Strategic Trading in Multiple Assets and the Effects on Market Volatility*

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Abstract

We study government policies designed to increase liquidity by extending government guarantees to fundamentally illiquid assets. We characterize the effects of such policies on equilibrium price dynamics, trading strategies and welfare. We build on the strategic trading framework of Brunnermeier and Pedersen (2005) and Carlin et al (2007) by adding multiple assets and by modeling all agents' utility functions. The assets in our model differ in their fundamental liquidity, i.e. the price reaction of the non-strategic (or "retail") traders to amounts sold by the strategic traders. Non-strategic traders are willing to accept greater amounts of the more liquid asset with less short-term price reaction. These additions allow us to consider the welfare implications of, for example, shifting some assets from the illiquid category to the liquid category, a proxy for government guarantees on a risky asset. As in other models of this type, the strategic players "predate" on each other when one becomes distressed and is forced to liquidate its holdings. As others have shown, the more liquid the asset, the cheaper it is to predate on a distressed firm. Our model features an additional channel between liquidity and predation: because of the cross-elaiticies of demand across assets, firms can create liquidity in one asset by shorting a complementary asset. We find that when firms begin with larger endowments in highly liquid assets, forced liquidation of those assets tends to result in higher price volatility than would otherwise be the case. For plausible parameter ranges, such a policy also results in lower welfare for the non-strategic traders. This finding suggests that market interventions designed to alleviate illiquidity in particular asset markets may instead unintentionally exacerbate price volatility.

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1 Introduction

Policymakers have considered several different policies to stabilize financial markets and institutions since the onset of the financial crisis in August 2007. A feature of the crisis has been a sharply decreased willingness of investors to accept illiquid assets, citing mark-to-market risk, difficulty in valuation, or simple preference for assets that can quickly be converted to cash. One policy response has been to harden the implicit government guarantee on certain assets important for basic intermediation, such as obligations of Fannie Mae and Freddie Mac.

The financial crisis has also been marked by forced sales of assets by distressed firms. Other firms, however, have specialized in strategic trading around such fire sales. Prices in related markets, such as private-label mortgage-backed securities backed by prime mortgages, have fallen when firms liquidated a different asset, such as MBS backed by subprime mortgages. Presumably, strategic traders have also been taking advantage of these correlated price movements.

We study equilibrium price dynamics in such an environment, and conduct a welfare analysis of extending government guarantees to less liquid securities. Our results also allow us to consider the optimal exit strategy for a government that accumulates assets on its balance sheet, either directly through purchases or indirectly by guaranteeing assets.

Specifically, we investigate the role of predatory trading and substitution effects across assets when a subset of the firms are forced to liquidate a portion of their portfolios. Following Brunnermeier and Pedersen (2005) and Carlin, Lobo, and Viswanathan (2007), we define "predation" in this context as trading that exploits "distressed" investors' need to alter their positions. Substitution effects arise whenever there is an incentive to actively trade assets other than ones that actually must be liquidated, and we propose one mechanism by which these incentives may arise.

An example of a scenario in which both predation and substitution across assets have been important determinants of trading behavior is the financial market turmoil of late 2008, during which banks attempted to liquidate holdings of mortgage-backed securities (MBS), resecuritizations of MBS, and other related assets, many of which lacked an informative market price due to the general cessation of trading. Illiquidity in markets for assets not directly related to mortgages provides evidence of substitution.

Our study relates to two main strands of theoretical research on market microstructure, as well as the current policy debate about the effect of government interventions designed to make certain assets more liquid. First, we extend the literature on the role of strategic trading when a subset of firms become distressed and are forced to liquidate their assets (Brunnermeier and Pedersen, 2005; Carlin, Lobo, and Viswanathan, 2007). Brunnermeier and Pederson introduce the notion that profit-maximizing behavior by the non-distressed firms can cause the price of the asset being liquidated to overshoot its "fundamental" value (*i.e.*, its long-term value after netting out the market impact of trading). Carlin, Lobo, and Viswanathan extend the model by setting it in the context of a repeated game and by allowing for trading to have market impact, such that prices depend upon not only the stock but also the flow of assets being traded in each instant. A key feature of both models is that non-distressed traders initially race to sell the asset at the same time that the distressed traders are trying to liquidate, and then buy back the asset at a discount toward the end of the trading window. The dynamics implied by this "race and fade" behavior leads to higher price volatility than what we would observe if the non-distressed firms were passive. As well, predation generates inefficiencies in the stage game and implies the existence of Pareto improvements that can be sustained through supergame strategies. Our key contribution to the existing literature comes from the introduction of multiple goods. While the race-and-fade phenomenon is preserved under heterogeneous goods, heterogeneity also leads to novel properties that are absent when there is just a single, homogeneous asset.

To understand the implications of having multiple assets, first note that when there is just a single asset, profit opportunities come from two sources. To begin with, by selling high and buying low (for expositional convenience, we refer to this behavior as the "predatory" component of the non-distressed firm's optimal response), non-distressed firms can exploit the declining fundamental value of the asset driven by the forced liquidation. Here, we define the fundamental value of the asset, at a moment, to be its price after netting out the liquidity premium or discount caused by the trading in that instant. Second, by causing the speed of trading to vary over time, predation also generates a competing incentive for non-distressed firms to make profits by supplying liquidity to distressed firms, i.e., purchasing the asset when the distressed firms are selling it at a faster rate, and selling when the distressed firms are selling at a slower rate. The first effect is a pure transfer across firms, and does not affect the total surplus summed over all of the strategic players. However, because traders do not internalize the effect of their own trading on the liquidity costs of others, the latter effect implies that equilibrium outcomes are socially inefficient relative to the cooperative outcome, which would entail setting a net trading speed that is constant over time.

When there are multiple assets, linkages in the fundamental values of different assets imply that forced liquidation in any one of the assets will also result in declines in the fundamental values of the remaining assets. Therefore, there are opportunities for non-distressed firms to obtain predatory profits through trading in any of the assets. Similar to the one-asset case, there is also a competing incentive to supply liquidity. However, because of cross-asset linkages in the short-term market impact of trading, the act of purchasing (selling) any of the assets to a greater or lesser degree also enhances liquidity for sellers (buyers) of all the other assets. In the one-asset case, the competing incentives to predate and to provide liquidity must necessarily involve the same asset, and the latter incentive tends to dampen the volatility-inducing effects of predation. By contrast, when there are multiple assets, non-distressed firms can predate in some assets (usually the ones in which distressed firms hold a large initial endowment) while generating liquidity cross-subsidies through trading in the other assets, which lowers the overall liquidity cost of predation.

An important implication is that overall market volatility will actually tend to be higher when the distressed firms hold a large initial endowment in relatively liquid assets. Because the short-term, own-price market impact of trading is—by definition—high for the illiquid assets, firms are less able to use liquidity cross-subsidies to buffer price volatility induced by predation in the liquid good. The opposite is true when distressed firms must dispose of large quantities of the relatively illiquid assets. Even though non-distressed firms also engage in predation in this latter case, the cost of providing the liquidity cross-subsidy—in terms of the short-term impact of trading in the liquid assets on their own price—is relatively low.

In other words, cross-elasticities in the fundamental values create an incentive to predate in assets other than the chief ones being liquidated, while cross-elasticities in the short-term market impact of trading provide opportunities to reduce the liquidity cost of predation. When the bulk of the assets being liquidated are relatively liquid, the overall impact of the cross-subsidy is relatively weak, implying that predation results in high price volatility.

Our work is also related to the incomplete-information approach to modeling strategic trading, in which market participants have private information about the fundamental value of an asset. Both Brunnermeier and Pedersen as well as Carlin, Lobo, and Viswanathan treat as exogenous the parameters determining the market-clearing price. By contrast, in a second group of papers (Kyle, 1985; Holden and Subrahmanyam, 1992; and Back, Cao and Willard, 2000), the market impact of trading arises endogenously because firms have private information about the fundamental value of the asset. Similar to the first group of authors, we do not attach a structural interpretation to the parameters of the demand system. However, we can think of them as being plausible reduced-form effects implied by a multi-asset extension of the incomplete-information literature. Namely, if there is correlation across either the asset values themselves or across agents' private signals about those values, then trading activity in any one asset is informative about the other assets. In equilibrium, such correlations would give rise to the cross-elasticities in our model.

