Rollover Risk and Market Freezes*

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Abstract

The sub-prime crisis of 2007 and 2008 has been characterized by a sudden freeze in the market for short-term, secured borrowing. We present a model that can explain a sudden collapse in the amount that can be borrowed against assets with little credit risk. The borrowing in this model takes the form of asset-backed commercial paper that has to be rolled over several times before the underlying assets mature and their true value is revealed. In the event of default, the creditors (holders of commercial paper) can seize the collateral. We assume that there is a small cost of liquidating the assets. The debt capacity of the assets (the maximum amount that can be borrowed using the assets as collateral) depends on how information about the quality of the asset is revealed. In one scenario, there is a constant probability that "bad news" is revealed each period and, in the absence of bad news, the value of the assets is high. We call this the "optimistic" scenario because, in the absence of bad news, the expected value of the assets is increasing over time. By contrast, in another scenario, there is a constant probability that "good news" is revealed each period and, in the absence of good news, the value of the assets is low. We call this the "pessimistic" scenario because, in the absence of good news, the expected value of the assets is decreasing over time. In the optimistic scenario, the debt capacity of the assets is equal to the fundamental value (the expected NPV), whereas in the pessimistic scenario, the debt capacity is below the fundamental value and is decreasing in the liquidation cost and frequency of rollovers. In the limit, as the number of rollovers becomes unbounded, the debt capacity goes to zero even for an arbitrarily small default risk. Our model explains why markets for rollover debt, such as asset-backed commercial paper, may experience sudden freezes. The model also provides an explicit formula for the haircut in secured borrowing or repo transactions.

J.E.L. Classification: G12, G21, G24, G32, G33, D8.

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1 Introduction

1.1 Motivation

One of the many striking features of the sub-prime crisis of 2007 and 2008 has been the sudden freeze in the market for the rollover of short-term debt. While the rationing of firms in the unsecured borrowing market is not uncommon and has a long-standing theoretical underpinning (see, for example, the seminal work of Stiglitz and Weiss, 1981), what is puzzling is the almost complete inability of financial institutions to issue (or roll over) short-term debt backed by assets with relatively low credit risk. From a theoretical standpoint, this is puzzling because the ability to pledge assets and provide collateral has been considered one of the most important tools available to firms in order to get around credit rationing (Bester, 1985). From an institutional perspective, the inability to borrow overnight against high-quality assets has been a striking market failure that effectively led to the demise of a substantial part of investment banking in the United States. More broadly, it led to the collapse in the United States, the United Kingdom, and other countries of banks and financial institutions that had relied on the rollover of short-term wholesale debt in the asset-backed commercial paper (ABCP) and overnight secured repo markets.

The first such collapse occurred in the Summer of 2007. On July 31, 2007, two Bear Stearns hedge funds based in the Cayman Islands and invested in sub-prime assets filed for bankruptcy. Bear Stearns also blocked investors in a third fund from withdrawing money. In the week to follow, more news of problems with sub-prime assets hit the markets. Finally, on August 7, 2007, BNP Paribas halted withdrawals from three investment funds and suspended calculation of the net asset values because it could not "fairly" value their holdings:[1]

> "[T]he complete evaporation of liquidity in certain market segments of the US securitization market has made it impossible to value certain assets fairly regardless of their quality or credit rating... Asset-backed securities, mortgage loans, especially sub-prime loans don't have any buyers... Traders are reluctant to bid on securities backed by risky mortgages because they are difficult to sell on... The situation is such that it is no longer possible to value fairly the underlying US ABS assets in the three above-mentioned funds."

This announcement appeared to cause investors in asset backed commercial paper (ABCP), primarily money market funds, to shy away from further financing of ABCP structures. These investors could no longer be guaranteed that there was minimal risk in ABCP debt. In particular, many ABCP vehicles had recourse to sponsor banks that set up these vehicles as off-balance-sheet structures but provided them with liquidity and credit enhancements

[1] Source: "BNP Paribas Freezes Funds as Loan Losses Roil Markets" (Bloomberg.com, August 9, 2008).

(Acharya and Schnabl, 2009a,b). If ABCP debt could not be rolled over, the sponsor banks would have to effectively take assets back onto their balance-sheets. But, given the assets had little liquidity, the banks' ability to raise additional finance – typically rollover debt in the form of financial CP – would be limited too. Money market funds thus faced the risk that the assets underlying ABCP would be liquidated at fire-sale prices. This liquidation and rollover risk produced a "freeze" in the ABCP market, raised concerns about counter-party risk amongst the banks, and caused LIBOR to shoot upwards. The sub-prime crisis truly took hold as the European Central Bank pumped 95 billion Euros in overnight lending into the market that same day in response to the sudden demand for cash from banks.

The failure of Bear Stearns in mid-March 2008 was the next example of a market freeze.[2] As an intrinsic part of its business, Bear Stearns relied day-to-day on its ability to obtain short-term finance through secured borrowing. Beginning late Monday, March 10, 2008 and increasingly through that week, rumors spread about liquidity problems at Bear Stearns and eroded investor confidence in the firm. Even though Bear Stearns continued to have high quality collateral to provide as security for borrowing,[3] counterparties became less willing to enter into collateralized funding arrangements with the firm. This resulted in a crisis of confidence late in the week, where counterparties to Bear Stearns were unwilling to make even secured funding available to the firm on customary terms. This unwillingness to fund on a secured basis placed enormous stress on the liquidity of Bear Stearns. On Tuesday, March 11, the holding company liquidity pool declined from \$18.1 billion to \$11.5 billion (see Figure 1). On Thursday, March 13, Bear Stearns' liquidity pool fell sharply and continued to fall on Friday. In the end, the market rumors about Bear Stearns' liquidity problems became self-fulfilling and led to the near failure of the firm. Bear Stearns was adequately capitalized at all times during the period from March 10 to March 17, up to and including the time of its agreement to be acquired by J.P. Morgan Chase. Even at the time of its sale, Bear Stearns' capital and its broker dealers' capital exceeded supervisory standards. In particular, the capital ratio of Bear Stearns was well in excess of the 10% level used by the Federal Reserve Board in its well-capitalized standard.

— Figure 1 about here —

In his analysis of the failure of Bear Stearns, the Federal Reserve Chairman Ben Bernanke observed:[4]

[2]The discussion that follows is based on the Security and Exchange Commission's Chairman Christopher Cox's Letter to the Basel Committee in Support of New Guidance on Liquidity Management, available at: http://www.sec.gov/news/press/2008/2008-48.htm

[3]This high quality collateral mainly consisted of highly rated mortgage-backed assets which had low but not inconsequential credit risk by this time in the sub-prime crisis.

[4]The quotes are based on Ben Bernanke's remarks to the Risk Transfer Mechanisms and Financial Stability Workshop at the Bank for International Settlements, May 29, 2008.

> "[U]ntil recently, short-term repos had always been regarded as virtually risk-free instruments and thus largely immune to the type of rollover or withdrawal risks associated with short-term unsecured obligations. In March, rapidly unfolding events demonstrated that even repo markets could be severely disrupted when investors believe they might need to sell the underlying collateral in illiquid markets. Such forced asset sales can set up a particularly adverse dynamic, in which further substantial price declines fan investor concerns about counterparty credit risk, which then feed back in the form of intensifying funding pressures. . . In light of the recent experience, and following the recommendations of the President's Working Group on Financial Markets (2008), the Federal Reserve and other supervisors are reviewing their policies and guidance regarding liquidity risk management to determine what improvements can be made. In particular, future liquidity planning will have to take into account the possibility of a sudden loss of substantial amounts of secured financing."

1.2 Model and results

Our paper is an attempt to provide a theoretical model of such market freezes. We consider the debt capacity of a finitely-lived asset when (i) the debt is short-term in nature and, hence, needs to be rolled over; (ii) there is the risk of a fire sale in the event the borrower defaults and the current lender needs to seize and liquidate the underlying assets; and (iii) the arrival of information about the quality of the assets may be "pessimistic" in nature (in a sense to be formalized below). These are essentially the features alluded to in the preceding discussion of the conditions surrounding the freeze in the market for ABCP and the collapse of Bear Stearns.

When debt is short-term in nature, it is natural to assume that uncertainty about the credit risk of the underlying asset will not be fully resolved by the date of the next rollover. In fact, debt may have to be rolled over several times before information about the asset is completely revealed. In the period of the 2007-2008 crisis, there were numerous examples of the slow release of information about the quality of assets held by banks or used as collateral for asset-backed securities. On the one hand, the difficulty of valuing complex securities or assets for which no markets existed led to repeated changes in the losses announced by financial institutions. On the other, these same institutions were loath to reveal all their holdings of troubled assets. For both these reasons, the true extent of the financial distress dribbled out over many months. In these circumstances, a lender that is unwilling or unable to extend credit until the assets mature has to take into account the risk that the borrower will not be able to find another lender, that is, to roll over his debt. Does such rollover risk diminish the debt capacity of an asset? The answer to this question depends crucially on the way in which information is released relative to the rate at which debt is being rolled over.

Although the preceding discussion makes it clear that the release of information is endogenous to the market, from the point of view of lenders it can be treated as an exogenous signal. We consider two different *information structures*, that is, two different assumptions about the signals that reach the commercial paper market. In each structure, there is a constant small hazard of news arriving in each time period, but the nature of the news is different. The first structure, which we call the "optimistic" structure, is illustrated in Figure 2a. In the optimistic information structure, at each rollover date, either "bad news" is released, revealing that the assets underlying the lending have no value, or no news is released. If bad news never arrives, the value of the asset is positive, so obviously no news is "good news". The information structure is "optimistic" in the sense that, as time passes and bad news has not arrived, the probability that the value of the assets will be positive is increasing, as illustrated in Figure 2b.

The standard result in efficient markets is that the debt capacity[5] of an asset is equal to its NPV or "fundamental" value. We show in the sequel that this result holds when the information structure is optimistic, the debt capacity of an asset is simply equal to the expected present value of the asset's cash flows at maturity. In other words, there is credit risk but no rollover risk.

— Figures 2a, 2b and 3a, 3b about here —

The standard result breaks down, however, when the information structure is "pessimistic." The pessimistic structure is illustrated in Figure 3a. In the pessimistic structure, there is a constant hazard at each rollover date that "good news" is released, revealing that the value of the asset is high. If good news never arrives, the value of the asset is zero, so in this case no news is "bad news." The structure is pessimistic in the sense that, as time passes and good news has not arrived, the probability that the asset will have positive value decreases, as illustrated in Figure 3b. Intuitively, this information structure is pessimistic in the sense that investors are waiting for good news and the outlook gets worse each time good news fails to arrive.

Our main objective in this paper is to characterize the rollover debt capacity[6] of an asset used as collateral for short-term borrowing. Using the methods developed in this paper, we can show that, under the pessimistic information structure, the rollover debt capacity of an asset is *always* smaller than its buy-to-hold debt capacity.[7] We also establish that

[5] The *debt capacity* is the maximum amount of debt that can be obtained using the asset as collateral.

[6] The *rollover debt capacity* is the maximum amount of debt that can be obtained when the debt has to be rolled over each period.

[7] The *buy-to-hold debt capacity* refers to the maximum amount of debt that could be obtained if the term of the debt guaranteed that the borrower could hold the assets to maturity without risk of liquidation.

the rollover debt capacity is declining in the rollover frequency, liquidation cost, and credit risk. In fact, our main result shows that the debt capacity of an asset *tends to zero as the number of debt rollovers grows without bound*. We call this phenomenon a "market freeze." This remarkable and perhaps counter-intuitive result holds for *arbitrarily small credit risks*, capturing the scenario that Bear Stearns experienced during its failure in March 2008.

The intuition for the "market-freeze" result can be explained as follows. In the pessimistic case, things are going to get worse unless "good news" arrives. When the rate at which good news is rationally expected to arrive is much slower than the rate at which the debt has to be rolled over, the borrower anticipates that he will still be in the pessimistic scenario with probability close to one at the next roll over date. Then debt capacity today is essentially determined by debt capacity tomorrow. As the final rollover date approaches, however, the fundamental value of the assets is zero in the pessimistic case, so the debt capacity is zero as well. By backwards induction the debt capacity at any preceding date in the pessimistic case must be close to zero as well.

A similar backward induction argument in the optimistic case leads to the opposite result, since the fundamental value – and hence the debt capacity – increases over time as the maturity date is approached in the optimistic case.

The two special information structures can be nested in a more general regime-switching model. More precisely, we assume there are two states, a pessimistic state and an optimistic state. The value of the asset is determined by the state of the economy in the final period when the asset matures. If the terminal state is optimistic, the value of the asset is positive; otherwise, it is zero. In each period, the economy switches, with some small probability, from the prevailing state to the other one.[8] We show that, as the rollover frequency becomes unbounded, the debt capacity in the optimistic state is bounded away from zero and in the pessimistic state tends to zero. In particular, a sudden switch in the market's expectation from the optimistic state to the pessimistic one can cause the market for rollover asset-backed debt to freeze.

A sudden market freeze arises in the model because of a change in investors' rational expectations. Instead of good news being the default state and bad news being a dreaded surprise, now bad news is the default state and good news the hoped for surprise. If good news is likely to arrive at a slower rate than the debt has to be rolled over, the effect on debt capacity may be catastrophic, even if the fundamental value of the asset (the probability distribution over terminal states) has not changed significantly.

Something like a switch from optimistic to pessimistic states or expectations may have

[8] An imperfect analogy is to think about an economy whose state is characterized by a regime-switching model. When it is more likely to be in the healthy state, the more it stays in that state, the less likely it is to switch to the recession state before the assets mature. Conversely, when it is more likely in a recession, the more it stays in that state, the less likely it is to switch to the healthy state before the assets mature. No news in the healthy state is good news, whereas no news in recession is bad news.

happened in the second quarter of 2007. The cause was growing awareness of the poor performance of securities backed by sub-prime mortgages and, more importantly, the failure of the existing valuation models to predict their market prices, as revealed by BNP Paribas in its August 9 announcement of suspension of NAV calculation for three of its funds invested in sub-prime assets. Normal risks are familiar and can be discounted with confidence. The realization that no one knew how to value these complex securities caused a fundamental change in attitudes. The future looked bleak, absent the arrival of some new information that would persuade investors that they could return to business as usual. In the event, such information did not arrive in time (perhaps was strategically held back) so that investors were not persuaded that they could carry on business as usual. In fact, many institutions, especially money market mutual funds, were persuaded that they should not be in the business of accepting commercial paper backed by these assets. The "freeze" was on. What occurred was an "adverse dynamic," to borrow Bernanke's phrase, whereby a fundamental change in information structures had reduced debt capacity through backward induction, in the limiting case diminishing the debt capacity of the assets to zero.

It is important to acknowledge that we take the short-term nature of debt and fire sales as given. That investment banks are (or used to be!) funded with rollover debt and that debt capacity can be higher with short-term debt under some circumstances for many underlying assets, are interesting facts in their own right. Indeed, there exist agency-based explanations in the literature (for example, Diamond, 1989, 1991, 2004, Calomiris and Kahn, 1991, and Diamond and Rajan, 2001a, 2001b) for the existence of short-term debt as optimal financing in such settings. Our model presents a counter-example to the claim that short-term debt maximizes debt capacity: when expectations are pessimistic, debt capacity through short-term borrowing may in fact be arbitrarily small, suggesting that institutions ought to arrange for this possibility by funding themselves also through sufficiently long-term financing. Providing a micro (for example, an agency-theoretic) foundation for debt maturity in a model where there is a switch between optimistic and pessimistic regimes is a fruitful goal for future research, but one that is beyond the scope of this paper.

