Maturity Rationing and Collective Short-Termism*

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April 22, 2013

Abstract

Financing terms and investment decisions are jointly determined. This interdependence links firms' asset and liability sides and can lead to short-termism in investment. In our model, financing frictions increase with the investment horizon, such that financing for long-term projects is relatively expensive and, potentially, rationed. In response, firms whose first-best investment opportunities are long-term may change their investments towards second-best projects of shorter maturities. This worsens financing terms for firms with shorter maturity projects, inducing them to change their investment as well. In equilibrium, investment is inefficiently short-term. Equilibrium asset-side adjustments by firms can amplify shocks and, while privately optimal, can be socially undesirable.

Keywords: short-termism, asymmetric information, debt maturity, asset maturity, credit rationing

*For helpful comments, we thank Viral Acharya, Heitor Almeida, Nittai Bergman, Patrick Bolton, Charlie Calomiris, Elena Carletti, Douglas Gale, Barney Hartman-Glaser, Zhiguo He, Florian Heider, Peter Kondor, Marcus Opp, Tano Santos, Elu von Thadden, and seminar participants at MIT Sloan, Columbia, ESSFM Gerzensee, the 2nd Tepper-LAEF Macro Finance Conference, the 4th meeting of the Finance Theory Group, the 2012 AEA meetings in Chicago, Mannheim, Stockholm School of Economics, ESMT Berlin, Binghamton, the 2012 OxFIT conference, the 2013 European Winter Finance Conference, University of Minnesota, University of Calgary, the New York Fed, and the NBER symposium on "Understanding the Capital Structures of Non-Financial and Financial Institutions."

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1 Introduction

Financing terms affect investment decisions and investment decisions affect financing terms. This interdependence creates an intimate link between firms' asset and liability sides. In particular, when financing for long-term projects is relatively expensive, firms may adjust their investment behavior towards shorter-term projects, even when those are less efficient. In fact, one important phenomenon in the recent financial crisis, and over the business cycle in general, is that maturities shorten both on the *liability side* and the *asset side* of firms' balance sheets.[1]

In this paper we develop an integrated equilibrium framework to study how financing frictions that arise on the liability side affect investments on firms' asset sides, and vice versa. In our model, contracting frictions due to limited commitment are more pronounced at longer horizons. This leads to less attractive financing terms, or even credit rationing, for long-term investment projects. Firms respond by adjusting their asset-side investments and search for alternative projects of potentially shorter maturity, even if those projects are second best. A central result of our paper is that these asset-side adjustments are self-reinforcing: An individual firm's asset-side decision endogenously determines the financing frictions faced by other firms influencing their investment decision, and in turn creating an externality. The competitive equilibrium exhibits inefficient short-termism in real investment relative to the constrained optimum. Moreover, due to their self-reinforcing nature, firms' equilibrium asset side adjustments can amplify shocks and, while privately optimal, they can

[1] See, for example, the recent studies by Mian and Santos (2011) and Chen, Xu, and Yang (2012) which document shortening of debt maturities on firms' liability sides during downturns. Dew-Becker (2011) documents that downturns are also associated with drops in the maturity (or duration) of the investments firms undertake on the asset side of their balance sheets.

be undesirable from a social perspective.

In our model, some firms have "good" projects which are risk-free and have positive NPV. Other firms have "bad" projects, which are risk-shifted versions of good projects – their riskiness, a mean-preserving spread, increases with the maturity of the project. Overall, these bad projects have negative NPV. The main friction is a limited commitment assumption in the spirit of Bolton and Scharfstein (1990) and Hart and Moore (1994): Ex post, successful bad firms can always pretend to be good firms and abscond with part of the cash flow. Firms seek financing from a financial sector that can observe the maturity of an investment project. Financing optimally occurs via a debt contract and, in order for the financier to break even, the interest rate on this debt contract has to increase with maturity in order to reflect larger limited-commitment frictions at longer horizons. This leads to less attractive financing terms for long-term projects. In fact, beyond a certain maturity, the limited commitment friction is so severe that financiers cannot break even, such that lending breaks down and *maturity rationing* arises.

The relatively unattractive funding terms (or non-availability of funding) for long-term projects generate the endogenous asset side adjustments central to this paper. In particular, firms whose first-best projects only receive funding at unfavorable terms (or cannot raise financing at all) react by searching for second-best projects of potentially shorter maturities, for which financing is cheaper. This action is unobservable to financiers and thus generates endogenous asymmetric information: The inflow of second-best projects worsens the pool of funded, shorter-maturity projects. This affects the terms of the debt contract offered by financiers, which takes the form of a pooling contract, leading to a negative externality: Funding terms for firms that up to now could receive financing worsen and, because the maximum funded maturity shortens, a number of formerly fundable firms is now rationed. Those firms respond by also searching for shorter term projects, inducing an additional inflow of second-best projects into the funded region. The process repeats and a *short-termism spiral*, illustrated in Figure 1, emerges. Taking into account the interdependence between the asset and liability sides, the equilibrium is thus given by a fixed point: Firms' investment decisions respond optimally to financing frictions on the liability side, while financiers take into account firm's investment decisions when extending financing.

When capital markets are competitive, the resulting equilibrium is *constrained inefficient*: Investment is inefficiently short-term (and surplus strictly lower) compared to the case in which financing is offered by a central planner who is subject to the same informational and limited commitment constraints as financiers. The inefficiency of the competitive equilibrium arises because, through their impact on the quality of the pool of firms seeking financing, the asset side adjustments made by individual firms affect the financing terms faced by *all* firms, generating an externality. In fact, when this negative externality from the adoption of second-best project is strong enough, the short-termism spiral can lead to a complete breakdown of financing across *all* maturities. When financing terms are offered by a planner, this negative feedback loop is mitigated. Specifically, a planner would subsidize long-term projects and tax short-term projects, in order to counteract the excessive short-termism of the competitive equilibrium.

