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Bailouts, Time Inconsistency and Optimal Regulation*

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ABSTRACT

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Recent experience has shown that governments can, will, and perhaps should intervene during financial crises. Such interventions typically occur because governments seek to minimize the spillover effects of bankruptcy and liquidation upon the broader economy. Such interventions during financial crises alter the incentives for firms and financial intermediaries ex ante. In this paper we ask how optimal regulation should be designed to maximize ex ante welfare taking into account the temptation for the government to intervene ex post.

The theme that we explore in this paper is that, by altering private contracts, the prospect of bailouts reduces ex ante welfare. We view the prescription that governments should refrain from bailing out potentially bankrupt firm as unrealistic in practice. Benevolent governments simply do not have the power to commit themselves to such a prescription. A pragmatic approach to policy dictates that we take as given the incentives of governments to undertake bailouts and design ex ante regulation to minimize the ex ante costs of these ex post bailouts.

In this paper we study an optimal contracting problem in which, in order to provide incentives, privately optimal contracts feature ex post inefficient outcomes. Specifically, we develop a model in which managers must be provided incentives to engage in effort. In our model such incentives are provided by ensuring that the firm declares bankruptcy if outcomes are sufficiently bad. In our model bankruptcy is costly in two ways: it reduces the output of the firm and it imposes nonpecuniary costs on the manager. We think of these nonpecuniary costs as arising both from stigma-like effects on the manager's career as well as loss of private benefits from operating the firm.

The optimal contract features bankruptcy when outcomes are bad in order to provide proper incentives to the managers. This contract is ex post inefficient in the sense that, once the manager has exerted effort, bankruptcy imposes costs on the owners of the firm and the manager. After the effort choice has been made, both parties have incentives to renegotiate the contract and allow the manager to continue.

To set the stage for further developments we begin with an environment in which both the private agent and the government can commit to future actions. We show that a competitive equilibrium in which firms and managers sign optimal contracts is efficient. In our environment, after the managers have chosen their effort level, firms, managers, and the government all have incentives to intervene, but under commitment they understand that such ex post interventions only reduce ex ante welfare. Therefore, all parties commit not to intervene and the equilibrium features ex post inefficiency.

Next, we consider an environment in which private agents cannot commit to avoiding renegotiation. We allow for trigger strategies that discipline the behavior of agents during renegotiation. Specifically, we show that the best privately sustainable equilibrium can be supported by trigger strategies under which if a manager ever agrees to renegotiate, future firms will believe that the manager will always renegotiate. Expectations of such renegotiations reduce ex ante welfare and can thereby support equilibrium outcomes with ex post inefficiency. Into this environment we introduce a Ramsey planner with commitment. We allow the Ramsey planner to dictate the terms of private contracts. We show that it is not optimal for such a Ramsey planner to interfere with private contracts.

We then consider an environment in which a (benevolent) bailout authority uses taxes and transfers to change the decisions of private agents. This bailout authority has no commitment in that it chooses its policies after managers have chosen their effort levels. We assume that the bailout authority must raise revenues in a distorting fashion. We show that welfare with such a bailout authority than without the authority.

In this environment we ask how private contracts could be optimally regulated, taking as given that the bailout authority will intervene if it finds that such intervention is ex post desirable. We show that such optimal regulation can take one of two forms. In one such regulation takes the form of a cap on the manager's compensation. In the other, such regulation takes the form of limits on the set of outcomes in which the firm can declare bankruptcy. We argue that such limits can be interpreted as limits on the debt-equity ratio of firms.

1. A Static Contracting Model

Consider a model in which each period has decisions made at a first stage, called the beginning of the period, and a second stage called the end of the period. There are two types of agents, called lenders and managers both of whom are risk neutral and consume at the end of the period. There is a measure 1 of managers and a measure 1 of lenders.

A. A Simplified Economy

There are two production technologies. The *storage* technology is available to all agents, which transforms one unit of endowments at the first stage into one unit of consumption goods at the second stage. The *corporate* technology specifies projects that require two inputs at the first stage: effort a of managers and an investment of 1 of goods. This technology transforms these inputs into capital goods. The capital goods then can be used to make stage two consumption goods. Effort a of managers is unobserved by lenders.

If the corporate technology is used the amount of capital goods produced in the second stage stochastically depends on the effort level a of the manager as well as an idiosyncratic exogenous shock representing the manager's current draw of ability. In particular, given effort level a and a draw of ε with probability $p_H(a)$ the high state is realized and $A_H(1+\varepsilon)$ units of capital goods are produced and with probability $p_L(a) = 1 - p_H(a)$ the low state is realized and $A_L(1+\varepsilon)$ units of capital goods are produced where $A_L < A_H$. We assume the $p(a)$ is an increasing, concave function of a and the distribution of ε is given by $G(\varepsilon)$ which has mean zero.

We think of the project as being undertaken by a firm. We think of managers as performing two tasks. The first task is to exert effort a and transform consumption goods from stage 1 into capital goods at stage 2. The second task is to transform capital goods stage 2 into final consumption goods.

