and $\sum_i y_i = 0$. Hence, we have $s_i + y_i = \psi_i^{-1}(i_\ell - i_m)$. Using the market clearing condition we obtain that i_m solves

$$\sum_i \psi_i^{-1}(i_\ell - i_m) = \sum_i s_i$$

Since ψ_i^{-1} is continuous and monotone for all i and $\psi_i^{-1}(0) = -\infty$ and $\psi_i^{-1}(i_\ell - i_d) = \infty$, there exists by the intermediate value theorem a unique $i_m \in [i_d, i_\ell]$ such that the above equality holds. ■

Note that if $B_i = B$ for all i, then we can derive a simple expression for i_m as

$$i_m = i_\ell - (i_b - i_d) F\left(\sum_i \frac{s_i}{n}\right) - (i_\ell - i_b) F\left(\sum_i \frac{s_i}{n} + B\right). \tag{2}$$

In this case, we also have that

$$y_i = \sum_i \frac{s_i}{n} - s_i. \qquad (3)$$

An important implication of this equation is that the size of the limit on the MLF does not affect trading on the interbank market, but only affects the equilibrium interbank market rate. Also, note that if $B_i = 0$ for all i, $i_m = i_\ell (1 - F(s+y)) + i_d F(s+y)$, which is the standard equilibrium equation in an environment with daily required reserves and no averaging.[8] In words, setting the MLF limits to zero, or equivalently eliminating the MLF, is similar to introducing a regime with daily required reserves and no averaging. Then in this case, y^k is given by

$$F(s+y) = \frac{i_\ell - i_m}{i_\ell - i_d}.$$

Also if $B_i = +\infty$, so that infinite recourse to the MLF is possible, then y is given by

$$F(s+y) = \frac{i_b - i_m}{i_b - i_d}.$$

Corollary 2 *For all S, there is $\bar{B}$ such that if $B_i > \bar{B}$ for all i, then $i_b \geq i_m$.*

Proof. Set B to a sufficiently large amount that $F(s+y+B_i) = 1$. Then equation (**??**) becomes $i_m = i_b - (i_b - i_d) F(s_i + y_i) < i_b$. By continuity, either $i_m < i_b$ for all $B \geq 0$, or there is $\bar{B}$ such that if $B_i > \bar{B}$ for all i, then $i_b \geq i_m$. ∎

Finally, under some mild assumptions, we show that there is an average reserve deficit S/n so that $i_m = i_b = (i_\ell + i_d)/2$.

Lemma 3 *Suppose the density function of the shock distribution is centered around its mean, $B_i = B$ for all i and $i_b = (i_\ell + i_d)/2$. If $S/n = -B/2$, then $i_m = (i_\ell + i_d)/2$.*

Proof. We showed that if $B_i = B$ for all i then i_m is

$$i_m = i_\ell - (i_b - i_d) F(S/n) - (i_\ell - i_b) F(S/n + B).$$

Replacing $i_m = i_b = (i_\ell + i_d)/2$, we obtain that S/n must solve

$$F(S/n) = 1 - F(S/n + B).$$

However, since F is centered around its mean, we have that $F(-x) = 1 - F(x)$ for all x. Hence, $S/n = -B/2$ solves this equation. ∎

Therefore, in order to hit the middle range of the corridor implied by the deposit and lending facilities rates, it is sufficient for the central bank to target an average liquidity deficit

[8] See for instance Gaspar et. al (2007).

that equals half the cap in the MLF. When this is the case, note that we can decompose each s_i in two parts: $s_i = -B/2 + A_i$, where $A_i = d_i + a_i$ can be itself decomposed between the uncertain portion of the aggregate liquidity deficit d_i and bank i idiosyncratic early shock, a_i. When n is large, the law of large number implies that $d_i/n \sim 0$, so that $\sum_{i=1}^{n} A_i/n \sim \sum_{i=1}^{n} a_i/n$, i.e. the number of banks impacts the extent to which the uncertainty about the aggregate liquidity shock is affecting individual banks relative to their idiosyncratic shock. As n become large, the interbank market is used in order to trade away only bank's idiosyncratic shock, while the aggregate liquidity deficit is covered by a sure recourse to the MLF. As larger caps are imposed, banks can also cover some of their individual schock at the MLF, which then reduces the variability of the interbank market rate.