Because of their self-reinforcing nature, firms' privately optimal asset side adjustments information can amplify shocks. For example, an increase in the difference in riskiness between good and bad projects can lead to significantly larger reductions in financed maturities and reductions in surplus in a setting where firms can adjust their asset side, relative to the

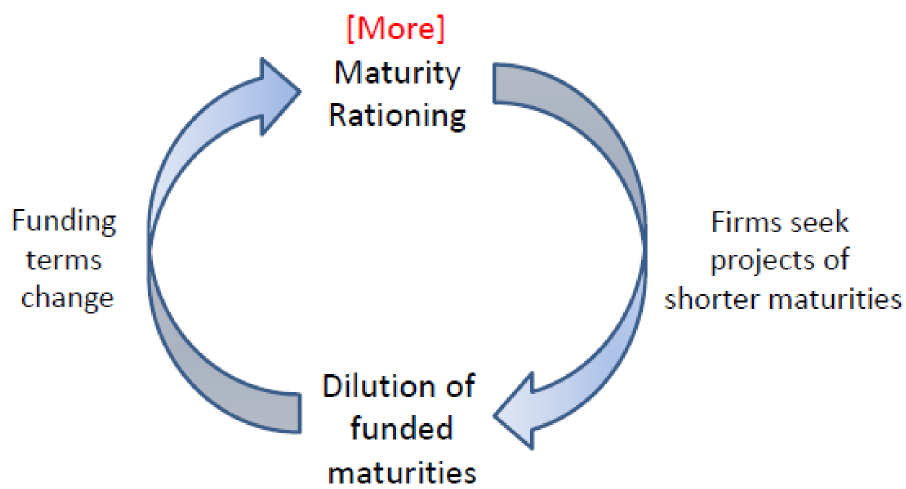

Figure 1: Illustration of the short-termism spiral that emerges from endogenous adjustments on the asset side in response to asymmetric information frictions on the liability side.

benchmark in which firms' asset sides are held fixed. We also show that whether firms' equilibrium asset-side adjustments increase or decrease surplus depends on the severity of the cross-firm externality. At one extreme, when second-best projects are (essentially) as good as first-best projects, the ability of firms to adjust their asset side investments increases surplus. In this case, firms that adopt shorter maturity projects do not impose an externality on other firms. The only consequence from their maturity adjustments is an increase in output, as formerly rationed firms find shorter maturity projects that can be funded. On the other hand, when second-best projects are worse than firms' original first-best projects, privately optimal asset-side adjustments can lead to an overall reduction in surplus. This reduction in surplus can occur even when second-best projects have positive NPV or when, as a result of firms' asset side adjustments, more projects get financed and thus total lending increases. The reason is that the drop in average project quality can outweigh the gains from increased lending.

Overall, our results underscore the importance considering firms' asset side decisions and their interaction with the pricing of funding instruments of different maturities, such as short-term and long-term debt. In particular, our analysis highlights a potentially important cost of short-term debt that has not received much attention in the literature. Most of the literature on short-term debt follows Diamond and Dybvig (1983) and Diamond (1991)[2] in focusing on early liquidation of (fixed) investment projects as the main cost of short-term debt. Our analysis draws attention to a different channel: Short-term financing changes firms' investment behavior and generates inefficient endogenous short-termism on the asset side. Unlike many standard explanations of short-termism that have focused on bad incentives and behavioral biases as sources of short-termism (such as labor market reputation building (Narayanan (1985)), concern with near-term stock prices (Stein (1989)), short-term investor horizons (Froot, Perold, and Stein (1992)) or short-term incentive schemes (Bolton, Scheinkman, and Xiong (2006))), in our framework (collective) short-termism arises as an equilibrium phenomenon in a fully rational setting.

In highlighting this link between short-term financing and short-termism in investment, our paper is related to von Thadden (1995) and Dewatripont and Maskin (1995), who study settings in which firms may adopt short-term projects in fear of being liquidated at an interim date. In contrast, in our model it is not the fear of early liquidation that leads to short-termism, but the fact that financing is either unavailable or only available at unattractive terms for long-term projects. More importantly, though, short-termism in von Thadden (1995) is part of a constrained efficient outcome: The gain from reducing liquidation risk outweighs the social cost of adopting less productive short-term projects. In contrast, because

[2]Among the recent papers in this vein are, for example, Acharya, Gale, and Yorulmazer (2011), He and Xiong (2012), and Brunnermeier and Oehmke (2013).

of the cross-firm externalities that arise in the presence of endogenous adverse selection, short-termism is constrained *inefficient* in our framework, which leads to different policy implications. For example, in our framework policy measures to subsidize long-term investment can raise welfare. In Dewatripont and Maskin (1995) a related inefficiency can arise. While in our model the unique competitive equilibrium is inefficiently short-term because of the relatively higher cost of funding long-term investment, in Dewatripont and Maskin (1995) an interim liquidation threat can lead to multiple Pareto-ranked equilibria. In the inferior equilibrium, firms with good long-term projects adopt short-term projects for fear of being liquidated. Another recent paper that stresses asset and liability side interactions is Cheng and Milbradt (2012), who develop a model in which a firm trades off liquidation costs arising from liability side frictions (specifically, debt runs) against asset side distortions that arise from managerial risk-shifting. However, they focus on a single firm in isolation, such that the cross-firm externalities, which are central to this paper, cannot emerge.[3] In highlighting how asymmetric information can lead to cross-firm externalities that can amplify the response of equilibrium prices and quantities to shocks, our paper is related to Eisfeldt (2004), Kurlat (2010) and Bigio (2011), who study amplification through asymmetric information in macroeconomic settings without maturity choice. Finally, as a point of departure, our paper builds on the extensive literature on credit rationing.[4]

[3]A recent paper that also highlights spillover effects among firms is Bebchuk and Goldstein (2011). In their setup, spillovers arise directly in project payoffs (projects become more attractive as more firms invest, leading to a payoff externality), while in our framework spillovers arise due to endogenous asymmetric information on the financing side (leading to an information externality).

[4]For a summary of this literature, a good starting point is the discussion in Bolton and Dewatripont (2005, Chapter 2), Freixas and Rochet (2008, Chapter 5), or the survey on financial contracting by Harris and Raviv (1992). The classic contributions on credit rationing are Jaffee and Modigliani (1969), Jaffee and Russell (1976), Stiglitz and Weiss (1981), and DeMeza and Webb (1987). Bester (1985) and Besanko and Thakor (1987) examine the role of collateral as a screening device in models with credit rationing. Suarez and Sussman (1997) develop an overlapping generations model in which credit rationing can lead to endogenous business cycles.