After the manager has completed the first task and a certain amount of capital has been produced the firm can choose to continue the project under the current manager or it can declare bankruptcy. If it continues then the project produces one unit of output for every unit of capital, so that the firm's output is

$$(1) \quad Y_{ci}(\varepsilon) = A_i(1+\varepsilon) \text{ for } i \in \{H, L\}$$

where c denotes *continue*. If the firm declares bankruptcy, the manager is removed, the firm incurs a direct output loss and the manager suffers a nonpecuniary cost. The direct output loss occurs because following bankruptcy the capital $A_i(1+\varepsilon)$ is used in an inferior technology, referred to as the *traditional* technology, that yields $R \leq 1$ consumption goods for every unit of capital invested so that the value of the output of the firm in the bankruptcy state is

$$(2) \quad Y_{bi}(\varepsilon) = RA_i(1+\varepsilon)$$

where b denotes *bankruptcy*. In the event of bankruptcy the manager suffers a nonpecuniary loss $-B$. This nonpecuniary cost is supposed to represent extra costs to the manager, such as a loss in reputation or a loss in nonpecuniary benefits from being employed as a manager that are incurred from a liquidation.

Lenders are endowed with e units of a consumption good in the first stage but cannot operate the corporate technology. Managers have no endowments of goods but can operate the corporate technology. Lenders choose whether to lend to firms that operate the corporate technology or to store their endowments.

We assume that $e > 1$. Since the economy has an equal measure of managers and lenders and since the corporate technology uses 1 unit of the endowment per manager the storage technology is always active and the rate of return to lending to the corporate technology is 1.

Let $c_i(\varepsilon)$ denote the consumption of the managers in state i given the realization ε and $d_i(\varepsilon)$ the return to the investor in a project when the state is i and the idiosyncratic shock is given by ε. Let B_i denote the set of idiosyncratic shocks ε such that the firms declares bankruptcy in state $i \in \{H, L\}$ and C_i denote the complementary sent in which the project is continued.

We assume that firms operate a continuum of projects. Given the symmetry of the expected returns across projects, firms will choose the same effort level for all managers. The profits generated by a firm which finds it optimal to operate the corporate technology at a positive level are

$$(3) \quad \sum_i p_i(a) \left[\int_{C_i} Y_{ci}(\varepsilon) dG(\varepsilon) + \int_{B_i} Y_{bi}(\varepsilon) dG(\varepsilon) - \int [c_i(\varepsilon) + d_i(\varepsilon)] dG(\varepsilon) \right]$$

Firms compete in offering contracts to managers and lenders. These contracts must attract investment by lenders so that they must offer a return to lenders of at least one. Thus, a contract must meet the following participation constraint for lenders

$$(4) \quad \sum_i p_i(a) \left[\int d_i(\varepsilon) dG(\varepsilon) \right] \geq 1$$

The contracts must also attract managers. Let $\bar{U}$ denote the value of the best alternative contract offered to a managers. Thus, a contract must meet a participation constraint for managers

$$(5) \quad \sum_i p_i(a) \left[\int c_i(\varepsilon) dG(\varepsilon) - B \int_{B_i} dG(\varepsilon) \right] - a \geq \bar{U}.$$

Since the effort choice a of managers is unobservable a contract must satisfy an incentive constraint given by

$$(6) \quad a \in \arg\max_a \sum_i p_i(a) \left[\int c_i(\varepsilon) dG(\varepsilon) - B \int_{B_i} dG(\varepsilon) \right] - a.$$

Finally, the consumption of managers must satisfy a nonnegativity constraint

(7) $c_i(\varepsilon) \geq 0$

The firms' contracting problem is then to choose a recommended effort level a, compensation schemes $c_i(\cdot)$, $d_i(\cdot)$ and bankruptcy and continuation sets B_i and C_i to maximize profits (3) subject to (4), (5), (6), and (7

Clearly the consumption level of a lender that lends 1 to firms and invests the rest in the storage technology is given by

(8) $$c^I = \sum_i p_i(a) \left[\int d_i(\varepsilon) dG(\varepsilon) \right] + e - 1$$

The resource constraint is

(9) $$\sum_i p_i(a) \left[\int c_i(\varepsilon) dG(\varepsilon) \right] + c^I \leq$$

$$\sum_i p_i(a) \left[\int_{C_i} Y_{ci}(\varepsilon) dG(\varepsilon) + \int_{B_i} Y_{bi}(\varepsilon) dG(\varepsilon) \right] + e - 1$$

An *allocation* is a collection $a, c_i(\cdot)$, $d_i(\cdot)$,c^I, C_i, B_i. A *competitive equilibrium* is an allocation together with a minimum utility level $\bar{U}$ such that

i) the allocations $a, c_i(\cdot)$, $d_i(\cdot)$, and sets C_i, B_i solve the contracting problem

ii) the minimum utility level $\bar{U}$ is such that firm profits are zero.

iii) the consumption of lenders satisfies (8).

iv) the resource constraint (9) holds.

Note here that $\bar{U}$ plays the role of a price. Note also that by Walras' Law the resource constraint is implied by zero profits of firms and the consumption of lenders (8).

Throughout we will restrict attention to environments in which the competitive equilibrium has an active corporate technology. A sufficient condition for such an equilibrium to exist is that A_H and $p'(0)$ are sufficiently large.

If a competitive equilibrium with a corporate technology does not exist then only the storage technology is used and the managers utility level $\bar{U} = 0$.

We turn the efficiency of a competitive equilibrium. Given a utility level of lenders $\bar{c}^I$, an allocation is *efficient* if it satisfies the following planning problem, namely to maximize the welfare of managers subject to (6), (7), (**??**), and

(10) $c^I \geq \bar{c}^I$.

Proposition 1. The competitive equilibrium is efficient.