4 How do MLF systems compare with Reserve Averaging systems?

In this section, we show that a system relying on the MLF can replicate the outcome of a reserve averaging system in the first days of the maintenance period, that is, in a reserve averaging system when the money market rate is the least volatile. We show that this is the case when we restrict the maintenance period to two days.[9]

It is relatively straightforward to derive the solution of the problem of a bank in the first day of a two-days maintenance period.[10] When bank i starts this day with balance $s_{i,1}$ on its current account, and faces a daily reserve requirement of R_i (or a reserve requirement of $2R_i$ over the whole maintenance period), bank i chooses to borrow $y_{i,1}$ on the interbank market, where $y_{i,1}$ satisfies

$$\begin{aligned} & i_d F\left(s_{i,1}+y_{i,1}-2R_i\right)+i_\ell\left[1-F\left(s_{i,1}+y_{i,1}\right)\right] \\ & +E_\lambda\left[i_{m,2}\right]\left[F\left(s_{i,1}+y_{i,1}\right)-F\left(s_{i,1}+y_{i,1}-2R_i\right)\right] \quad = \quad i_{m,1} \qquad (4) \end{aligned}$$

Intuitively, a bank chooses y_1 so as to equate its marginal cost $i_{m,1}$, and its marginal benefit. The marginal benefit from borrowing is composed of three parts. First, when the bank current account balance $s_{i,1} + y_{i,1}$ is insufficient to cover the liquidity shock, the additional unit borrowed allows the bank to borrow an additional unit less at the lending facility, which has a cost i_ℓ. This event has probability $1 - F(s_{i,1} + y_{i,1})$. Since the bank

[9] It is possible to extend the results to an n-day maintenance period.

[10] See for instance Whitesell (2006b).

is required to hold a daily *average* of R_i, it does not have to hold R_i and therefore it can use all of its account balance to buffer its shock. Second, when the bank current account is large enough to cover both its liquidity shock and its reserve requirement *for the whole maintenance period* $2R_i$, then it can deposit the rest at the deposit facility and earn return i_d on the additional unit borrowed. This event has probability $F(s_{i,1} + y_{i,1} - 2R_i)$. Finally, for moderate shocks, the bank will neither access the lending nor the deposit facility, and the additional unit borrowed can be used on the interbank market in the last day of the maintenance period, which has an expected return of $E_\lambda[i_{m,2}]$ (where $i_{m,2}$ is the money market rate in the second period of the maintenance period). This event has probability $F(s_{i,1} + y_{i,1}) - F(s_{i,1} + y_{i,1} - 2R_i)$. Now, in the last day of the maintenance period, the bank will set y_2 so that

$$i_m = i_\ell [1 - F(s_{i,2} + y_{i,2} - R_{i,2})] + i_d F(s_{i,2} + y_{i,2} - R_{i,2}) \tag{5}$$

where s_2 and R_2, are respectively the account balance and the reserve deficiency of the bank at the start of the last day of the maintenance period. We can now state the main result of this section.

Proposition 4 *Any equilibrium* $\{y_i\}_{i=1}^n$ *in an average reserve requirement system is an equilibrium allocation in a MLF system. However, there are equilibrium allocations in a MLF system that are not implemented with an average reserve requirement system.*

Proof. To replicate the allocation of an averaging system, it is enough to show that appropriately chosen non-stochastic component of the account balance s_i, borrowing limits B_i at the MLF and the MLF rate i_b, can generate the exact same borrowing level for bank i, for all i. If this is the case then the equilibrium rate will be identical across the two systems. Since the averaging system is dynamic, the MLF system will only be able to replicate it if we allow the borrowing limit to be time dependent. Now, given $s_{i,1}$, R_i, and $E_\lambda[i_{m,2}]$ for bank i in the average reserve requirement system, set s_i, $B_{i,1}$ and i_b such that:

$$s_i = s_{i,1} - 2R_i \qquad B_{i,1} = 2R_i \qquad i_b = E_\lambda[i_{m,2}]$$

Then it is obvious that equation (1) defining y_i^{MLF} in the MLF system is equivalent to (4) defining y_i^{RA} in the reserve averaging system. Since there is a unique equilibrium in the RA and MLF systems, we obtain $y_i^{MLF} = y_i^{RA}$ for all i and $i_m^{MLF} = i_{m,1}$. Now, given $s_{i,2}$ and $R_{i,2}$ for bank i in the average reserve requirement system, set s_i, $B_{i,1}$ and i_b such that: $s_i = s_{i,2} - R_{i,2}$ and $B_i = 0$. Then it is clear that equation (1) defining y_i^{MLF} is equivalent to (5) defining $y_{i,2}^{RA}$. To show the second part of the proposition, it is enough to set $B_i > 0$ and $i_b \in (i_d, i_\ell)$ for all period. Indeed, in such a case, since there is no buffer function from reserves in the last day of the maintenance period, it will be the case that $y_i^{RA} \neq y_i^{MLF}$, for some i. ■

Another way to state the above proposition is that the variability of the overnight rate will be smaller in the MLF system. The reason is that on the last day of a maintenance period, required reserves do not play their buffer function anymore, while the MLF can always act as a buffer as long as $B > 0$. Therefore, given B is set to replicate the allocation in the first days of a maintenance period (when the volatility of the overnight rate is the smallest), the rate under a reserve averaging system will always be more volatile than under the MLF scheme, as the end of the maintenance period gets closer.