The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 derives the competitive equilibrium of asset-side choices and liability-side funding terms and contrasts the resulting equilibrium in which the asset side is held fixed. Section 4 demonstrates that, relative to the allocation implemented by a constrained central planner, the competitive equilibrium exhibits excessive short-termism. Section 5 discusses the importance of explicitly taking into account firms' asset-side responses to financing frictions on the liability side, highlighting in particular the role of asset side adjustments in the amplification of shocks and pointing out that those asset side adjustments, while privately optimal, may be socially undesirable. Section 6 concludes. All proofs are in the appendix.

2 Model Setup

There is a continuum of firms, each of which is born with an (initial) investment project of a maturity t, which is drawn uniformly from the interval $[0, \overline{T}]$. The maturity of a project indicates how long it takes for the project to pay off: A project of maturity $t \in [0, \overline{T}]$ generates cash flow only at date t and no cash flows beforehand (or after). Firms seek financing from a financial sector composed of a continuum competitive, risk-neutral financiers with deep pockets.

While project maturity is commonly observable (such that there is no uncertainty about when a particular project pays off), the quality of projects is not observable ex-ante (either to firms or financiers). Some firms are born with positive NPV projects ("good" projects), while others are born with negative NPV (or "bad") projects. Financiers and firms merely know that, out of the initial projects, a fraction β has positive NPV, while a fraction $1 - \beta$

has negative NPV.

Both good and bad projects cost 1 dollar to set up. Good projects are risk-free and have positive NPV. Specifically, they pay off a certain amount R at maturity t. For simplicity, we set the risk-free rate to $r = 0$,[5] such that good projects, irrespective of their maturity, have a constant positive NPV of

$$NPV_G = R - 1 > 0. \tag{1}$$

Bad projects, on the other hand, are risky and have negative NPV — bad projects pay off only with probability $\Delta e^{-\lambda t}$. However, conditional on success, bad projects pay off $e^{\lambda t}R$, which is more than R, the payoff from a good project. The interpretation of these cash flows is that the manager of a bad firm can undertake some amount of risk shifting *at each moment in time*, which generates a natural link between project maturity and project risk. The parameter $\lambda > 0$ can be interpreted as a constant intensity of default that results from risk shifting, while $\Delta < 1$ is the probability that the projects fails instantaneously after being undertaken. Taken together, bad projects have a constant negative NPV of[6]

$$NPV_B = \Delta R - 1 < 0. \tag{2}$$

The cash flows that are realized at maturity are private information to the firm and are not contractible, which introduces a limited commitment friction in the spirit of Bolton and

[5]It would be straightforward to allow for $r > 0$. In this case, to keep NPV of the good project constant irrespective of its maturity, the payoff would have to be adjusted to $e^{rt}R$. None of our results would change.

[6]Our results are robust to a number of variations in these assumptions. For example, it is not necessary to assume that the drift of bad projects exactly compensates for the default intensity λ. However, this assumption is convenient because it guarantees that the NPV of bad projects is independent of the project maturity. Hence, our results are driven by differences in financial frictions (arising from limited commitment) across different maturities as opposed to differences in NPV across different maturities.

Scharfstein (1990) and Hart and Moore (1994). Specifically, we assume that at maturity it can only be verified whether or not a project succeeded, but not which exact cash flow has realized. Hence, a successful firm with a bad project, which receives $e^{\lambda t}R$ at maturity, can always pretend to have only received R, and pocket the difference. This contracting friction thus limits the amount financiers can extract in case of project success to R.

While firms are born with a first-best project of some maturity t drawn uniformly from $\left[0, \overline{T}\right]$, we allow them to adjust their investment decisions in response to financing frictions. If, for example, firms with long-term projects expect financing terms for long-term projects to be unattractive, they can alter their asset side investment decision and search for a new project of potentially shorter maturity. However, this asset side adjustment comes at a cost: When a firm searches for a new project, this reduces the attractiveness of the investment in the sense that this new project is only second-best. This assumption captures that it is costly for firms to adjust their investments away from their first-best investment opportunities. One interpretation of this second-best project is that the firm literally searches for a new, second-best investment project that it will undertake instead of the first-best project it was born with. An alternative interpretation is that the firm attempts to implement a rushed version of the original project, in which the firm speeds up the required research and construction or cuts corners in the implementation of the project.

We make the following assumptions to make the analysis of firms' asset side adjustments tractable: After a firm learns the maturity of its first-best project, it can decide to search for a new project with a different and potentially shorter maturity. To search for a new project, firms effectively play a bandit: If a firm decides to change its project maturity, the firm receives a new project drawn from the original maturity distribution (i.e., uniform on

$[0, \overline{T}]$).[7] However, this new project is second best, in the sense that the new project is of high quality only with probability $\alpha\beta$, where $\alpha \in [0, 1]$. Drawing a new project has no direct cost for firms (i.e., the firm can play the bandit for free), but it has an implicit cost since the firm loses access to its original project.

The potential adjustments on firms' asset sides change the distribution of average project quality as a function of maturity. This is depicted in Figure 2: The left panel depicts the initial distribution of projects and their quality across the maturity spectrum. Suppose now that all firms that have projects with a maturity of six years or more search for a new project. The resulting distribution of projects and their quality is given in the right panel of the figure. Whereas before redrawing (left panel) all projects are good with probability β, after redrawing of projects we obtain a step-function (right panel). To the left of the redrawing cutoff, the average project quality is a mixture of qualities β and $\alpha\beta$, whereas to the right of the cutoff all projects have an average quality of $\alpha\beta$.