Our model assumes that when a lender needs to seize assets, these can only be sold to another buyer who must also finance the assets with short-term debt. Further, the maximum liquidation price the lender can obtain is proportional to, but smaller than, the new debt capacity of assets. This assumption captures the idea of a "fire sale" that is rooted either in the specificity of assets to current owners or in finance constraints for arbitrageurs or both.[9]

[9]There is a large body of literature in finance and economics justifying, verifying or employing fire sales of assets during periods of industry- or economy-wide shocks. On the theoretical front, Williamson (1988) and Shleifer and Vishny (1992) link this to the notion of specificity of assets, that is, the non-transferability of assets across industries. On the empirical front, Pulvino (1998), Krugman (1998), Aguiar and Gopinath (2005), Coval and Stafford (2006), Acharya, Bharath and Srinivasan (2007), and Acharya, Shin and Yorulmazer (2007) have provided evidence of fire sales in real and financial markets in a variety of

From the standpoint of our paper, these two features – short-term borrowing and fire sales – imply that market freezes due to changes in expectations about the rollover risk are most likely when the borrowing and/or lending horizon is short-term and the underlying assets are "crowded" in the sense that most financial institutions are on one side of the market for these assets. The first feature generates rollover risk and the second feature generates fire sales in asset liquidations.

1.3 Applications

Our results can alternatively be stated in terms of the so-called "haircut" of an asset when it is pledged for secured borrowing or used in a repo transaction. Measuring the haircut as one minus the ratio of the debt capacity of an asset to its expected value (or the debt capacity for buy-to-hold debt), our model shows that the haircut can be calculated simply, based on three inputs: the credit risk of the asset (the hazard rate of default), the number of times the debt must be rolled over before the asset matures, and the fire-sale discount incurred in the event of liquidation of the assets by the lender. Under the optimistic information structure, the haircut is zero whereas, under the pessimistic structure, the haircut is positive and reaches 100% in the limit as the rollover frequency becomes unbounded. More generally, when there is the possibility of a switch between the optimistic and the pessimistic scenarios, the haircut is positive but small in the optimistic case and can switch to a very large number in the pessimistic case.[10] Interestingly, while some of the collateralized debt and loan obligations (CDO and CLO) have had no secured borrowing capacity at all during the sub-prime crisis, equities – which are in principle riskier assets – had only around a 20% haircut.[11] This is consistent with our model since, when there is rollover risk, it is not the underlying risk of asset's cash flows but its rollover risk which primarily affects its debt capacity.

Overall, our model and results apply to several institutional settings. The most natural candidate, as we have discussed, is the practice of taking assets off-balance-sheet, putting enough capital and liquidity/credit enhancements to make them "bankruptcy-remote" and AAA-rated, and then borrowing short-term against these assets. Such structures, charac-

settings. And, on an applied front, a large literature on financial crises (starting with Allen and Gale 1994, 1998) has employed fire sales as a key modeling device.

[10] Shin (2008), for example, documents based on data from Bloomberg, that the typical haircuts on treasuries, corporate bonds, AAA asset-backed securities, AAA residential mortgage-backed securities and AAA jumbo prime mortgages are respectively, less than 0.5%, 5%, 3%, 2% and 5%, whereas, in March 2008, these haircuts respectively rose to between 0.25% and 3%, 10%, 15%, 20% and 30%. Gorton and Metrick (2009) also show that the hair-cuts on borrowing against a number of securitization tranches (backed by mortgages, auto loans and corporate bonds and loans) were close to zero during the first half of 2007, but rose gradually to levels as high as 35% between July 2007 and January 2009. See also the discussion in Brunnermeier and Pedersen (2005) on widening of haircuts in stress times.

[11] See Box 1.5 from Chapter 1, Page 42 of IMF (2008).

terized by a maturity mismatch between assets and liabilities, were prevalent in many forms ("structured investment vehicles" or SIVs, "conduits," and others) in the period leading up to the sub-prime crisis (Crouhy, Jarrow and Turnbull, 2007).

Yet another candidate is the commercial paper market accessed primarily by financial institutions, but also by highly-rated industrial corporations, where rollover at short maturities is a standard feature. Consider the balance-sheet of a financial institution (such as that of Northern Rock or of the investment banks Bear Sterns and Lehman Brothers) where the funding model inherently resembles that of a SIV, that is, long-term risky assets such as mortgages are funded by short-term, asset-backed commercial paper. These markets experienced severe stress during the sub-prime crisis and froze (100% haircut) for many days at a stretch once the expectations about the quality of mortgage assets became pessimistic, even though, prior to this period, they appeared to be the cheapest form of financing available (near-zero haircut).

Overall, the sub-prime crisis of 2007 and 2008 illustrates our key assumptions: Assets were funded with short-term rollover debt; once the crisis broke out, long-term financing was difficult to obtain for any institution; and there have been substantial discounts in the sale of assets in SIVs and conduits.[12]

The rest of the paper proceeds as follows. Section 2 presents a general discrete-time framework that contains the optimistic and pessimistic information structures as special cases (subsections 2.3 and 2.4 respectively) show how to derive the debt capacity in each case; subsection 2.6 extends this analysis to the general case in which the economy can switch between optimistic and pessimistic regimes. A number of comparative static results for cases in which there is a closed form solution for the debt capacity, including a continuous-time model, are gathered together in Section 3. Section 4 discusses the merits of some potential interventions to help unfreeze rollover markets. Section 5 discusses the related literature. Section 6 concludes with some ideas for further work. Appendix I contains proofs. Appendix II extends the discrete-time model to allow for more general information structures.

[12]See, for instance, "SIV restructuring: A ray of light for shadow banking," *Financial Times, June 18 2008*; and "Creditors find little comfort in auction of SIV Portfolio assets," *Financial Times, July 18 2008*, which both report that net asset values due to asset fire sales have fallen below 50% of paid-in capital. As the first article reports: "[W]hen defaults on US subprime mortgages rose last summer, ABCP investors stopped buying [short-term ABCP] notes – creating a funding crisis at SIVs. . . . This situation prompted deep concern about the risk of a looming firesale of assets. The prospect was deemed so alarming that the US Treasury attempted to organize a so-called "super-SIV" last autumn, which was supposed to purchase SIV assets."

2 Model

In the introduction, we described two different information structures, a "pessimistic" one and an "optimistic" one. In this section we develop a framework that nests the two cases of optimistic and pessimistic expectations. There are two states, one of which can be identified with the optimistic regime and the other with the pessimistic regime. The evolution of the economy is governed by a stationary, finite, Markov chain that switches randomly between the two regimes (Figure 4). The two structures described in the introduction can be obtained as limiting cases.

— Figure 4 about here —

2.1 Time

Figure 5 illustrates the timeline.

— Figure 5 about here —

For the sake of illustration, we consider a SIV attempting to raise asset-backed finance. The SIV is set up at date $t = 0$ with a collection of assets as collateral. The sponsoring bank's objective is to maximize the value of the assets transferred to the SIV. This is equivalent to maximizing the amount of debt finance that can be raised by the SIV using the assets as collateral since the sponsoring bank can extract the full amount as payment for the assets. In a perfectly competitive asset market, the sponsoring bank will make zero profit after expenses, because the price of the assets in equilibrium will be bid up until they equal the amount of finance available, i.e., the debt capacity of the assets. So, in equilibrium, finding the maximum debt capacity is equivalent to determining the market value of the assets.

The assets backing the SIV have a finite life (e.g., mortgages) and we normalize units of time so that the assets mature at date $t = 1$. The SIV issues commercial paper with a maturity denoted by $0 < \tau < 1$, which implies that the SIV must rollover its debt exactly N times, where

$$N + 1 = \frac{1}{\tau}.$$

The unit interval $[0, 1]$ is divided into intervals of length τ by a series of dates denoted by t_n and defined by

$$t_n = n\tau,\ n = 0, 1, ..., N + 1,$$

where t_0 is the date the SIV is set up, t_n is the date of the n-th rollover (for $n = 1, ..., N$), and t_{N+1} is the final date at which the assets mature and their terminal value is realized. For the time being we treat the maturity of the commercial paper τ and hence the number of rollovers as fixed. Later we will be interested to see what happens when the number of rollovers needed to span the interval $[0, 1]$ increases without bound.

2.2 Information

There are two states denoted by s_1 and s_2, where s_1 is the "pessimistic" state and s_2 is the "optimistic" state. Transitions between states occur at the dates t_n and are governed by a stationary transition probability matrix

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} p & 1-p \\ 1-q & q \end{bmatrix},$$

where a_{ij} is the probability of a transition from state s_i at date t_n to state s_j at date t_{n+1}. We assume $q > 1-p$. If the economy is in state s_1 at date t_n, it remains in the same state at date t_{n+1} with probability p and switches to state s_2 at date t_{n+1} with probability $1-p$. So, in terms of the story we told in the introduction, p is the probability of no news (which is in fact bad news here) and $1-p$ is the probability of good news. Similarly, in state s_2, the probability of no news (which is in fact good news) is q and the probability of bad news is $1-q$.

The optimistic and the pessimistic information structures illustrated in Figures 2 and 3 are special cases of this more general structure. For example, the pessimistic structure (Figure 3) is obtained by assuming that the initial state at date t_0 is s_1 and that state s_2 is an absorbing state. This requires that $q = 1$ and $0 < p < 1$. Similarly, we obtain the optimist information structure (Figure 2) by assuming that the initial state is s_2 and that s_1 is an absorbing state, that is, $p = 1$ and $0 < q < 1$. We shall use these special cases to illustrate the theory, but our main results hold for the more general regime switching model.

The terminal value of the asset is a random variable $\tilde{V}$, which is defined by

$$\tilde{V} = \begin{cases} 0 & \text{if the state is } s_1 \text{ at the terminal date } t = 1 \\ V > 0 & \text{if the state is } s_2 \text{ at the terminal date } t = 1. \end{cases}$$

For simplicity, we assume that the asset has a current yield of zero and the risk-free interest rate is 0. The market is assumed to be risk neutral.

If the SIV is forced to default and liquidate the assets, we assume that the assets fetch a fraction $\lambda \in [0,1]$ of the *maximum amount of finance that could be raised by the SIV as a going concern*. It is important to note that the recovery rate λ is applied to the maximum debt capacity rather than to the fundamental value of the assets. If the buyer of the assets were a wealthy investor who could hold the assets until maturity, the fundamental value would be the relevant benchmark and the investor might well be willing to pay some fraction of the fundamental value, only demanding a discount to make sure that he does not mistakenly overpay for the assets. What we are assuming here is that the buyer of the assets is another financial institution that must also issue short term debt in order to finance the purchase. Hence, the buyer is constrained by the same forces that determined the debt capacity in the first place. While the debt capacity provides an upper bound on the purchase price, there is no reason to think that the assets will reach this value. In fact, we assume $\lambda < 1$ in what follows.

2.3 Debt capacity in the optimistic structure

Our problem is to determine the maximum amount that can be borrowed by the SIV using only the assets as collateral. We denote the maximum value of debt in state i at date t_n as B_n^i when the SIV is solvent. As a benchmark, we consider first the debt capacity in the optimistic information structure, that is, the special case where $p = 1$ and $0 < q < 1$. We can do this by backward induction, beginning with the period before the asset matures. There are two possible situations to consider. Either (i) bad news has arrived, in which case the value of the asset is 0 for certain, or (ii) bad news has not yet arrived, in which case the value of the asset remains uncertain.

Suppose that bad news has not yet arrived, that is, the economy is in state s_2, at the penultimate date t_N and the SIV issues debt with a face value of D. If $D > V$, the SIV defaults next period regardless of the value of the asset and the expected value of the debt is

$$q\lambda V + (1-q) \times 0 = q\lambda V.$$

On the other hand, if $D \leq V$, then the SIV will only default in state s_1 and the expected value of the debt is

$$qD + (1-q) \times 0 = qD.$$

Then the value of debt is given by the formula

$$\max\left\{q\lambda V, qD\right\},$$

which is maximized by setting $D = V$. If we let B_n^i denote the maximum debt that can be raised in state s_i at date t_n, then we have shown that $B_N^2 = qV$, that is, the maximum debt that can be raised at date t_N in state s_2 is qV.

Now let's move back one period and assume again that good news has not arrived. If the face value of the debt issued is D, the value of the debt in the ante-penultimate period is

$$\begin{cases} q\left(\lambda qV\right) + (1-q) \times 0 & \text{if } D > qV \\ qD + (1-q) \times 0 & \text{if } 0 \leq D \leq qV. \end{cases}$$

It is clear that the value of the debt issued is maximized by setting $D = qV$ and hence, in the notation introduced earlier, $B_{N-1}^2 = q^2 V$.

Continuing in this way, we can calculate the maximum value of the debt that can be raised against the SIV's collateral at any date t_n and find that it is given by the formula

$$B_n^2 = q^{N-n+1} V.$$

Now q^{N-n+1} is the probability that $\tilde{V} = V$, conditional on reaching t_n without receiving bad news. So the debt capacity at any date t_n is simply the fundamental value of the assets. This could be considered the "normal" result, where there is no haircut in secured borrowing the absence of risk aversion and asymmetric information.

2.4 Debt capacity in the pessimistic structure

Now consider pessimistic information structure (Figure 3) in which $q = 1$ and $0 < p < 1$. At any date, there are two possible situations to consider depending on the state the economy. Either (i) good news has arrived, in which case the value of the asset is V for certain, or (ii) good news has not yet arrived, in which case the value of the asset remains uncertain. Again, we can analyze the debt capacity by backward induction, beginning with the penultimate date t_N.

Suppose that goods news has not yet arrived. In the penultimate period, the SIV issues debt with a face value of D. If $D > V$, the SIV defaults next period regardless of the value of the asset and the expected value of the debt is

$$(1-p)\lambda V + p \times 0 = (1-p)\lambda V.$$

On the other hand, if $D \leq V$, then the expected value of the debt is

$$(1-p) D + p \times 0 = (1-p) D.$$

Then the value of debt is given by the formula

$$\max\{(1-p)\lambda V, (1-p) D\},$$

which is maximized by setting $D = V$. Then the maximum debt that can be raised is $(1-p) V$ or, in the notation introduced earlier, $B_N^1 = (1-p) V$.

Now let's move back one period and assume again that good news has not arrived. If the face value is D, the value of the debt in the ante-penultimate period is

$$\begin{cases} (1-p)\lambda V + p\lambda B_N^1 & \text{if } D > V \\ (1-p) D + p\lambda B_N^1 & \text{if } B_N^1 < D \leq V \\ D & \text{if } D \leq B_N^1. \end{cases}$$

Noting that $V > B_N^1$, it is clear that the value of the debt is maximized by setting $D = V$ and the maximum value of the debt is $B_{N-1}^1 = (1-p) V + p\lambda (1-p) V = (1+p\lambda)(1-p) V$. Unlike in the optimistic structure, the debt capacity at date t_{N-1} is not equal to the fundamental value, which is $(1-p^2) V$. The reason for the divergence is interesting. The value of the asset is higher in state s_2 than in state s_1 but the market value of the debt will not reflect this higher asset value unless the face value of the debt is higher than the debt capacity in state s_1, i.e., $D > B_N^1 = (1-p)\,V$. So in order to maximize the value of its debt, the SIV has to choose a face value that will force it to default in state s_1 and lose a fraction $(1-\lambda)$ of the asset's value, thus ensuring that the market value of the debt is below the fundamental value.