Proof. Since profits are zero in a competitive equilibrium, we can use duality to rewrite the contracting problem as one of maximizing the utility of managers subject to the constraint the firm profits be nonnegative. Substituting for the consumption of lenders from (8) into firms' profits (3) yields the resource constraint. Clearly, the rewritten contracting problem coincides with the planning problem.

Proposition 2. If $A_L < 1$ then the competitive equilibrium with privately observed effort information has strictly lower effort level a and welfare than the competitive equilibrium with publicly observed effort.

Proof. In the competitive equilibrium with publicly observed effort it is straightforward to show that the optimal effort level solves

$$p'_H(a)(A_H - A_L) = 1$$

and the liquidation sets B_H and B_L are empty. The first order condition for effort in the private information economy is

$$\sum_i p'_i(a)\left[\int c_i(\varepsilon)dG(\varepsilon) - B\int_{B_i} dG(\varepsilon)\right] = 1$$

A moment's reflection makes clear that the only way to support the allocations with publicly observed effort is to pay the manager $A_H - A_L$ in the high state and zero in the low state. But, since $A_L < 1$ it is not feasible to do so and still play the lender's 1 per unit borrowed. Q.E.D.

From here onwards the term competitive equilibrium refers to competitive equilibrium with privately observed effort.

Next, we will say that allocations are *ex post inefficient* if either of the sets B_H or B_L is nonempty. If either of these sets is nonempty then after the effort choices are made, clearly all agents in this economy can be made better off by continuing the project in all states. Nonetheless, committing to ex post inefficient allocations may be desirable as a way of providing the manager with stronger incentives for providing high effort and thereby raising ex ante welfare.

Proposition 3. Under condition A, B_L is nonempty. That is, supporting ex ante efficient allocations requires ex post inefficiency.

(We are working on the exact specification of condition A. A sufficient condition is that the first best effort level, defined by

$$p'_H(a)(A_H - A_L) = 1$$

has $p_H(a) = 1$. Then bankruptcy only takes place off the equilibrium path as a threat and has no costs but it improves incentives. We don't need something this strong.)

We now show that the contracting problem reduces to a simpler one under the condition that $A_L < 1$. We will show that in any competitive equilibrium the optimal contracting problem can be reduced to the following: Choose c_H, a, and ε^* to solve

$$\text{(11)} \quad \max p_H(a)c_H - p_L(a)BG(\varepsilon^*) - a$$

subject to

$$\text{(12)} \quad p'_H(a)\left[c_H + BG(\varepsilon^*)\right] = 1$$

$$\text{(13)} \quad p_H(a)c_H + 1 \leq p_H(a)A_H + p_L(a)A_L\left[\int_{\varepsilon^*}^{\bar{\varepsilon}}(1+\varepsilon)dG(\varepsilon) + R_2\int_{\underline{\varepsilon}}^{\varepsilon^*}(1+\varepsilon)dG(\varepsilon)\right]$$

To establish this result we first note that if $A_L < 1$ the incentive constraint is always binding. Hence an optimal contract must reward the manager only in the high state and set the consumption of managers in the low state to be zero for all ε, that is, $c_L(\varepsilon) = 0$. The intuition for this result is that as long as consumption is positive in the low state, manager's incentives can be improved by shifting consumption from the low state to the high state.

Since the manager cares only about expected consumption the optimum can be achieved by setting consumption in the high state to be a constant so that $c_H(\varepsilon) = c_H$.

Second, note the only role of bankruptcy is to improve incentives so that it never optimal to declare bankruptcy in the high state. In the low state, the optimal bankruptcy rule has a cutoff form: declare bankruptcy for $\varepsilon \leq \varepsilon^*$ and continue otherwise. This result follows because the output loss from bankruptcy is $(1-R)A_L(1+\varepsilon)$ which is smaller the lower is ε and that manager only cares about the probability of bankruptcy in the low state. More formally, if the optimal contract had bankruptcy for a high realization ε and continuation for a low realization of ε, then the output loss could be reduced by rearranging the set of realizations for which there is bankruptcy and the incentives for the managers maintained.

Third, note that (12) is the incentive constraint written in first order condition form.

Fourth, in any competitive equilibrium profits are zero. Hence, we can use duality to write the optimal contracting problem as maximizing the utility of the manager subject to a nonnegativity constraint on profits. Note that we write the nonnegativity constraint on profits as (13) using the assumption that the expected value of ε is zero along with the other features of the optimal contract derived above.

We summarize this discussion in a proposition.

Proposition 4. If $A_L < 1$ the optimal contracting problem in a competitive equilibrium can be written as (11).

B. The Benchmark Static Economy

The benchmark economy we will consider differs in two ways from the simplified economy considered above. First, we assume that managers stochastically lose their ability to convert capital goods into consumption goods. Specifically, with probability α_0 the capital goods produced in stage 2 can no longer be managed by the incumbent manager and must instead be used in the traditional technology. Second, we replace the traditional technology which previously was simply described by the constant $R < 1$ with a constant returns to scale production technology $F(k_1, k_2)$ where k_1 denotes that capital invested in this technology in stage 1 by the lenders and k_2 denotes the capital invested in this technology in stage 2. We refer to this event as *liquidation.* We assume that F is concave and has diminishing marginal products. We also assume that the incumbent managers are more productive in converting capital goods to consumption goods than is the traditional technology. That is, we assume that marginal product of k_2 in the traditional technology is always less than the marginal product of capital in the corporate sector. Formally, $F_2(k_1, 0) = 1$ so that $F_2(k_1, k_2) \leq 1$ for all k_1, k_2.