Finally, our paper relates to the ongoing debate over the effect of policies intended to increase the liquidity of particular assets. For example, our analysis suggests that the extension of government guarantees to particular classes of illiquid assets—which reduce the price impact of trading by effectively decreasing the degree of asymmetric information about the assets' payoffs—may actually increase price volatility by making it "cheaper" (in terms of trading costs) to predate.

In effect, the cross-elasticities among assets work against orderly trading if firms' initial endowments contain a high proportion of liquid assets. The analysis thus suggests that government interventions may be more effective at stemming market disruptions if they focus on relieving the distress of financially troubled firms, as opposed to extending government guarantees to illiquid assets.

The remainder of this paper proceeds as follows. The remainder of this section discusses a motivating example. In the next section, we set up a model of strategic interaction among n firms trading in m assets. In Section 3, we characterize the equilibrium outcome as well as the cooperative solution that maximizes total surplus for the strategic players. Note that we do not explicitly model the preferences or behavior of the retail investors. As an alternative, in Section 4 we use a non-structural approach to capture the effect of strategic trading on the welfare of the retail investors. Using a hazard specification, we investigate the mapping from model primitives to the probability of financial distress among retail traders. Section 5 discusses potential policy implications, and Section 6 concludes.

An example

To fix ideas, consider the following example: two firms, A and B, own and trade in two assets: private-label residential mortgage-backed securities (RMBS) and Treasury bonds. Firms A and B act strategically, trading in the short run in response to perceived deviations from fundamental values. A large number of non-strategic or retail investors follow a "buy and hold" strategy in RMBS and bonds as well. For concreteness, assume that A and B each initially holds \$ 200 billion in RMBS and zero bonds, and that the retail investors hold the remaining amounts of each asset, where outstanding quantities total \$1,000 and \$5,000 billion respectively.

Firm A encounters unmodeled financial difficulties and must liquidate its entire portfolio of RMBS to satisfy its creditors within a relatively short amount of time, say, a week. Firm B knows this. As a convenient normalization, we will require that Firm A and B hold the same amount of RMBS and bonds at the end of the week as they did at the beginning (before A became distressed), with the exception that A's holdings of RMBS run down to zero. Thus at the end of the period, A will hold \$0 of RMBS and \$0 in bonds, B will hold \$200 billion in RMBS and \$0 in bonds, and the retail investors will hold \$800 billion in RMBS and \$5,000 billion in bonds. During the week of trading, the strategic traders may go long or short in bonds. (The trajectories of prices and quantities for this example are shown in figure 5.)

The retail investors thus, on net, increase their holdings of RMBS and will have to be compensated by higher yields (lower prices) on RMBS. They may also have to be compensated by higher yields on bonds, depending on the degree to which RMBS and bonds are seen as similar assets by retail investors. These changes in yields are the long-run or fundamental change. However, as trading proceeds within the week, the strategic firms A and B may quickly buy and sell large volumes of each asset in response to actions by the other. The retail investors will require extra compensation the faster A or B trade because they have only limited capacity to evaluate each asset as it is sold to them. They require less extra compensation for the same volume of bond sales because bonds are largely homogeneous. They require extra compensation for RMBS sales because each security differs.

Importantly, they will demand more compensation on bonds even if sales of bonds do not increase but sales of RMBS do. This cross-elasticity can be motivated in a variety of ways: retail investors have only limited capacity to evaluate any kind of asset within a short time frame or they are reluctant to rebalance their portfolios on short notice. So-called "buy and hold investors" or "real money accounts" might view their holdings of financial assets as one part of a broader portfolio that might contain real assets and other risk exposures correlated with financial asset returns, limiting their willingness and ability to quickly switch positions. Strategic traders, by contrast, are driven primarily by the returns available on the financial assets in the model.

Thus, there will be spillover effects from the standard strategic "racing" and "fading" effects. With multiple assets there are further effects at work. Distressed firms face an incentive to "create liquidity" for themselves by trading in opposite directions in each asset. In turn, non-distressed firms will use these cross-asset price effects to further predate on the distressed firm. For relatively small cross-elasticities, this liquidity creation motive is relatively muted and the optimal trading strategies are much the same as those in Brunnermeier and Pedersen (2005) or Carlin at al (2007). For extremely large cross-elasticities, for example, cross-elasticities greater than unity, the optimal trading strategies look very different as firms predate on one another by trading in the non-distressed asset. We do not focus on such cases in this paper, in part because they are often inconsistent with standard conditions required for uniqueness. However, one could imagine assets whose prices are correlated in this manner; for example, changes in the value of a junior tranche of a structured asset would be larger than changes to the senior tranche of the same asset. Thus, one could imagine modeling the equity tranches of ABS as the illiquid asset and the investment-grade tranches as the liquid asset. In such a case, large cross elasticities would be perfectly plausible.

The main predatory effect is that Firm B sells RMBS at the same time that Firm A is forced to.

Firm B then buys the RMBS back at distressed prices, booking a profit.

Note that neither firm starts with bonds or faces any requirement to trade in them. However, the fundamental value of bonds will decrease because the retail investors will demand greater compensation to hold a permanently higher amount of RMBS. This is due to the long-run cross-elasticity between the two assets, which can be different from the short-run cross-asset effects. Thus, there is a standard motive for bond trading.

In addition, Firm A can provide itself some liquidity by buying bonds from the retail investors while selling them RMBS. In effect, Firm A mitigates the price decrease in RMBS by temporarily buying bonds from the retail investors. Later, when trading volumes subside, Firm A unwinds its position in bonds. However, knowing that Firm A will engage in such behavior, Firm B predates against Firm A.

We model retail traders' utility as a function of their probability of financial distress; this, in turn, is driven by the fluctuations in the prices of the assets they hold. Thus sudden asset price drops, even if followed by recoveries, will harm retail investors. For example, retail investors could be thought of us mutual fund managers who are fired if a fund's NAV falls too much.

The government could attempt to protect retail investors and generally lower volatility by providing guarantees on the illiquid asset. In our example, this would correspond to transforming some portion of RMBS outstanding into government bonds. For example, the government might guarantee all RMBS issued by a particular firm. Imagine that the government guarantees 10 percent of outstanding RMBS. Now, Firm A and Firm B each begins the period holding \$180 billion in RMBS and \$20 billion in bonds. The retail investors hold \$540 billion in RMBS and \$5,060 in bonds at the start of the period.

Effectively, then, such guarantees change the endowments of all parties prior to the start of trading. We study the effects on asset price volatility and retail traders' financial distress of such a change in endowments. We find that increasing holdings of the liquid asset increases price volatility. For plausible parameter values, this increases the probability of financial distress among retail investors, decreasing their welfare. Volatility increases because of the standard effect–predation is cheaper when assets are more liquid–but also because of the spillover effects. Increased predation in one asset leads to increased trading in the other asset. (We show the probability of financial distress among retail investors in figures 6 and 7.)

2 Model

We consider an economy in which there are m assets traded in continuous time and n strategic players. In addition, we assume that there is a continuum of nonstrategic "retail" investors who comprise the residual demand for the assets. The incomplete-information approach to modeling strategic trading actually derives the residual demand based on the information available in the market about the fundamental value of the assets. However, for simplicity we treat the residual demand function as exogenously given. Each strategic firm maximizes returns given the actions of its competitors. On the other hand, the retail investors are "unaware" of the temporary liquidity needs in the market. Instead, the residual demand is determined mechanistically as a function of the total stocks of assets held by the strategic traders as well as the current volume of trading activity.