2.4.1 The general formula for the pessimistic case

Continuing in this way, we can calculate the maximum value of the debt that can be raised against the SIVs collateral for any date t_n. The general formula is proved by induction. We want to show that debt capacity at any date t_n is given by the formula

$$B_n^1 = \left(\sum_{i=0}^{N-n} p^i \lambda^i \right) (1-p)\,V. \tag{1}$$

This formula agrees with the results obtained above, namely, $B_N^1 = (1-p)\,V$ and $B_{N-1}^1 = (1+p\lambda)\,(1-p)\,V$, so we have already proved that (1) holds for n equal to N and $N-1$.

Now suppose as an induction hypothesis that (1) holds at t_n and consider what happens in the preceding period. Note first that the debt capacity B_{n-1}^1 must satisfy

$$B_{n-1}^1 = \max\left\{ (1-p)\,V + p\lambda B_n^1, B_n^1 \right\},$$

depending on whether the face value of the debt is set equal to V or to B_n^1, and that

$$B_{n-1}^1 = (1-p)\,V + p\lambda B_n^1 \tag{2}$$

if and only if

$$(1-p)\,V + p\lambda B_n^1 \geq B_n^1$$

which is equivalent to

$$\begin{aligned} B_n^1 &\leq \frac{(1-p)\,V}{1-p\lambda} \\ &= \left(1 + p\lambda + p^2\lambda^2 + \cdots\right)(1-p)\,V \\ &= \left(\sum_{i=0}^{\infty} p^i \lambda^i \right) (1-p)\,V. \end{aligned}$$

Our induction hypothesis guarantees that this inequality is satisfied and hence that (2) is satisfied. Then using (2) and the induction hypothesis (1), we calculate

$$\begin{aligned} B_{n-1}^{1} &= (1-p)\,V + (1-p)\,\lambda B_{n}^{1} \\ &= (1-p)\,V + p\lambda \left(\sum_{i=0}^{N-n} p^{i}\lambda^{i} \right) (1-p)\,V \\ &= \left(\sum_{i=0}^{N-n+1} p^{i}\lambda^{i} \right) (1-p)\,V, \end{aligned}$$

as required.

Thus, we have proved by induction that,

Lemma 1 *For any date t_n, regardless of the number of rollovers that have taken place, the maximum amount of finance that can be raised is given by the formula (1).*

2.4.2 Market freezes

We say that the market for short-term borrowing by the SIV is experiencing a "freeze" if the debt capacity of the SIV goes to zero. To characterize a market freeze, we examine the effect of the number of rollovers N on the maximum finance B_n^1 that can be raised at each date t_n. There are various ways of interpreting the changes in the model that lead to an increase in the number of rollovers. We could think of this as involving a shortening of the maturity of the commercial paper (reducing the length of the time period). Alternatively, we could think of this as involving an increase in the time horizon. Either way, it is convenient simply to track the number of rollovers to maturity and treat N as the exogenous variable in our comparative static analysis. There should, of course, be a ceteris paribus assumption that the fundamental value of the asset remains constant as we vary N. The natural way to achieve this is to assume that the time period τ between successive dates t_n and t_{n+1} shrinks while the probability of a switch between states remains constant per unit of time.

Let p' be the credit risk of the asset, given by the equation $p' = p^{N+1}$. We wish to evaluate the debt capacity at the initial date B_0^1, holding p' constant, as $N \to \infty$ or, what is the same, as $\tau \to 0$. Now,

$$\begin{aligned} B_{0}^{1} &= \left(\sum_{i=0}^{N} p^{i}\lambda^{i} \right) (1-p)V \\ &\leq (1-p)V \sum_{i=0}^{\infty} p^{i}\lambda^{i} \\ &= (1-p)V \frac{1}{1-p\lambda} \\ &\leq \frac{1}{1-\lambda}(1-p)V. \end{aligned}$$

Holding $p' = p^N > 0$ constant implies that $p \to 1$ as $N \to \infty$. Then the inequality above implies that $B_0^1 \to 0$ as $N \to \infty$. Thus, we have shown our main result, the existence of a "market freeze".

Proposition 2 *Under the pessimistic information structure, the debt capacity of the asset B_0^1 tends to zero as the number of rollovers becomes unbounded, holding constant the credit risk of the asset held to maturity, provided $p' > 0$ and $\lambda < 1$.*

What is striking about the result is that it holds for *arbitrarily small credit risks*, capturing the scenario that Bear Stearns experienced during its failure in March 2008. A corollary of this result is that, as credit risk increases, fewer debt rollovers are needed to make the debt capacity of an asset fall below some arbitrary threshold.

2.4.3 Intuitive explanation of the market freeze

Since the result on market freeze is somewhat surprising, we provide an informal description of why it arises.

It is clear that, as we approach the maturity date in state s_1, the fundamental value of the asset converges to zero. So the debt capacity, which is bounded above by the fundamental, must also converge to zero as we approach the maturity date in state s_1.

At earlier dates, the debt capacity in state s_1 may be greater than zero, because of the possibility of switching to state s_2. This raises the important question of how different factors, such as rollover risk and credit risk, limit the debt capacity when there is a substantial amount of time left before the assets mature.

One key determinant of the debt capacity is the rate at which the debt is rolled over *relative to* the arrival of good news, i.e., a switch to state s_2. The faster the debt has to be rolled over, other things being equal, the smaller the probability that good news will arrive before the next rollover date. In other words, with high probability, there is no (good) news before the next roll over date. Then the main determinant of current debt capacity in state s_1 is the future debt capacity in state s_1.

Consider the relationship between the face value of the debt and its market value. Intuitively, a high face value increases the market value of the debt because of the possibility that the optimistic state, s_2, is realized; but the more likely outcome is that the economy remains in the pessimistic state, s_1, in which case the high face value leads to default and liquidation. To be more precise, suppose that the debt capacity at the next rollover date is B. If we issue debt with face value B, the market value of the debt is also B. If we issue debt with a higher face value, we gain at most $V - B$ if the state switches to s_2 before the next rollover date, but we lose the default cost $(1-\lambda) B$ if there is no switch. The market value of the debt increases if and only if

$$(1-p)(V-B) - p(1-\lambda) B \geq 0$$

or

$$B \le \frac{(1-p)\,V}{1-p\lambda}.$$

Since this inequality holds at each rollover date, it shows that the debt capacity, starting at zero at the terminal date 1, can never rise above the right hand bound, no matter how many periods are left before the maturity date. Since the expression on the right converges to zero as the probability of good news, $1-p$, converges to zero, we have the market freeze result.

2.5 Factors driving market freezes

Before we move on to the general case, it may be useful to review the various components of the model to identify the key drivers of the market freeze phenomenon in our setup.

- **Credit risk** While some credit risk is necessary for the market freeze result, an arbitrarily small value of $p' > 0$ is sufficient. The assumption of some (small) credit risk is natural when one thinks of asset-backed commercial paper and non-government bond repo transactions.

- **Liquidation risk** Similarly, as long as there is a positive liquidation cost $1-\lambda > 0$, the market freeze result holds. Conversely, if there is zero liquidation cost, for example, because the underlying asset is fully liquid and requires no asset-specific skills on the part of the borrower, or there are enough alternative, efficient buyers who do not require rollover financing, then regardless of credit risk, rollover risk and information structure, the debt capacity of the asset is equal to the fundamental value of the asset (set $\lambda = 1$ in formula 1).

- **Rollover frequency** Even under the pessimistic information structure, regardless of the credit risk and the liquidation cost, the debt capacity of buy-to-hold debt is equal to the fundamental value of the asset. Hence, rollover risk is critical to obtaining the market freeze result.

- **Information structure** The information structure is also crucial for the market freeze result. In the optimistic structure, the debt capacity is always equal to the fundamental value, regardless of the other parameters of the model, whereas, in the pessimistic structure, the debt capacity falls to zero as the number of rollovers increases without bound, holding constant the credit risk p'.

We conclude that the critical features of the model are the necessity of frequent rollovers and the information structure, specifically, the existence of the pessimistic state.

While an arbitrarily small liquidation cost is sufficient for the market freeze result, the magnitude of $1-\lambda$ is an important determinant of debt capacity for a given number of rollovers. We see from the formula in (1) that the debt capacity in the pessimistic state is increasing in λ. A small value of λ is most likely where the likely acquirers of assets are all on one side of the market. That is, when potential acquirers are likely to be hit by a common shock or the underlying trade is "crowded", then its liquidation will lead to fire sales. An alternative interpretation is that a market freeze is likely in assets that are complex and where owners of assets have expertise or management skills which are not transferable when lenders try to seize and liquidate the asset (Williamson, 1988, Shleifer and Vishny, 1992). A fitting example here might be mortgage-backed securities since prepayment and default risk of households on home loans are borne by the financial sector as a whole, but the risk gets repackaged also within the financial sector through these securities. Hence, a common shock to the financial sector would render this asset class relatively illiquid, as witnessed briefly during the Long Term Capital Management crisis of 1998 and more extensively during the sub-prime crisis of 2007-08.

The credit risk has a direct impact on the debt capacity because it determines the fundamental value of the asset. But it also has an indirect effect that reduces debt capacity relative to the fundamental value. In other words, the higher the credit risk, the higher the "haircut," as we show formally in Section 3.

2.6 Debt capacity in the general case

We now show that the result on market freezes derived under the pessimistic information structure of Figure 3 can also be derived under the more general information structure of Figure 4.

In the general case, the debt capacities in the two states, s_1 and s_2, are simultaneously determined and this prevents us from obtaining a closed form expression for the debt capacity. As in the special cases with absorbing states, we calculate the maximum value of the debt that can be issued using the assets as collateral by backward induction, beginning with the period before the asset matures. The arguments are similar to those used in the special cases discussed above and are relegated to the appendix. The information we need is contained in the following proposition.

Proposition 3 *At the penultimate date t_N, the borrowing capacity is $(1-p)V$ in state s_1 and qV in state s_2. For all dates $t_n < t_N$, the borrowing capacities in state s_1 and state s_2 are denoted by B_n^1 and B_n^2, respectively, and defined recursively by the equations*

$$\begin{aligned} B_n^1 &= \max\left\{B_{n+1}^1, p\lambda B_{n+1}^1 + (1-p)\, B_{n+1}^2\right\} \\ B_n^2 &= \max\left\{B_{n+1}^1, (1-q)\,\lambda B_{n+1}^1 + qB_{n+1}^2\right\}. \end{aligned}$$

The presence of the max operator in these expressions is inconvenient, but we can show that it is often not binding. Let us first note the following facts.

Lemma 4 *Debt capacity in the pessimistic state (B_n^1) and the optimistic state (B_n^2) satisfies the following properties:*
1. *For each $n = 0, 1, ..., N$, $B_n^2 > B_n^1$.*
2. *$B_{n-1}^2 < B_n^2$ for every $n = 1, ..., N$.*

In words, Lemma 4 says that (1) the debt capacity is always higher in the optimistic state s_2 than in the pessimistic state s_1, and (2) the debt capacity in the optimistic state s_2 is a strictly increasing function of the date t_n. The first result is intuitive. In the penultimate state, the probability that $\tilde{V} = V$ is higher in state s_2 than in state s_1 (because $q > 1 - p$). This immediately tells us that debt capacity is higher in state s_2 than in state s_1 at date t_N. Then, assuming that the result holds for some date t_n, we can show that it holds for date t_{n-1} using the formulae in Proposition 3. The second result is explained by the fact that the probability that $\tilde{V} = V$, conditional on being in state s_2, increases as one gets closer to the horizon.

The evolution of debt capacity in the pessimistic state s_1 follows an interesting pattern. For sufficiently large dates t_n the debt capacity must be declining, but there may also be an initial period during which it is constant, as the following result shows.

Proposition 5 *Suppose that $q + p\lambda > 1$. Then, there exists a critical value $n^* < N$ such that $B_n^1 > B_{n+1}^1$ for every $n \geq n^*$ and $B_n^1 = B_{n+1}^1$ for every $n < n^*$.*

If the likelihood of good news in the pessimistic state is sufficiently small relative to that in the optimistic state ($p\lambda > 1 - q$), then debt capacity in the pessimistic state (weakly) decreases as the asset matures, i.e., as t_n increases. Note that this assumption will always be satisfied if the time between rollover dates is sufficiently small.

2.7 Limiting value of debt capacity as $\tau \to 0$

We want to explore the effect of increasing the number of roll overs, holding constant the total probability of ending up in the bad state. The most elegant way of doing this is to reduce the length of the period, τ, holding constant the probability of switching states *per unit of time*. In what follows, we denote the debt capacity in state s_i by $B_n^i(\tau)$ to make clear the dependence on τ and let $p(\tau) = e^{-\alpha\tau}$ and $q(\tau) = e^{-\beta\tau}$ for fixed $\alpha > 0$ and $\beta > 0$.

Consider the behavior of $B_n^1(\tau)$ as $\tau \to 0$. Since $B_n^1(\tau)$ is non-increasing in n for fixed τ it is sufficient to consider what happens to the debt capacity $B_0^1(\tau)$ at the time of the original debt issue. We obtain the following formal proposition.

Proposition 6 *As the period length $\tau \to 0$, $B_0^1(\tau) \to 0$.*

In other words, debt capacity in the pessimistic state goes to zero in the limit as number of rollover become unbounded, *even when there is some chance of switching from the pessimistic state to the optimistic state*. In this sense, the market freeze associated with the pessimistic state is especially perverse.

By contrast, when we consider the behavior of $B_n^2(\tau)$, the debt capacity in the optimistic state, we obtain that in this case, debt capacity is bounded from below by a finite quantity even as the number of rollovers becomes unbounded. In particular, this is true *in spite of the fact that we hold constant the probability of ending up in the good state* s_2.

Proposition 7
1. *For any* n *and* τ, $B_n^2(\tau) \geq q(\tau)^{N-n+1} V$, *where* $\tau = \frac{1}{N+1}$.
2. *As* $\tau \to 0$ *and* $n\tau \to t \in (0,1)$, $B_n^2(\tau) \geq e^{-\beta(1-t)} V$.
3. *If* $n\tau \to t \in (0,1)$ *as the period length* $\tau \to 0$, *then* $B_n^2(\tau) \to e^{-\beta(1-t)} V$ *as* $\tau \to 0$.

We want to find out the effect of letting the number of rollovers increase holding everything else constant. In other words, we want to hold constant the flow of information about the final state while increasing the relative speed at which debt is rolled over. To make sure that the information flow is kept constant, we need to examine the transition probabilities between fixed points in time, rather than between successive periods. The general formula for the k-step transition probability matrix is given by

$$\begin{aligned} P^k &= \begin{bmatrix} p_{11}^{(k)} & p_{12}^{(k)} \\ p_{21}^{(k)} & p_{22}^{(k)} \end{bmatrix} \\ &= \frac{1}{2-(p+q)} \begin{bmatrix} 1-q & 1-p \\ 1-q & 1-p \end{bmatrix} + \frac{(q+p-1)^k}{2-(p+q)} \begin{bmatrix} 1-p & p-1 \\ q-1 & 1-q \end{bmatrix} \end{aligned}$$

where $p_{ij}^{(k)}$ is the probability of a transition from s_i to s_j in k steps. The derivation can be found in a standard reference such as Feller (1968). The first term in the expression is the ergodic distribution. To see this, simply note that, if q and p are both strictly less than 1, then $(q+p-1)^k \to 0$ as $k \to \infty$ and

$$\lim_{k\to\infty} P^k = \frac{1}{2-(p+q)} \begin{bmatrix} 1-q & 1-p \\ 1-q & 1-p \end{bmatrix}.$$

In other words, the probability of being in either state is independent of the initial distribution for large n.

