

# Leverage, Moral Hazard and Liquidity

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# Introduction

- ▶ We present a model wherein risk-shifting problem tied to leverage limits the funding liquidity of trading-based financial intermediaries.
- ▶ We consider pledging of cash collateral (resulting from asset sales) as a means to relax this borrowing constraint.
- ▶ We endogenize liquidity “shocks” as arising due to asset-liability mismatch in an incomplete contracts set-up:
  - ▶ Ex-post lender control is optimal to maximize ex-ante borrowing capacity.
  - ▶ Given asset-shock uncertainty, liquidity shocks are thus determined by optimal leverage structure.
  - ▶ Capital structure matters!

## Introduction – continued

- ▶ Key result: The model revolves around exactly one parameter – the maximum borrowing allowable due to ex-post risk shifting.
  - ▶ It affects funding liquidity, market liquidity, and asset prices.
  - ▶ It affects ex-ante borrowing capacity and thereby the distribution of future liquidity shocks.
  - ▶ It provides one possible explanation for why liquidity crises that follow good times seem to be more severe.
  - ▶ In good times, balance-sheets of institutions are levered up, so that in case of an adverse shock, there is not much spare debt capacity in the system.

# Motivation

- ▶ Adverse shocks that follow good times seem to produce deeper liquidity crises:
- ▶ For example, Paul McCulley asks in the Investment Outlook of PIMCO during the sub-prime crisis of Summer of 2007:

*“Where did all the liquidity go? Six months ago, everybody was talking about boundless global liquidity supporting risky assets, driving risk premiums to virtually nothing, and now everybody is talking about a global liquidity crunch, driving risk premiums half the distance to the moon. Tell me, Mac, where did all the liquidity go?”*

- ▶ Our paper is an attempt to provide some answers to these questions based on the central role played by leverage in affecting asset prices.

# Overview of Setup

- ▶ Timeline (Figure 0 / Figure 5).
- ▶ At time 0, agents have differing borrowing needs  $s$  for a project that payoffs at date 2.
- ▶ To finance this project they issue roll over debt  $\rho(s)$ , this rollover debt is due at date 1.
- ▶ At date 1, state  $\theta_2$  realizes (aggregate state) – more on this later.
- ▶ At date 1, lenders demand  $\rho(s)$  – They can either agree to roll over  $\rho(s)$  or insist that investors pay back  $\rho(s)$ .

<b>t = 0</b>	<b>t = 1</b>		<b>t = 2</b>
<b>FIRST ROUND OF BORROWING</b>	<b>DEBT DUE</b>	<b>MARKET FOR ASSETS</b>	<b>MORAL HAZARD PROBLEM</b>
	<b>ADDITIONAL DEBT FINANCING</b>	<b>AT PRICE <math>p(\theta_2)</math></b>	
	<b>REALIZATION OF ASSET QUALITY <math>\theta_2</math></b>		
	<ul style="list-style-type: none"> <li>• Firms with low <math>\rho_i</math> :               <ul style="list-style-type: none"> <li>- Borrow to rollover debt and potentially buy assets</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>- Buy assets</li> </ul>	<ul style="list-style-type: none"> <li>• Choose between safe and risky asset</li> <li>• Assets pay off, debt is due</li> </ul>
<ul style="list-style-type: none"> <li>• Firm <math>i</math> differ in financing needs. Firms with financing need <math>s_i</math> raise capital with debt of face value <math>\rho_i</math>; Firms with very high <math>s_i</math> are rationed</li> </ul>	<ul style="list-style-type: none"> <li>• Firms with moderate <math>\rho_i</math> :               <ul style="list-style-type: none"> <li>- Credit-rationed</li> <li>- Borrow to raise <math>(\rho_i - \alpha_i p)</math></li> </ul> </li> <li>• Firms with high <math>\rho_i</math> :               <ul style="list-style-type: none"> <li>- Credit-rationed</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>- “De-lever” by liquidating <math>\alpha_i</math> assets</li> </ul>	<ul style="list-style-type: none"> <li>• Choose between safe and risky asset</li> <li>• Assets pay off, debt is due</li> </ul>
		<ul style="list-style-type: none"> <li>- Are entirely liquidated</li> </ul>	

**Figure 5:** Timeline of the augmented model.

# Setup and risk-shifting problem

- ▶ Timeline (Figure 1).
- ▶ The short term debt that is due at date 1 of  $\rho(s)$  constitute the endogenous liquidity or margin needs in our model.
- ▶ For now, we focus on date 1 and take these liquidity “shocks” at time 1 as given – we are effectively working backwards.
- ▶ Liquidity shocks at date 1:  $\rho \sim g(\rho)$  over  $[\rho_{\min}, \rho_{\max}]$ .
- ▶ We also fix an aggregate state  $\theta$  of the world at date 1.

<b>t = 1</b>		<b>t = 2</b>	
<b>LIQUIDITY</b>	<b>DEBT FINANCING</b>	<b>MARKET FOR ASSETS</b>	<b>MORAL HAZARD PROBLEM</b>
<b>SHOCKS</b>		<b>AT PRICE <math>p</math></b>	
	<ul style="list-style-type: none"> <li>• Firms with low <math>\rho_i</math> :               <ul style="list-style-type: none"> <li>- Borrow to rollover debt and potentially buy assets</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>- Buy assets</li> </ul>	<ul style="list-style-type: none"> <li>• Choose between safe and risky asset</li> <li>• Assets pay off, debt is due</li> </ul>
<ul style="list-style-type: none"> <li>• Firm <math>i</math> has liability outstanding of <math>\rho_i</math></li> </ul>	<ul style="list-style-type: none"> <li>• Firms with moderate <math>\rho_i</math> :               <ul style="list-style-type: none"> <li>- Credit-rationed</li> <li>- Borrow to raise <math>(\rho_i - \alpha_i p)</math></li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>- “De-lever” by liquidating <math>\alpha_i</math> assets</li> </ul>	<ul style="list-style-type: none"> <li>• Choose between safe and risky asset</li> <li>• Assets pay off, debt is due</li> </ul>
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**Figure 1:** Timeline of the benchmark model.



# The risk-shifting problem

- ▶ Asset-substitution problem: Asset 2 is better but asset 1 is riskier and may be desirable from a risk-shifting standpoint.
  - ▶  $\theta_1 < \theta_2, y_1 > y_2, \theta_1 y_1 \leq \theta_2 y_2$ .
  - ▶  $\rho_{\min} \equiv \theta_1 y_1 \leq \rho_i$ .
  - ▶  $\theta_2 y_2 \leq \rho_{\max}$ .
- ▶ All agents are risk-neutral and risk-free rate is zero.
- ▶ Precursors – Stiglitz and Weiss (1981), Diamond (1989, 1993)

# Moral hazard induced rationing

This introduces the concept of ex-post debt capacity

- ▶ Incentive compatibility:

$$\theta_2(y_2 - f) > \theta_1(y_1 - f). \quad (1)$$

- ▶ Simplifies to an upper bound on the face value of new debt:

$$f < f^* \equiv \frac{(\theta_2 y_2 - \theta_1 y_1)}{(\theta_2 - \theta_1)}. \quad (2)$$

**Lemma 1:** Firms with liquidity need  $\rho_i$  at date 0 that is greater than  $\rho^* \equiv \theta_2 f^*$  are credit-rationed in equilibrium.