We let the vector $x_i(t) \in \mathbb{R}^m$ denote player i's position in each of the m assets at time t, and correspondingly let the stacked vector $x(t) \equiv [x_1(t) \dots x_n(t)]$ denote all of the players' positions.

Each strategic firm i begins with initial endowments $x_i(0) = x_{0i}$. We assume that for exogenous reasons, the traders must reach positions $\{x_{1i}\}_{i=1 \dots n}$ by the end of the trading period, normalized to $t = 1$. For example, setting x_{1i} to zero implies that firm i must liquidate all of its assets by the end of trading.

The trading speeds at time t are given by the vector $y(t) \equiv dx(t)/dt$. Demand for assets is imperfectly elastic and is described by the following system of inverse demand equations:

$$p(t) = u + \Gamma X(t) + \Lambda Y(t) \tag{1}$$

where $X(t) \equiv \sum_{i=1}^{n} x_i(t)$ captures the strategic traders' total positions and $Y(t) \equiv \sum_{i=1}^{n} y_i(t)$ captures the net instantaneous speed of trading. Λ and Γ, the elasticity matrices, are positive-definite by assumption. Here, we treat the demand functions as primitives. However, as discussed in the introduction, a similar relationship arises endogenously under the assumption that traders have differing degrees of private information and there is a risk-neutral market-maker who sets prices in order to make zero profits (e.g., Kyle, 1985). Alternatively, without explicitly specifying the information structure or the price-setting process of our model, we can also interpret the dependence of prices on $X(t)$ through the matrix of elasticities Γ as reflecting heterogeneity in nonstrategic traders' valuation of the assets. The dependence of prices on $Y(t)$ through the matrix of elasticities Λ captures the reduced-form effect of the instantaneous speed of trading: the more quickly traders are buying or selling the assets, the higher the "liquidity" costs.

If the elasticity matrices Γ and Λ are diagonal, the markets for each of the m assets are completely segmented. In this case, the greater the j'th diagonal term of Λ is, the greater is the market impact of trading in asset j. Crucially, our model allows for nonzero off-diagonal terms in Γ and Λ, reflecting cross-elasticities in the permanent and short-term effects of trading. The importance of these off-diagonal terms is explored in the following section.

3 Equilibrium Analysis

3.1 Competitive Equilibrium

Similar to both Brunnermeier and Pedersen (2005) as well as Carlin, Lobo, and Viswanathan, we focus on open-loop equilibria for the sake of tractability. That is, we assume that strategies are deterministic, are chosen at $t = 0$, and are not conditioned on subsequent actions. Under this framework, each trader i solves the following optimal control problem (where we use a superscripted T to indicate the transpose of a vector or matrix):

$$\begin{gathered} \max_{x_i(t), y_i(t)} - \int_{t=0}^{1} p(t)^T y_i(t)\, dt \\ s.t. \\ y_i(t) = dx_i(t)/dt, \\ x_i(0) = x_{0i}, \quad x_i(1) = x_{1i} \end{gathered} \tag{2}$$

In the above expression, $p(t)^T y_i(t)$ is firm i's instantaneous revenue at time t; $y_i(t) = dx_i(t)/dt$ are the equations of motion governing i's position $x_i(t)$; $x_i(0) = x_{0i}$ and $x_i(1) = x_{1i}$ are the boundary conditions. The latter boundary condition captures the notion that some firms become distressed. In general, we assume that for nondistressed firms, $x_{1i} = x_{0i}$ while for distressed firms, $x_{1i} < x_{0i}$ (in the vector sense). In practice, our later examples focus on the specific case in which $x_{1i} = 0$, but the general intuition holds even without assuming this particular endpoint.

Substituting in the price equations (1), the Hamiltonian for trader i can be expressed as $H_i = [-u - \Gamma X(t) - \Lambda Y(t)]^T y_i(t) + z_i(t)^T y_i(t)$, where $z_i(t) \in \mathbb{R}^m$ are the shadow costs of the equations of motion. Following Carlin, Lobo, and Viswanathan, we limit the set of admissible strategies to differentiable trading profiles satisfying $\int_0^T (y_i(t))^2 < \infty$. The necessary conditions defining a competitive equilibrium are as follows:

$$\begin{array}{rl} -\frac{\partial H_i(t)}{\partial x_i(t)} = \Gamma^T y_i(t) = \frac{dz_i(t)}{dt} & \\ -\frac{\partial H_i(t)}{\partial y_i(t)} = u + \Gamma X(t) + \Lambda Y(t) + \Lambda^T y_i(t) - z_i(t) = 0, & \quad i = 1 \ldots n \end{array}$$

Differentiating the second equation with respect to t and substituting in the first equation yields

$$\Gamma Y(t)dt + \Lambda dY(t) + \Lambda^T dy_i(t) - \Gamma^T y_i(t)dt = 0, \quad i = 1 \ldots n$$

We can reexpress the above system in matrix form by defining $\Upsilon(t) = [y_1(t) \ldots y_i(t) \ldots y_n(t)]$ (i.e., $y(t) = vec(\Upsilon(t))$):

$$\begin{array}{rcl} \Gamma Y(t)dt\mathbf{1}^T + \Lambda dY(t)\mathbf{1}^T + \Lambda^T d\Upsilon(t) - \Gamma^T \Upsilon(t)dt & = & \\ \Gamma \Upsilon(t)dt\mathbf{1}\mathbf{1}^T + \Lambda d\Upsilon(t)\mathbf{1}\mathbf{1}^T + \Lambda^T d\Upsilon(t) - \Gamma^T \Upsilon(t)dt & = & 0 \end{array}$$

where $\mathbf{1}$ denotes an $n \times 1$ vector of ones. Γ and Λ are symmetric by assumption,[1] implying

$$\begin{array}{rcl} & \Lambda d\Upsilon(t)[I + \mathbf{1}\mathbf{1}^T]\Gamma\Upsilon(t)dt[I - \mathbf{1}\mathbf{1}^T] & \\ & \Longleftrightarrow & \\ & d\Upsilon(t) = \Lambda^{-1}\Gamma\Upsilon(t)dtA & \\ & \Longleftrightarrow & \\ dy(t) & = & vec(\Lambda^{-1}\Gamma\Upsilon(t)dtA) \\ & = & (A^T \otimes \Lambda^{-1}\Gamma)vec(\Upsilon(t)dt) \\ & = & (A^T \otimes \Lambda^{-1}\Gamma)y(t)dt \end{array} \tag{3}$$

where $A \equiv [I - \mathbf{1}\mathbf{1}^T][I + \mathbf{1}\mathbf{1}^T]^{-1}$.[2]

The general solution to the system of differential equations given by (3) is $y(t) = exp(tA^T \otimes \Lambda^{-1}\Gamma) \cdot c_0$. Integrating $y(t)$ over t yields the following:

[1] If $\Lambda = 0$ and the demand system is linear, symmetry of Γ is implied by the symmetry of the Slutsky matrix.

[2] The final expression comes from applying the vec operator to both sides of the equation and using the fact that for any three matrices A, B, and C, $vec(ABC) = c^T \otimes A\ vec(B)$.

Proposition 1 *There is a unique open-loop equilibrium to the trading game, in which the strategic traders' asset positions have the following trajectory:*

$$x(t) = \int y(t)dt = (A^T \otimes \Lambda^{-1}\Gamma)^{-1} exp(tA^T \otimes \Lambda^{-1}\Gamma) \cdot c_0 + c_1 \tag{4}$$

Here, "$exp(\cdot)$" is the matrix exponential operator and the constants c_0 and c_1 are implied by the boundary conditions.[3] Uniqueness follows from the fact that each firm's objective function is globally concave in the control function[4] and from the linearity of the equation of motion.[5] Thus, there is a unique trading profile that satisfies the necessary conditions.