This is not the ceteris paribus exercise we want to carry out, however, since here we have held p and q fixed while varying k, which implies that more and more information is accruing during the interval $[0,1]$. Instead, we want to explore the effect of increasing the number of roll overs, holding constant the credit risk, that is, the probability of ending up in state s_1.

The most elegant way of doing this is to reduce the length of the period, holding constant the probability of switching states *per unit of time*. So let $p = e^{-\alpha\tau}$ and $q = e^{-\beta\tau}$, where τ is the period length. Then as $\tau \to 0$, we can approximate p and q by $1-\alpha\tau$ and $1-\beta\tau$, respectively. Substituting these into the $N+1$ step transition probability matrix P^{N+1} representing the transition probabilities between date 0 and date 1 and setting $\tau = \frac{1}{N+1}$, we obtain

Proposition 8 *Let $p_{ij}^{(\infty)}$ denote the probability of a transition from state s_i at time $t=0$ to state s_j at time $t=1$ in the limit as $\tau \to 0$ and let $P^{\infty} = \left[p_{ij}^{(\infty)}\right]_{2\times 2}$. Then*

$$P^{\infty} = \lim_{\tau \to 0} P^{\frac{1}{\tau}} = \frac{1}{(\alpha+\beta)} \begin{bmatrix} \beta & \alpha \\ \beta & \alpha \end{bmatrix} + \frac{e^{-\alpha-\beta}}{(\alpha+\beta)} \begin{bmatrix} \alpha & -\alpha \\ -\beta & \beta \end{bmatrix}.$$

Let $\kappa = \alpha + \beta$ and $\alpha = \gamma\kappa$. Then we can rewrite the expression above as

$$P^{\infty} = \begin{bmatrix} 1-\gamma & \gamma \\ 1-\gamma & \gamma \end{bmatrix} + e^{-\kappa} \begin{bmatrix} \gamma & -\gamma \\ -(1-\gamma) & 1-\gamma \end{bmatrix}.$$

The first term is the ergodic distribution. We can think of γ as measuring the ratio of α to β, which determines the probability of the two states in the ergodic distribution, and κ as measuring the amount of information that arrives over the period $[0,1]$. Holding κ and γ constant ensures the ceteris paribus assumption is satisfied.

The parameters κ and γ have a natural interpretation in terms of the credit risk faced by the lenders in states s_1 and s_2, respectively. The probability of ending up in state s_1 is $1-\gamma\left(1-e^{-\kappa}\right)$ conditional on being in state s_1 at t_0 and it is $(1-\gamma)\left(1-e^{-\kappa}\right)$ conditional on being in state s_2 at t_0. The difference in these probabilties is

$$1-\gamma\left(1-e^{-\kappa}\right)-(1-\gamma)\left(1-e^{-\kappa}\right) = e^{-\kappa}.$$

Thus, $e^{-\kappa}$ measures the difference in credit risk in states s_1 and s_2 at the initial date. The same is true of the probabilities of ending up in state s_2 after starting in state s_1 or state s_2, respectively. Thus, by assuming $\kappa = \alpha+\beta$ large enough, we can ensure that the transition probabilities from different initial states are as close as we like. Although this is not a particularly plausible case, it demonstrates that the results in Propositions 6 and 7 are not driven by different probabilities of ending up in the bad state.

3 Comparative statics

Although the regime switching model allows us to prove a quite general result about market freezes, it is convenient to have closed form solutions in order to derive comparative static properties. For this purpose, we revert to the special case of the pessimistic structure, in which $q = 1$ and the good state s_2 becomes an absorbing state. Our first exercise is to use the formula derived in Section 2.4.1 to derive the impact of the parameters on the so-called "haircut" when an asset is used as collateral.

3.1 A formula for haircuts

Results on debt capacity can be stated in terms of the equivalent haircut of an asset. In secured borrowing or repo transactions, the haircut on an asset is the proportion of (some notion of) its fundamental value that the investor cannot borrow against. We define the fundament value of the asset for as the debt capacity when the debt is held to maturity, that is, $(1 - p')V$. Recall that p' is the credit risk of the asset, given by the equation $p' = p^{N+1}$. Thus, we can measure the haircut H_0 in the model as one minus the ratio of B_0^1 to $(1-p')V$. The haircut is then given by the formula:

$$H_0 = 1 - \frac{1-p}{1-p'} \left(\sum_{i=0}^{N} p^i \lambda^i \right). \tag{3}$$

The haircut can thus be calculated simply based on three inputs:

1. the credit risk of the asset, p';
2. the number N of debt rollovers, or its rollover risk; and,
3. the recovery rate λ or, equivalently, the fire-sale discount $(1 - \lambda)$ incurred in case of liquidation of the asset by a lender.

Then, it follows from Lemma 1 and Proposition 6 that

Corollary 9 *As long as $p' > 0$ and $\lambda < 1$, the haircut of an asset is strictly positive and tends to 100% as its rollover risk N becomes unbounded.*

Brunnermeier and Pedersen (2005) and Shin (2008) discuss in detail the sharp widening of haircuts in stress times, even for relatively high quality assets such as AAA-rated asset pools. The table from Shin (2008) is included in footnote 10. Our model provides an explanation for why stress times – when credit risk, rollover risk and liquidation risk rise – are associated with such large swings in haircuts. Figures 6a and 6b provide some illustrative calibrations to generate haircuts of the magnitude described by Shin (2008), summarized in footnote 10.

— Figures 6a, 6b about here —

Even more strikingly, Box 1.5 from Chapter 1 of IMF's Global Stability Report (2008) shows that while some of the collateralized debt obligations (CDO) have had no secured borrowing capacity at all during the sub-prime crisis, equities and high-yield bonds – which are in principle riskier assets – had only around 20% and 25 to 40% haircuts, respectively. This is also consistent with our model since, whenever there is rollover risk, it is not the underlying risk of an asset's cash flows but its liquidation risk that primarily determines the debt capacity.

Finally, the fact that haircut tends to 100% as the rollover frequency increases without bound provides a potential explanation for why Bear Stearns experienced a complete inability in March 2008 to obtain *any* overnight rollover financing against its even high quality assets.

3.2 A continuous-time model

We can obtain more tractable, closed-form solutions if we assume that time is continuous and that both information and rollover dates arrive according to independent Poisson processes. This allows us to consider not only variations in the relative rates of information arrival and rollovers, but also uncertainty about the number of rollovers that may be necessary before the underlying asset matures. Below, we provide a closed-form solution for the pessimistic information structure.

Let α denote the arrival rate of good news (i.e., a switch from state s_1 to the absorbing state s_2) and let ρ denote the arrival rate of the rollover date. For simplicity, we assume that the information arrival process is independent of the rollover process (the correlated case can be analyzed analogously). Then the probability that a rollover and an information event occur in the same period of length Δt is simply the product of the two probabilities $\alpha\Delta t$ and $\rho\Delta t$, i.e.,

$$(\alpha\Delta t)(\rho\Delta t) = \alpha\rho(\Delta t)^2 = o(\Delta t).$$

Let $B(t)$ denote the maximum amount of debt that can be raised at time t assuming that good news has not yet arrived and that the SIV is solvent.[13] Then $B(t)$ must satisfy the difference equation

$$B(t) = (1 - \alpha\Delta t - \rho\Delta t) B(t + \Delta t) + \alpha\Delta t V + \rho\Delta t\lambda B(t + \Delta t) + o(\Delta t). \tag{4}$$

With probability $(1 - \alpha\Delta t - \rho\Delta t)$, no information arrives, the debt is not re-financed in the period $(t, t + \Delta t)$ and the pledgeable value of the asset is $B(t + \Delta t)$. With probability $\alpha\Delta t$, good news arrives and the pledgeable value is V. And with probability $\rho\Delta t$ it is necessary to re-finance the debt and, assuming the face value of the debt is $V > B(t + \Delta t)$, the asset has to be sold at a fire sale and realizes a sale price of $\lambda B(t + \Delta t)$.

[13] We have dropped the superscript 1 as we are only focusing on the pessimistic information structure here (Figure 3).

Taking limits as $\Delta t \to 0$, we obtain the differential equation

$$B'(t) = -\alpha V + (\alpha + \rho(1-\lambda)) B(t).$$

Solving this differential equation using standard methods yields

Lemma 10 *The debt capacity of the asset at time t is denoted by $B(t)$ and defined by*

$$B(t) = \frac{\alpha V}{\alpha + \rho(1-\lambda)} \left(1 - e^{(\alpha+\rho(1-\lambda))(t-1)}\right). \tag{5}$$

The maximum finance that can be raised at the initial date is obtained by setting $t = 0$ in the formula above. Recall that the probability that good news does not arrive in the interval $[0, 1]$, that is, the probability that the value of the asset is 0, is $e^{-\alpha}$. Hence, the fundamental value of the asset is $(1 - e^{-\alpha}) V$. Thus, the haircut at date 0 can be denoted by $H(0)$ and defined by

$$\begin{aligned} H(0) &= 1 - \frac{B(0)}{(1-e^{-\alpha}) V} \\ &= 1 - \frac{\alpha\left(1 - e^{-(\alpha+\rho(1-\lambda))}\right)}{[\alpha + \rho(1-\lambda)](1-e^{-\alpha})}. \end{aligned}$$

The following proposition provides comparative statics results on the debt capacity and the haircut.

Proposition 11 *Using the formula for the haircut $H(0)$ obtained above, we can derive the following comparative statics results:*
(i) The debt capacity $B(0)$ is decreasing in the rollover frequency, ρ, and the liquidation cost, $(1-\lambda)$, and is increasing in α.
(ii) The haircut $H(0)$ is increasing in the rollover frequency, ρ, and the liquidation cost, $(1-\lambda)$, and is decreasing in α.

Figure 7 illustrates the results from the above proposition. In particular, we vary ρ to get a sense of the role of the relative speeds of information arrival and re-financing on debt capacity (Figure 7a) and haircut (Figure 7b). We choose values of α such that the asset's credit risk, $e^{-\alpha}$, takes values 0.001, 0.01 and 0.1,[14], we normalize $V = 1$, and we use $\lambda = 0.7$ to capture the liquidation cost.

— Figures 7a, 7b about here —

[14] The corresponding α-values are 6.9078, 4.6052, and 2.3026, respectively.

Note that, as ρ approaches 0, the debt capacity of the asset approaches its fundamental value, and, in turn, the required haircut approaches 0, for all values of the credit risk. Furthermore, the rollover risk (captured by ρ) and the credit risk (captured by α) act as substitutes, that is, for lower levels of credit risk, higher levels of rollover risk are needed to get the same levels of debt capacity and haircut. Since ρ is equal to the expected number of rollovers in the unit interval, we can see that there is a substantial haircut even when the number of rollovers is reasonably small and the level of credit risk is extremely small.

4 Policy implications

It is tempting to ask the question: Can a regulator do something to unfreeze the market? Note that our model is partial equilibrium. It shows how the nature of information arrival affects rollover debt capacity. It does not endogenize either the short-term nature of debt or the liquidation cost in case of default. Hence, any policy implications we draw below must also be viewed as partial equilibrium ones, narrowly focused at alleviating the market freeze without attention to any other related efficiency issues or unintended consequences.

We discuss two possible policy interventions.

4.1 Improving the liquidation value of assets

Recall that in our model, debt capacity shrinks with rollovers under the pessimistic information structure since in case of default, asset must be sold to another player also constrained to borrow using rollover debt. If the maturity of borrowing of alternative buyers of assets lengthens, then the haircut in borrowing would fall and so would liquidation costs. During a systemic crisis, however, there are few private agents with long-term horizons because the ultimate providers of finance - the households - themselves become short-term focused. A regulator such as the Central Bank or the Treasury can, in principle, directly attempt to improve the liquidation value (λ) of affected assets by lending against the asset as a collateral based on its full buy-and-hold value.

This argument could provide a rationale for the wide variety of lending facilities created by the Federal Reserve during the sub-prime crisis to lend to a large number of borrowers, against a wide variety of collateral, at minimal (if any) haircut.[15] In practice, such facilities

[15] For example, in addition to the traditional tools the Fed uses to implement monetary policy (e.g., Open Market Operations, Discount Window, and Securities Lending program), five new programs were implemented during August 2007 to March 2008: 1) Term Discount Window Program (announced 8/17/2007) - extended the length of discount window loans available to institutions eligible for primary credit from overnight to a maximum of 90 days; 2) Term Auction Facility (TAF) (announced 12/12/2007) - provides funds to primary credit eligible institutions through an auction for a term of 28 days; 3) Single-Tranche OMO (Open Market Operations) Program (announced 3/7/2008) - allows primary dealers to secure funds for a

had a temporary effect on most markets they intervened in, but they appear to have failed to resolve the market freezes completely. While the reason behind this failure remains an important puzzle, one possible explanation could be the following. Our model suggests that for haircuts to vanish or become minimal, at *each* rollover date until maturity of the asset, the lender should anticipate *guaranteed* regulatory lending against the assets to potential buyers. The Federal Reserve facilities were, however, newly introduced and their horizon of provision was announced as short-term, with discretionary extension in future. The residual uncertainty left by such discretion may have been a factor that prevented a complete thawing of asset-backed debt markets in spite of substantial short-term interventions.

4.2 Requiring higher "capital" in asset-backed finance

An alternative policy implication of our model is that while short-term rollover debt entails little financing cost (if any) in good times, its availability can dry up suddenly when unexpected or hitherto unexperienced events happen. The excessive reliance on rollover finance thus exposes borrowers (such as SIV's and conduits) to low likelihood but high magnitude funding risk. It may be more prudent for such borrowers to account for such funding risk and complement rollover debt in their capital structure with forms of capital such as long-term debt and equity capital that face lower rollover risk.

Of course, the very information frictions that cause a switch in investors' expectations from pessimistic to optimistic ones may also preclude availability of such long-term finance once a crisis erupts. Hence, the reduced reliance on rollover debt must be a part of prudential capital structure choice in good times rather than being undertaken during bad times. While financial institutions should have privately recognized the risks of rollover finance, it cannot be ruled out that such risks were not yet fully understood. Going forward, a prudential regulator could play the role of a watchdog, looking out for excessive reliance on rollover finance, and encouraging in such cases a greater reliance on long-term capital, through moral suasion or rule-based policies. Since long-term capital may be infeasible beyond a point for opaque, off-balance-sheet structures such as SIV's and conduits, such regulatory push will most likely reduce their incidence in the first place.

term of 28 days. These operations are intended to augment the single day repurchase agreements (repos) that are typically conducted; 4) Term Securities Lending Facility (TSLF) (announced 3/11/2008) - allows primary dealers to pledge a broader range of collateral than is accepted with the Securities Lending program, and also to borrow for a longer term — 28 days versus overnight; and, 5) Primary Dealer Credit Facility (PDCF) (announced 3/16/2008) - is an overnight loan facility that provides funds directly to primary dealers in exchange for a range of eligible collateral.