The capital k_2 invested in the traditional technology comes from two sources: the exogenously liquidated capital and the capital from bankrupt firms and is given by

$$(14) \quad k_2 = \alpha_0 \left[\sum p_i(a) \int A_i(1+\varepsilon) dG(\varepsilon) \right] + \alpha_1 \left[\sum p_i(a) \int_{B_i} A_i(1+\varepsilon) dG(\varepsilon) \right]$$

Here competitive firms operate the traditional technology and choose k_1 and k_2 to maximize

$$F(k_1, k_2) - R_1 k_1 - R_2 k_2$$

The first order conditions are

(15) $F_1(k_1, k_2) = R_1$

(16) $F_2(k_1, k_2) = R_2$

The lenders in this economy choose how much of their endowment e to invest in the corporate technology, k_c, at rate R_c how much to invest in the storage technology, k_s, at rate 1 and how much to invest in the traditional technology, k_1 at rate R_1. That is, lenders solve

(17) $c_I = \max R_c k_c + k_s + R_1 k_1$

subject to

(18) $k_c + k_s + k_1 \leq e.$

We will assume that all three technologies are used in equilibrium. A set of sufficient conditions are the following. First, e is sufficiently large, so that the storage technology is always used. Second, that the corporate technology is sufficiently productive in that A_H is large enough and that $p'_H(0)$ is sufficiently large, so that it is always used. Finally, that $F_1(0, k_2) > 1$ for all $k_2 > 0$, so that the traditional technology is always used. Under these assumptions we have that

(19) $R_c = R_1 = 1$ and $k_c = 1$

and we will impose these conditions from now on.

With Commitment

We now set up the contracting problem for this economy. Following the logic of Proposition 4, in a competitive equilibrium the contracting problem solves

$$(20)\quad \max \alpha_1 \left[p_H(a)c_H - p_L(a)BG(\varepsilon^*)\right] - a$$

subject to

$$(21)\quad \alpha_1 p_H'(a)\left[c_H + BG(\varepsilon^*)\right] = 1$$

$$(22)\quad \alpha_1 p_H(a)c_H + 1 \leq \alpha_1 \left[p_H(a)A_H + p_L(a)A_L \int_{\varepsilon^*}^{\bar{\varepsilon}} (1+\varepsilon)dG(\varepsilon)\right] + R_2 k_2$$

where

$$(23)\quad k_2 = \alpha_0 \left[\sum p_i(a)A_i\right] + \alpha_1 p_L(a) \int_{\underline{\varepsilon}}^{\varepsilon^*} (1+\varepsilon)dG(\varepsilon).$$

Note that we have suppressed the nonnegativity constraint for c_H.

The resource constraint for this economy is

$$(24)\quad \alpha_1 p_H(a)c_H + c^I \leq \alpha_1 \left[p_H(a)A_H + p_L(a)A_L \int_{\varepsilon^*}^{\bar{\varepsilon}} (1+\varepsilon)dG(\varepsilon)\right] + F(k_1, k_2) + k_s$$

A *competitive equilibrium with commitment* is an allocation $c_H, a,\ \varepsilon^*, k_1, k_2,\ R_2$ such that

i) given R_2, the allocations solve the contracting problem (20)

ii) given $R_1 = 1$ and R_2, k_1 and k_2 satisfy (15) and (16)

iii) given $R_c = R_1 = 1$ and $k_c = 1$, the consumption of lenders satisfies (17).

iv) the resource constraints (24) and (18) hold.

Let $x^C = \{c_H, a,\ \varepsilon^*, k_1, k_2,\ R_2\}$ denote a competitive equilibrium with commitment.

An allocation is efficient, given a utility level of lenders $\bar{c}^I$, if it solves the following planning problem, namely to maximize the welfare of managers subject to (21), and the resource constraints (18), (23) and (24).

Proposition 5. The competitive equilibrium is efficient.

Proof. The logic is similar to that in Proposition 1. The social planner's problem is to maximize the manager's utility subject to the incentive constraint and the resource constraints. Substituting for k_s from (18) into (24) and using the fact that $k_c = 1$ we obtain

$$\alpha_1 p_H(a)c_H + c^I \leq \alpha_1 \left[p_H(a)A_H + p_L(a)A_L \int_{\varepsilon^*}^{\bar{\varepsilon}} (1+\varepsilon)dG(\varepsilon) \right] + F(k_1, k_2) + e - 1 - k_1$$

Any interior solution to the planner's problem must satisfy $F_1(k_1, k_2) = 1$. Since F has constant returns to scale, it follows that $F(k_1, k_2) = F_1(k_1, k_2)k_1 + F_2(k_1, k_2)k_2$. Furthermore, the value of k_1/k_2 is the same as the in the competitive equilibrium because the condition $F_1(k_1, k_2)$ pins down the ratio of k_1/k_2 uniquely. Thus $F_2(k_1, k_2)$ in the planning problem coincides with R_2 in the competitive equilibrium Since $c^I = e$, the rewritten resource constraint in the planning problem coincides with the budget constraint in the contracting problem in the competitive equilibrium. Q.E.D.

Without Commitment

Suppose now that the agents in this economy cannot commit to contracts. Specifically, suppose that after the action a has been taken, but before the state and the realization of ε have occurred, firms and managers can renegotiate their contracts if both parties agree. Clearly, all projects will be continued in order to avoid the output and the nonpecuniary costs of bankruptcy.