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# Collateral

- ▶  $(f_i, k_i)$  where  $k_i$  is the amount of collateral to pledge.
- ▶ Units sold:  $\alpha_i = k_i/p$ .
- ▶ Incentive compatibility:

$$\theta_2 [k + (1 - \alpha)y_2 - f] > \theta_1 [k + (1 - \alpha)y_1 - f]. \quad (3)$$

- ▶ This limits the face value of debt and funding liquidity of the asset:

$$\rho < \rho^{**}(k) \equiv [\alpha p + (1 - \alpha)\rho^*], \quad (4)$$

## Collateral – continued

### Optimal collateral requirement or asset sales

**Proposition 1:** If the liquidation price  $p$  is greater than  $\rho^*$  (as will be the case in equilibrium), then collateral requirement relaxes credit rationing for firms with  $\rho \in (\rho^*, p]$ , and takes the form

$$k(\rho) = \alpha(\rho)p = \frac{(\rho - \rho^*)}{(p - \rho^*)} p. \quad (5)$$

The collateral requirement  $k(\rho)$  is increasing in liquidity shock  $\rho$  and decreasing in liquidation price  $p$ , and the proportion of firms for which credit rationing is relaxed,  $[\rho - \rho^*]$ , is increasing in liquidation price  $p$ .

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## Market for asset sales

- ▶ Essentially, an industry equilibrium approach.
- ▶ Non-rationed firms buy assets (“arbitrageurs”), rationed and collateralizing firms sell assets.
- ▶ With ability to purchase assets, non-rationed firms’ debt capacity is even greater.

$$\theta_2[(1 + \alpha)y_2 - f] > \theta_1[(1 + \alpha)y_1 - f], \quad (6)$$

- ▶ This requires that the interest rate  $f$  satisfy the condition:

$$f < \frac{(1 + \alpha)\rho^*}{\theta_2}. \quad (7)$$

so the non-rationed firm can borrow up to  $(1 + \alpha)\rho^*$



## Market for asset sales – continued

- ▶ Liquidity available with firm  $i$  for asset purchase is thus

$$l(\alpha, \rho) = [\theta_2 f^*(\alpha) - \rho] = [(1 + \alpha)\rho^* - \rho]. \quad (8)$$

- ▶ No buyer will pay more than  $\bar{p} = \theta_2 y_2$ .

- ▶ For  $p > \bar{p}$ , demand is  $\hat{\alpha} = 0$ .

- ▶ For  $p \leq \bar{p}$ :

$$p \hat{\alpha} = l(\hat{\alpha}, \rho), \quad (9)$$

which simplifies to

$$\hat{\alpha}(p, \rho) = \frac{(\rho^* - \rho)}{(p - \rho^*)}. \quad (10)$$

## Market for asset sales – continued

- ▶ Overall demand for assets:

$$D(p, \rho^*) = \begin{cases} \int_{\rho_{\min}}^{\rho^*} \frac{(\rho^* - \rho)}{(p - \rho^*)} g(\rho) d\rho & \text{if } \rho^* \leq p \\ \left[ 0, \int_{\rho_{\min}}^{\rho^*} \frac{(\rho^* - \rho)}{(p - \rho^*)} g(\rho) d\rho \right] & \text{if } p = \bar{p} \end{cases} \quad (11)$$

- ▶ Overall supply of assets:

$$S(p, \rho^*) = \int_{\rho^*}^p \frac{(\rho - \rho^*)}{(p - \rho^*)} g(\rho) d\rho + \int_p^{\rho_{\max}} g(\rho) d\rho. \quad (12)$$

- ▶ Market-clearing determines the equilibrium price  $p^*$ : Either there is POSITIVE excess demand for all  $p < \bar{p}$  and  $p^* = \bar{p} = \theta_2 y_2$ , or

$$D(p, \rho^*) = S(p, \rho^*). \quad (13)$$

## Equilibrium price and its properties

- ▶ Excess demand can be expressed in the simple form for  $p < \bar{p}$ :

$$E(p, \rho^*) = -1 + \frac{1}{(p - \rho^*)} \int_{\rho_{\min}}^p G(\rho) d\rho. \quad (14)$$

$$\implies p = \rho^* + \int_{\rho_{\min}}^p G(\rho) d\rho \quad (15)$$

- ▶ If the solution to this equation exceeds  $\bar{p}$ , then we have  $p^* = \theta_2 y_2$ .
- ▶ “Cash-in-the-market” pricing as in (Allen and Gale, 1994, 1998).
- ▶ Proposition 2: Cash in the market pricing is inversely related to funding liquidity Figures 2, 4.
- ▶ Proposition 3: Secondary market sales are inversely related to funding liquidity – Figure 3.

Figure 2: Equilibrium price  $p^*$  as a function of (inverse) moral hazard intensity

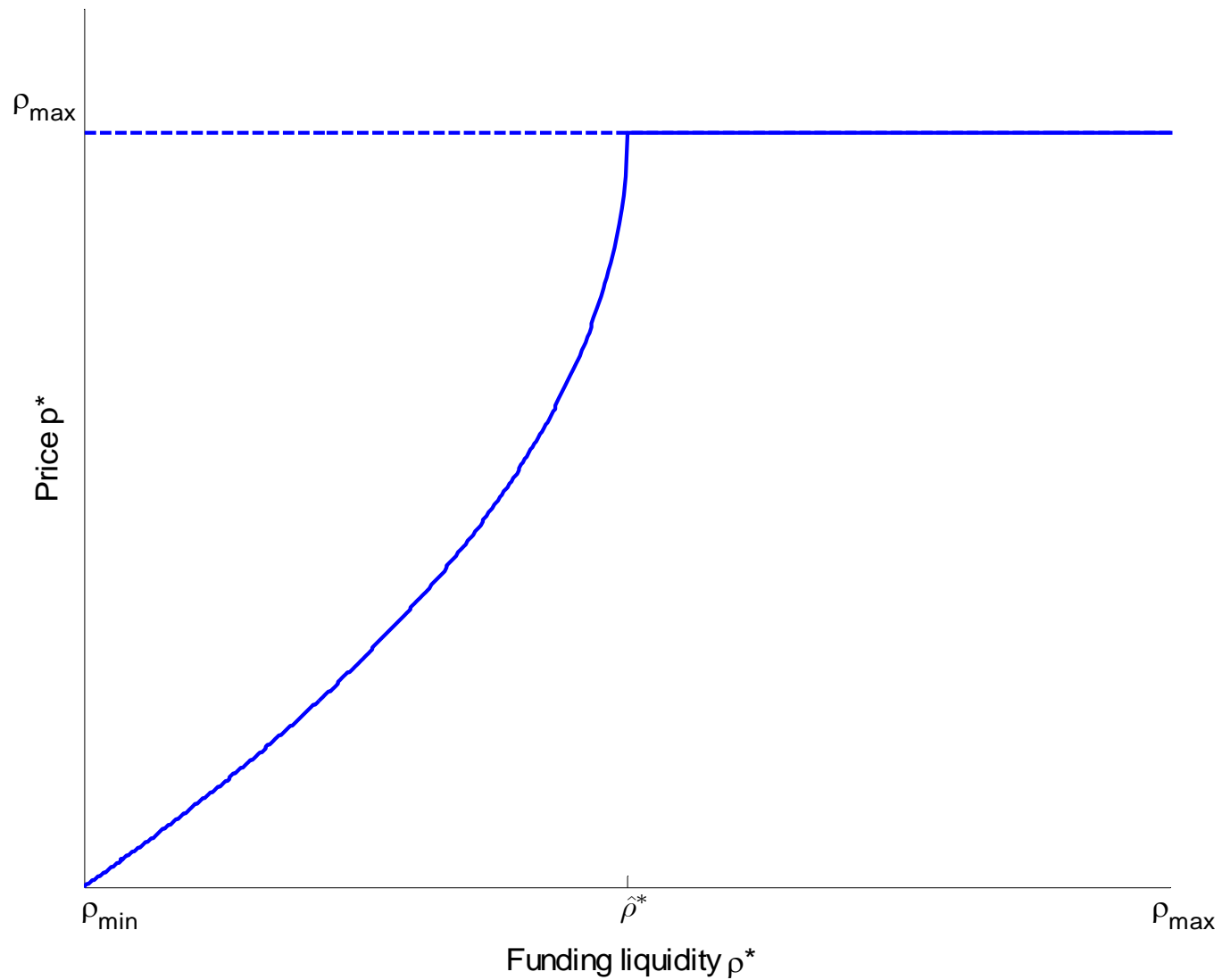


Figure 3: Equilibrium de-leveraging or asset-sale proceeds as a function of leverage  $\rho$