5 Related literature

Our paper is related to the literature on haircuts, freezes and runs in financial markets. Rosenthal and Wang (1993) use a model where owners occasionally need to sell their assets for exogenous liquidity reasons through auctions with private information. Because of the auction format, sellers may not be able to extract the full value of the asset and this liquidation cost gets built into the market price of the asset, making the market price systematically lower than the fundamental value. In our model, the source of the haircut is not the private information of potential buyers. Furthermore, we show that under the optimistic information structure, even with forced sales (roll over in our case), the asset can generate its full value. And our regime switching model provides a more general analysis to flesh out the specific information structures (pessimistic case) that lead to haircuts and market freezes.

He and Xiong (2009) consider a model of dynamic bank runs in which bank creditors have supplied debt maturing at differing maturities and each creditor faces the risk at the time of rolling over that fundamentals may deteriorate before remaining debt matures causing a fire sale of assets. In their model, volatility of fundamentals plays a key role in driving the runs even when the average value of fundamentals has not been affected. Our model of freeze or "run" of short-term debt is complementary to theirs, and somewhat different in the sense that *both* average value and uncertainty about fundamentals are held constant in our model, but it is the nature of revelation of uncertainty over time – whether good news arrives early or bad news arrives early – that determines whether there is rollover risk in short-term debt or not.

Huang and Ratnovski (2008) model the behavior of short-term wholesale financiers who prefer to rely on noisy public signals such as market prices and credit ratings, rather then producing costly information about the institutions they lend to. Hence, wholesale financiers run on other institutions based on imprecise public signals, triggering potentially inefficient runs. While their model is about runs in the wholesale market as is ours, their main focus is to challenge the peer-monitoring role of wholesale financiers, whereas our main focus is the role of rollover and liquidation risk in generating such runs.

An alternative modelling device to generate market freezes is to employ the notion of Knightian uncertainty (see Knight, 1921) and agents' overcautious behavior towards such uncertainty. Gilboa and Schmeidler (1989) build a model where agents become extremely cautious and consider the worst-case among the possible outcomes, that is, agents are uncertainty averse and use maxmin strategies when faced with Knightian uncertainty. Dow and Werlang (1992) apply the framework of Gilboa and Schmeidler (1989) to the optimal portfolio choice problem and show that there is an interval of prices within which uncertainty-averse agents neither buy nor sell the asset. Routledge and Zin (2004) and Easley and O'Hara (2005, 2008) use Knightian uncertainty and agents that use maxmin strategies to generate widening bid-ask spreads and freeze in financial markets. Caballero and Krishnamurthy

(2007) also use the framework of Gilboa and Schmeidler (1989) to develop a model of flight to quality during financial crises: During periods of increased Knightian uncertainty, agents refrain from making risky investments and hoard liquidity, leading to flight to quality and freeze in markets for risky assets.

As opposed to these models, in our model agents maximize their expected utility and the main source of the market freeze is rollover and liquidation risk which become relevant when the rate of arrival of good news is slower than the rate at which debt is being rolled over. While both types of models can generate market freezes, we believe our model, by emphasizing the rollover and liquidation risk, better captures important features of the recent crisis where the market for rollover debt completely froze while equities - typically considered to be more risky - continued to trade with haircuts of only 20%.

6 Conclusion

In this paper, we have attempted to provide a simple information-theoretic model for freezes in the market for secured borrowing against finitely lived assets. The key ingredients of our model were rollover risk, liquidation risk, and switch in investors' expectations from optimistic to pessimistic, that is, for the waiting-for-good-news type. Such a switch was shown critical to obtaining the result on market freezes. This potentially explains why in response to events not witnessed before by market participants (such as the Long Term Capital Management episode or the sub-prime crisis), markets for rolling over short-term asset-backed debt experience a sudden drying-up.

In future work, it would be interesting to embed an agency-theoretic role for short-term debt, which we assumed as given, and see how the desirability of such rollover finance is affected when information problems can lead to complete freeze in its availability. While we took the pattern of release of information about the underlying asset as either ordained by nature or determined by investors' expectations, it seems worthwhile to reflect on its deeper foundations, and thereby assess whether a strategic disclosure of information by agents in charge of the asset can alleviate (or aggravate) the problem of freezes to some extent.

Appendix I: Proofs

Proof of Proposition 3: At date t_N, we are either in state s_1 or in state s_2. Suppose the SIV is in state s_1 and issues debt with a face value of D. By the usual argument, we can show that it is never optimal to choose $D > V$ and, if $D \leq V$ the the value of debt is $(1-p)\,D$, which is maximized by setting $D = V$. Let $B_N^1 = (1-p)\,V$ denote the debt capacity at date t_N in state s_1.

Similarly, in state s_2 it is never optimal to choose $D > V$ and, if $D \leq V$, the value of the debt is qD, which is maximized by setting $D = V$. Let $B_N^2 = qV$ denote the borrowing capacity at date t_N in state s_2.

Now let B_n^1 and B_n^2 be the borrowing capacities in states s_1 and s_2, respectively, when at date t_n, in the period when the n-th roll over occurs. If D is the face value of the debt issued at date t_{n-1}, then the value of the debt in state s_1 is

$$\begin{cases} p\lambda B_n^1 + (1-p)\lambda B_n^2 & \text{if } D > B_n^2 \\ p\lambda B_n^1 + (1-p) D & \text{if } B_n^1 < D \leq B_n^2 \\ D & \text{if } D \leq B_n^1. \end{cases}$$

Clearly, $D > B_n^2$ is never optimal, so we choose between $D = B_n^1$ and $D = B_n^2$ and the borrowing capacity is

$$B_{n-1}^1 = \max\left\{B_{1,n}, p\lambda B_n^1 + (1-p) B_n^2\right\}$$

Similarly, in state s_2, the value of the debt is

$$\begin{cases} (1-q)\lambda B_n^1 + q\lambda B_n^2 & \text{if } D > B_n^2 \\ (1-q)\lambda B_n^1 + qD & \text{if } B_n^1 < D \leq B_n^2 \\ D & \text{if } D \leq B_n^1. \end{cases}$$

Again, $D > B_n^2$ is never optimal, the choice is between $D = B_n^1$ and $D = B_n^2$ and the borrowing capacity is

$$B_n^2 = \max\left\{B_n^1, (1-q)\lambda B_n^1 + qB_n^2\right\}.$$

Proof of Lemma 4:

1. This is certainly true for $n = N$ and assuming it is true for n it follows that

$$\begin{aligned} B_{n-1}^2 &= \max\left\{B_n^1, (1-q)\lambda B_n^1 + qB_n^2\right\} \\ &\geq \max\left\{B_n^1, p\lambda B_n^1 + (1-p) B_n^2\right\} = B_{n-1}^1 \end{aligned}$$

since $q > 1-p$ and, by hypothesis, $B_n^2 > B_n^1$. $\diamondsuit$

2. This follows from the fact that $B_n^2 > B_n^1$ and hence

$$\begin{aligned} B_{n-1}^2 &= \max\left\{B_n^1, (1-q)\lambda B_n^1 + qB_n^2\right\} \\ &< B_n^2. \end{aligned}$$

This proves the stated claim. $\diamondsuit$

Proof of Proposition 5: Certainly, for $n = N - 1$ we have

$$\begin{aligned} B^1_{N-1} &= \max\left\{B^1_N, p\lambda B^1_N + (1-p)\, B^2_N\right\} \\ &= \max\left\{(1-p)\, V, p\lambda\,(1-p)\, V + (1-p)\, qV\right\}. \end{aligned}$$

Then $B^1_{N-1} > B^1_N$ if and only if

$$q + p\lambda > 1,$$

which we assumed to hold. Let n^* denote the smallest value of n such that $B^1_n > B^1_{n+1}$ for all $n \geq n^*$. Then for $n = n^*$, we have $B^1_{n-1} = B^1_n$ and

$$\begin{aligned} B^1_{n-2} &= \max\left\{B^1_{n-1}, p\lambda B^1_{n-1} + (1-p)\, B^2_{n-1}\right\} \\ &\leq \max\left\{B^1_n, p\lambda B^1_n + (1-p)\, B^2_n\right\} \\ &= B^1_{n-1}, \end{aligned}$$

since $B^1_{n-1} = B^1_n$ and $B^2_n > B^2_{n-1}$.

But the same argument could be extended indefinitely, since it only depends on the assumption that $B^1_{n-1} = B^1_n$ and $B^2_n > B^2_{n-1}$. This completes the proof. $\diamondsuit$

Proof of Proposition 6: We consider two cases in turn. Suppose first that $n^*(\tau) >> 0$ for infinitely many values of $\tau \to 0$. For fixed τ, we know that $B^1_n(\tau) \leq B^1_{n^*}(\tau)$ for any $n \leq N$. From the definition of $n^*(\tau)$,

$$\begin{aligned} 0 &= B^1_{n^*+1}(\tau) - B^1_{n^*}(\tau) \\ &= (1 - p(\tau))\, B^2_{n^*}(\tau) + p(\tau)\,\lambda B^1_{n^*}(\tau) - B^1_{n^*}(\tau) \\ &= (1 - p(\tau))\, B^2_{n^*}(\tau) - (1 - p(\tau)\,\lambda)\, B^1_{n^*}(\tau) \\ &= \left(1 - e^{-\alpha\tau}\right) B^2_{n^*}(\tau) - \left(1 - e^{-\alpha\tau}\lambda\right) B^1_{n^*}(\tau). \end{aligned}$$

Then

$$B^1_{n^*}(\tau) \leq \frac{(1 - e^{-\alpha\tau})}{(1 - e^{-\alpha\tau}\lambda)} B^2_{n^*}(\tau).$$

Since the right hand side is bounded and $e^{-\alpha\tau} \to 1$ as $\tau \to 0$, we see that $B^1_{n^*}(\tau) \to 0$ as $\tau \to 0$, which means that $B^1_0(\tau) \to 0$ as $\tau \to 0$.

Now suppose that $n^*(\tau) = 0$ and $B^1_0(\tau) >> B^1_1(\tau)$ for infinitely many values of $\tau \to 0$. Then

$$\begin{aligned} B^1_0(\tau) &= p(\tau)\,\lambda B^1_1(\tau) + (1 - p(\tau))\, B^2_1(\tau) \\ &\leq p(\tau)\,\lambda B^1_0(\tau) + (1 - p(\tau))\, B^2_1(\tau) \end{aligned}$$

which implies

$$\begin{aligned} B_0^1(\tau) &\leq \frac{1-p(\tau)}{1-p(\tau)\lambda} B_1^2(\tau) \\ &= \frac{1-e^{-\alpha\tau}}{1-e^{-\alpha\tau}\lambda} B_1^2(\tau). \end{aligned}$$

Since $B_1^2(\tau)$ is bounded and $e^{-\alpha\tau} \to 1$, as $\tau \to 0$, we see that $B_0^1(\tau) \to 0$ as $\tau \to 0$. $\diamondsuit$

Proof of Proposition 7:

1. For each n,

$$\begin{aligned} B_n^2(\tau) &= \max\left\{B_{n+1}^1(\tau), q(\tau) B_{n+1}^2(\tau) + (1-q(\tau))\lambda B_{n+1}^1\right\} \\ &\geq q(\tau) B_{n+1}^2(\tau) \\ &= q(\tau)\max\left\{B_{n+2}^1(\tau), q(\tau) B_{n+2}^2(\tau) + (1-q(\tau))\lambda B_{n+2}^1\right\} \\ &\geq q(\tau)^2 B_{n+2}^2(\tau) \\ &\quad \ldots \\ &= q(\tau)^{N-n} B_N^2(\tau) = q(\tau)^{N-n+1} V. \end{aligned}$$

   This proves the stated claim. $\diamondsuit$

2. From the preceding result,

$$\begin{aligned} B_n^2(\tau) &\geq q(\tau)^{N-n+1} V \\ &= \left(e^{-\beta\tau}\right)^{N-n+1} V \\ &= \left(e^{-\beta\tau}\right)^{\frac{1-n\tau}{\tau}} V \\ &= e^{-\beta(1-n\tau)} V. \end{aligned}$$

   Then

$$\lim_{\tau\to 0} e^{-\beta(1-n\tau)} = e^{-\beta(1-t)}$$

   follows from the fact that $n\tau \to t$ as $\tau \to 0$. $\diamondsuit$

3. For fixed τ,

$$\begin{aligned} B_n^2(\tau) &= q(\tau) B_{n+1}^2(\tau) + (1-q(\tau))\lambda B_{n+1}^1 \\ &= q(\tau)^2 B_{n+2}^2 + (1-q(\tau))\lambda B_{n+1}^1 + q(\tau)(1-q(\tau))\lambda B_{n+2}^1 \\ &= q(\tau)^{N-n+1} V + \sum_{i=1}^{N-n} (1-q(\tau))\lambda q(\tau)^{i-1} B_{n+i}^1(\tau) \\ &\leq q(\tau)^{N-n+1} V + \sum_{i=1}^{N-n} (1-q(\tau))\lambda q(\tau)^{i-1} \max_i B_{n+i}^1(\tau) \\ &= q(\tau)^{N-n+1} V + \lambda\left(1-q(\tau)^{N-n}\right) \max_i B_{n+i}^1(\tau). \end{aligned}$$

Now let $N+1=\frac{1}{\tau}$ and $\tau\to 0$. As $\max_i B^1_{n+i}(\tau)$ converges to zero and $q(\tau)^N$ converges to $e^{-\beta(1-t)}$, the left hand side converges to $e^{-\beta(1-t)}V$. $\diamondsuit$

Proof of Proposition 8:

Substituting the expressions for p and q into P^{N+1} and setting $\tau=\frac{1}{N+1}$ gives us

$$\begin{aligned} P^{\frac{1}{\tau}} &= \frac{1}{(\alpha+\beta)\tau}\begin{bmatrix}\beta\tau & \alpha\tau\\ \beta\tau & \alpha\tau\end{bmatrix}+\frac{(1-(\alpha+\beta)\tau)^{\frac{1}{\tau}}}{(\alpha+\beta)\tau}\begin{bmatrix}\alpha\tau & -\alpha\tau\\ -\beta\tau & \beta\tau\end{bmatrix}\\ &= \frac{1}{(\alpha+\beta)}\begin{bmatrix}\beta & \alpha\\ \beta & \alpha\end{bmatrix}+\frac{(1-(\alpha+\beta)\tau)^{\frac{1}{\tau}}}{(\alpha+\beta)}\begin{bmatrix}\alpha & -\alpha\\ -\beta & \beta\end{bmatrix}.\end{aligned}$$

Now

$$\lim_{\tau\to 0}(1-(\alpha+\beta)\tau)^{\frac{1}{\tau}}=e^{-\alpha-\beta}$$

so

$$P^{\infty}=\lim_{\tau\to 0}P^{\frac{1}{\tau}}=\frac{1}{(\alpha+\beta)}\begin{bmatrix}\beta & \alpha\\ \beta & \alpha\end{bmatrix}+\frac{e^{-\alpha-\beta}}{(\alpha+\beta)}\begin{bmatrix}\alpha & -\alpha\\ -\beta & \beta\end{bmatrix}.$$

Proof of Corollary 9: From Proposition 6, when $p'>0$ and $\lambda<1$, $B^1_0<(1-p')V$ for $n>1$, and $B^1_0\to 0$ as $n\to\infty$. In turn, the haircut H_0 is always positive under these conditions and goes to 100% as number of rollovers become unbounded. $\diamondsuit$

Proof of Lemma 10: Consider the difference equation (4). To see that it is optimal to set the face value of the debt equal to V, consider the effect of choosing $D<V$ as the face value. Clearly, there is no point choosing $D>B(t+\Delta t)$ or $D<B(t+\Delta t)$ so consider $D=B(t+\Delta t)$ for some period Δt. Then the value of the debt with face value D is

$$\begin{aligned}&(1-\alpha\Delta t-\rho\Delta t)B(t+\Delta t)+\alpha\Delta tB(t+\Delta t)+\rho\Delta tB(t+\Delta t)+o(\Delta t)=\\ &\qquad B(t)+\alpha\Delta t(B(t+\Delta t)-V)+(1-\lambda)\rho\Delta tB(t+\Delta t)+o(\Delta t).\end{aligned}$$

Now

$$\alpha(B(t+\Delta t)-V)+(1-\lambda)\rho B(t+\Delta t)+\frac{o(\Delta t)}{\Delta t}<0$$

for Δt sufficiently small if

$$\alpha\left(1-\frac{V}{B(t+\Delta t)}\right)+\rho(1-\lambda)<0. \qquad (6)$$

We assume this condition is satisfied for the time being and check it later.