3.2 Features of Equilibrium Outcome

Figure 1a plots the firms' asset positions $x_i(t)$ as a function of time for the case of two firms ($n = 2$) and two assets ($m = 2$), and specific values of the elasticities Λ and Γ. By assumption, asset 2 is more "liquid" than asset 1, in the sense of having a lower market impact of trading (i.e., $\lambda_{22} < \lambda_{11}$, where λ_{ij} indicates element i, j of Λ). The graphs show what happens when firm 1 becomes distressed at time 0 and must liquidate all of its assets.

Figure 1b plots the corresponding asset prices as a function of time. The example is instructive because it shows that even though the fundamental values of both assets are expected to fall, only trading in asset 2 (the more liquid asset) exhibits the "race and fade" effect, which can be seen in the convexity of the non-distressed firm's trajectory in that asset. By contrast, the non-distressed firm's trajectory in asset 1 is concave: toward the beginning of the trading window, the firm actually purchases asset 1, providing liquidity for the disposition of the distressed firm's portfolio.

Several competing effects are at play. To begin with, ignoring the cross-elasticities (for the time being), note that the non-distressed firm's trading activity in the distressed asset is driven by two competing incentives. Just as in Carlin, Lobo, and Viswanathan's single-asset case, there is an

[3] To compute the exponent of a matrix, first express the matrix in terms of its orthogonal factorization PVP^{-1}, where P is a matrix of eigenvectors and V is a diagonal matrix of eigenvalues $v_1 \ldots v_n$. $B^m = PV^mP^{-1}$ for any power m, and because the exponential operator is defined as a power series of B, we have $exp(B) = Pexp(V^m)P^{-1}$, where $exp(V^m)$ can be computed element-by-element. The constants are determined by the boundary conditions $x(0) = x_0$ and $x(1) = x_1$. Defining $B = A^T \otimes \Lambda^{-1}\Gamma$, we have

$$\begin{bmatrix} B^{-1} & I \\ B^{-1}exp(B) & I \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} x_0 \\ x_1 \end{bmatrix}$$

[4] This assertion relies on the assumption that Λ is positive-definite.

[5] See Newcomb (1970).

incentive to predate (by selling rapidly when the fundamental price is high and then "fading" back when the fundamental price is low). Of course, the distressed firm best-responds by racing the non-distressed firm to sell early, resulting in faster net selling toward the beginning of trading and slower net selling toward the end of trading, relative to the rate that would be realized if firms traded at a uniform speed over time. However, because the speed of trading affects the liquidity discount or premium, the incentive to predate is partially mitigated by a desire to smooth the speed of trading over time. In effect, the predatory incentive to race and fade simultaneously generates a second, countervailing incentive to provide liquidity by buying the asset being predated on at a discount during the "race" phase of the game—when it is being sold more rapidly than the average over time—and selling the asset at a premium during the "fade" phase, when it is being sold more slowly than the average over time. Moreover, because the individual players internalize less of the liquidity impact of predation when there are more firms, the trading speed deviates more and more from a uniform rate as the number of firms $n \to \infty$ (Figure 2a). As shown in Figure 2b, in the limit as the number of competing firms, n, goes to ∞, prices immediately jump down to the final level at the very start of trading and then remain constant thereafter.[6]

In addition to the previous effects, which our model shares in common with the single-asset case, having multiple assets introduces two novel features. First, cross-elasticities in the permanent values of assets (off-diagonal terms of Γ) imply that *forced liquidation of any one asset puts downward pressure on the fundamental values of all remaining assets, creating incentives to trade strategically even in assets that do not have to be liquidated.* Trading activity in assets that do not need to be liquidated reflects the fact that the goods are imperfect substitutes. We can isolate the effect of cross-elasticities in Γ by examining the dynamics when only Λ is diagonal (Figures 3a and 3b). In this case, we observe the race-and-fade phenomenon in both assets, with accelerated early sales for both the asset held in the distressed firm's initial portfolio as well as for the other asset. Note that relative to the case shown in Figures 1a and 1b, the extent of racing and fading in the liquid asset (asset 2) is weaker: in the former case, the cross-subsidization effect essentially reduces the liquidity cost of trading, encouraging more predation.

Clearly, when declines are expected in the fundamental values of all assets, race-and-fade behavior must be manifested to some extent for at least a subset of the assets. At the same time, the cross-elasticities in the short-term impact of trading (Λ) introduce a second effect: *firms have the opportunity to provide liquidity "cross-subsidies" to themselves.* Except when Λ is diagonal, firms can lower their overall trading costs at each moment by trading in opposite directions in the various assets. In particular, purchasing activity in one asset increases the price premium on all other assets.

[6]Intuitively, because under perfectly competitive markets each individual trader is a price-taker, fluctuations in prices during the trading window would imply the presence of arbitrage opportunities. Thus, prices must be constant throughout the trading window when $n = \infty$.

Therefore, racing and fading in a particular asset (due to an expected decline in its fundamental value) simultaneously generates an incentive to make purchases in other assets during the "race" phase and then to sell back those other assets during the "fade" phase. Figures 4a and 4b isolate the self-subsidization effect for the two-asset case by setting Γ to be a diagonal matrix, such that only Λ has off-diagonal terms. Similar to Figures 1a and 1b, in this example, the non-distressed firm tends to trade in opposite directions in each of the assets at every moment.

In the examples discussed so far, the non-distressed firm predates in the liquid asset and trades in the opposite direction in the illiquid asset. However, this need not be the case in general, and it is possible to generate examples for which the reverse is true. Because of the cross-elasticities in Λ and Γ, both the incentive to predate and the incentive to provide liquidity exist for all assets. In particular, the trading profile will be concave for certain assets when the incentive to provide liquidity through those assets actually outweighs the incentive to predate on those assets. In other words, whether the trajectory of a given asset position exhibits race-and-fade behavior, or instead the opposite, depends upon the net liquidity costs of trading in that asset, relative to the expected decline in its fundamental value.

For example, if firms hold a large initial endowment in the *illiquid* asset (Figures 5a and 5b), the non-distressed firm's trading profiles in both assets are convex, with early selling and later buying-back of the assets. Furthermore, relative to the case in which the strategic firms begin with a liquid endowment (Figures 1a and 1b), overall trading takes place at a much more constant speed and prices fall more gradually. Intuitively, when the distressed firm must dispose of a more illiquid endowment, there is a weaker incentive to predate because the liquidity costs associated with racing and fading in the illiquid asset are high relative to the expected decline in its fundamental value. Also, for this particular case, the non-distressed firm's incentive to race and fade in the liquid good (asset 2) outweighs the incentive to provide liquidity, though the overall trading volume is light.

3.3 Monopoly Case and Cooperative Outcome

In the monopoly case or when firms behave cooperatively, agents entirely internalize the liquidity impact of trading. Note that in the monopoly case, the total trading costs associated with liquidating the portfolio are minimized by selling each asset at a uniform rate. The minimized trading costs amount to $\frac{1}{T}(x(1) - x(0))^T \Lambda (x(1) - x(0))$. There is effectively no cross-subsidization across assets, because unlike the competitive case—in which firms race and fade—the degree of illiquidity for a given asset is uniform over time, yielding no benefits to trading in assets other than the ones that actually must be liquidated.

Similarly, if there are multiple firms, the first-best cooperative outcome is attained by setting *net* trading speed to be constant over time. The trading profile that achieves the cooperative optimum is not unique, and may be any trading profile such that net trading occurs at a uniform speed, so long as the boundary conditions are met. In particular, all trading could be assigned to a single firm, and then at the last moment firms "clear accounts" by trading infinitely fast to attain the required endpoint positions.

4 Effect on Nonstrategic Investors

A key issue of policy relevance is the effect of strategic trading on the nonstrategic, "retail" investors. The preferences and behavior of the retail traders are not explicitly modeled in the setup. However, common intuition suggests that investors are harmed when asset prices decline suddenly, relative to the more gradual declines that would be observed if assets were liquidated at a constant rate.