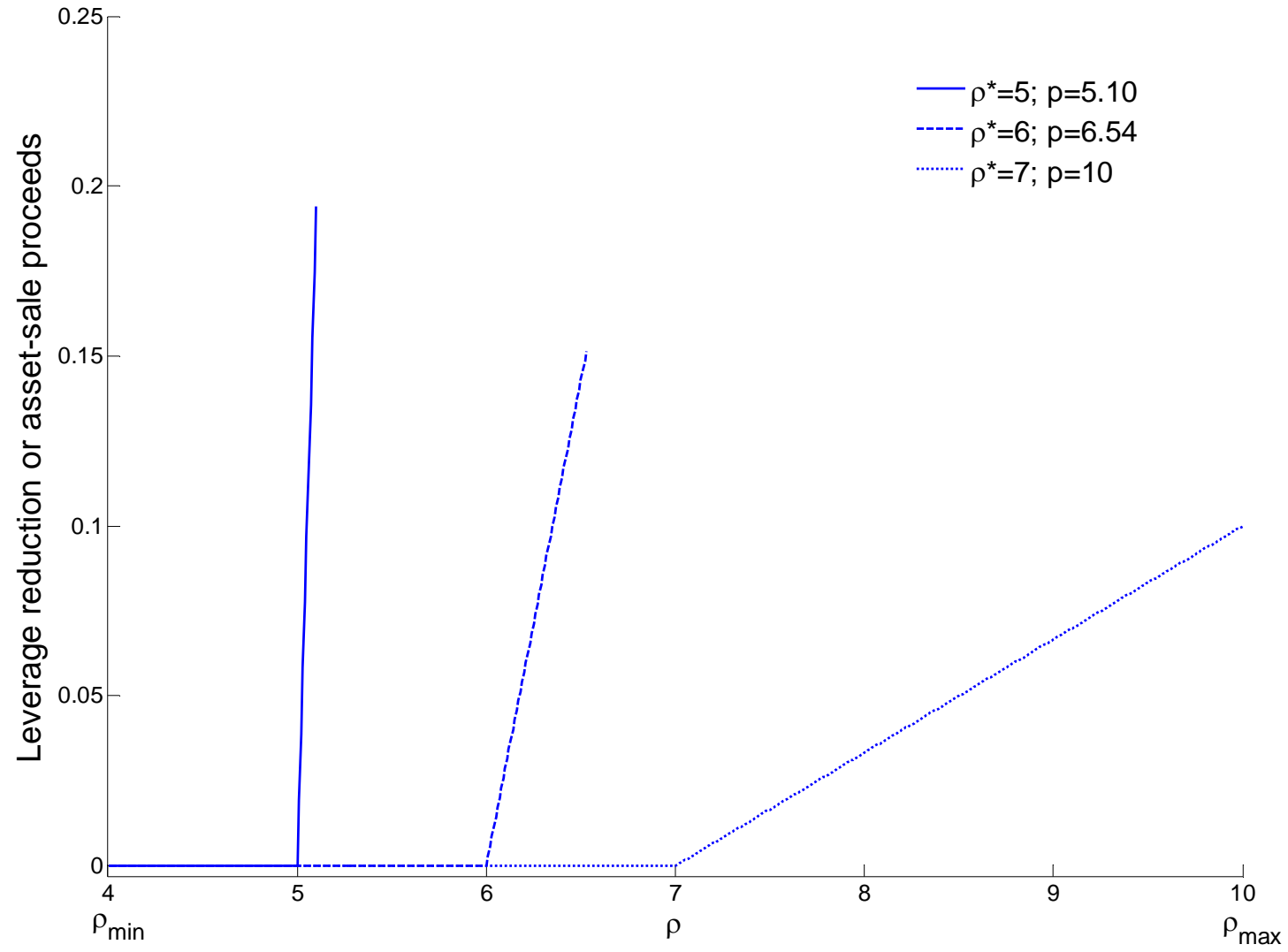
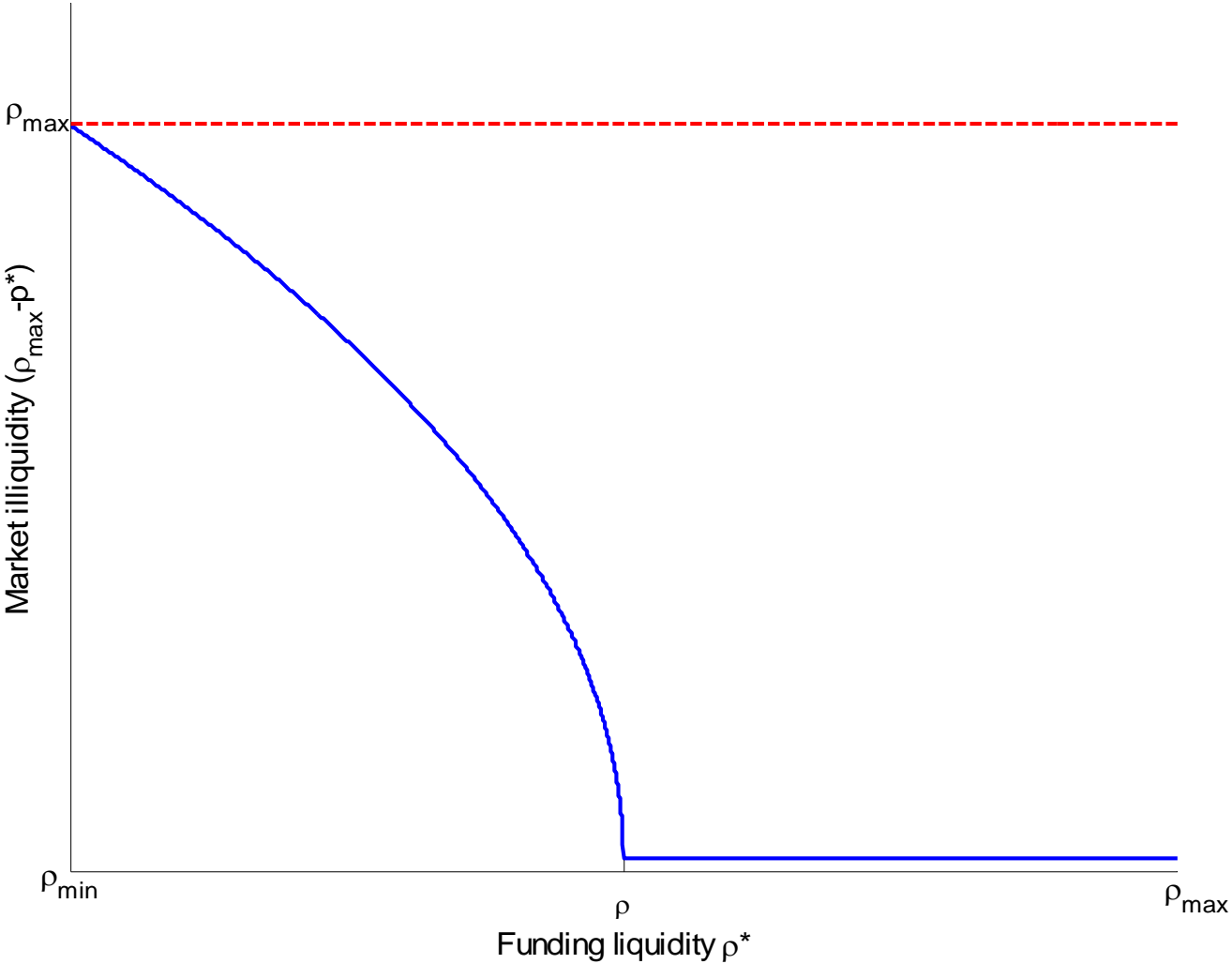


Figure 4: The relationship between market (il)liquidity and funding liquidity



## Related literature

- ▶ Rationing: Stiglitz and Weiss (1981), Bester (1985), Diamond (1989, 1993).
- ▶ Fire sales: Shleifer and Vishny (1992), Allen and Gale(1994, 1998).
- ▶ Exogenous collateral constraints: Gromb and Vayanos (2002), Brunnermeier and Pedersen (2005), Plantin and Shin (2006), Anshuman and Viswanathan (2006).
- ▶ Land is collateralizable, production is not: Kiyotaki and Moore (1997), Caballero and Krishnamurthy (2001), Krishnamurthy (2003).
- ▶ **Holmstrom and Tirole (1998)**: Important differences.