Re-arranging the terms of (4), we obtain

$$\frac{B(t+\Delta t)-B(t)}{\Delta t}=-\alpha V+(\alpha+\rho(1-\lambda))B(t+\Delta t)+\frac{o(\Delta t)}{\Delta t}.$$

Taking limits as $\Delta t\to 0$, we obtain the differential equation

$$B'(t)=-\alpha V+(\alpha+\rho(1-\lambda))B(t).$$

This can be solved to yield a general solution of the form

$$B(t)=\frac{\alpha V}{\alpha+\rho(1-\lambda)}+Ce^{(\alpha+\rho(1-\lambda))t}$$

where C is an undetermined coefficient. Setting $t=1$ and using the boundary condition $B(1)=0$, we get

$$0=\frac{\alpha V}{(\alpha+\rho(1-\lambda))}+Ce^{(\alpha+\rho(1-\lambda))}$$

or

$$C=-\frac{\alpha V}{(\alpha+\rho(1-\lambda))}e^{-(\alpha+\rho(1-\lambda))}.$$

Then the solution is as given in equation (5).

It is clear that $B(0)>B(t)$ for any $t>0$, so in order to confirm condition (6) it is sufficient to show that it holds for $t=0$. Then

$$\alpha\left(1-\frac{V}{B(0)}\right)+(1-\lambda)=\alpha\left(1-\frac{\alpha+\rho(1-\lambda)}{\alpha\left(1-e^{-(\alpha+\rho(1-\lambda))}\right)}\right)+\rho(1-\lambda)<0,$$

as required. ◊

Proof of Corollary 11:

(i) We already showed that

$$B(0)=\frac{\alpha V}{\alpha+\rho(1-\lambda)}\left(1-e^{-(\alpha+\rho(1-\lambda))}\right).$$

Let $x=\alpha+\rho(1-\lambda)$ so that $\frac{dx}{d\rho}>0$, $\frac{dx}{d(1-\lambda)}>0$, and $\frac{B(0)}{\alpha V}=\frac{1}{x}\left(1-e^{-x}\right)$. Hence, $sign\left(\frac{\partial B(0)}{\partial\rho}\right)=sign\left(\frac{\partial B(0)}{\partial(1-\lambda)}\right)=sign\left(\frac{\partial\left(\frac{B(0)}{\alpha V}\right)}{\partial x}\right)$. We can show that

$$\frac{\partial\left(\frac{B(0)}{\alpha V}\right)}{\partial x}=-\frac{1}{x^2}\left(1-e^{-x}\right)+\frac{1}{x}e^{-x}=\frac{1}{x^2}\left((1+x)e^{-x}-1\right).$$

Let $f(x) = (1+x)e^{-x} - 1$. Note that $f(0) = 0$ and $f'(x) = -xe^{-x} < 0$ for $x > 0$. Hence, we obtain $\frac{\partial B(0)}{\partial \rho} < 0$ and $\frac{\partial B(0)}{\partial (1-\lambda)} < 0$.

Next, we show $\frac{\partial B(0)}{\partial \alpha} > 0$. We obtain

$$\begin{aligned} \frac{\partial B\left(0\right)}{\partial \alpha} &= \left(\frac{V\left[\alpha+\rho\left(1-\lambda\right)\right]-\alpha V}{\left[\alpha+\rho\left(1-\lambda\right)\right]^{2}}\right)\left(1-e^{-(\alpha+\rho(1-\lambda))}\right)+\frac{\alpha V}{\alpha+\rho\left(1-\lambda\right)}e^{-(\alpha+\rho(1-\lambda))}. \\ &= \frac{V}{x}\left(1-\frac{\alpha}{x}\right)\left(1-e^{-x}\right)+\frac{\alpha V}{x}e^{-x} > 0, \end{aligned}$$

since $\alpha < x = \alpha + \rho(1-\lambda)$. $\diamondsuit$

(ii) We can show that

$$\frac{\partial H\left(0\right)}{\partial x} = -\frac{\alpha}{x^{2}\left(1-e^{-\alpha}\right)} f(x) < 0,$$

since $f(x) = (1+x)e^{-x} - 1 < 0$ for $x > 0$. Hence, we obtain $\frac{\partial H(0)}{\partial \rho} > 0$ and $\frac{\partial H(0)}{\partial (1-\lambda)} > 0$.

Next, we show $\frac{\partial H(0)}{\partial \alpha} < 0$. We can write $H(0) = 1 - \frac{g(\alpha,y)}{g(\alpha,0)}$ where $y = \rho + (1-\lambda)$, and $g(\alpha, y) = \frac{1-e^{-(\alpha+y)}}{\alpha+y}$. We can write

$$\frac{dH\left(0\right)}{d\alpha} = \frac{g(\alpha,y)}{g(\alpha,0)}\left[\frac{g_{\alpha}(\alpha,0)}{g(\alpha,0)} - \frac{g_{\alpha}(\alpha,y)}{g(\alpha,y)}\right],$$

where

$$\frac{g_{\alpha}(\alpha,y)}{g(\alpha,y)} = \frac{e^{-(\alpha+y)}}{1-e^{-(\alpha+y)}} - \frac{1}{\alpha+y} = \frac{1}{e^{(\alpha+y)}-1} - \frac{1}{\alpha+y}.$$

Let $h(y) = \frac{g_{\alpha}(\alpha,y)}{g(\alpha,y)}$, so that

$$\frac{dH\left(0\right)}{d\alpha} = \frac{g(\alpha,y)}{g(\alpha,0)}\left[h(0)-h(y)\right].$$

Hence, $\frac{dH(0)}{d\alpha} < 0$ if and only if $g(0) < g(y)$. We can write

$$g'(y) = -\frac{e^{(\alpha+y)}}{\left(e^{(\alpha+y)}-1\right)^{2}} + \frac{1}{\left(\alpha+y\right)^{2}} = -\frac{1}{e^{(\alpha+y)}-2+e^{-(\alpha+y)}} + \frac{1}{\left(\alpha+y\right)^{2}}.$$

Using the power series expansions for $e^{(\alpha+y)}$ and $e^{-(\alpha+y)}$, we get

$$g'(y) = -\frac{1}{2\left(\sum\limits_{i=1}^{\infty}\frac{(\alpha+y)^{2i}}{(2i)!}\right)} + \frac{1}{\left(\alpha+y\right)^{2}} > 0 \text{ for all } y.$$

Hence, we obtain $\frac{dH(0)}{d\alpha} < 0$. $\diamondsuit$

Appendix II: General information structures

The binomial example that has been used extensively in this paper is insightful, but it is also a rather special case. To establish the robustness of these results, we consider a more general class of information structures in this appendix. We assume for simplicity that re-financing occurs at a fixed sequence of regular intervals.

Suppose there are $N+1$ periods, where the period length corresponds to the maturity of the commercial paper issued. The information revealed in each period is represented by a random variable with a finite number of values. Without loss of generality we assume there are S different outcomes in period n and denote them by $\omega_n = 1, ..., S$. The information set in period n can be identified with the vector $\omega^n = (\omega_1, ..., \omega_n)$ and the states of nature can be identified with the vector $\omega = (\omega_1, ..., \omega_{N+1}) \in \Omega = \{1, ..., S\}^{N+1}$. The probability of state ω is denoted by $P[\omega]$.

Again, we assume the assets have a terminal value $V_{N+1}(\omega)$ in period $N+1$ but no yield in periods $n = 1, ..., N$ and for simplicity we assume that the short-term interest rate is 0.

Our task is to calculate the maximum amount of finance that can be raised in the initial period, before any information has been released. We compute this amount by backward induction, starting with the last period before the assets' value is realized. The assets are worth $V(\omega)$ at in state ω in period $N+1$ so, assuming the face value of the debt is D, the SIV is solvent if and only if

$$D \leq V(\omega).$$

The *payoff function* of the SIV is denoted by $B_{N+1}(\omega; D)$ and defined by

$$B_{N+1}(\omega; D) = \begin{cases} D & \text{if } D \leq V(\omega) \\ \lambda V(\omega) & \text{if } D > V(\omega) \end{cases},$$

for any state ω and face value D. Then the expected value of the debt in period N, conditional on the face value D and the information set ω^N, is

$$\mathbf{E}\left[B_{N+1}(\omega; D) \mid \omega^N\right].$$

The maximum pledgeable value of the asset is denoted by $V_N\left(\omega^N\right)$ and defined by

$$V_N\left(\omega^N\right) = \max_{D \geq 0} \mathbf{E}\left[B_{N+1}(\omega; D) \mid \omega^N\right].$$

Now suppose that we have defined $B_n(\omega^n; D)$ and $V_n(\omega^n)$ for $n = N-k, ..., N$. For $n = N-k-1$ we define $V_n(\omega^n)$ by putting

$$V_n(\omega^n) = \max_{D \geq 0} \mathbf{E}\left[B_{n+1}\left(\omega^{n+1}; D\right) \mid \omega^n\right].$$

Then we can define $B_n(\omega^n; D)$ in the obvious way, putting

$$B_n(\omega^n; D) = \begin{cases} D & \text{if } D \leq V_{n+1}(\omega^{n+1}) \\ \lambda V_{n+1}(\omega^{n+1}) & \text{if } D > V_{n+1}(\omega^{n+1}) \end{cases}.$$

By induction, we have defined $V_n(\omega^n)$ for every period $n = 1, ..., N$ and every information set ω^n. V_0 is the maximum amount of finance that can be raised at the first date.

We make two assumptions that are counterparts to the parametric assumptions in the text.

A.1 The terminal value of the asset $V(\omega)$ is bounded by $\bar{V} < \infty$ for every terminal state $\omega \in \Omega$.

A.2 There exists a terminal state $\bar{\omega} = (\bar{\omega}_1, ..., \bar{\omega}_{N+1})$ such that $V(\bar{\omega}) = 0$ and, for every period n, $P[\bar{\omega}_{n+1}|\bar{\omega}^n] \geq 1 - p > 0$.

Under the maintained assumptions we can show that

$$V_0 \leq p\bar{V}.$$

To prove this, we begin by considering the information set $\bar{\omega}^N$ and note that, under the maintained assumptions $A.1$ and $A.2$, the maximum finance that can be raised must be bounded by

$$\mathbf{E}\left[V\left(\omega^{N+1}\right) \mid \bar{\omega}^N\right] \leq (1-p) \times 0 + p\bar{V} = p\bar{V}.$$

Thus, $V_N\left(\bar{\omega}^N\right) \leq p\bar{V}$. Now suppose that

$$\begin{aligned} V_n(\bar{\omega}^n) &\leq \left[1 + (1-p)\lambda + \cdots + (1-p)^{N-n}\lambda^{N-n}\right] p\bar{V} \\ &= \left[\sum_{k=0}^{N} (1-p)^k \lambda^k\right] p\bar{V} \end{aligned}$$

and use the maintained assumptions to conclude that the inequality holds for $n-1$.

Conversely, we can show that there is a lower bound to the amount of finance that can be raised.

A.3 For some constants $\bar{v} > 0$ and $p > 0$, $P\left[V(\omega) \geq \bar{v} \mid \omega^N\right] \geq \pi$, for any information set ω^N.

Then, under Assumption $A.3$, it can be shown that

$$V_n(\omega^n) \geq p\bar{v},$$

for any information set ω^n. The proof is obvious.

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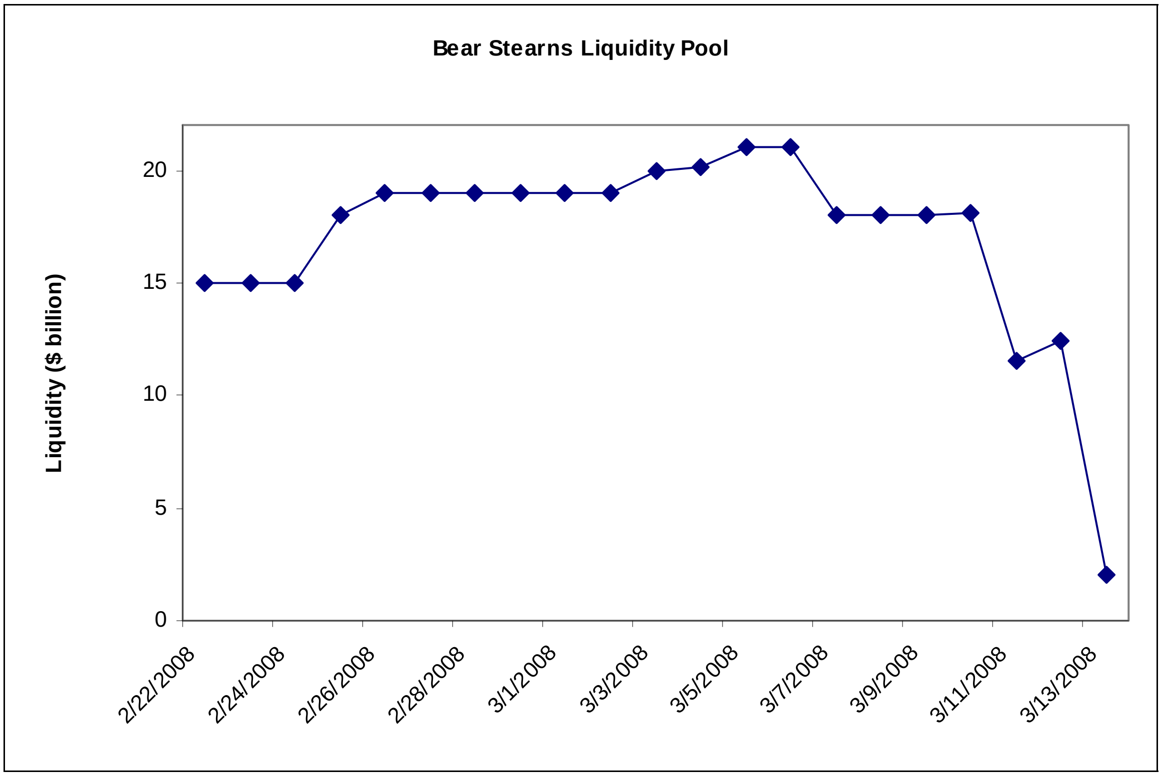

Figure 1: Bear Stearns' liquidity pool just before its near failure (Source: SEC Chairman Christopher Cox's letter to Basel Committee in support of New Guidance on Liquidity Management, available at: http://www.sec.gov/news/press/2008/2008-48.htm).