For reasons already discussed in Subsection 3.2, when the distressed firm begins with a relatively more illiquid endowment, prices may actually fall more gradually than when it is initially endowed with relatively liquid assets. In order to construct a summary measure of the distress that sudden drops in price are likely to create among retail investors, we consider the following scenario. Suppose there is a "representative" retail investor who begins with a vector of endowments, x_{0r}. At time t, the investor's net worth, $w_r(t)$, is given by:

$$w_r(t) = p(t)'x_r(t) - \int_{\tau=0}^{t} p(\tau)y_r(\tau)\, d\tau \tag{5}$$

where $x_r(t)$ is the retail investor's portfolio at time t and $-\int_{\tau=0}^{t} p(\tau)y_r(t)\, d\tau$ is the cash position generated by her trading activity between time 0 and t. (Note that summing over the retail investors and the strategic players yields zero net trading.)

At each point in time, the investor's hazard of becoming "distressed," $h(t)$, depends upon her net worth as a proportion of initial wealth. While it is inessential how we actually interpret the event of becoming distressed, as a conceptual matter, we can think of distress as the prospect of facing a margin call.[7] Specifically, we assume that the probability of the retail investor becoming distressed before the end of the trading period (at time T) is given by:

[7] We abstract from any potential price impact of the retail investors having to liquidate.

$$
\begin{aligned}
&1 - \int_0^T e^{h(t)} dt \\
h(t) &= -log[min(1, w_r(t)/w_r(0))]
\end{aligned} \tag{6}
$$

The functional form of $h(t)$ implies that the retail investor only faces a nonzero hazard of distress if her net worth falls below the initial level. Figures 6a and 6b graph the retail investor's probability of survival as a function of the strategic traders' initial asset allocation for the two-good case. Each curve in the graph corresponds to a particular value of γ_{12}, the cross-price elasticity in the permanent values of the assets. Figure 6a assumes that the retail traders have an initial endowment in the illiquid asset while Figure 6b assumes that the retail traders have an initial endowment in the liquid asset. Similarly, each curve in Figures 7a and 7b corresponds to a particular value of λ_{12}, the cross-price elasticity in the short-term market impact of trading.[8] There are four main points to take from these graphs:

- Shifts along each of the curves indicate that as we increase the initial proportion of the strategic players' endowment in whichever asset is held by the retail traders, the retail traders are less likely to survive. Because trading by the strategic traders depresses the prices of the assets being liquidated, when the two types of traders hold the same types of assets, the retail traders are more likely to become distressed.

- As shown by the flatness of the curves corresponding to high values of γ_{12}, as the cross-price elasticity of the permanent effect increases, survival of the retail investors becomes less sensitive to the initial allocation of the strategic traders. Essentially, higher γ_{12} implies that the two assets are closer substitutes, meaning that retail investors are less affected by changes in the composition of the portfolio that must be liquidated.

- Shifts across the curves corresponding to values of λ_{12} indicate that as the short-term cross-elasticity increases, the retail investor's probability of survival generally increases over most ranges of the initial endowments for the strategic traders. To understand why, note that a higher cross-elasticity strengthens the cross-subsidization effect, making it cheaper (in terms of overall price impact) for the non-distressed firms to predate. The reduction in price impact has not only a direct impact but also indirect effects. (The situation is loosely analogous to the effect of a decrease in the price of a good in standard consumer theory: the overall expenditure share may either increase or decrease, depending on the relative strength of the income and substitution effects). In particular, higher values of λ_{12} encourage the non-distressed firms to engage in more racing and fading with respect to the asset being liquidated

[8]In the graphs, values of γ_{12} and λ_{12} are limited to ranges such that Γ and Λ are positive-definite.

by the distressed firm, while their trajectory in the other asset becomes more concave. The overall change in price volatility is theoretically ambiguous. However, when the strategic firms' initial endowment is not too heavily skewed toward the liquid good, the direct price effect dominates, and higher values of λ_{12} tend to reduce price volatility and increase the probability of survival for the retail investor.

- On the other hand, if the strategic traders' initial allocation in the liquid good is sufficiently close to 1.0, the indirect effect of λ_{12} on the extent of predation in the more liquid asset predominates, which may actually decrease the retail investor's probability of survival. To see why, note that the trading trajectories for the illiquid good are relatively insensitive with respect to λ_{12}. On the other hand, high values of λ_{12} make it less costly for the non-distressed firm to predate in the liquid asset, with the overall effect of increasing price volatility.

To summarize, increasing the share of the liquid good is likely to benefit the retail traders only under certain conditions: the retail traders must not also hold large endowments of the liquid good, and the assets must not be close substitutes (as indicated by the cross-elasticities of the permanent effects). Moreover, increasing the share of the liquid good is less likely to benefit the retail traders when there are strong cross-elasticities in the market impact of trading various assets.

5 Policy Implications

Our model suggests that a constant trading speed is a feature of the first-best outcome (defined as the trading profile that maximizes the strategic players' total surplus). This feature corresponds to the idea of an "orderly resolution" of distressed or insolvent firms. The concept is recognized by bankruptcy laws, which give judges broad latitude to wind down firms in a manner that tries to conserve value for liability holders. However, at times the government has taken actions to intervene in the liquidation of large and complex distressed firms with many counterparties, based on the concern that standard corporate bankruptcy law and practice are insufficient to avoid market disruptions.

Most recently, the government has contemplated or engaged in four different types of actions with regard to banks and other large financial institutions: (1) providing liquidity facilities for illiquid assets (financing collateralized by the illiquid assets), (2) injecting equity into firms so that they do not need to liquidate assets in the first place, (3) buying assets from distressed firms, and (4) extending government guarantees to certain classes of asset.

In the context of our model, the first three of these policies are similar in the sense that each reduces the overall quantity of illiquid assets that must be sold by the distressed firms. As the amount of assets that must be liquidated by distressed firms declines (say, from more widespread capital injections), trading in the markets becomes more orderly in all assets. Similarly, as the holdings of the illiquid assets by distressed firms decline (either because the government buys the asset or is willing to finance the asset for a time longer than the trading period), trading becomes more orderly across asset markets.

In contrast, when the government increases the *relative* proportion of liquid assets by guaranteeing some of the illiquid assets, asset price volatility and predation increase. There are two channels for this increase. First, increasing the proportion of liquid assets directly increases predatory behavior because the strategic players find it cheaper to trade against one another in the liquid asset. Second, this increased predation leads to increased trading in the illiquid asset because of the cross-elasticities between the two. Thus, our analysis suggests that government actions to relieve disorderly trading conditions should focus on relieving the distress of financially troubled firms rather than extending government guarantees to illiquid assets.

6 Conclusion

The current financial crisis has been marked by a strong preference by investors for liquid assets amid heightened price volatility. One policy response has been to increase liquidity by extending government guarantees. We study the effects of such a policy on equilibrium prices and welfare in an endogenous liquidity model containing both strategic and non-strategic traders. We augment the standard model by adding multiple assets and by modeling the utility of the non-strategic traders.

We find that extending guarantees, modeled as shifting assets from the illiquid to the liquid category, increases predatory trading when one of the strategic traders becomes distressed. In part this is because of the decreased cost of predation when assets are more liquid, but in part it reflects the linkages between prices in the two markets. For broad and plausible parameter ranges, the increase in volatility from such a policy leads to a decrease in the welfare of the non-strategic traders.

This model can also be used to consider the policy choices facing a government wishing to unwind its position in the liquid asset. After the financial crisis abates, government officials may want to reduce public holdings or guarantees on assets. Policymakers would then be in a position analogous to that of a distressed firm in our analysis. However, they presumably would also want to minimize price volatility, and associated welfare costs, further exposing themselves to predation.

Finally, we can conjecture about what the financial system would look like following a widespread increase in government guarantees. Such a post-crisis system would feature a smaller amount of private assets, and these assets would trade in thinner, less-liquid markets with associated increased price volatility. Restarting issuance of risk securities such as consumer ABS in such a market would seem particularly difficult.