The optimal contracting problem now solves (20) with $\varepsilon^* = 0$. Let U^N denote the value of the contracting problem without commitment. Clearly, welfare is lower without commitment, so that $U^N < U^*$.

For later use we will show that the equilibrium value of R_2 is the same in the economies with and without commitment. To show this result note that in both economies $F_1(k_1, k_2) = 1$ and hence since F has constant returns to scale, this implies that $F_1(k_1/k_2, 1) = 1$ so that k_1/k_2 is the same value, say $\tilde{k}$ in both economies. Since $R_2 = F_2(k_1, k_2) = F_2(\tilde{k}, 1)$ we know R_2 is also the same in both economies. We will use this result to show that the government's sustainability constraint is tighter than the private sustainability constraint. We record this result in the following lemma.

Lemma 1. The equilibrium value of R_2 is the same in the economies with and without commitment.

Note that in this static model without commitment the incentive to renegotiate is so strong that the equilibrium has no ex post inefficiency.

2. A Dynamic Contracting Model

Consider now an infinitely repeated version of the benchmark static economy. We are interested in models without commitment and to the extent to which government policy aimed at alleviating ex post inefficiency ends up lowering welfare. In contrast to the static model, in a dynamic model firms and managers who renegotiate may be perceived as being willing to renegotiate in the future. This perception can make the welfare levels associated with future contracts lower if firms and managers renegotiate today than if they do not.

A. Without Commitment by Private Agents

We formalize this idea by requiring that the contracts managers and firms enter into must be self enforcing. We say that a contract is self enforcing if, after the manager has chosen the effort level, the payoff from continuing with the contract is at least as large as the payoff from the best one-shot deviation followed in all subsequent periods by the static per period payoff U^N.

More formally, suppose now that a manager has taken an action a but uncertainty has not yet been realized. Consider the outcomes if the firm and the manager agree to renegotiate. We model the renegotiation as follows. The manager makes a take it or leave it offer to the firm. If the firm takes the offer that offer is implemented, while if the firm rejects the offer the existing contract is implemented. Clearly, the firm will accept any offer which yields nonnegative profits. Thus, the best take it or leave it offer is one that maximizes the manager's payoff subject to the constraint that profits are nonnegative. Since the action a has already been taken, it is optimal for the manager to set $\varepsilon^* = 0$ and avoid bankruptcy. Since firms profits associated with an accepted offer must be nonnegative, the maximum expected consumption the manager can receive is determined by (22) with $\varepsilon^* = 0$. Hence, the maximum expected payoffs to the manager are

$$(25)\quad \hat{U}(a) = \alpha_1 p_H(a)\hat{c}_H - a = \alpha_1\left[p_H(a)A_H + p_L(a)A_L\right] + R_2\hat{k}_2 - 1 - a$$

where $\hat{c}_H$ is the consumption associated with the renegotiated contract and

$$(26)\quad \hat{k}_2 = \alpha_0 \sum p_i(a)A_i.$$

For some given contract a, ε^* if the manager does not renegotiate then expected consumption is determined from (22) and the manager's payoffs are given by

$$(27)\quad U(a,\varepsilon^*) = \alpha_1 p_H(a)c_H - \alpha_1 BG(\varepsilon^*) - a$$

$$= \alpha_1\left[p_H(a)A_H + p_L(a)A_L\int_{\varepsilon^*}^{\bar{\varepsilon}}(1+\varepsilon)dG(\varepsilon)\right] + R_2k_2 - \alpha_1 BG(\varepsilon^*) - 1 - a$$

where

$$(28)\quad k_2 = \alpha_0\left[\sum p_i(a)A_i\right] + \alpha_1 p_L(a)\int_{\underline{\varepsilon}}^{\varepsilon^*}(1+\varepsilon)dG(\varepsilon)$$

We say that a contract x is *privately sustainable* if

$$(29)\quad U(a,\varepsilon^*) + \frac{\beta}{1-\beta}U(a,\varepsilon^*) \geq \hat{U}(a) + \frac{\beta}{1-\beta}U^N.$$

The *optimal contracting problem without commitment* is now to maximize the manager's utility (20) subject to (21), (22), (23) and (29). The privately sustainable equilibrium is defined analogously to the competitive equilibrium with commitment, except that condition (i) is replaced by the requirement that the contract be the best sustainable contract.

A *privately sustainable equilibrium* is an allocation $c_H, a, \varepsilon^*, k_1, k_2$ and a price R_2 such that

i) given R_2, the allocations solve the optimal contracting problem without commitment.

ii) given $R_1 = 1$ and R_2, k_1 and k_2 satisfy (15) and (16)

iii) given $R_c = R_1 = 1$ and $k_c = 1$, the consumption of lenders satisfies (17).

iv) the resource constraints (24) and (18) hold.

Let $x^N = \{c_H, a, \varepsilon^*, k_1, k_2, R_2\}$ denote a privately sustainable equilibrium.

One rationalization for our formalization of the optimal contracting problem without commitment is that manager and firm behavior is disciplined by trigger strategies. Under this rationalization, the optimal contracting problem finds the best trigger strategy equilibrium in a game between the manager and firms, holding fixed the prices in a competitive equilibrium. A standard result in the game theory literature is that the best equilibrium can be supported by a trigger strategy which prescribes the worst equilibrium continuation payoff following any deviation. In our economy, the worst equilibrium is the infinite repetition of the static equilibrium without commitment. This infinite repetition has per period value U^N.