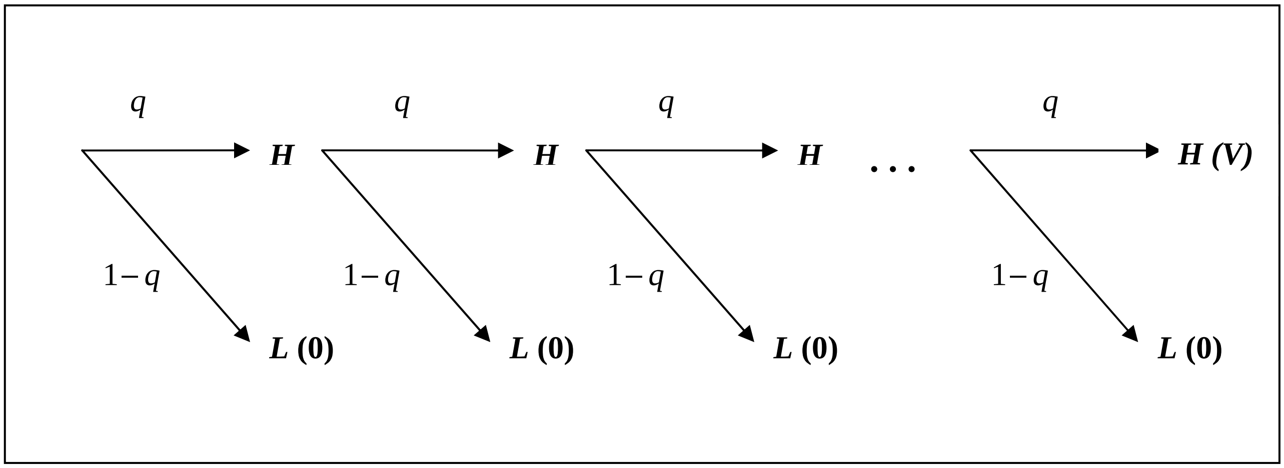

Figure 2a: Optimistic information structure

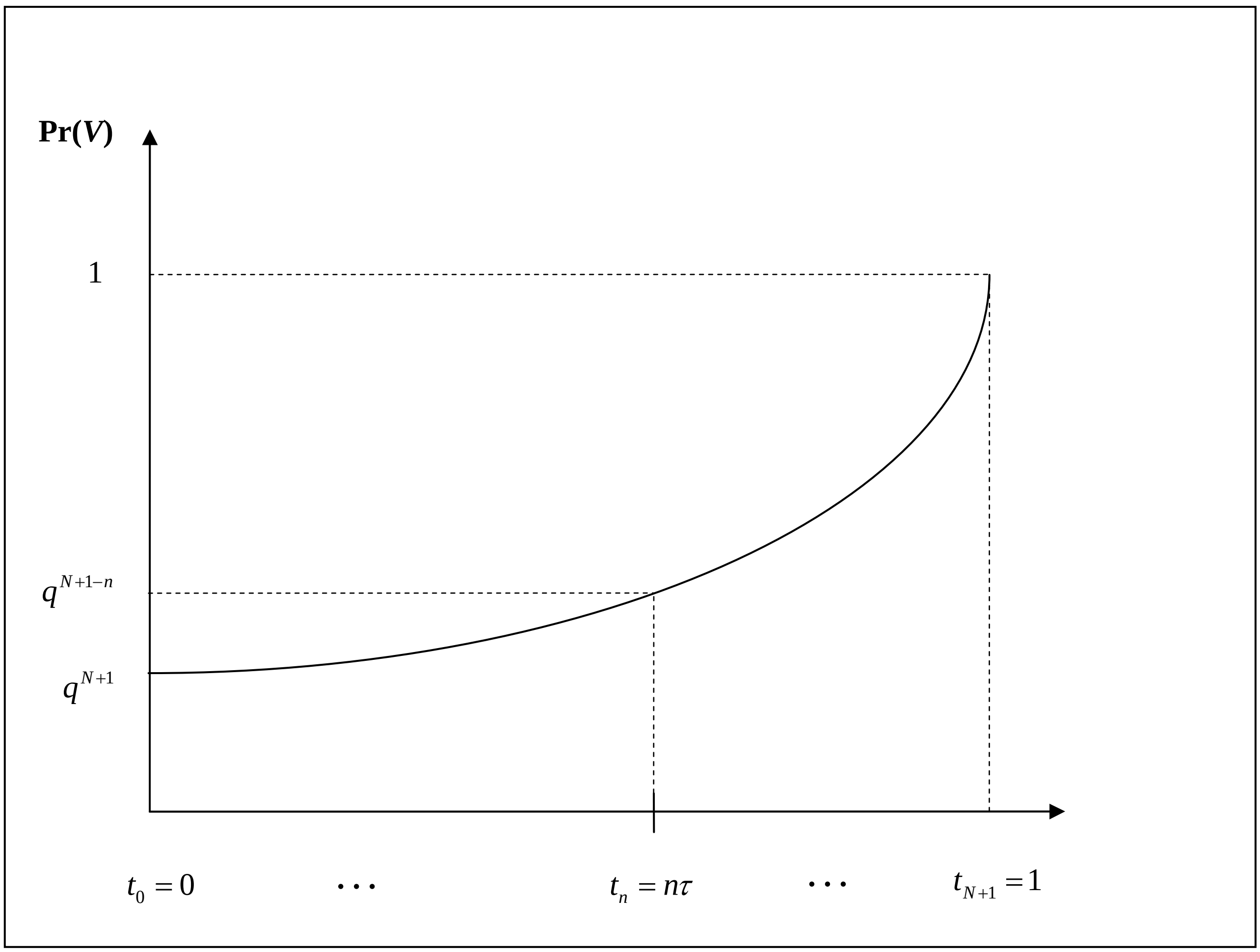

Figure 2b: Probability of the high return *V* over time in the optimistic information structure conditional on no news.

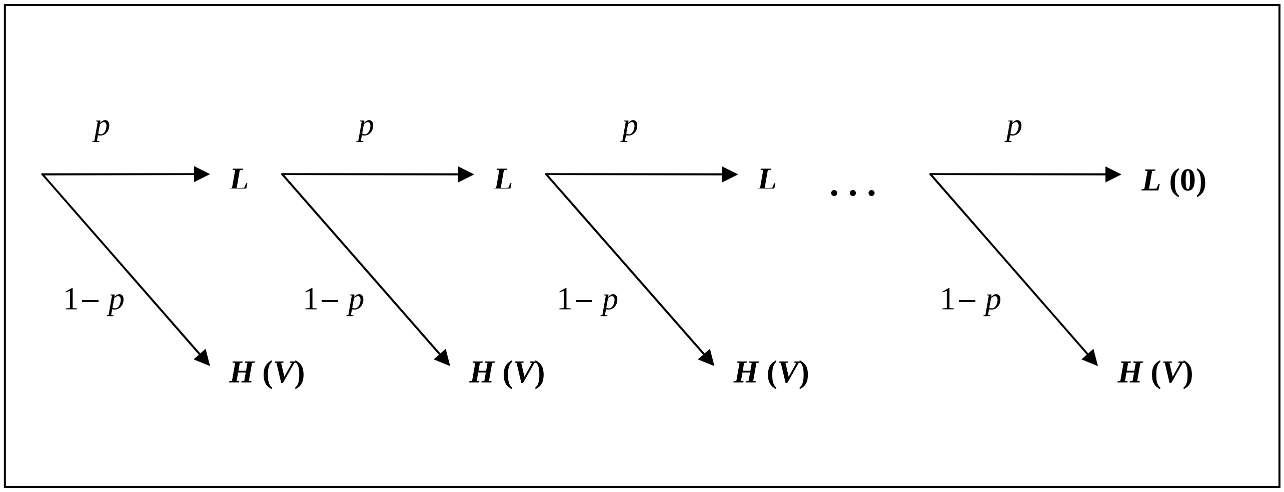

Figure 3a: Pessimistic information structure

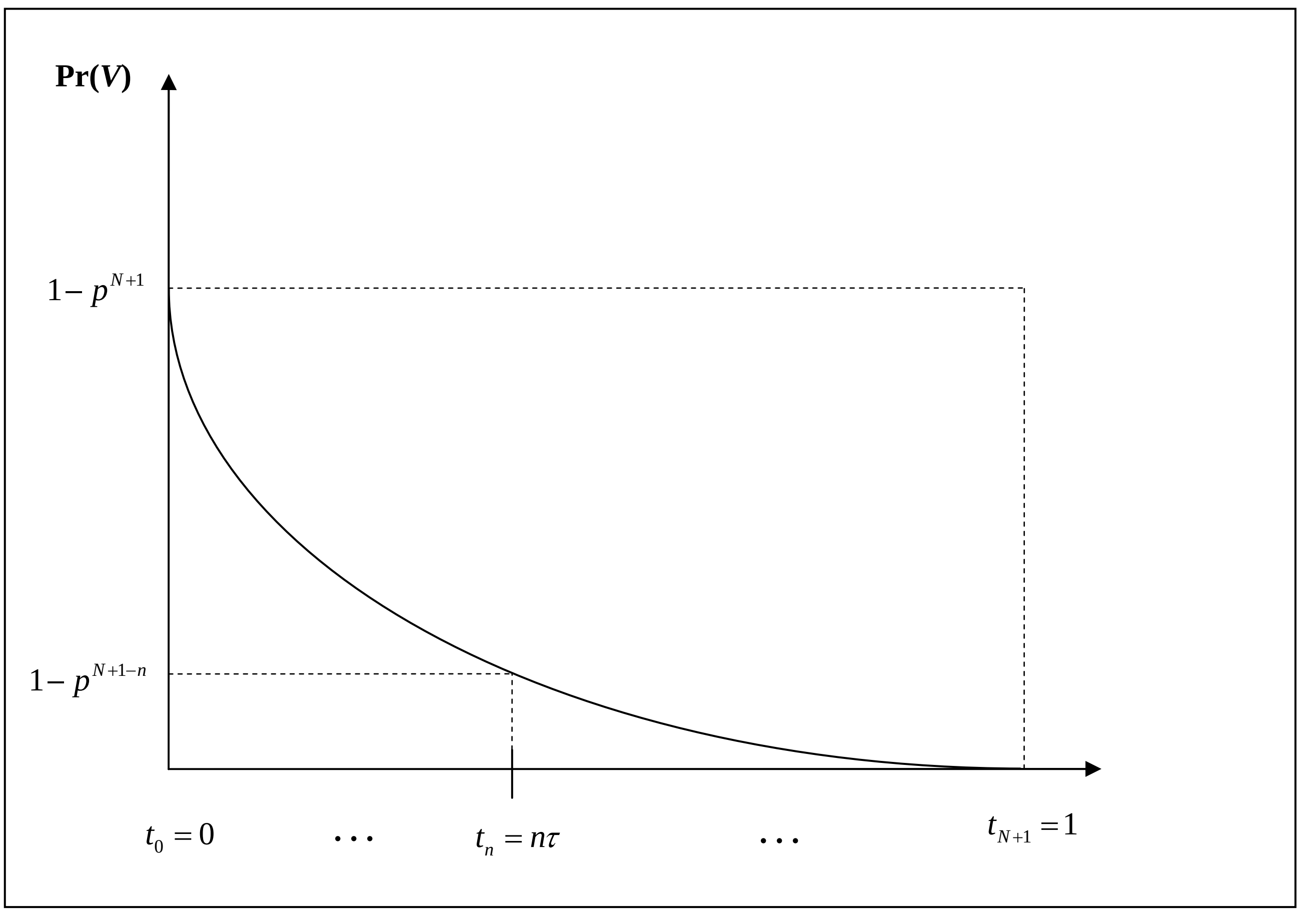

Figure 3b: Probability of the high return V over time in the pessimistic information structure conditional on no news.

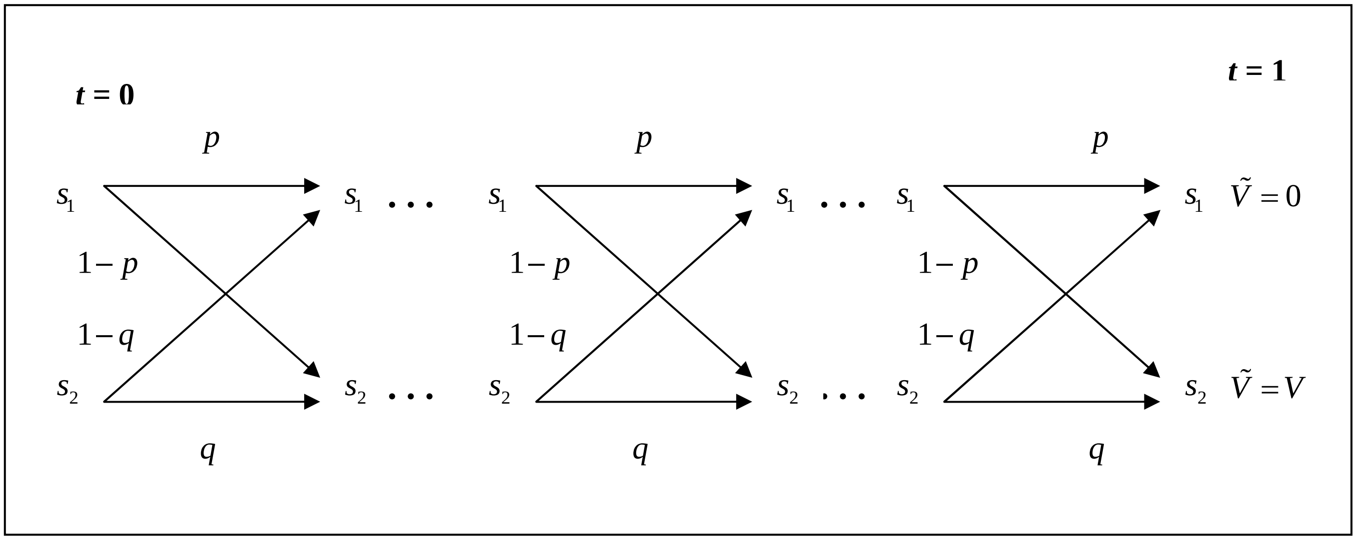

Figure 4: State transitions in the model with nested information structures.

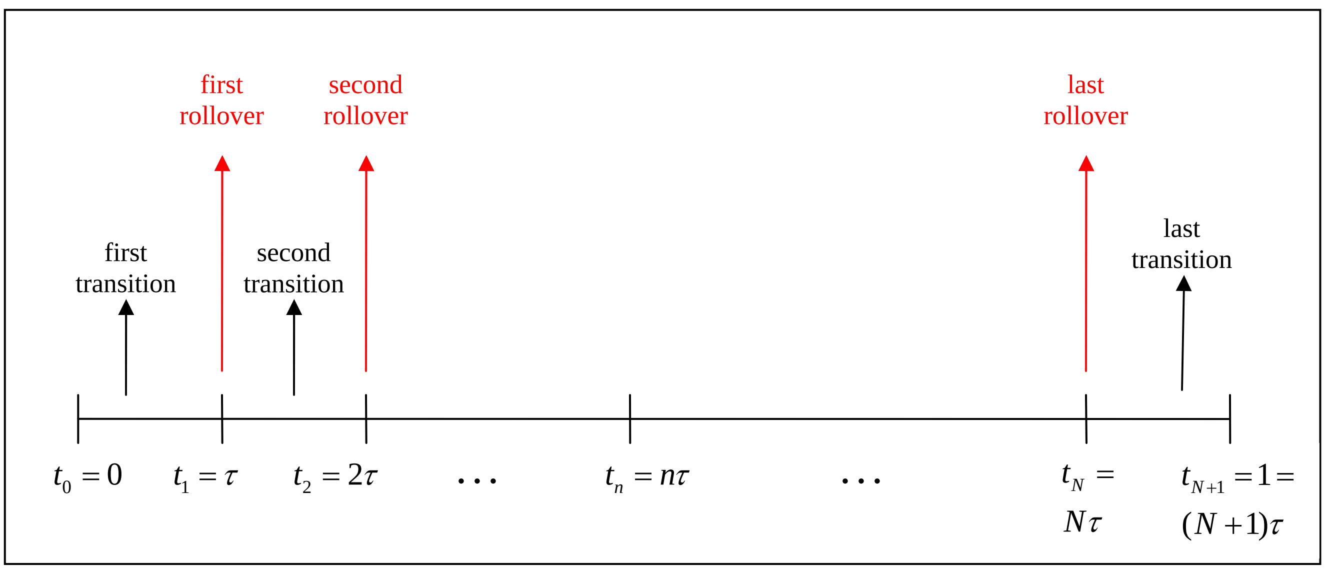

Figure 5: Timeline (illustrating *N*+1 state transitions and *N* rollovers).