5 Transaction costs

In this section we extend our theoretical framework to introduce transaction costs. When banks decide to be active on the interbank market, they have to pay a fixed nominal cost $\xi > 0$ which is independent of the size of their transactions. Therefore, when bank i accesses the interbank market, we will denote its trade by y_i. If $y_i < 0$ then bank i lends balances, while if $y_i > 0$, bank i borrows balances. However, its balance at the closing of the interbank market is either $s_i + y_i - \xi$ if bank i was active on the interbank market, or s_i, otherwise. What differentiate the fixed cost from trading is that there is no interest being paid/receive on the fixed cost. The decision of bank i is now whether to become active or inactive on the interbank market and if so, which level of activity to choose. We will denote by $z_i (s_i, i_m) \in \{0, 1\}$, the decision of bank i to become active ($z_i = 1$) or inactive ($z_i = 0$) on the interbank market given its initial account balance s_i and the market rate i_m. Therefore, a bank now solves,

$$Z(s) = \max_{y, z(s, i_m) \in \{0,1\}} V(s + z(s, i_m)(y - \xi), z(s, S)\, y)$$

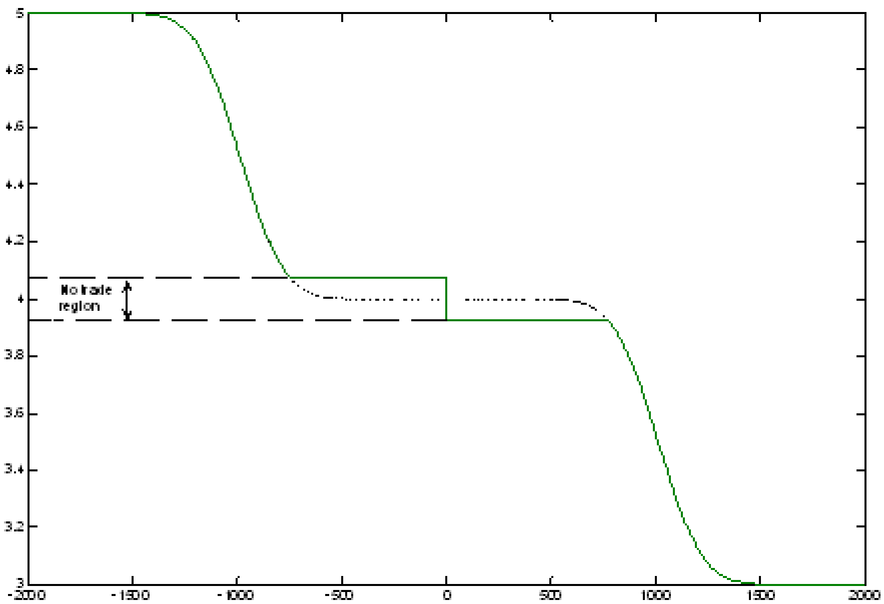
Hence, bank i will access the interbank market whenever

$$V(s_i - \xi + y_i, y_i) \geq V(s_i, 0).$$

We therefore obtain the following result, which proof has been relegated to the Appendix.

Lemma 5 *Given i_m and B_i, bank i does not access the interbank market if $s_i \in (\underline{s}_i, \bar{s}_i)$ and otherwise, sets $y_i = y_i^*$ such that*

$$(i_b - i_d) F(s_i - \xi + y_i^*) + (i_\ell - i_b) F(s_i - \xi + y_i^* + B_i) = i_\ell - i_m \tag{6}$$



The above figure shows the demand function for a typical bank when it faces transaction costs $\xi = 10$. The dotted line shows the same demand function when there is no transaction costs. Under the chosen parameters, a bank will not access the interbank market ($y_i = 0$) for rates that are close to the MLF rate. The money market will therefore only be used to accomodate large shocks to the banking sector, while small shocks will be accomodated by the MLF.

When there are transaction costs, the use of the MLF is welfare enhancing as it reduces the costs of trading relative to a reserve averaging system. Indeed, on the last day of the maintenance period of a reserve averaging system, banks will have to cover their early shock by accessing the market, since the buffer role of required reserves is not any more present. However, in this case, they have to bear transaction costs.