# Ex-ante debt capacity and liquidity shocks

Where do liquidity needs come from? What do they depend on?

- ▶ We consider ex-ante (date 0) financial liabilities.
- ▶ The distribution of ex-ante liabilities depends on the liquidation price, and thus on anticipated distribution of asset quality at date 1.
  - ▶ The distribution of asset quality is effectively the distribution of moral hazard intensity in future.
- ▶ But the liquidation price depends on the distribution of ex-ante liabilities in the system.
- ▶ This leads to an important feedback between the distribution of asset quality and financial liabilities at date 1.



## Ex-ante debt capacity – continued

- ▶ The augmented time-line is specified in Figure 5.
- ▶ A continuum of firms, each of which has a financing shortfall  $s_i$
- ▶ Cdf of  $s_i$  is  $R(s_i)$  over the support  $[\theta_1 y_1, I]$ .
- ▶ Firms raise debt of face value  $\rho_i$ , assumed to be hard and payable at date 1.

## Ex-ante debt capacity – continued

- ▶ Note that  $\theta_1 < \theta_2$ ,  $y_1 > y_2$ , and  $\theta_1 y_1 < \rho_i < \theta_2 y_2$ .
- ▶ Viewed from date 0,  $\theta_2$  is uncertain:
  - ▶  $\theta_2$  has cdf  $H(\theta_2)$  and pdf  $h(\theta_2)$  over  $[\theta_{\min}, \theta_{\max}]$ ;
  - ▶  $\theta_{\min} y_2 \geq \theta_1 y_1$ , that is, the worst-case expected outcome for the safer asset is no worse than that for the riskier asset.
- ▶ In fact we impose that

$$\theta_{\min} = \frac{\theta_1 y_1}{y_2} \left[ 1 + \sqrt{1 - \frac{y_2}{y_1}} \right], \quad (16)$$

- ▶ This ensures that  $\rho^* > \theta_1 y_1$ .

# Feedback in the model

We *jointly* solve for the distribution of liquidity shocks (face value of debt) and prices.

Interaction between leverage and fundamentals:

- ▶ Creditor recoveries, and therefore, ex-ante face values, depend on future prices – funding liquidity affects market liquidity.
- ▶ Future prices depend on the ex-ante distribution of face values – funding liquidity depends on the expectation of market liquidity.
- ▶ Viewed in a different way, the industry equilibrium notion of market-clearing prices leads to an industry equilibrium notion of debt capacities, and vice-versa.

**Definition:** An equilibrium of the ex-ante borrowing game is:

- ▶ (i) a pair of functions  $\rho(s_i)$  and  $p^*(\theta_2)$ , which respectively give the promised face-value for raising financing  $s_i$  and equilibrium price given quality of assets  $\theta_2$ ,
- ▶ (ii) a truncation point  $\hat{s}$ , which is the maximum amount of financing that a firm can raise in equilibrium, such that  $\rho(s_i)$ ,  $p^*(\theta_2)$  and  $\hat{s}$  satisfy the following fixed-point problem;

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## Definition – continued

1. For every  $\theta_2$ , prices are determined by the industry equilibrium condition of Proposition 3:

$$p^*(\theta_2) \leq \bar{p}^*(\theta_2) + \int_{\rho_{\min}}^{p^*(\theta_2)} \hat{G}(u) du, \quad (17)$$

where compared to equation (15), we have replaced distribution of liquidity shocks  $G(\cdot)$  with the induced distribution  $\hat{G}(\cdot)$  and also substituted the variable of integration  $\rho$  with  $u$  to avoid confusion with the function  $\rho(s_i)$ .

$\hat{G}(u)$  is the truncated equilibrium distribution of liquidity shocks given by  $\hat{G}(u) = \text{Prob}[\rho(s_i) \leq u | s_i \leq \hat{s}] = \frac{R(\rho^{-1}(u))}{R(\hat{s})}$ .

As in case of equation (15), a strict ( $<$ ) inequality leads to  $p^*(\theta_2) = \bar{p}(\theta_2) = \theta_2 y_2$ .

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## Definition – continued

2. Given the price function  $p^*(\theta_2)$ , for every  $s_i \in [0, \hat{s}]$ , the face value  $\rho$  is determined by the requirement that lenders receive in expectation the amount that is lent:

$$s_i = \int_{\theta_{\min}}^{p^{*-1}(\rho)} p^*(\theta_2) h(\theta_2) d\theta_2 + \int_{p^{*-1}(\rho)}^{\theta_{\max}} \rho h(\theta_2) d\theta_2. \quad (18)$$

3. The truncation point  $\hat{s}$  for maximal financing is determined by the condition

$$\hat{s} \leq \theta_1 y_1 + \int_{\theta_{\min}}^{\theta_{\max}} p^*(\theta_2) h(\theta_2) d\theta_2, \quad (19)$$

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# Equilibrium and comparative statics

Rewriting the above conditions as integro-differential equations, the main theorem solves for the endogenous distribution of firms that get financed, their leverage and equilibrium prices.

## Proposition 4:

There exists a unique equilibrium of the ex-ante borrowing game. In fact, it is a contraction mapping leading to easy numerical computations.

Comparative static exercise does not lead to unambiguous results because the marginal borrower who is financed changes (the distribution of firms financed is endogenous)

- ▶ Numerical examples: Vary the distribution of fundamentals at date 1 keeping the distribution of wealth at date 0 constant.

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- ▶ Numerical examples: Vary the distribution of fundamentals at date 1 keeping the distribution of wealth at date 0 constant.

## Numerical Example 1.

- ▶ Assume that  $y_1 = 4$ ,  $y_2 = 1$ ,  $\theta_1 = 0.05$ ,  $\theta_1 y_1 = 0.2$ . We assume that the borrowing  $s$  has support  $[0.2, 1]$ .

- ▶ Let  $t = 0.8$ . The distribution of borrowing at date 0 is uniform:

$$R(s) = \frac{s - 0.2}{t} \quad (20)$$

- ▶ Suppose also that  $H(\theta)$  has support  $[\theta_{\min}, \theta_{\max}]$  where  $\theta_{\min} = 0.1(2 + \sqrt{3})$  and  $\theta_{\max} = 0.9$ .

- ▶ We suppose that  $H(\theta)$  is given by the following distribution:

$$H(\theta) = 1 - \left(1 - \frac{\theta - \theta_{\min}}{\theta_{\max} - \theta_{\min}}\right)^{1/\gamma}, \quad \gamma > 0. \quad (21)$$

- ▶ A higher value of  $\gamma$  implies first-order stochastic dominance (FOSD): Hopenhayn (1993) calls this monotone conditional order (MCD).