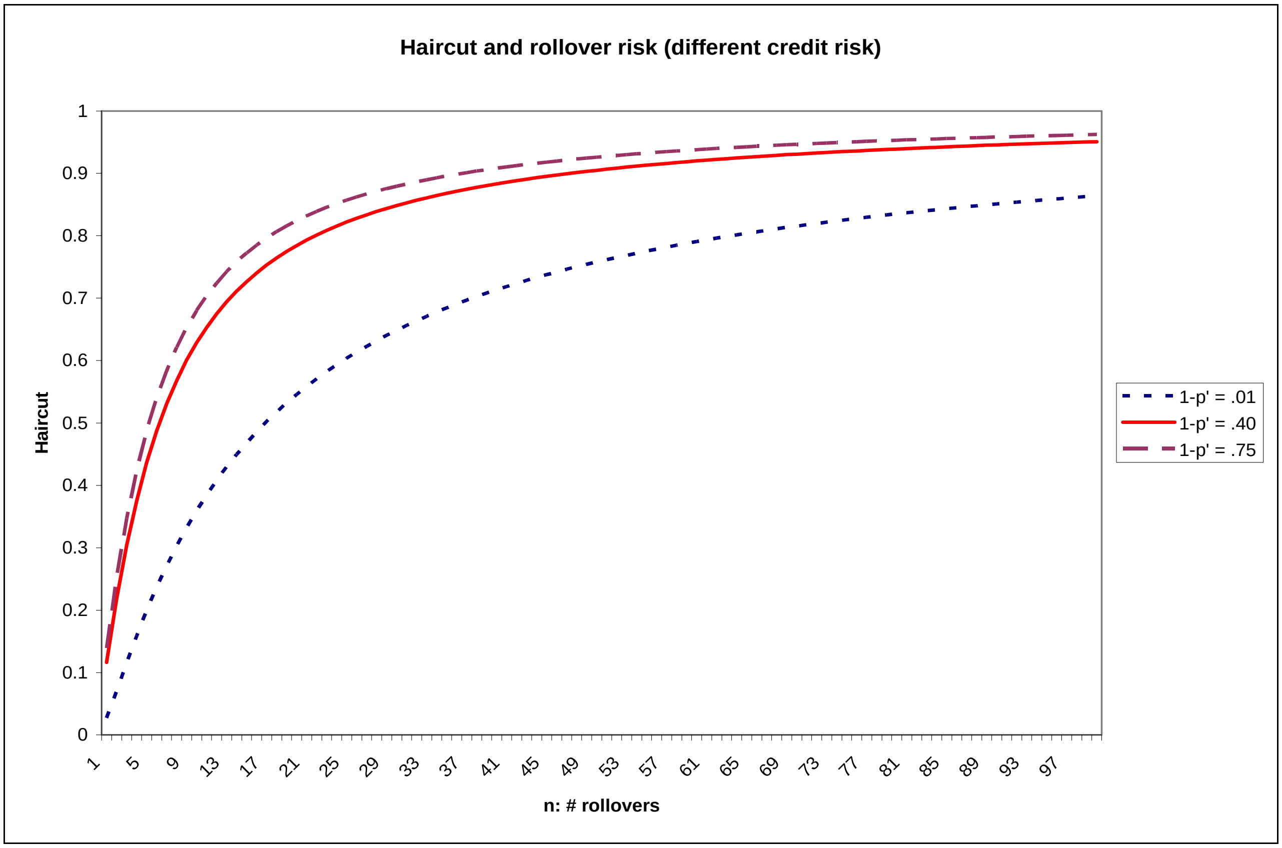

Figure 6a: Haircut as a function of *n* for different levels of credit risk (1-*p'*) for $\lambda = 0.7$.

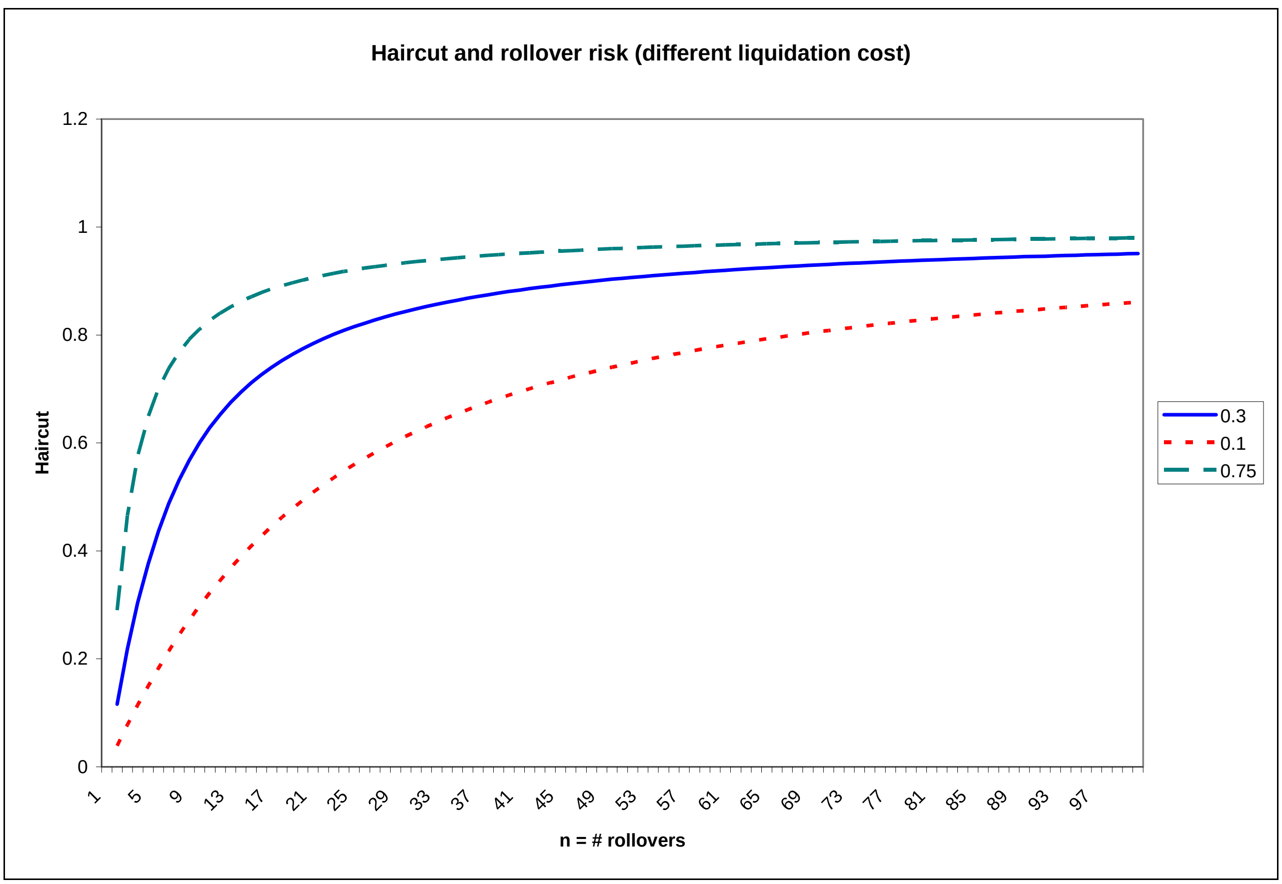

Figure 6b: Haircut as a function of *n* for different levels of liquidation risk (1-λ) for *p'*= .60.

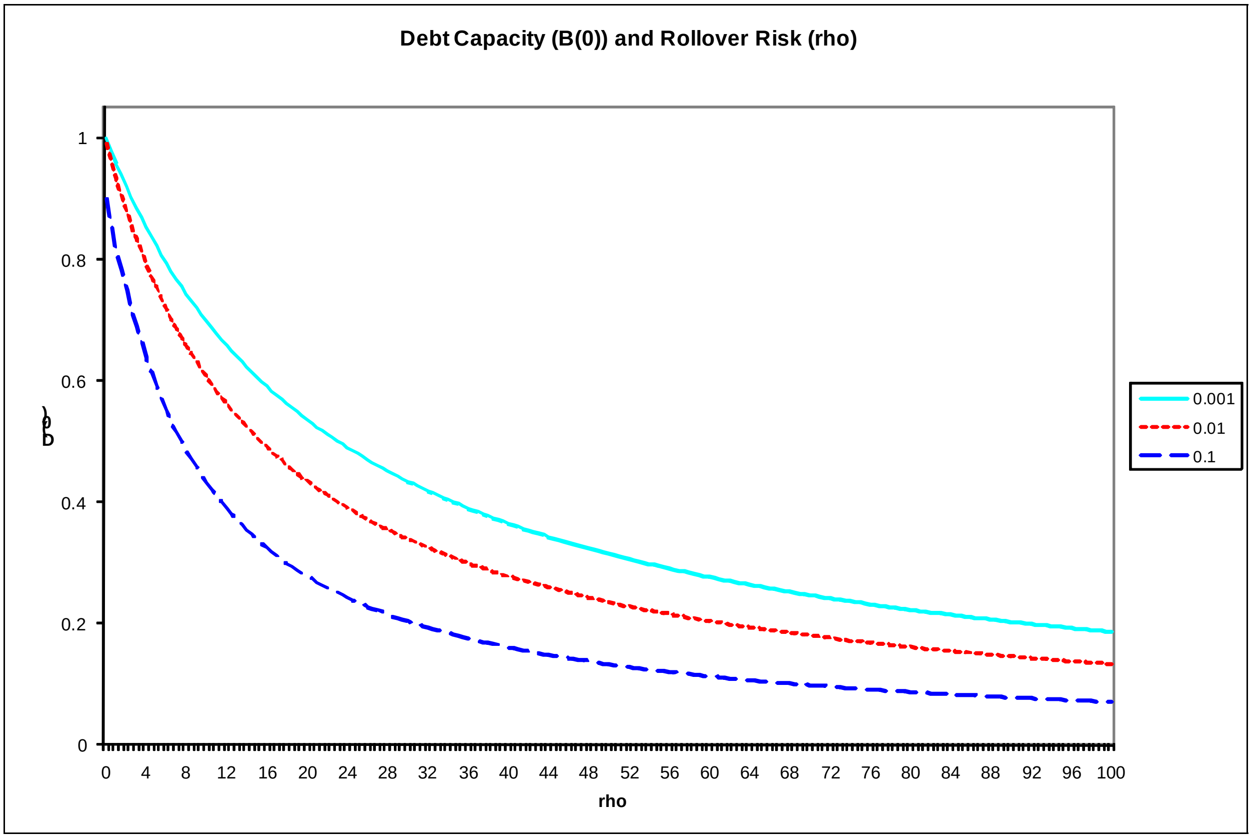

Figure 7a: Debt capacity as a function of ρ for different levels of credit risk ($e^{-\alpha} = 0.001,\ 0.01$, and 0.1) for $V = 1$, and $\lambda = 0.7$.

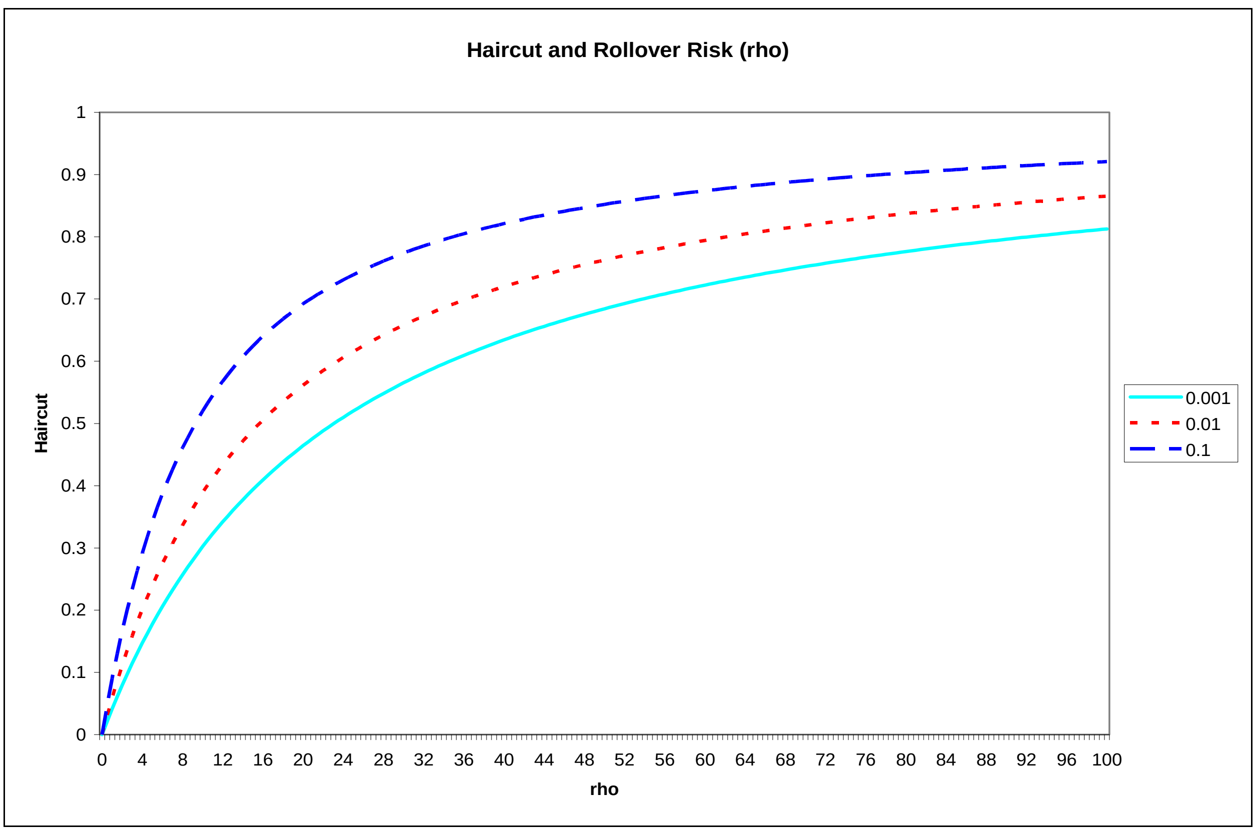

Figure 7b: Haircut as a function of ρ for different levels of credit risk ($e^{-\alpha} = 0.001,\ 0.01$, and 0.1) for $V = 1$, and $\lambda = 0.7$.