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Figure 1a: Asset position trajectories for baseline case with 2 assets and 2 firms

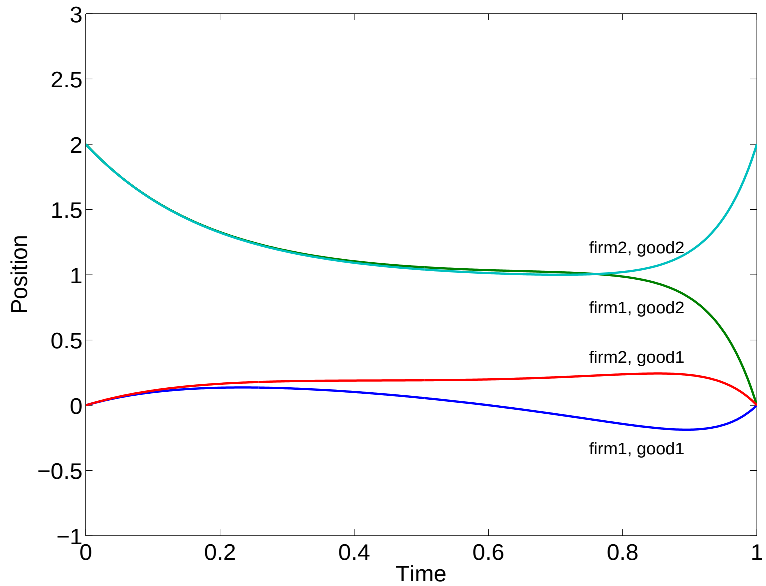
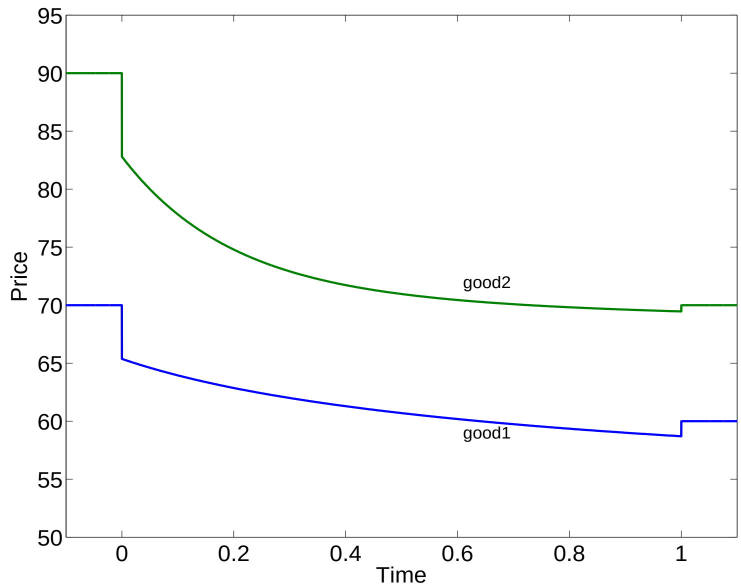


Figure 1b: Equilibrium price trajectories for baseline case with 2 assets and 2 firms



Figures generated under the following assumptions: $\Gamma = \begin{bmatrix} 10 & 5 \\ 5 & 10 \end{bmatrix}$, $\Lambda = \begin{bmatrix} 3 & 1.25 \\ 1.25 & 1 \end{bmatrix}$, both strategic players begin with endowments $(0, 2)$, and player 1 must liquidate all assets by $t = 1$.

Figure 2a: Asset position trajectories with 50% of firms distressed, varying number of firms N

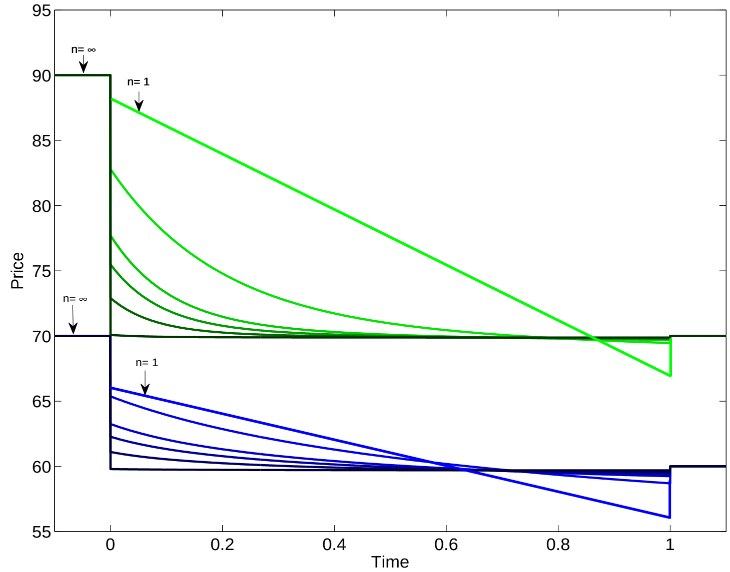
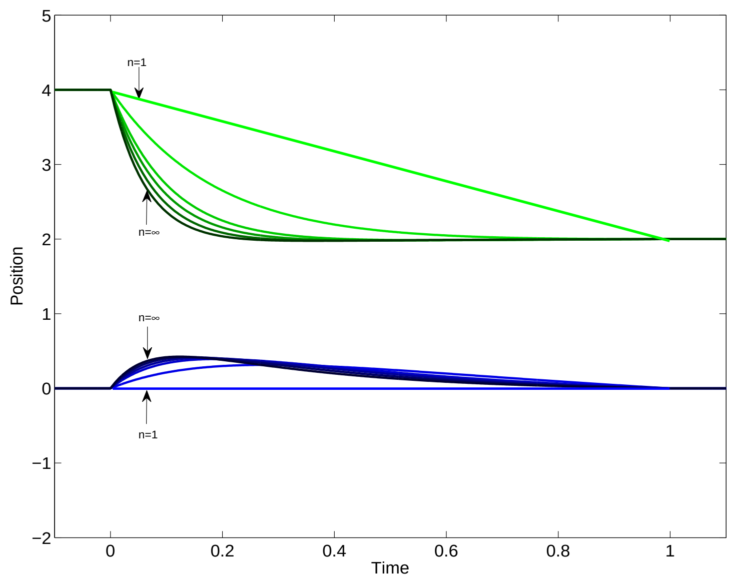


Figure 2b: Equilibrium price trajectories with 50% of firms distressed, varying number of firms N



Figures generated under the following assumptions: $\Gamma = \begin{bmatrix} 10 & 5 \\ 5 & 10 \end{bmatrix}$, $\Lambda = \begin{bmatrix} 3 & 1.25 \\ 1.25 & 1 \end{bmatrix}$, all firms begin with endowments $(0, 2/N)$, and *half* of all firms must liquidate all assets by $t = 1$.