Under this rationalization, we assume that managers are infinitely-lived but that the only publicly observed outcome from a given period is whether or not the manager renegotiated. This assumption keeps the manager's incentive constraint static and allows us to focus on the incentives to renegotiate. Consider the following trigger strategies: if a manager ever renegotiates, then all firms believe that the manager will always renegotiate so that bankruptcy will never be declared in the future. Since this continuation yields the worst payoffs, it follows that the best equilibrium for the game between managers and firms, holding fixed market prices, solves the optimal contracting problem.

We emphasize that our notion of equilibrium does not depend on this rationalization. Formally, our optimal contracting problem is similar to that in the literature on models with enforcement constraints.

If the private sustainability constraint is binding in the contracting problem, the privately sustainable equilibrium yields lower welfare than the competitive equilibrium under commitment. To see this result, we will show that the only difference between the contracting problem in the two economies is that the contracting problem without commitment has an additional constraint compared to the one with commitment. Since $R_1 = 1$ in both the economy with commitment and that without commitment, it follows that k_1/k_2 is the same in both economies so that R_2 is the same in both economies. Thus, market prices are the same in both economies. It follows that the only difference between the contracting problems in the two economies is the private sustainability constraint.

The private sustainability constraint is binding if the discount factor β is not too large. We denote by $\bar{\beta}$ the critical value of the discount factor such that the the private sustainability constraint just binds at the commitment allocations. That is $\bar{\beta}$ satisfies

(30a) $$U(a^c, \varepsilon^{*c}) + \frac{\bar{\beta}}{1-\bar{\beta}} U(a^c, \varepsilon^{*c}) = \hat{U}(a^c) + \frac{\bar{\beta}}{1-\bar{\beta}} U^N$$

where a^c, ε^{*c} denote the contract in a competitive equilibrium with commitment. Clearly, if $\beta \geq \bar{\beta}$, the commitment outcomes are privately sustainable, and if $\beta < \bar{\beta}$,the commitment outcomes are not sustainable.

Obviously, the competitive equilibrium is inefficient if we think the planner solves the same problem as in the environment where the private agents can commit. We think of this comparison as uninteresting.

B. Adding a bailout authority with commitment

We now consider adding a bailout authority with a limited set of instruments. We show that such an authority cannot improve upon the best privately sustainable equilibrium. The basic idea is that the authority cannot loosen any of the constraints in the optimal contracting problem, specifically the private sustainability constraint.

The bailout authority can make transfers or levy taxes on firms contingent on the state and the realization of the idiosyncratic shock ε. We assume that taxes are costly to collect. Formally, we assume that the bailout authority can levy proportional taxes τ on firms in the high state and make transfers to firms in the low state. If the authority levies a tax rate of τ, on a firm with one unit of capital, revenues raised are $\lambda\tau$ where $\lambda < 1$ The bailout authority's budget constraint is

$$(31) \quad \lambda p_H(a)\tau A_H = p_L(a)A_L \int_{\varepsilon^*}^{\bar{\varepsilon}} T_L(\varepsilon)dG(\varepsilon)$$

A *policy* for a bailout authority consists of a tax rate τ and transfers $T_L(\varepsilon)$. A policy induces a competitive equilibrium as follows. Given a policy, the budget constraint of the financial intermediary becomes

$$(32)\quad \alpha_1 p_H(a)c_H + 1 \leq \alpha_1 \left[p_H(a)(1-\tau)A_H + p_L(a)A_L \int_{\varepsilon^*}^{\bar{\varepsilon}} (1+\varepsilon+T_L(\varepsilon))dG(\varepsilon) \right] + R_2 k_2.$$

The *optimal contracting problem with a bailout policy* is to choose a contract c_H and ε^* to maximize the utility of the manager (20) subject to the incentive constraint for the manager (21), the private sustainability constraint (29), and the budget constraint of the financial intermediary (32) where k_2 is given by (23)

A *competitive equilibrium with a bailout policy* consists of an allocation $c_H, a, \varepsilon^*, k_1, k_2,$ R_2, and a policy τ, $T_L(\varepsilon)$ such that

i) given R_2, the allocations solve the optimal contracting problem with policy

ii) given $R_1 = 1$ and R_2, k_1 and k_2 satisfy (15) and (16)

iii) given $R_c = R_1 = 1$ and $k_c = 1$, the consumption of lenders satisfies (17).

iv) the resource constraints (24) and (18) hold.

v) the government's budget constraint (31) holds.

Given the set of competitive equilibria associated with various bailout policies, let the Ramsey equilibrium with bailouts denote the one with the highest welfare.

Clearly, given the instruments available to the bailout authority, a competitive equilibrium with a bailout policy can only yield lower welfare than the privately sustainable equilibrium. To see this note that any use of the tax transfer instrument throws away resources. We then have the following proposition.

Proposition 6. Any competitive equilibrium with a bailout policy in which $\tau > 0$ has lower welfare then in the privately sustainable equilibrium. Thus, the Ramsey equilibrium with bailouts has $\tau = 0$.