From the financiers' point of view, firms with second-best projects are contractually indistinguishable from firms with first-best projects (in our model, first-best firms have no way to separate themselves from second-best firms, because there is no scope for signaling). Hence, financing at any maturity t is only possible if the financier can break even on a pooling contract. We assume that financiers maximize profits and compete by simultaneously offering take-it-or-leave-it funding schedules contingent on the project maturity $t \in [0, \overline{T}]$, taking into

[7] While we assume that firms receive a new project from the original maturity distribution $[0, \overline{T}]$, one could also assume that a firm with an initial project of maturity t can search for a new project on $[0, t]$. In this alternative specification, search for new projects is directed, in the sense that each firm looking for a new project will receive a project of shorter maturity. An advantage of our formulation is that shortening is not a primitive feature of the redrawing technology but arises endogenously in the equilibrium. Moreover, the alternative specification with redrawing on $[0, t]$ requires additional restrictions on the funding strategy space to guarantee a unique equilibrium.

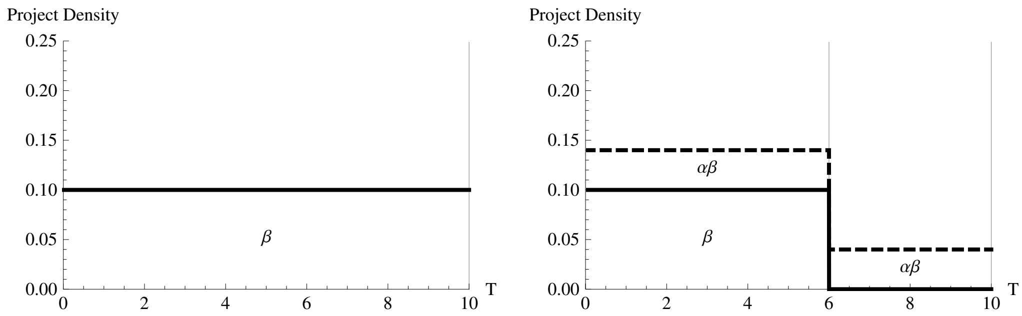

Figure 2: **Density of average project quality** as a function of the project maturity. **Left panel:** The average quality distribution before any redrawing is flat. **Right panel:** After redrawing at $T = 6$, the projects that were originally on $(6, 10]$ are evenly distributed over $[0, 10]$ with average quality $\alpha\beta$, depicted by the dashed line. The original projects that did not get redrawn are depicted by the solid black line.

account firms' equilibrium asset side decisions and the resulting quality distribution as a function of project maturity. After the schedules of financing terms have been posted, firms make their redrawal decisions and fund themselves at the best rate they can find if funding is available. Note that this implies that firms respond to deviations by investors when making their project redrawal decisions. Competition among financiers implies that they have to break even for every funded maturity. Note that this rules out cross-subsidization across maturities.

Given our assumption on the verifiability of cash flows, the optimal financial contract that pools good and bad types takes the from of a debt contract.[8] We focus on debt contracts that match the maturity of the project. However, we show in the appendix that this is without loss of generality: Our results extend to a setting where firms are allowed to finance their investment through sequences of rollover debt contracts.[9]

[8]Note that because we set the low cash flow to zero, debt contracts can, strictly speaking, also be structured as equity contracts. However, if the payoff in the low state was $L > 0$, then a debt contract becomes strictly optimal: To relax the incentive constraint in the high state, it is then optimal to extract as much as possible in the low state.

[9]We can focus on debt contracts that match the maturity of the project because, in our model, firms are indifferent between financing with a debt contract that matches maturities and any sequence of rollover

3 Competitive Equilibrium

We now derive the Bayesian Nash equilibrium of funding decisions and investment decisions given the setup introduced in Section 2. A Bayesian Nash equilibrium is given by a a set of project redrawal decisions by firms and funding terms offered by financiers that satisfy the following conditions:

Definition 1 *A Bayesian Nash equilibrium is given by (i) project redrawing decisions by firms and (ii) funding conditions offered by the financier sector, such that:*

1. *Given the funding terms offered, firms that are funded at their original maturity have no incentive to redraw the maturity of their projects (incentive compatibility (IC) constraint).*
2. *Investors break even at each funded maturity (individual rationality (IR) constraint).*

For simplicity, we restrict the funding strategies to strategies that do not allow partial rationing within a given maturity t. This means that, for any funded maturity t, all firms can obtain unlimited funding at the funding terms offered by financiers.

3.1 Benchmark: Fixed Asset Side

Before we characterize the full equilibrium with endogenous asset side, it is instructive to briefly consider the equilibrium under the assumption that the asset side is fixed (i.e., firms cannot redraw their project). In this case, we can neglect the IC constraint and the equilibrium is solely driven by the IR constraint. The limited commitment friction implies that the

debt contracts. Intuitively, because project maturity is observable and because there are no intermediate signals about project quality (except the event of default), such rollover contracts do not add anything to the contracting environment.

maximum financiers can extract from a successful project is given by R: Raising the face value of debt above R leaves the amount paid back to financiers unchanged because firms with a payoff higher than R would simply pretend to have received R. Thus, the maximum "effective" face value on the debt contract is given by R, and a project of maturity t can be financed if and only if the break-even face value D of the pooling debt contract satisfies

$$D(t, \beta) \leq R. \tag{3}$$

Under our assumption of competitive capital markets, given a pool quality of β, the face value of debt for each project maturity t is determined by the financiers' break-even condition:

$$\beta D + (1 - \beta)\,\Delta e^{-\lambda t} D = 1. \tag{4}$$

This break-even condition implies a face value of debt of

$$D\left(t, \beta\right) = \frac{1}{\beta + (1 - \beta)\,\Delta e^{-\lambda t}}. \tag{5}$$

Note that the break-even face value (5) is increasing in project maturity, $D_t\left(t, \beta\right) > 0$. This reflects the increased riskiness of bad projects at longer horizons (even though their NPV is fixed). Given the constraint $D\left(t, \beta\right) \leq R$, this implies that funding is available up to some maximum maturity T_B. Beyond T_B, no projects are financed and maturity rationing arises, as illustrated in Figure 3.