It is also interesting to know how trading activity in the presence of trading costs is affected by a change in the borrowing limit at the MLF. However, while it is possible (and actually easy) to derive the effect of a change in B_i on the demand schedule of a given bank (the no-trade region as depicted above decreases), it is more involved to derive the general equilibrium effects of an increase in MLF borrowing limits, since it will also affect the equilibrium interest rate. We therefore leave this issue for future work.

6 Setting the MLF limits

To implement the proposed scheme, a central bank will have to decide how it sets the limits on the MLF. In this section, we propose three possibilities to set limits on the MLF for each bank.

Average reserve requirements As shown in Section 4, setting the MLF limits to the level of required reserves over an hypothetical maintenance period would surely achieve low interest rate variability, as long as the central bank targets a sufficiently large liquidity deficit. The implementation of the MLF limits would be very similar to the functioning of the reserve averaging system: based on data from their balance sheets, the central bank would calculate banks' MLF limits valid for, say, the forthcoming month. Each month, a new value for the MLF limits would be calculated. Aside from the calculation methods of the MLF limits, all other ingredients of the average reserve scheme would be removed from the implementation framework.

Auctioning limits The previous methods of calculation only relies on a rule imposed by the central bank. However, there is little theoretical grounding for the optimal level of required reserves. In fact, many central banks now do not impose required reserves any more, but rather trust bank in holding the appropriate amount of voluntary reserves. Also, the MLF limits as calculated on the basis of required reserves could be interpreted as being a subsidy to banks eligible to access the central bank facilities. Rather the central bank could decide on an *aggregate* amount of borrowing at the MLF and auction this amount to eligible banks. Details of the auction (fixed or variable rate tender, etc.) should be carefully studied. The auction would introduce an interesting element as those banks in needs for larger limits would price their needs themselves. Also, this would allow the central bank to control the overall maximum amount to be borrowed from the MLF, and as such would facilitate the targeting of the liquidity deficit.

Buying limits (Whitesell) Finally, an alternative to auctioning limits, which may bring problems of its own, is for the central bank to sell, on demand, MLF limits for a fee. The fee could just be a couple of basis points or a more complex pricing function. The main difference with auctioning limits is that the central bank would then not be able to set the aggregate amount available at the MLF.

7 Conclusion

In this paper, we analyzed the behaviour of interbank market rates, when the implementation framework does not use reserve averaging over a maintenance period, but offer a buffer through lending at a main lending facility. This facility offers banks the possibility to borrow at a rate which is the average of the deposit and lending facilities rates. However, the central bank can impose a limit on the amount borrowed at the main lending facility. Within this framework, we showed that rates are less volatile than under an implementation framework that uses reserve averaging, and this without affecting the functionning of the interbank market. These results are promising, but further analysis should now be devoted to explore the optimal limit size of the main lending facility.

8 'Technical' Appendix

8.1 Derivation of V_s and V_y

From the definition of V we have

$$\begin{aligned} V\left(s_i, y_i\right) &= \int_{-\infty}^{s_i} \left[-\left(1+i_m\right) y_i + \left(1+i_d\right)\left(s_i - \lambda_i\right)\right] f\left(\lambda\right) d\lambda \\ &+ \int_{s_i}^{s_i+B_i} \left[-\left(1+i_b\right)\left(\lambda_i - s_i\right) - \left(1+i_m\right) y_i\right] f\left(\lambda\right) d\lambda \\ &+ \int_{s_i+B_i}^{+\infty} \left[-\left(1+i_b\right) B_i - \left(1+i_m\right) y_i - \left(1+i_\ell\right)\left(\lambda_i - \left(s_i + B_i\right)\right)\right] f\left(\lambda\right) d\lambda. \end{aligned}$$

This can be easily simplified to

$$\begin{aligned} V\left(s_i, y_i\right) &= -\left(1+i_m\right) y_i + \left(1+i_d\right) s_i F\left(s_i\right) - \left(1+i_d\right) \int_{-\infty}^{s_i} \lambda_i f\left(\lambda\right) d\lambda \\ &+ \left(1+i_b\right) s_i \left[F\left(s_i + B_i\right) - F\left(s_i\right)\right] - \left(1+i_b\right) \int_{s_i}^{s_i+B_i} \lambda_i f\left(\lambda\right) d\lambda \\ &- \left[\left(i_b - i_\ell\right) B_i - \left(1+i_\ell\right) s_i\right]\left[1 - F\left(s_i + B_i\right)\right] - \left(1+i_\ell\right) \int_{s_i+B_i}^{+\infty} \lambda_i f\left(\lambda\right) d\lambda. \end{aligned}$$