## Numerical example 1 – continued

- ▶ We let  $\gamma$  take values 0.5, 5.0.
- ▶ Figures 6, 7: The distributions of  $\rho(s)$  and  $\rho(\theta)$ .
- ▶ Figure 8: The cdf of  $\rho$  and  $p$ .
- ▶ Note that the price function has the counterintuitive property that in adverse states, prices are in fact lower with better fundamentals.
  - ▶ Better fundamentals lead to higher leverage ex ante.
  - ▶ In turn, this leads to lower prices, in case shocks are adverse ex post.
  - ▶ This seems to describe well some recent liquidity crises.

Figure 6:  $\rho(s)$  for various  $\gamma$

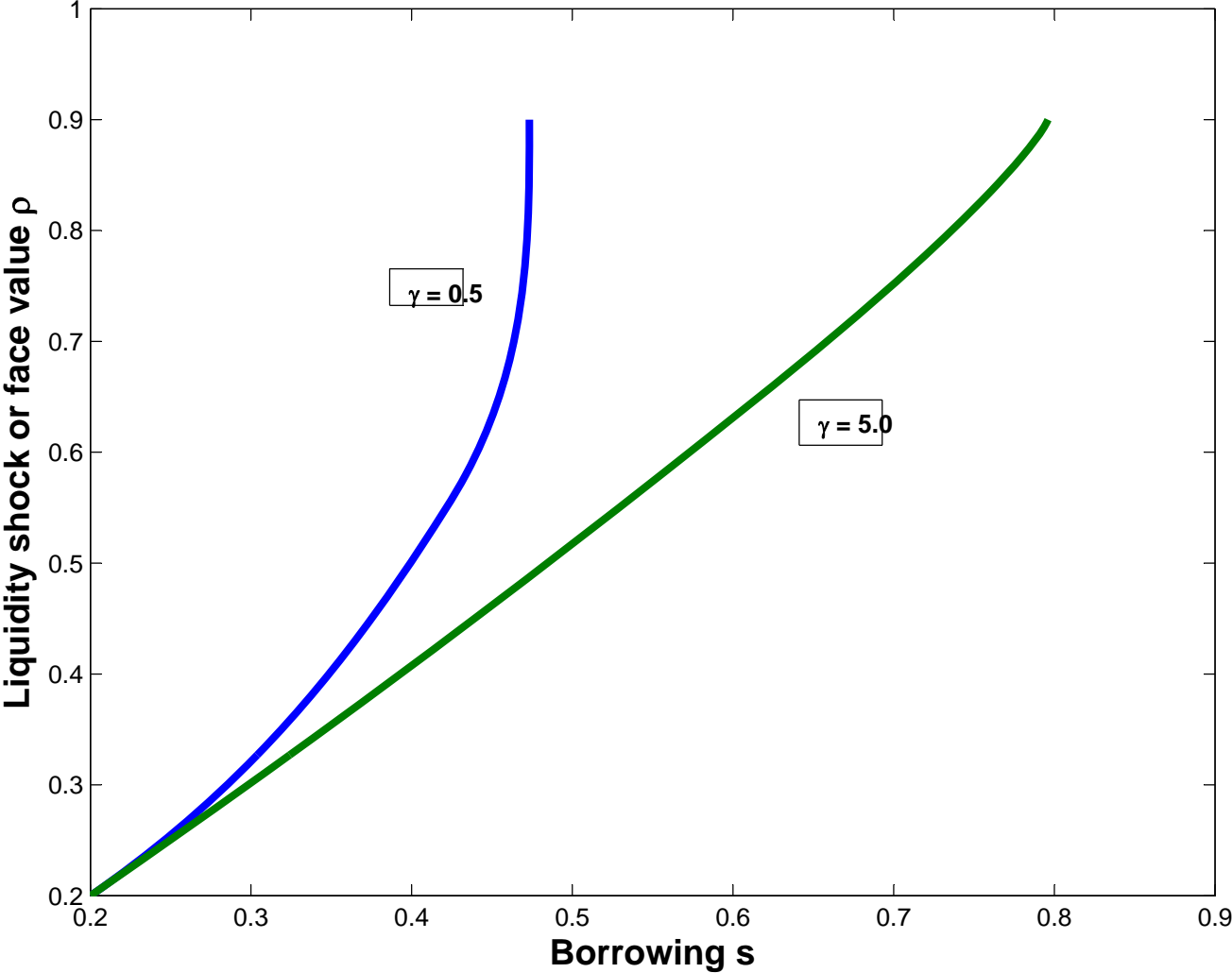




Figure 7:  $p(\theta)$  for various  $\gamma$

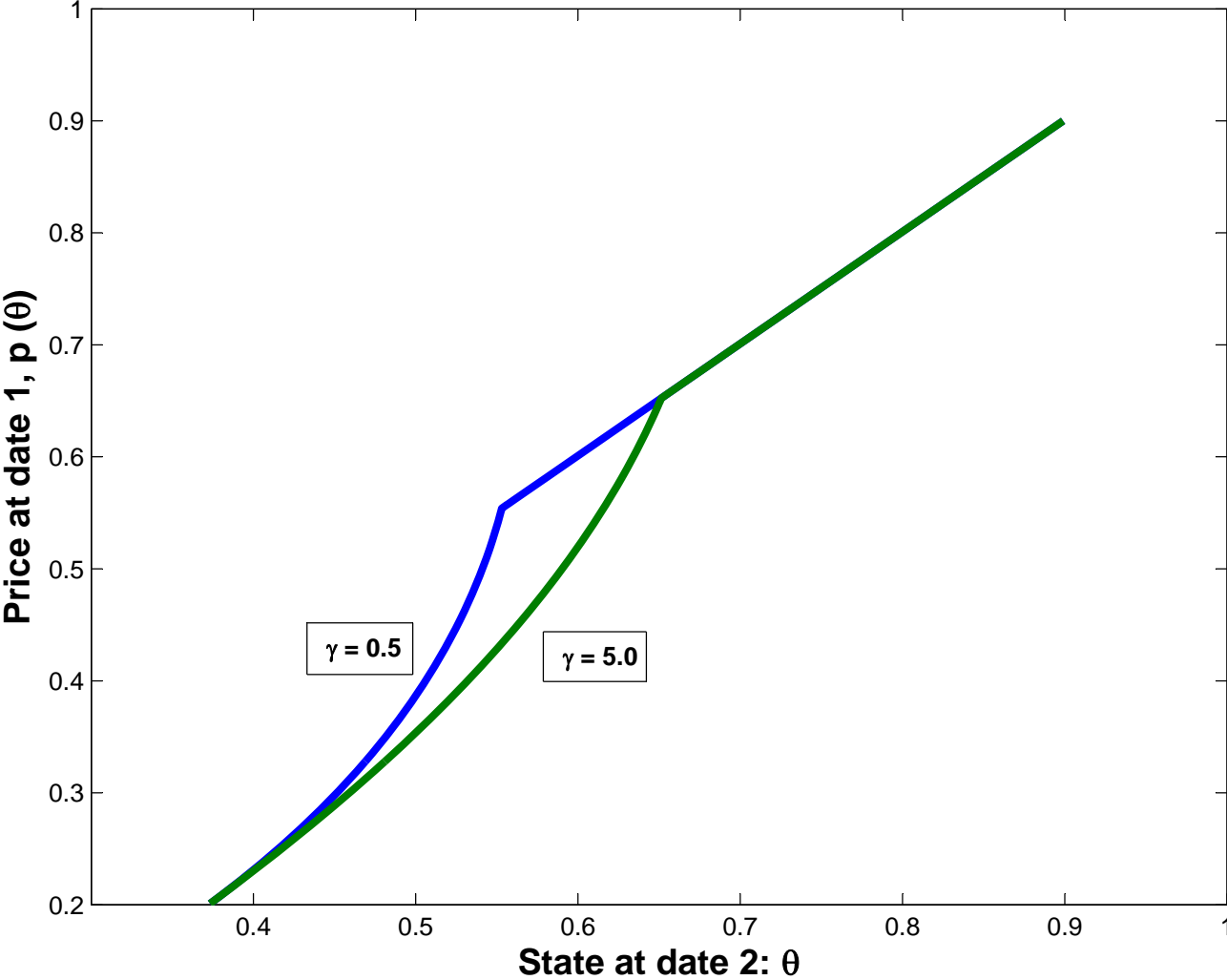


Figure 8a: CDF of  $\rho(s)$  in equilibrium

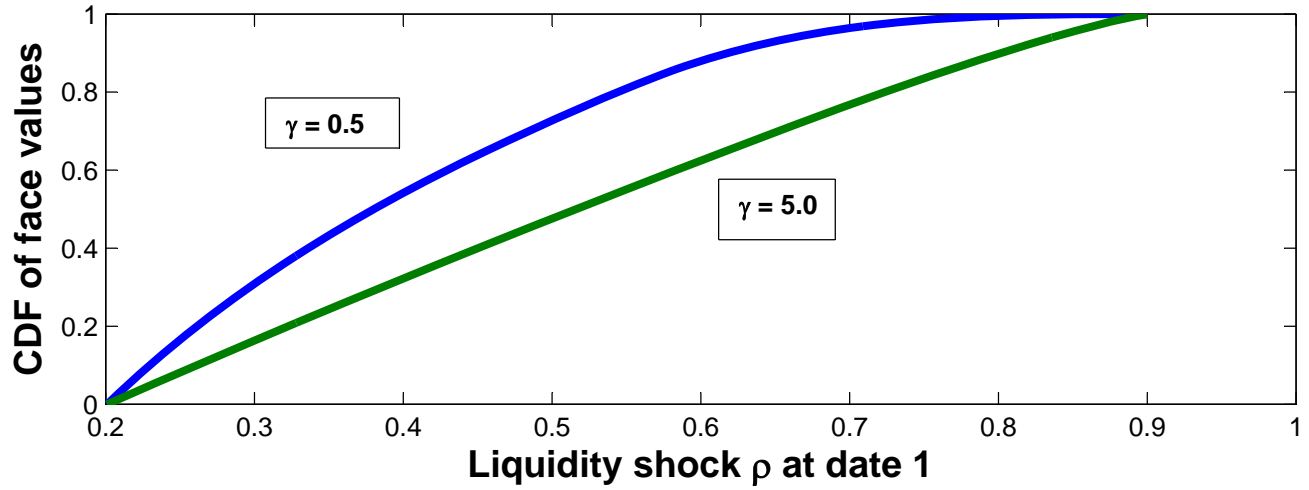


Figure 8b: CDF of prices in equilibrium

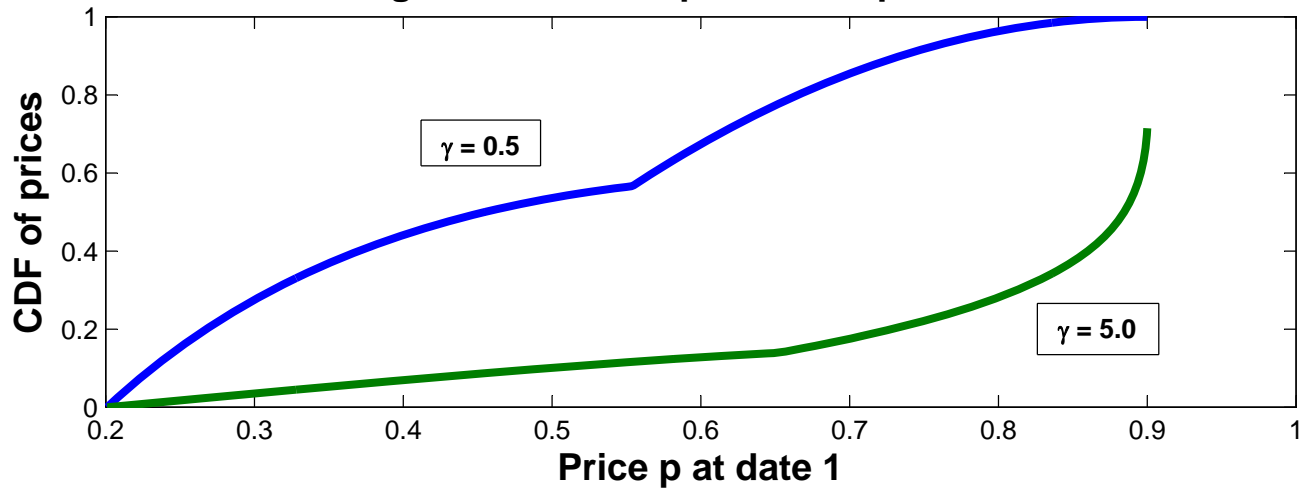
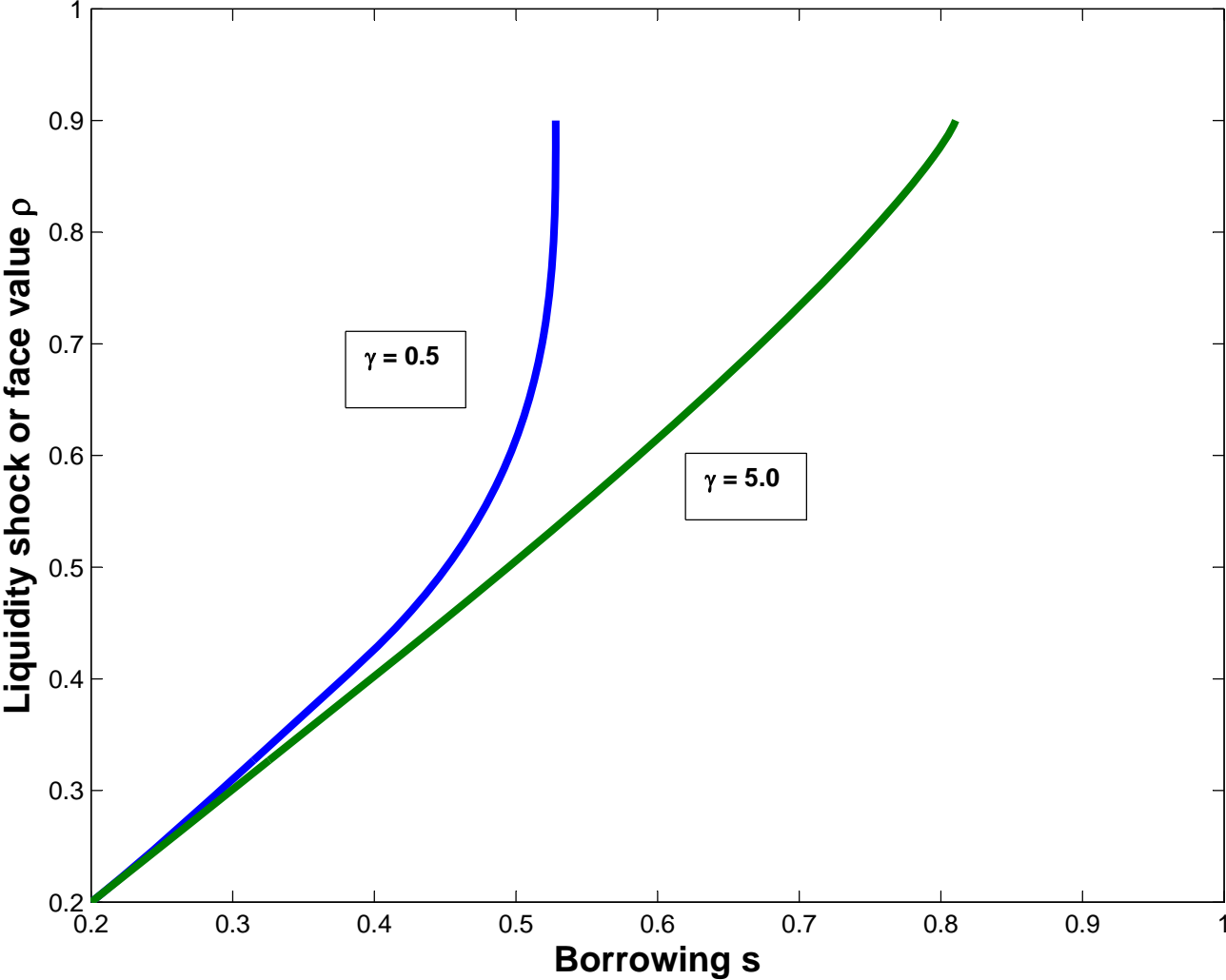