The main takeaway from this benchmark case with fixed asset side is that, because financing frictions increase with project maturity, funding terms for longer maturity projects

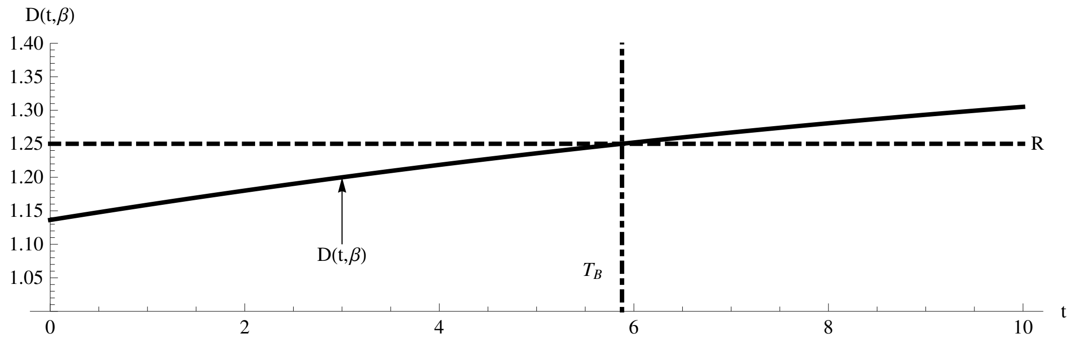

Figure 3: **Fixed Asset Side: Face value and maturity.** The figure illustrates the benchmark case with a fixed asset side. For maturities below 5.9 years, the required face value $D(t, \beta)$ lies below R, such that a pooling equilibrium exists. Beyond a maturity of 5.9 years, the required face value exceeds R, such that good firms drop out and no pooling equilibrium exists. These maturities cannot be funded in equilibrium.

are less favorable than those for shorter maturities. When $T_B < \overline{T}$, maturity rationing arises and some long-term projects cannot be funded. In the full model with endogenous asset side, this means that firms with long-term investment projects may look for alternative projects, either to gain funding or to gain funding at better terms.

3.2 Full Model: Endogenous Asset Side

We now solve for the full equilibrium with endogenous asset side. This involves making sure that both the IC constraint (firms funded at their original maturity have no incentive to redraw their project) and the IR constraint (investors break even at each funded maturity) are satisfied. In contrast to the model with fixed asset side, the quality of projects at a given maturity is now endogenous, because it depends on firms' asset side adjustments, as illustrated in Figure 2.

As we show in the appendix, the unique (pure-strategy) Bayesian Nash equilibrium takes the form of a cut-off equilibrium, in the sense that firms with initial maturities beyond this cutoff search for new projects, while firms with initial project maturities below this cutoff stick with their initial project. Given the cutoff nature of the equilibrium, we denote by $\hat{\beta}(T)$ the average pool quality on $[0, T]$ under the assumption that firms on $\left(T, \overline{T}\right]$ redraw their maturities. For example, in Figure 2, the cutoff is given by $T = 6$, such that $\hat{\beta}(6)$ is the mixture of average qualities β and $\alpha\beta$ on the interval $[0, 6]$. Given our assumption of a uniform distribution, is is straightforward to show that

$$\hat{\beta}(T) = \beta \frac{\overline{T} + \alpha\left(\overline{T} - T\right)}{2\overline{T} - T} \leq \beta. \tag{6}$$

Moreover, when firms on $\left(T, \overline{T}\right]$ redraw their maturities, the average pool quality on $(T, \overline{T}]$ is simply given by $\alpha\beta < \hat{\beta}(T)$, because the only firms in this interval are ones who, after searching for a second-best project, again ended up on $\left(T, \overline{T}\right]$.

Let us now consider the IC and IR constraints. To derive the IC constraint, consider an individual firm's incentive to change its maturity. Unfunded firms always have an incentive to search for a new project since doing so is costless and hence constitutes a free option. In determining the IC constraint, we can thus concentrate on the incentive for funded firms to redraw their maturities. To determine the incentive for funded firms, we define the expected profit of a firm of type $\theta \in \{\alpha\beta, \beta\}$ with a project of maturity t that accepts competitive funding based on an average project quality of $\hat{\beta}$ as

$$\begin{aligned} \pi\left(\theta, t, \hat{\beta}\right) &= \theta \overbrace{\left[R - D\left(t, \hat{\beta}\right)\right]}^{\text{Exp. payoff good}} + (1-\theta)\overbrace{\Delta\left[R - e^{-\lambda t} D\left(t, \hat{\beta}\right)\right]}^{\text{Exp. payoff bad}} \\ &= \left[\theta + (1-\theta)\Delta\right] R - \frac{D\left(t, \hat{\beta}\right)}{D(t, \theta)} \end{aligned} \tag{7}$$

The intuition behind this equation is as follows: With probability θ, the firm has a good project and receives the expected payoff of a good firm given a pool quality of $\hat{\beta}$. With probability $1-\theta$, the firm has a bad project and thus receives the expected payoff for a bad firm given a pool quality of $\hat{\beta}$. Then, the incentive constraint not to redraw at maturity t is given by

$$\pi\left(\beta, t, \hat{\beta}\right) \geq \frac{1}{\overline{T}} \int_{[0,T]} \pi\left(\alpha\beta, s, \hat{\beta}\right) ds, \tag{8}$$

where the right hand side is the expected payoff of drawing a new project of quality $\alpha\beta$ from the original maturity distribution. For a given funded set $[0,T]$ and average quality $\hat{\beta}$ on that funded set, we can then define the *net value of redrawing* for a firm with a project of maturity t as

$$NVR\left(t, \hat{\beta}\right) \equiv \frac{1}{\overline{T}} \int_{[0,T]} \pi\left(\alpha\beta, s, \hat{\beta}\right) ds - \pi\left(\beta, t, \hat{\beta}\right). \tag{9}$$

Among funded firms, the incentive to redraw the maturity of the project is largest for the firm with the highest maturity project. This follows directly from the observation that $NVR_t\left(t, \hat{\beta}\right) > 0$. The intuition for this finding is as follows. A firm that has not redrawn its project knows that it has a good project with probability β. The market, however, treats this firm as being drawn from a pool of $\hat{\beta}(T) < \beta$. A firm of quality β thus has to pay a financing rate that is higher than the fair rate on a pool of quality β. The gap between the required rate and the fair rate for a pool of quality β increases with project maturity t, such that the benefits of searching for a new project increase with the maturity of the firm's original project.