or

$$\begin{aligned} V(s_i, y_i) &= -(1+i_m)\, y_i - i_d \int_{-\infty}^{s_i} \lambda_i f(\lambda)\, d\lambda - i_b \int_{s_i}^{s_i+B_i} \lambda_i f(\lambda)\, d\lambda - i_\ell \int_{s_i+B_i}^{+\infty} \lambda_i f(\lambda)\, d\lambda \\ &\quad + [(1+i_d)\, s_i - (1+i_b)\, s_i]\, F(s_i) \\ &\quad + [(1+i_b)\, s_i + (i_b - i_\ell)\, B_i - (1+i_\ell)\, s_i]\, F(s_i + B_i) \\ &\quad - (i_b - i_\ell)\, B_i + (1+i_\ell)\, s_i \end{aligned}$$

which is equivalent to

$$\begin{aligned} V(s_i, y_i) &= -(1+i_m)\, y_i - (i_b - i_\ell)\, B_i + (1+i_\ell)\, s_i \\ &\quad - (i_b - i_d)\, s_i F(s_i) - (i_\ell - i_b)\,(s_i + B_i)\, F(s_i + B_i) \\ &\quad - i_d \int_{-\infty}^{s_i} \lambda_i f(\lambda)\, d\lambda - i_b \int_{s_i}^{s_i+B_i} \lambda_i f(\lambda)\, d\lambda - i_\ell \int_{s_i+B_i}^{+\infty} \lambda_i f(\lambda)\, d\lambda \end{aligned}$$

Therefore we obtain, using Leibniz Rule:

$$\begin{aligned} V_y(s_i, y_i) &= -(1+i_m) \\ V_s(s_i, y_i) &= (1+i_\ell) - (i_b - i_d)\, F(s_i) - (i_b - i_d)\, s_i f(s_i) - (i_\ell - i_b)\, F(s_i + B_i) \\ &\quad - (i_\ell - i_b)\,(s_i + B_i)\, f(s_i + B_i) \\ &\quad - i_d s_i f(s_i) + i_b s_i f(s_i) - i_b (s_i + B_i)\, f(s_i + B_i) + i_\ell (s_i + B_i)\, f(s_i + B_i) \\ &= (1+i_\ell) - (i_b - i_d)\, F(s_i) - (i_\ell - i_b)\, F(s_i + B_i) \end{aligned}$$

8.2 Transaction costs

Given bank i choses to access the interbank market, it solves the same problem as before, with first order condition gives $V_y + V_s = 0$. From the definition of V, we have (see the Appendix for details),

$$\begin{aligned} V_y(s_i + y_i - \xi, y_i) &= -(1+i_m) \\ V_s(s_i + y_i - \xi, y_i) &= (1+i_\ell) - (i_b - i_d)\, F(s_i - \xi + y_i) - (i_\ell - i_b)\, F(s_i - \xi + y_i + B_i) \\ &= (1+i_d)\, F(s_i - \xi + y_i) + (1+i_b)\,[F(s_i - \xi + y_i + B_i) - F(s_i - \xi + y_i)] \\ &\quad + (1+i_\ell)\,[1 - F(s_i - \xi + y_i + B_i)] \end{aligned}$$

so that bank i trading activity is given by

$$(i_b - i_d)\, F(s_i - \xi + y_i) + (i_\ell - i_b)\, F(s_i - \xi + y_i + B_i) = i_\ell - i_m$$

Bank i expected payoff in this case is

$$V(s_i-\xi+y_i,y_i) = (1+i_m)(s_i-\xi)-(i_b-i_\ell)B_i\left[1-F(s_i-\xi+y_i+B_i)\right]-\mu -i_d\int_{-\infty}^{s_i-\xi+y_i}\lambda_i f(\lambda)\,d\lambda - i_b\int_{s_i-\xi+y_i}^{s_i-\xi+y_i+B_i}\lambda_i f(\lambda)\,d\lambda - i_\ell\int_{s_i-\xi+y_i+B_i}^{+\infty}\lambda_i f(\lambda)\,d\lambda.$$

Note that in this case, given B_i and i_m, bank i exists the interbank market with the same level of account balance $s_i-\xi+y_i=A_i$, independent of s_i.