Figure 3a: Asset position trajectories for zero cross-elasticities in short-term market impact

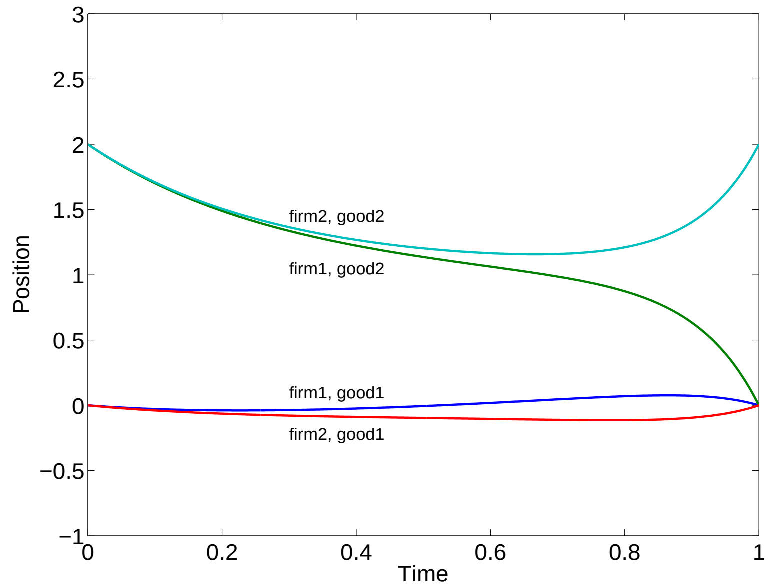
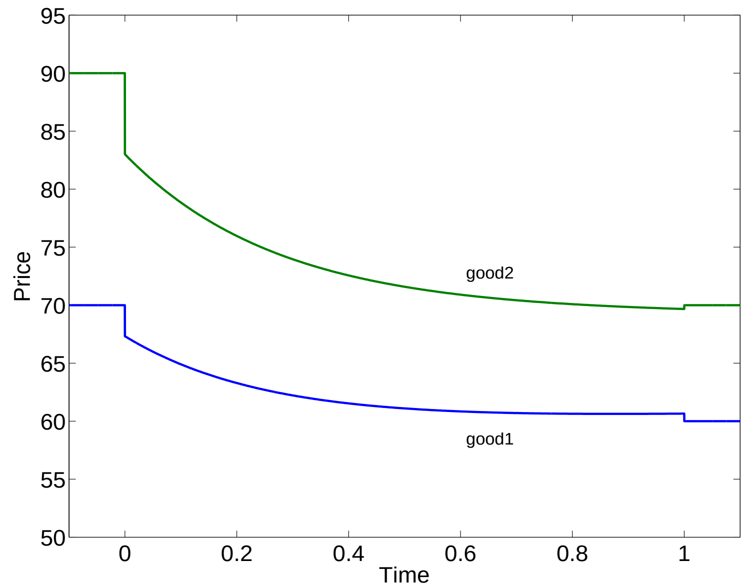


Figure 3b: Equilibrium price trajectories for zero cross-elasticities in short-term market impact



Figures generated under the following assumptions: $\Gamma = \begin{bmatrix} 10 & 5 \\ 5 & 10 \end{bmatrix}$, $\Lambda = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$, both strategic players begin with endowments $(0, 2)$, and player 1 must liquidate all assets by $t = 1$.

Figure 4a: Asset position trajectories for zero cross-elasticities in fundamental values

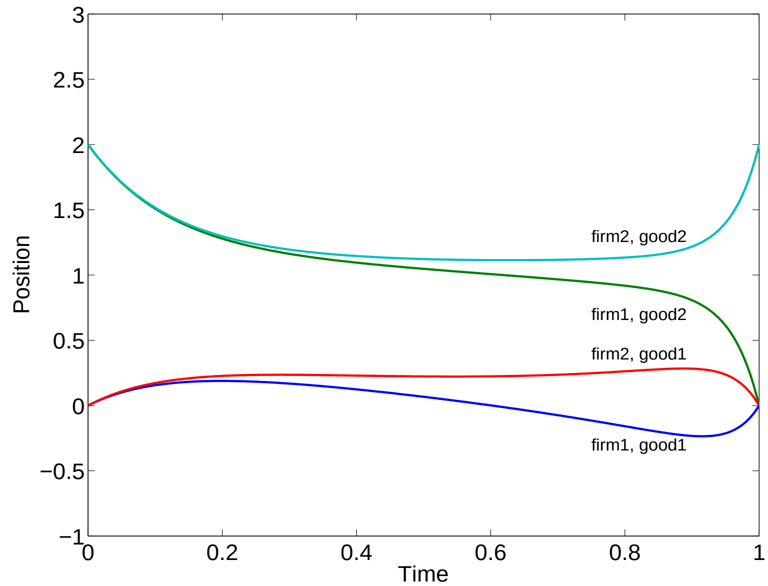
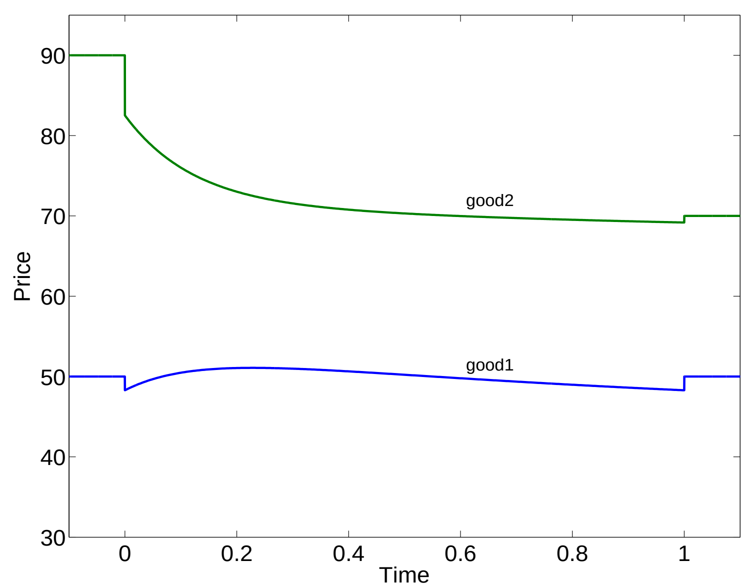


Figure 4b: Equilibrium price trajectories for zero cross-elasticities in fundamental values



Figures generated under the following assumptions: $\Gamma = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$, $\Lambda = \begin{bmatrix} 3 & 1.25 \\ 1.25 & 1 \end{bmatrix}$, both strategic players begin with endowments $(0, 2)$, and player 1 must liquidate all assets by $t = 1$.

Figure 5a: Asset position trajectories with more illiquid initial endowment

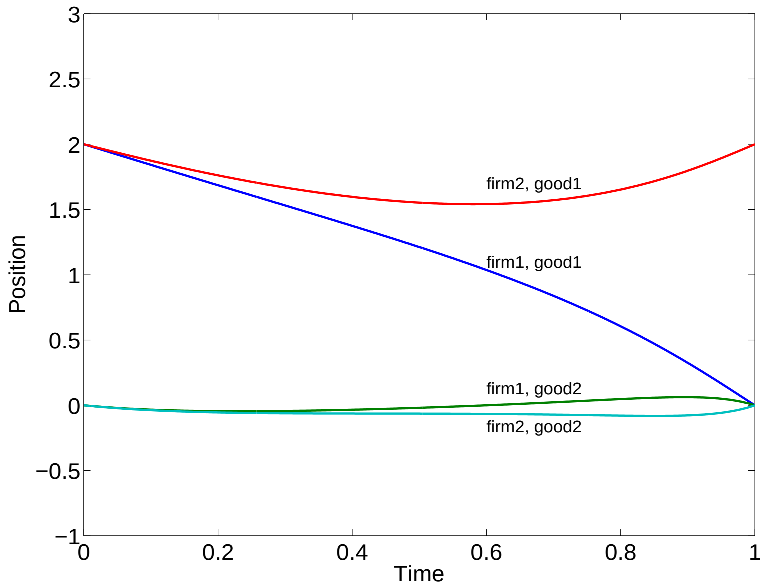
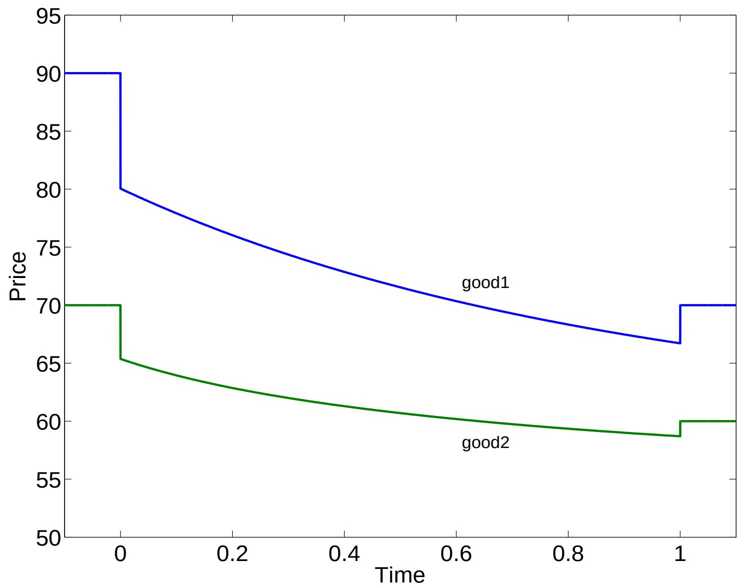


Figure 5b: Equilibrium price trajectories with more illiquid initial endowment



Figures generated under the following assumptions: $\Gamma = \begin{bmatrix} 10 & 5 \\ 5 & 10 \end{bmatrix}$, $\Lambda = \begin{bmatrix} 3 & 1.25 \\ 1.25 & 1 \end{bmatrix}$, both firms begin with endowments $(2, 0)$, and firm 1 must liquidate all assets by $t = 1$.