Proof. Using the same logic as in Lemma 1, the level of R_2 is the same in the competitive equilibrium with a bailout policy and in the privately sustainable equilibrium. Inspecting the contracting problems in the environment with a bailout policy and the environment without one, we see that the only difference is the budget contracts. Inspecting the budget constraints (22) and (32) we see that any allocation that is budget feasible with taxes is also budget feasible without taxes. Since taxes are distorting allocations that are budget feasible without taxes are not budget feasible with taxes. Since in the both equilibrium the budget constraints hold with equality welfare is lower with $\tau > 0$ than with $\tau = 0$. Thus, a Ramsey planner will choose $\tau = 0$. *Q.E.D.*

C. Adding a regulatory authority with commitment

We now consider expanding the powers of the government to allow for regulation of private contracts as well as taxes and transfers. We can think of these powers as being the combination of those of an ex ante regulator and a bailout authority with commitment. Here a *policy* for the combined authority consists of a choice of a compensation contract c_H^P, a liquidation level ε^P, a tax rate τ, transfers $T_L(\varepsilon)$. A policy induces a competitive equilibrium as follows.

The *optimal contracting problem with combined policy* is now to choose a contract c_H and ε^* to maximize the utility of the manager (20) subject to the incentive constraint for the manager (21), the private sustainability constraint (29), and the budget constraint of the financial intermediary (32) where k_2 is given by (23) and subject to the policy induced constraints

(33) $c_H = c_H^P$ and $\varepsilon^* = \varepsilon^P$.

A *competitive equilibrium with policy* consists of an allocation c_H, a, ε^*, k_1, k_2, R_2, and a policy c_H^P, ε^P, τ, $T_L(\varepsilon)$ is defined similarly to a competitive equilibrium with tax policy.

Again let the Ramsey equilibrium with combined policy denote the competitive equilibrium with the highest welfare.

Clearly, given the instruments available to the Ramsey planner, a competitive equilibrium with policy can only yield lower welfare than the privately sustainable equilibrium. To see this note that any use of the tax transfer instrument, throws away resources and any choice of liquidation or managerial compensation can at, best, constrain the optimal contracting problem. We then have the following proposition.

Proposition 7. The Ramsey equilibrium outcomes with combined policy and the privately sustainable equilibrium outcomes coincide.

D. Without Commitment by the Bailout Authority or by Private Agents

Suppose now that the bailout authority, as well as private agents, cannot commit to their future actions. We will show that this lack of commitment leads to lower welfare than in the best privately sustainable equilibrium. We do so by showing that any equilibrium without commitment by the bailout authority must satisfy a government sustainability constraint which is tighter than the private sustainability constraint.

The government's per period payoff is given by the sum of consumption of all agents in the economy. The government makes its policy decision after the managers have chosen their actions but before the realization of either the state, H or L or the shocks ε. The instruments available to the bailout authority are as before, namely tax rate τ in the high state and the lump sum transfers $T_L(\varepsilon)$ in the low state. Once the bailout authority has chosen its policy, each manager makes a take it or leave it offer to firms.

We assume that government behavior is disciplined by trigger strategies. The trigger strategies for the government are similar to those for private agents. As in the environment without commitment by private agents, we begin by characterizing the equilibrium in which after any deviation, agents believe that all future contracts will be renegotiated and hence revert to an equilibrium with $\varepsilon^* = 0$. The reversion equilibrium has per period value U^N as before. The only subtlety to keep in mind is that R_2 has the same value in the static economy with commitment. The reason is similar to that discussed previously, in both economies $F_1(k_1, k_2) = 1$ and hence since F has constant returns to scale, this implies that $F_1(k_1/k_2, 1) = 1$ so that k_1/k_2 is the same value, say $\tilde{k}$ in both economies. Since $R_2 = F_2(k_1, k_2) = F_2(\tilde{k}, 1)$. (Note that if we discarded the storage technology then we not be assured that $F_1 = 1$ and the reversion equilibrium would have a different value associated with some other R_2.)

Next we develop the bailout authority's sustainability constraint. To develop this constraint, suppose that the bailout authority chooses to deviate. It is clearly optimal for the authority to set the bankruptcy sets to be empty. In such a case, given some value of k_1, the sum of consumption is given by

(34) $$\hat{U}^G(a) = \alpha_1 \left[p_H(a) A_H + p_L(a) A_L \right] + \hat{R}_2(k_1, \hat{k}_2)\hat{k}_2 - 1 - a$$

where

$$(35)\quad \hat{k}_2 = \alpha_0 \sum p_i(a) A_i,$$

and where

$$\hat{R}_2(k_1, \hat{k}_2) = F_2(k_1, \hat{k}_2).$$

Note that the payoff to the government from a deviation differs from the payoff to private agents from deviation given (25) only if $\hat{R}_2(k_1, \hat{k}_2)$ differs from R_2. These returns (or prices) differ if F_2 depends nontrivially on k_1. If the government choose not to deviate from some contract x,

$$(36)\quad U(x) = \alpha_1 \left[p_H(a) A_H + p_L(a) A_L \int_{\varepsilon^*}^{\bar{\varepsilon}} (1+\varepsilon) dG(\varepsilon) \right] + R_2 k_2 - \alpha_1 p_L(a) B G(\varepsilon^*) - 1 - a$$

where

$$(37)\quad k_2 = \alpha_0 \left[\sum p_i(a) A_i \right] + \alpha_1 p_L(a) \int_{\underline{\varepsilon}}^{\varepsilon^*} (1+\varepsilon) dG(\varepsilon)$$

Note that the continuation payoff if the government chooses not to deviate is the same as that in (27).