However, if bank i choses to remain inactive, its expected payoff is

$$V(s_i) = (1+i_\ell)s_i-(i_\ell-i_b)s_iF(s_i+B_i)-(i_b-i_d)s_iF(s_i)-(i_b-i_\ell)B_i\left[1-F(s_i+B_i)\right]-\mu -i_d\int_{-\infty}^{s_i}\lambda_i f(\lambda)\,d\lambda - i_b\int_{s_i}^{s_i+B_i}\lambda_i f(\lambda)\,d\lambda - i_\ell\int_{s_i+B_i}^{+\infty}\lambda_i f(\lambda)\,d\lambda$$

Hence, bank i will access the interbank market whenever

$$V(s_i-\xi+y_i,y_i)\geq V(s_i,0)\,,\text{ or}$$

$$(i_m-i_\ell)s_i+(i_b-i_d)s_iF(s_i)+(i_\ell-i_b)s_iF(s_i+B_i)-(i_b-i_\ell)B_i\left[F(s_i+B_i)-F(s_i-\xi+y_i+B_i)\right] \geq (1+i_m)\xi+i_d\left[\int_{-\infty}^{s_i-\xi+y_i}\lambda_i f(\lambda)\,d\lambda-\int_{-\infty}^{s_i}\lambda_i f(\lambda)\,d\lambda\right]+i_b\left[\int_{s_i-\xi+y_i}^{s_i-\xi+y_i+B_i}\lambda_i f(\lambda)\,d\lambda-\int_{s_i}^{s_i+B_i}\lambda_i f(\lambda)\,d\lambda\right] +i_\ell\left[\int_{s_i-\xi+y_i+B_i}^{+\infty}\lambda_i f(\lambda)\,d\lambda-\int_{s_i+B_i}^{+\infty}\lambda_i f(\lambda)\,d\lambda\right]$$

Since, given i_m and B_i, the account balance level after trade on the interbank market does not depend on s_i (the trading activity y_i, will!), we can rewrite this inequality as

$$-(i_\ell-i_m)s_i+(i_b-i_d)s_iF(s_i)+(i_\ell-i_b)s_iF(s_i+B_i)-(i_b-i_\ell)B_i\left[F(s_i+B_i)-F(A_i+B_i)\right] -i_d\left[\int_{-\infty}^{A_i}\lambda_i f(\lambda)\,d\lambda-\int_{-\infty}^{s_i}\lambda_i f(\lambda)\,d\lambda\right]-i_b\left[\int_{A_i}^{A_i+B_i}\lambda_i f(\lambda)\,d\lambda-\int_{s_i}^{s_i+B_i}\lambda_i f(\lambda)\,d\lambda\right] -i_\ell\left[\int_{A_i+B_i}^{+\infty}\lambda_i f(\lambda)\,d\lambda-\int_{s_i+B_i}^{+\infty}\lambda_i f(\lambda)\,d\lambda\right]\geq(1+i_m)\xi \quad (7)$$

Proposition 6 *Given B_i and i_m, there are two unique levels of account balance $\underline{s}_i$ and $\bar{s}_i$, such that $z(s_i,i_m)=0$ if $s_i\in(\underline{s}_i,\bar{s}_i)$, and $z(s_i,i_m)=1$, otherwise.*

Proof. When $s_i = A_i$, the left hand side of the inequality is zero. So the inequality does not hold and bank i will not pay the fixed cost to enter the interbank market. When $s_i \to +\infty$, the left hand side tends to

$$(i_m - i_d)\, s_i - (i_b - i_\ell)\, B_i \left[1 - F\left(A_i + B_i\right)\right] - i_d \left[\int_{-\infty}^{A_i} \lambda_i f(\lambda)\, d\lambda - \mu\right] - i_b \int_{A_i}^{A_i+B_i} \lambda_i f(\lambda)\, d\lambda - i_\ell \int_{A_i+B_i}^{+\infty} \lambda_i f(\lambda)\, d\lambda$$

which itself tends to $+\infty$ since $i_m > i_d$. When $s_i \to -\infty$, the left hand side tends to

$$-(i_\ell - i_m)\, s_i + (i_b - i_\ell)\, B_i F\left(A_i + B_i\right) - i_d \int_{-\infty}^{A_i} \lambda_i f(\lambda)\, d\lambda - i_b \int_{A_i}^{A_i+B_i} \lambda_i f(\lambda)\, d\lambda - i_\ell \left[\int_{A_i+B_i}^{+\infty} \lambda_i f(\lambda)\, d\lambda - \mu\right]$$

which itself tends to $+\infty$, since $i_\ell > i_m$. Therefore, by continuity, there is $\underline{s}_i$ and $\bar{s}_i$, such that $z(s_i, i_m) = 0$ if $s_i \in (\underline{s}_i, \bar{s}_i)$, and $z(s_i, i_m) = 1$, otherwise. Now we prove that $\underline{s}_i$ and $\bar{s}_i$ are unique for each i. To show this, it suffices to show that the left side of inequality (7) is monotone. Taking its derivative of the left hand side of (7) and simplifying, we obtain

$$-(i_\ell - i_m) + (i_b - i_d)\, F(s_i) + (i_\ell - i_b)\, F(s_i + B_i) \tag{8}$$