Hence, in order to check the IC constraint, we can restrict our attention to the firm located at the conjectured switching cutoff T. The firm located at T has no incentive to redraw whenever $\hat{\beta}(T)$, the induced average quality on $[0,T]$, is such that the net value of redrawing is (weakly) negative (i.e., $NVR\left(T, \hat{\beta}(T)\right) \leq 0$).

Lemma 1 *Suppose firms on* $\left(T, \overline{T}\right]$ *redraw maturities. Then among the firms that have not redrawn their maturity, the firm at the cutoff* T *has the strongest incentive to redraw. The IC constraint can thus be written as* $NVR\left(T, \hat{\beta}(T)\right) \leq 0$.

We denote the set off redrawing cutoffs that satisfy this incentive constraint by the non-redrawing set

$$\mathcal{IC} \;=\; \left\{T \in [0, \overline{T}] : NVR\left(T, \hat{\beta}(T)\right) \leq 0\right\}. \tag{10}$$

We now turn to the financiers' IR constraint. As we saw in our analysis of financing with exogenous asset side, the maximum "effective" face value is equal to R which means that projects of maturity t can be funded if and only if the break-even face value satisfies $D\left(t, \hat{\beta}(T)\right) \leq R$. Because, given a fixed average project quality $\hat{\beta}$, the face value of debt is increasing in maturity, $D_t\left(t, \hat{\beta}\right) > 0$, it is again sufficient to check this condition at the cutoff T. Hence, we can define the set of cutoff strategies T that can be funded under the conjecture that everyone on $\left(T, \overline{T}\right]$ decides to draw a new project as

$$\mathcal{IR} = \left\{T \in [0, \overline{T}] : D\left(T, \hat{\beta}(T)\right) \leq R\right\}. \tag{11}$$

In words, for each $T \in \mathcal{IR}$, we know that the conjectured cutoff T implies a pool quality $\hat{\beta}(T)$ such that at the highest funded maturity, the required break-even face value of debt does not exceed R.[10]

[10]Note that the set $\mathcal{IR}$ can have holes. For example, while a conjectured cutoff of nine years may be

To see the different effects at play, it is instructive to consider how the face value at the maximum funded maturity, $D\left(T, \hat{\beta}(T)\right)$, changes as the funding threshold T changes. Writing out the derivative, we have

$$\frac{dD\left(T, \hat{\beta}(T)\right)}{dT} = D(T,x)^2 \left[\underbrace{\Delta\lambda e^{-\lambda T}\left(1-\hat{\beta}(T)\right)}_{\text{Maturity effect}} - \underbrace{\left(1-\Delta e^{-\lambda T}\right)\hat{\beta}'(T)}_{\text{Dilution effect}}\right], \tag{12}$$

where $\hat{\beta}'(T) > 0$, because a higher funding threshold means that fewer firms draw second-best projects. Thus, reducing the maximum funded maturity T has two countervailing effects. The maturity effect leads to a decrease in the required face value because, all else equal, financing frictions are less severe at shorter maturities. The dilution effect, on the other hand, implies that reducing the maximum funded maturity leads to more redrawing and thus lowers the average project quality on the funded interval. This leads to an increase in the face value required to break even.

Taking into account both the IR and IC constraints, we are now in a position to characterize the equilibrium with endogenous asset side.

Proposition 1 *The unique pure-strategy Bayesian Nash equilibrium under competitive capital markets is given by a funding cutoff strategy of the form* $[0, T^*]$ *with*

$$T^* = \max\{\mathcal{IR} \cap \mathcal{IC}\}. \tag{13}$$

able to receive funding ($9 \in \mathcal{IR}$ so $[0,9]$ could be funded) a conjectured cutoff of six years may not be in the funded set ($6 \notin \mathcal{IR}$ so $[0,6]$ will not be funded), while a cutoff of three years may again be fundable ($3 \in \mathcal{IR}$). The reason is that the dilution of the pool that results when more firms redraw may make the cutoff strategy $T = 6$ not fundable, even if $T = 9$ and $T = 3$ are in the funded set. In the appendix we provide a more detailed characterization of the funded set $\mathcal{IR}$.

If $\mathcal{IR} \cap \mathcal{IC} = \emptyset$, no funding is provided in equilibrium.

Proposition 1 states that, in equilibrium, funding is provided up to the longest maturity for which both the IC and the IR constraints are satisfied. The equilibrium always exists.[11] If at the maximum maturity $\overline{T}$ both constraints are satisfied, funding is provided for all maturities ($T^* = \overline{T}$) and no redrawing of projects occurs. If at $\overline{T}$ at least one of the constraints is violated, some firms redraw their projects leading to the feedback effects discussed above. In this case, the equilibrium funding cutoff is strictly less than the maximum maturity (i.e., $T^* < \overline{T}$). If the IR and IC constraints cannot be satisfied simultaneously ($\mathcal{IR} \cap \mathcal{IC} = \emptyset$), no funding is provided in equilibrium and funding completely unravels. We will discuss this case in more detail in Section 3.3.

The result that financing is provided up to the largest maturity that satisfies both the IR and IC constraints follows from competition among financiers. Specifically, for any conjectured funding cutoff below $\max\{\mathcal{IR} \cap \mathcal{IC}\}$, there is a profitable deviation in which a financier increases the range of funded maturities he offers. As a result of this deviation, fewer firms redraw their maturity, such that the average project quality the financier is facing increases. This allows the financier to charge a lower interest rate and thereby undercut the other financiers, attracting all firms at a funding rate that is strictly profitable. Financiers thus compete in this fashion until all maturities that satisfy both the IR and the IC constraints are offered.

If the firms' IC constraint is binding at the equilibrium cutoff (i.e, T^* is on the boundary

[11]Existence of the equilibrium is guaranteed because all that is necessary is that the sets $\mathcal{IR}$ and $\mathcal{IC}$ are well-defined, which is always the case. The equilibrium is unique because firms' redrawing decisions do not exhibit strong enough strategic complementarities: Intuitively speaking, as more firms redraw and the conjectured funding cutoff shortens, redrawing effectively becomes more costly for firms, because the probability of ending up in the unfunded region increases. This effect is strong enough to guarantee uniqueness.