Figure 9:  $\rho(s)$  for various  $\gamma$



## Numerical example 2.

- ▶ We repeat the example above with a different distribution for borrowing shocks, we now use:

$$R(s) = 1 - \left(1 - \frac{s - 0.2}{t}\right)^{1/\zeta}, \quad (22)$$

with  $\zeta = 0.05$ .

- ▶ A higher  $\zeta$  implies lower capital levels and more borrowing at date 0 in a FOSD sense.
- ▶ This distribution has much thinner density in the right tail, reducing the effect of entry.
- ▶ Figures 9, 10, 11: Figure 11a shows the more “benign” distribution of leverage in this example, which results in prices being higher with higher fundamentals in Figure 10.

Figure 9:  $\rho(s)$  for various  $\gamma$

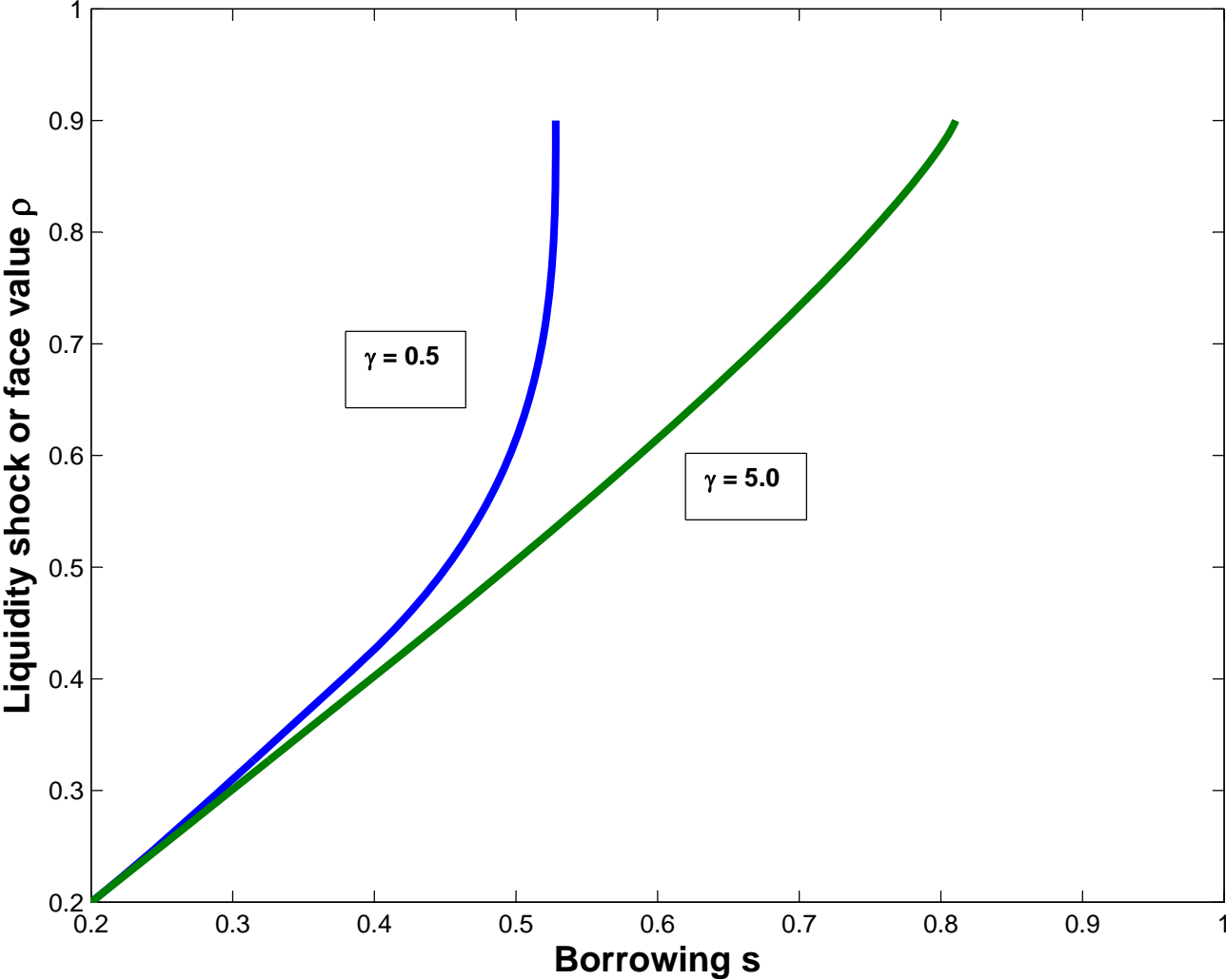


Figure 10:  $p(\theta)$  for various  $\gamma$

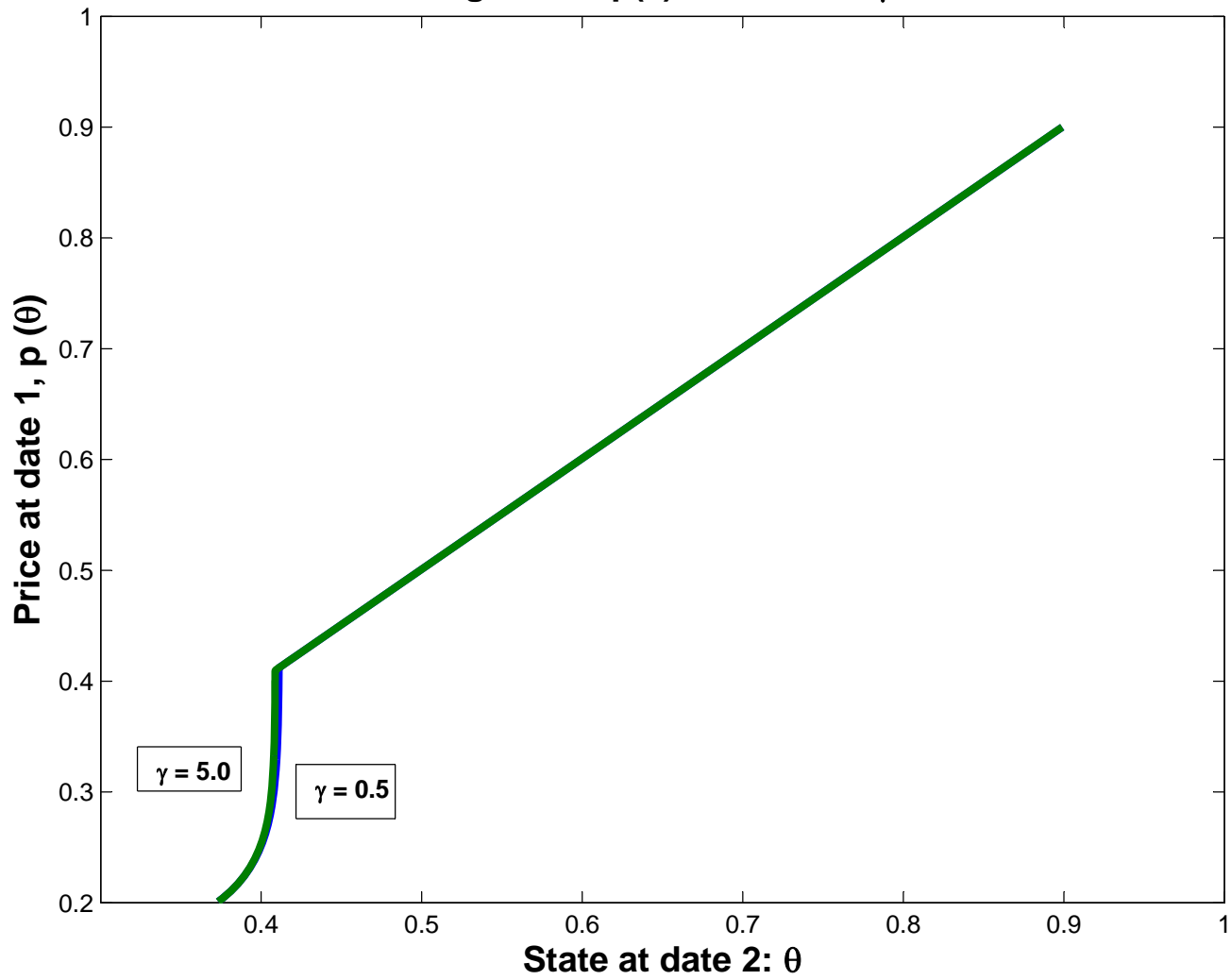


Figure 11a: CDF of  $\rho(s)$  in equilibrium

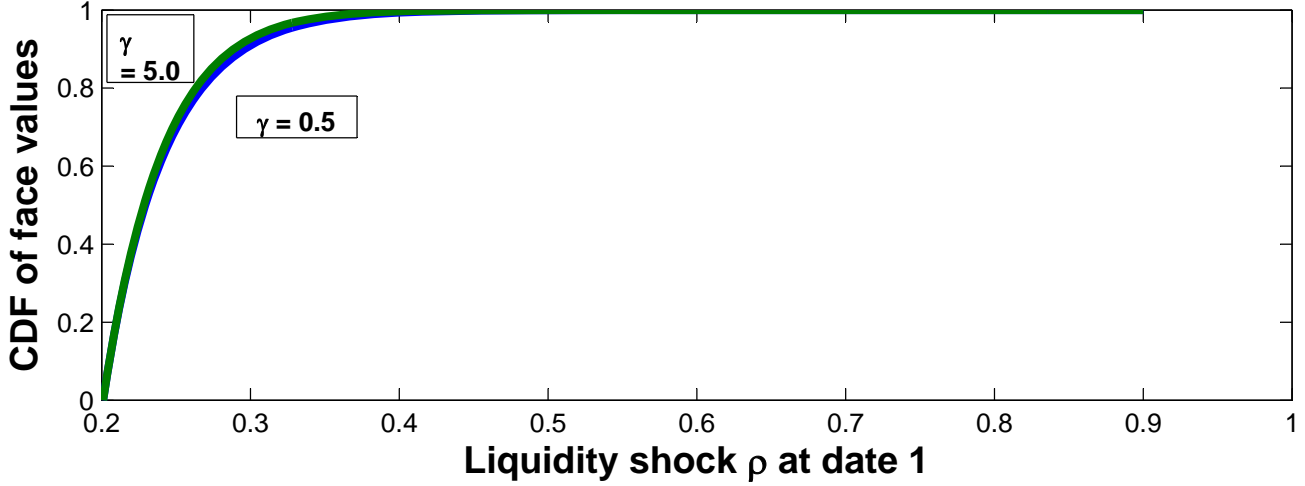
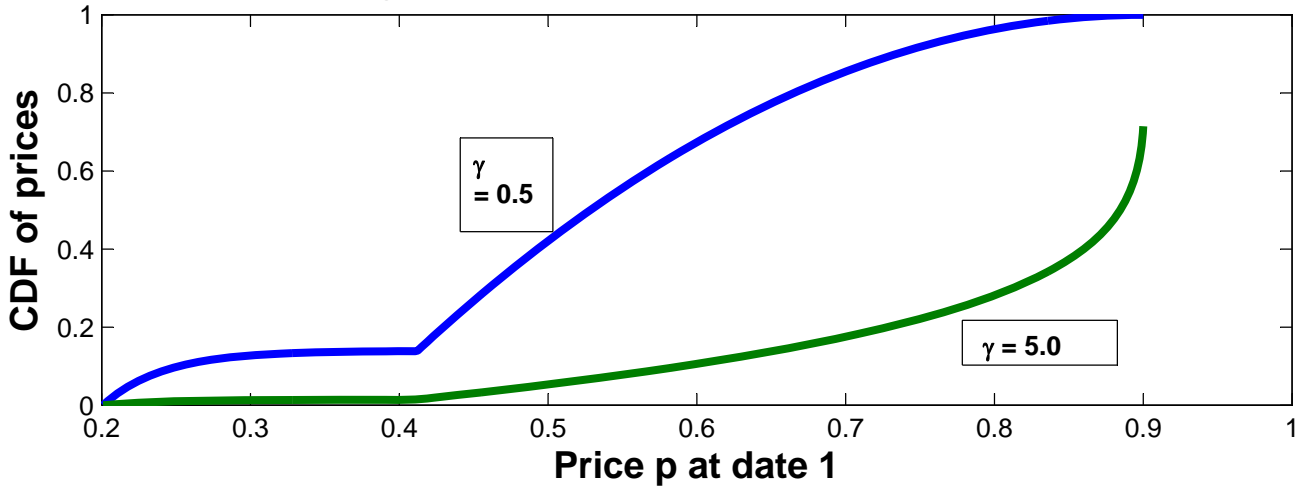


Figure 11b: CDF of prices in equilibrium



# Optimality of debt contracts with the lender's right to call

When can short-term debt contracts be optimal in our setup?

- ▶ **Assumption C1:** Courts can verify whether the state 0 occurs or whether  $\{y_1, y_2\}$  occurs, however they cannot distinguish between states  $\{y_1, y_2\}$ .
- ▶ **Assumption C2:** While the interim state  $\theta_2$  is observable, it is not contractible.