This depicts the marginal gains from trade, given initial account balance s_i. Given the definition of A_i, (8) is obviously zero at $s_i = A_i$. Also, (8), is strictly positive for $s_i > A_i$, and (8) is strictly negative for $s_i < A_i$. This proves that $\underline{s}_i$ and $\bar{s}_i$ are unique for each i. ■

8.3 Simulations

We simulate the model using similar assumption as in GPR (2007). In this way, their results can be used as a benchmark for our analysis. Therefore, there will be a very closed link between this section and the same section in GMP.

We simulate an economy where the lending rate at the residual lending facility is $i_\ell = 5$ percent and the deposit rate is $i_d = 3$ percent. This leaves a 200 basis point corridor, as in the Eurosystem. The main lending facility rate is set to $i_b = 4$ percent, the middle of the corridor. We assume that the cap is the same for all banks and we simualte the economy for different levels of cap $B = 0, 100, 230, 500, 1000$ and 1500.

For each economy, the equilibrium outcome can be characterized by a single interest rate each day and a distribution of quantities traded. This distribution and the interest rate depends on the heterogeneity of banks, that is the level of reserves they start day t with, $s_i(t)$, after the early shock hit. We model the early shock λ_i^e as a random draw from a normal distribution with mean $-B/2$ and standard deviation $\sigma = 20$. All banks face the same structure of early shocks.

To solve for the equilibrium prices and quantities we simulate $Z = 5000$ days identical to the one described at the beginning of the previous section. We construct late shocks as follows. We assume that λ_i represents the uncertainty about the changes in the current account by the end of the day because of clerical errors, etc. As in GPR (2007), we define $\lambda_i(k)$ as the random transfer of funds between bank i and bank k. This transfer is assumed to be distributed according to a normal distribution with mean 0 and standard deviation $\sigma = 20$. If the shock is positive, funds are moved from bank k to bank i and inversely. We assume $\lambda_i(i) = 0$. The shock λ_i is therefore

$$\lambda_i = \sum_{k=1}^{n} \lambda_i(k).$$

Hence, λ_i follows a normal distribution with zero mean and standard deviation $\sigma_i = \sigma\sqrt{12-1}$. Also, since late shocks sum up to zero across banks, no balances enter or leave the system. Moreover these shocks have co-variances and correlations equal to

$$E(\lambda_i \lambda_k) = -E\left(\lambda_i(k)^2\right) = -\sigma^2 \text{ and } \rho(\lambda_i \lambda_k) = \frac{E(\lambda_i \lambda_k)}{\sigma_i \sigma_k} = -\frac{1}{12-1}.$$

In our simulation, the standard deviation of individual transfers is $\sigma = 20$ which implies that banks are subject to shocks which are jointly normal with standard deviation of $\sigma_j = \sigma(12-1)^{1/2} = 66.33$ and a correlation between shocks of $\rho(\lambda_i, \lambda_k) = -0.09$. Even in this simple case with no shocks that change the overall supply of reserves, GPR (2007) show that the statistical properties of prices and quantities traded of their model follow the pattern found in the data for the EONIA.

Table 1 reports descriptive statistics for the state of the system defined by the initial levels of reserves s_i, and trade y_i. for $n = 12$. The column $mean(s)$ correspond to the following statistics

$$mean(s) = \frac{1}{Z} \sum_{z=1}^{Z} \frac{1}{n} \sum_{i=1}^{n} s_i(z)$$

where $s_i(z)$ is the initial reserves for bank i after the early shock is realised in the z^{th} simulation. The column "$\sigma(s)$" is the standard deviation of the variable s across simulations and is computed as

$$\sigma(s) = \sqrt{\frac{1}{Z} \sum_{z=1}^{Z} \left(\frac{1}{n} \sum_{i=1}^{n} [s_i(z) - mean(s)] \right)^2}.$$

The computation of the statistics for y is similar, although we restricted our attention to borrowing (i.e. to the observations such that $y > 0$). For convenience we also report the statistics computed in GPR (2007) for the same variable.

	$\lambda^e \sim N(-B/2, 20)$				Gaspar et. al. (2007)		
B	$mean(\mathbf{s})$	$\sigma(\mathbf{s})$	$mean(\mathbf{y})$	$\sigma(\mathbf{y})$		$mean(\mathbf{y})$	$\sigma(\mathbf{y})$
0	0.1	19.2	303.7	69.1	$t = 2$	441.1	82.3
100	-49.9	19.2	302.6	67.8	$t = 3$	687.4	50.4
230	-115.4	19.2	302.6	69.9			
500	-250	19.2	304.9	69.5			
1000	-499.3	18.8	304.2	68.5			
1500	-750	19.3	304.3	70.1			

Table 1: Average and standard deviation of banks' level of early reserves and borrowing levels.