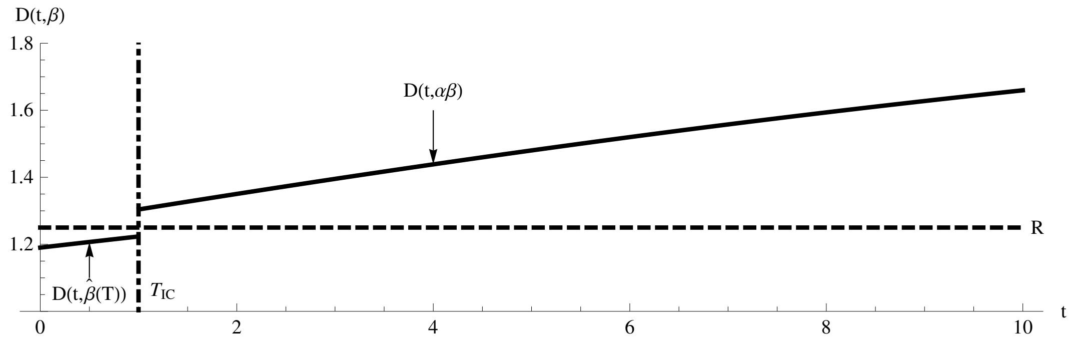

Figure 4: An example of an IC-driven equilibrium, with $R > D\left(T_{IC}, x\left(T_{IC}\right)\right)$.

of $\mathcal{IC}$), we refer to the equilibrium as *IC driven*. Figure 4 depicts such an equilibrium. Compared to the benchmark case in Figure 3, we see that the face-value function now depends on the average quality induced by firms' asset side adjustments. Moreover, when $\alpha < 1$, the required face value is discontinuous at the redrawing cutoff T: it jumps from $D\left(t, \hat{\beta}\left(T\right)\right)$ to $D\left(t, \alpha\beta\right)$ as t crosses the cutoff T. Note also that, in an IC-driven equilibrium, the face value of debt is below R at the cutoff $t = T$. This means that financiers would be willing to fund beyond T if firms did not redraw the maturity of their projects once the cutoff T is crossed. However, for firms it is privately optimal to redraw beyond T, because the possibility of improved funding conditions at shorter maturities outweighs certain funding at relatively bad terms that would be available were they not to redraw.[12]

If the financiers' IR constraint is binding at T^* (i.e., T^* is on the boundary of $\mathcal{IR}$), we call the equilibrium *IR driven*. Figure 5 depicts such an equilibrium. Also here, compared to Figure 3, the face-value function is discontinuous at the cutoff T. In contrast to the IC driven equilibrium in Figure 4, the IR driven equilibrium depicted in Figure 5 exhibits a binding IR constraint at $t = T$, i.e., $D\left(T, \hat{\beta}\left(T\right)\right) = R$.

[12]While not shown in Figure 4, it is possible that in the IC driven equilibrium some firms beyond T_{IC} are financed. This is the case when $D\left(T_{IC}, \alpha\beta\right) < R$, such that some firms of quality $\alpha\beta$ can be financed. Although theoretically possible, this case does not arise in any of our numerical examples.

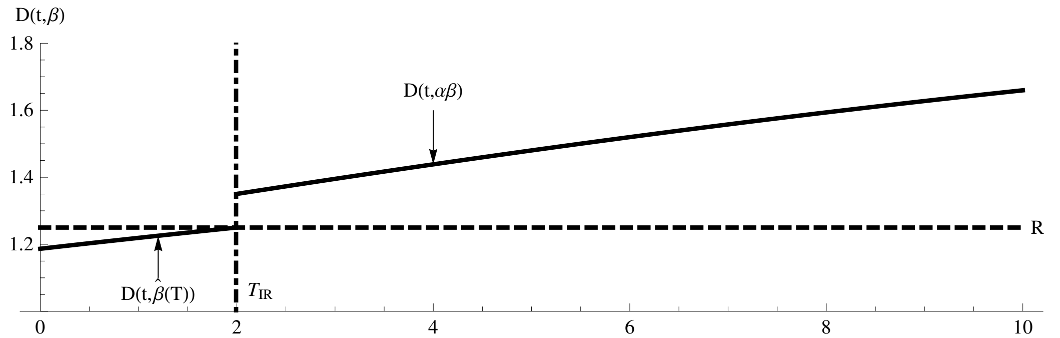

Figure 5: An example of an IR-driven equilibrium, with $R = D(T_{IR}, x(T_{IR}))$.

The intuition behind the equilibrium is as follows: Although financiers compete locally (i.e., maturity by maturity), they take into account that any deviation strategy that offers funding to a range of maturities that was previously unfunded affects the incentives of *all* firms. This is because such a deviation changes the IC constraint at every maturity, and therefore the pool quality. However, even though financiers internalize the effect of their funding decisions on the average project quality, they are restricted in their strategies because of competition at each maturity. Firms, on the other hand, ignore the impact of their individual investment decision on the aggregate outcome because they take the average pool quality as given. This difference between firms and financiers is driven by the scale of their impact: Each firm can only undertake one infinitesimal project and thus cannot affect the aggregate. Financiers, on the other hand, can affect the aggregate: Even though they are also infinitesimal, they offer contracts to and thereby affect the behavior of a mass of firms.

3.3 Complete Unraveling of Funding

One interesting implication of Proposition 1 is that the dilution effect of second-best projects can be so strong that funding markets can completely unravel. Complete unraveling results when, after taking into account firms' equilibrium redrawing behavior and financiers' equilibrium funding adjustments, the funded interval is empty: $\mathcal{IC} \cap \mathcal{IR} = \emptyset$. The necessary and sufficient condition for complete unraveling is that the dilution parameter α is low enough relative to the funding interval that is implied if only the IC constraint were imposed, i.e., $T^*_{IC} \equiv \max \mathcal{IC}$. The intuition is that, when α is sufficiently low, a reduction in the funding cutoff, say from T to T', does not relax the IR constraint: The reduction in the funding cutoff induces more firms to adopt second-best projects and, for low enough α, this leads to a dilution in project quality that prevents financiers from breaking even at the lower cutoff T'. In other words, in the spirit of equation (12), the dilution effect outweighs the maturity effect sufficiently on $[0, T]$ so that break-even funding is never possible. The following proposition summarizes this result on complete unraveling of funding.