We say that a contract x is *sustainable to bailouts* if

$$(38)\quad U(x) + \frac{\beta}{1-\beta} U(x) \geq \hat{U}^G(a) + \frac{\beta}{1-\beta} U^N.$$

A *sustainable equilibrium with bailout policy* is a competitive equilibrium with bailout policy which is sustainable to bailouts. The privately sustainable equilibrium with policy is the sustainable equilibrium which yields highest welfare. We then have the following proposition.

Proposition 8. Suppose the dicount factor β is strictly less than the threshold $\bar{\beta}$ given by (30a) at which the private sustainability constraint is binding. Then, any sustainable equilibrium with bailout policy has bailouts in equilibrium, in the sense that $\tau > 0$. Furthermore, any such equilibrium yields strictly lower welfare than the privately sustainable equilibrium.

Proof. Suppose the privately sustainable equilibrium with bailout policy has no bailouts in equilibrium, so that $\tau = 0$. Consider the solution to the optimal contracting problem with bailouts. Clearly, this problem coincides with that in the privately sustainable equilibrium, hence the solution must also.

We show that the allocations from a privately sustainable equilibrium are not sustainable to bailouts. To do that, note that when the bailout authority contemplates a deviation it realizes that by lowering the measure of bankruptcies, it raises the value R_2 of the capital that is transfered from the corporate sector to the traditional sector. In contrast, when a private firm contemplates a deviate it takes the value R_2 as given. Thus, the right side of the private sustainability constraint is lower than the right side of the sustainability to bailout constraint. Formally, we show that

$$\hat{U}^G(a) > \hat{U}(a)$$

To show this result we subsititute for both sides and reduce the inequality to

$$\hat{R}_2(k_1, \hat{k}_2)\hat{k}_2 > R_2(k_1, k_2)\hat{k}_2$$

This inequality follows because $\hat{k}_2 < k_2$ and the production function F has diminishing marginal products. This proves that the allocations from a privately sustainable equilibrium violate (**??**). Thus the any sustainable equilibrium with bailouts must use bailouts in equilibrium. From Proposition 6 it follows that welfare is strictly lower in the bailout equilibrium. *Q.E.D.*

E. Can an ex ante regulator improve welfare?

Consider the situation described in the previous section in which neither the bailout authority nor the private agents can commit to their actions. We show that a regulatory authority armed with the ability the dictate the terms of the private contract, namely the compensation contract c_H^R and the liquidation level ε^R, can improve on ex ante welfare. Such a regulator must take into account the incentives of the bailout authority to intervene.

Define a *regulatory equilibrium* as a competitive equilibrium with policy that is sustainable to bailouts. The *best regulatory equilibrium* is a regulatory equilibrium that maximizes the manager's welfare.

Consider the *regulator's problem* defined as follows:

$$(39)\quad \max \alpha_1 \left[p_H(a)c_H - p_L(a)BG(\varepsilon^*)\right] - a$$

subject to the manager's incentive constraint

$$(40) \quad \alpha_1 p_H'(a)\left[c_H + BG(\varepsilon^*)\right] = 1$$

the resource constraint

$$(41) \quad \alpha_1 p_H(a) c_H + c^I \leq \alpha_1 \left[p_H(a)A_H + p_L(a)A_L \int_{\varepsilon^*}^{\bar{\varepsilon}} (1+\varepsilon)dG(\varepsilon)\right] + F(k_1, k_2) + k_s$$

where k_2 is given by

$$(42) \quad k_2 = \alpha_0 \left[\sum p_i(a)A_i\right] + \alpha_1 p_L(a) \int_{\underline{\varepsilon}}^{\varepsilon^*} (1+\varepsilon)dG(\varepsilon)$$

voluntary savings by lenders

$$(43) \quad F_1(k_1, k_2) = 1$$

the bailout authority's sustainability constraint

$$(44) \quad U(x) + \frac{\beta}{1-\beta}U(x) \geq \hat{U}^G(a) + \frac{\beta}{1-\beta}U^N$$

Proposition 9. The best regulatory equilibrium solves the regulator's problem.

Proof. Note that any competitive equilibrium must satisfy the (40)-(43) and must satisfy (44) if it is sustainable to bailouts. Clearly, the best regulatory equilibrium must maximize manager's welfare subject to these constraints. Any solution to the regulator's problem can clearly be implemented by imposing constraints of the form (33) on the contracting problem. *Q.E.D.*

Next, we have

Proposition 10. Suppose $\beta < \bar{\beta}$. The solution to the regulator's problem yields higher welfare than the best sustainable equilibrium with bailouts.

Proof. From Proposition 8, we have that the best sustainable equilibrium with bailouts has positive taxes. Supoose that this equilibrium has some specific action $\bar{a}$ and bankruptcy cutoff $\bar{\varepsilon}^*$. This action and bankruptcy cutoff is feasible for the regulator's problem and involves no distortionary taxes. Hence, welfare is higher. *Q.E.D.*

Proposition 11. Under condition B, the solution to the regulator's problem can be implemented with either a cap on managerial compensation c_H or a cap on ε^*.

(We are still working on sufficient conditions for this to hold. We think that a sufficient condition is that the price effect $(\hat{R}_2 - R_2)\hat{k}_2$ is small. In that case one can show that the gain to a one shot deviation for the government is decreasing in effort and increasing in the cutoff ε^*. It is then pretty straightforward to replace a policy that imposes $c_H = c_H^r$ with a policy of the form $c_H \leq c_H^r$.)