Note that introducing the MLF does not reduce market activity when limits increase. This should not come as a surprise, as equation (3) shows that the level of borrowing does not depend on the size of the limit at the MLF. That banks' behaviour on the interbank market is not affected by the liquidity deficit, is explained by the fact that banks trade away their idiosyncratic shock on the interbank market, as when there is no aggregate liquidity deficit, and access the MLF to cover the aggregate liquidity deficit. However, market activities remain relatively more subdued that in a framework with reserves. The reason is that along the maintenance period, heterogeneity across banks increases, given more ground for trade. It is also important to note that in our framework, although the size of the MLF limit increases, the activity on the interbank market remains as important as when there is no MLF ($B = 0$) - that is when we would impose a daily reserve requirement and no reserves averaging. Table 2 below shows that the equilibrium interest rate decreases as the limit on the MLF loosens. The downward pressure on the interest rate comes from the MLF limit itself, since the higher this limit, the less likely banks are to access the residual lending facility.

	$\lambda^e \sim N(-B/2, 20)$		Gaspar et. al. (2007)	
B	$mean(\mathbf{i}_m)$	$d(\mathbf{i}_m)$		$d(\mathbf{i}_m)$
0	3.99	0.099	$t = 2$	0.0145
100	3.99	0.059	$t = 3$	0.0272
230	4	0.0133		
500	4	0		
1000	4	0		
1500	4	0		

Table 2: Average interbank rate and dispersion meaures.

Table 2 reports some statistics on the interbank market rate. We compute the average interest rate for this economy. Then we compute a measure of the dispersion of the interbank market rate as the square of the change between the aggregate interest rate in day t and its value at $t-1$, as in GPR (2007). The average of that series is shown in the column labelled $d(\mathbf{i}_m)$, while we report the standard deviation of i_m in the column $\sigma(\mathbf{i}_m)$. Both numbers give an indication of how volatile the aggregate interest rate is in the time series dimension. The interbank market rate volatility is decreasing with the limit on the MLF, and when $B \geq 500$, the variability disappears contrary to what GPR (2007) find. Interbank rate variability remains in a reserve averaging framework, as banks also take into account future shocks against which they may want to insure. This dynamic aspect creates some dispersion in interbank rates. As our simulation shows, we obtain approximately the same variability as in GPR (2007), when $B = -230$, which is approximately twice the size of their daily reserve requirements for a 2 days maintenance period.[11] Finally, we report the probability to access the three standing facilities in Table 3 below.

B	$\Pr RLF$	RLF	$\Pr MLF$	MLF	$\Pr DF$	DF
0	0.5	27.5	NaN	NaN	0.5	33.1
100	0.23	9.4	1	26.9	0.005	19.6
230	0.05	1.6	1	102.8	0	6.7
500	0	0	1	249.9	0	0.04
1000	0	0	1	499.3	0	0
1500	0	0	1	750	0	0
GPR (2007)			-	-		
t=1	0.0658	29.02	-	-	0.0013	18.71
t=2	0.0990	30.08			0.0993	29.54
t=3	0.1369	31.34	-	-	0.4705	56.52

Table 3: Probability and expected amount at each standing facility.

Tables 3 presents the use of the standing facilities for different levels of borrowing limits at the MLF. GPR (2007) computes the probabilities in the following way: for each realisation for the Monte Carlo simulation, the probability of going to a facility is computed for each bank given the distribution of shocks and then those probabilities are averaged over banks and simulations. In our (repeated, but static) framework, the probabilities are entirely defined by B and $mean(s)$, for each bank, since $s + y = mean(s)$ and we do not need to average across all banks. Table 3 includes the expected use that one bank will

[11] GPR (2007) effectively study a two-days reserve averaging system, where banks receive an early shock only in the morning of the first day and no shock otherwise.

make of the facilities (multiply by n to get the aggregate expected recourse to a facility). As one would expect, in our framework without reserves averaging, banks cannot postpone the access to the standing facilities by using the dynamics inherent to a maintenance period. However, introducing the MLF greatly reduces the likelihood of accessing the residual lending facility. Also, note that although there may be more systematic recourse to the facilities, the expected amount borrowed from these facilities is comparable to GPR (2007), and are actually lower when we compare our results to the last day of their maintenance period.

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