- ▶ **Assumption C3:** Payments at date 2 (ex-post states) cannot be bigger than the maximum payoff in that state or smaller than 0.



## Intuition for the optimality of hard debt contract

- ▶ With borrower control, the borrower can threaten the lender that he will risk shift, so that the lender can never get more than  $\rho^*$ .
- ▶ Ex ante, this is not desirable as the borrower wants to commit to returning to the lender as much as possible, especially if he wants to borrow more.
- ▶ Hence, it is preferable ex ante to give the lender control to call the loan at time 0.
- ▶ Collateral requirement is also desirable as it raises prices and allows the borrower to commit to higher repayments.

# Conclusions

- ▶ We have attempted to provide a tractable, agency-theoretic foundation to funding constraints with the goal of linking liquidity issues in financial markets directly to underlying agency problems.
  - ▶ Model revolves around a simple risk-shifting technology.
- ▶ We endogenized both the debt market and the asset market, allowing for collateral and examining its implications for prices and efficiency.
- ▶ We argued that hard debt contracts give lenders control and collateral requirements raise prices and lender recoveries, so that both may be desirable ex ante for raising debt capacities.
- ▶ Model easy to extend.
  - ▶ Composition effect in lending and credit boom and burst.
  - ▶ Risky collateral.
  - ▶ Opaqueness of hedge funds and prime brokerage.