Figure 6a: Probability of retail-trader distress vs. proportion of strategic traders' initial allocation in the illiquid asset. Retail traders have an initial endowment in the **illiquid** asset.

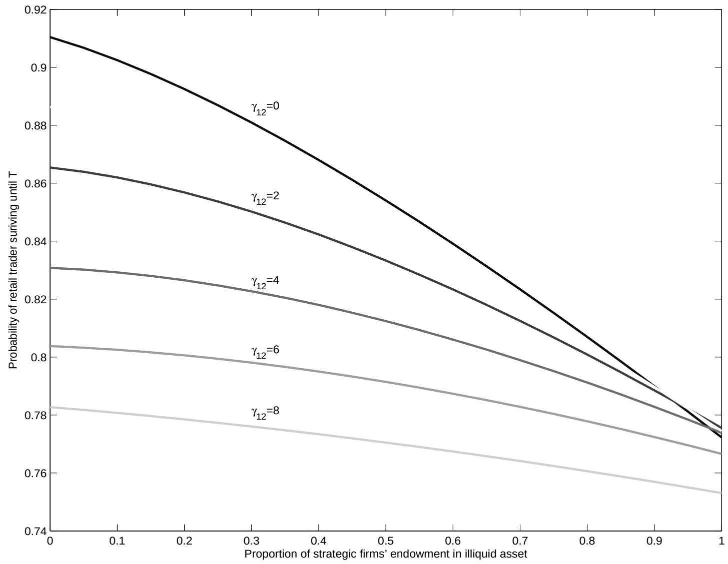
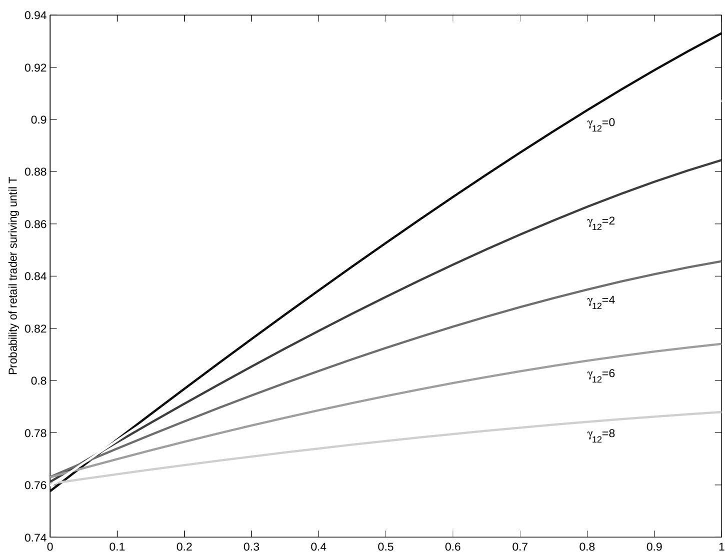


Figure 6b: Probability of retail-trader distress vs. proportion of strategic traders' initial allocation in the illiquid asset. Retail traders have an initial endowment in the **liquid** asset.



Figures generated under the following assumptions: $\Gamma = \begin{bmatrix} 10 & \gamma_{12} \\ \gamma_{12} & 10 \end{bmatrix}$, $\Lambda = \begin{bmatrix} 3 & 1.25 \\ 1.25 & 1 \end{bmatrix}$, each strategic player has a total endowment of 2 units (whose allocation between the two assets is captured by the x-axis), and player 1 must liquidate all assets by $t = 1$.

Figure 7a: Probability of retail-trader distress vs. proportion of strategic traders' initial allocation in the illiquid asset. Retail traders have an initial endowment in the **illiquid** asset.

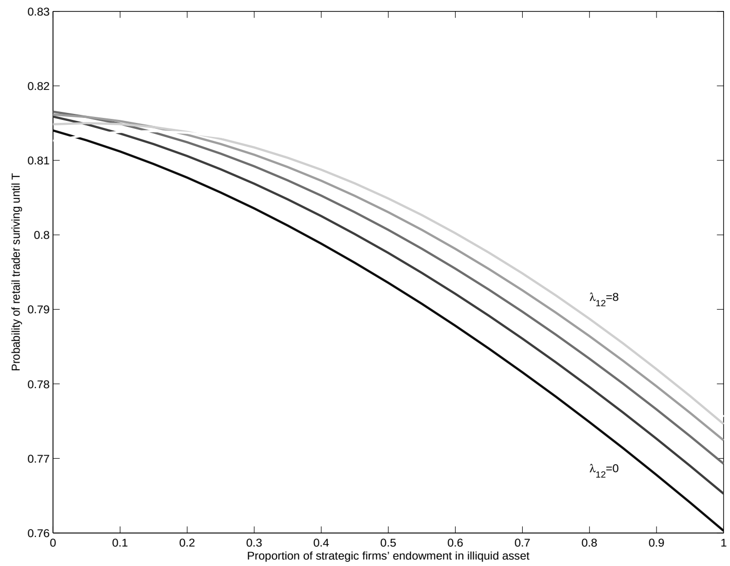
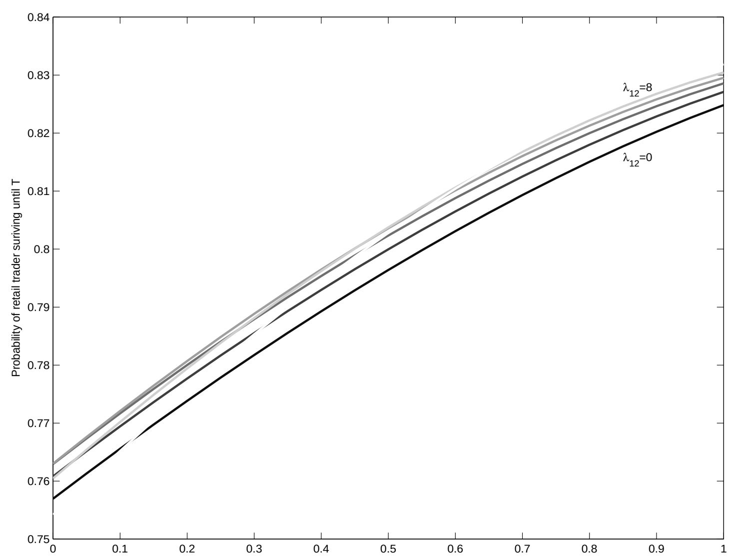


Figure 7b: Probability of retail-trader distress vs. proportion of strategic traders' initial allocation in the illiquid asset. Retail traders have an initial endowment in the **liquid** asset.



Figures generated under the following assumptions: $\Gamma = \begin{bmatrix} 10 & 5 \\ 5 & 10 \end{bmatrix}$, $\Lambda = \begin{bmatrix} 3 & \lambda_{12} \\ \lambda_{12} & 1 \end{bmatrix}$, each strategic player has a total endowment of 2 units (whose allocation between the two assets is captured by the x-axis), and player 1 must liquidate all assets by $t = 1$.