Proposition 2 $[0, T] \notin \mathcal{IR}$ *if and only if* $\alpha < \overline{\alpha}(T)$. *Thus, funding completely unravels when* $\alpha < \overline{\alpha}(T^*_{IC})$.

4 Central Planner Equilibrium

We now contrast the competitive equilibrium derived above with the allocation that would be implemented by a central planner facing the same informational constraints as the financiers (and the same restriction to pure strategies). The main difference between the solution to the constrained planner's problem and the competitive equilibrium is that, in contrast to the competitive financiers, the planner can cross-subsidize across maturities: While the planner also has to break even, he only faces a break-even constraint over the entire funded interval $[0, T_{cp}]$. Competitive financiers, on the other hand, have to break even for every maturity t

separately, which rules out cross-subsidization. As we show below, the central planner optimally taxes short-term projects and subsidizes long-term projects. This allows more projects to be financed and reduces firms' incentive to search for second-best projects, thus leading to a higher average project quality. Because the constrained planner can improve upon the competitive allocation, the competitive equilibrium is constrained inefficient. In contrast to some of the standard explanations of short-termism as resulting from bad incentives (e.g., reputation building as in Narayanan (1985) or concern with near-term stock prices as in Stein (1989)), in our framework, short-termism emerges as an equilibrium phenomenon in a fully rational setting.

The central planner's objective is to pick a funding cutoff T_{cp} and a sequence of face values $D_{cp}(t)$ to maximize aggregate NPV subject to three constraints. First, the planner has to break even (on average) over all funded maturities. Second, firms in the funded region must be content not to change the maturity of their project. Third, as before, the effective face value on the funded interval cannot exceed R. The following proposition establishes the optimal central planner funding policy:

Proposition 3 *The central planner optimally sets*

$$D_{cp}\left(t|\hat{t}, T_{cp}\right) = \begin{cases} \frac{C}{\beta+(1-\beta)\Delta e^{-\lambda t}} & t \leq \hat{t} \\ R & t > \hat{t} \end{cases},$$

where (i) $C = R\left[\beta + (1-\beta)\Delta e^{-\lambda\hat{t}}\right]$, *(ii)* T_{cp} *is the largest cutoff on* $\left[0, \overline{T}\right]$ *that fulfills the break-even condition, and (iii)* $\hat{t}$ *is the point at which face value reaches* R. *If* $\hat{t} > T_{cp}$, *the maximum face value* R *is never charged.*

We now discuss the main differences between the allocation implemented by the planner and the competitive equilibrium. Effectively, the planner picks the highest funding cutoff T_{cp} that satisfies all three constraints. This is done by picking a schedule of face values $D_{cp}(t)$ that, at each $t \in [0, T_{cp}]$, makes firms just indifferent between retaining their original project and searching for a new project. An exception to this policy arises in cases where the face value that makes firms indifferent between retaining their original project and searching for a new project would be larger then R, which happens at $\hat{t}$. Beyond $\hat{t}$ the planner sets the face value to the maximum effective face value R. Such a funding schedule is called the least-cost implementation for the funding set $[0, T_{cp}]$ as described in the appendix. Compared to the competitive equilibrium, the planner thus extracts more surplus at short maturities (effectively imposing a tax), which allows the planner to extend funding at maturities that cannot be funded in competitive equilibrium (effectively introducing a subsidy for long-term projects).[13]

The solution to the planner's problem is illustrated in Figure 6. The top panel depicts the case in which the planner makes all funded firms indifferent between staying at their original maturity in redrawing and picks as the funding cutoff the highest T such that he breaks even across all maturities. The bottom panel depicts the case in which the planner makes firms indifferent between staying and redrawing until the required face value reaches R. Beyond that point, the planner keeps the face value at R and funds up to the point where he just breaks even across all maturities. In both cases, the planner charges higher face values

[13] An interesting question is whether a monopolist (or, more generally, an imperfectly competitive capital market) would also find it optimal to subsidize long-term projects. We characterize the monopolist's problem in the appendix. There, we show that it is possible that the monopolist cross subsidizes and funds some loss-making longer-maturity projects in order to extract more from shorter maturities. However, depending on parameters, it is also possible that the monopolist finds it optimal not to cross subsidize and ends up choosing a funding cutoff where the marginal project is still strictly profitable.

for lower maturities than competitive financiers and—through this cross-subsidization—is able to fund a larger set of maturities than would be funded in competitive equilibrium. In competitive equilibrium such cross-subsidization is ruled out by maturity-by-maturity competition among financiers, a point that is similar to the impossibility of cross-subsidization between types in Rothschild and Stiglitz (1976), or the inability of a bank to cross-subsidize between patient and impatient consumers in the Diamond and Dybvig (1983) model when those consumers have access to the underlying asset markets (Jacklin (1987)). Contrasting with the competitive equilibrium, we see that it is the inability of the competitive firms to cross-subsidize across maturities that leads to deviations from the central planner solution.[14]

5 Discussion

5.1 Amplification of shocks through collective short-termism

One important implication of our model is that endogenous asset side adjustments may amplify shocks, compared to the benchmark case without asset side adjustments. Below, we illustrate this amplification by investigating the comparative statics of our equilibrium in response to changes in λ. Recall that λ, which parametrizes the riskiness of the bad project, is a proxy for the severity of the the financing friction arising from limited commitment: An increase in λ increases the amount successful bad firms can abscond with.

The comparative statics with respect to λ are illustrated in Figure 7. The left panel

[14] Note that cross-subsidization can increase the efficiency of the allocation even in the benchmark case with fixed asset side. In that case, the planner would set $D_{cp} = R$ for all maturities, thereby extracting maximum surplus at shorter maturities, and set the funding cutoff T_{cp} such that he breaks even across all funded maturities. The planner in the full model faces the additional constraint that he has to ensure that firms in the funded range do not redraw their maturity, meaning that at least over some interval $D_{cp} < R$.

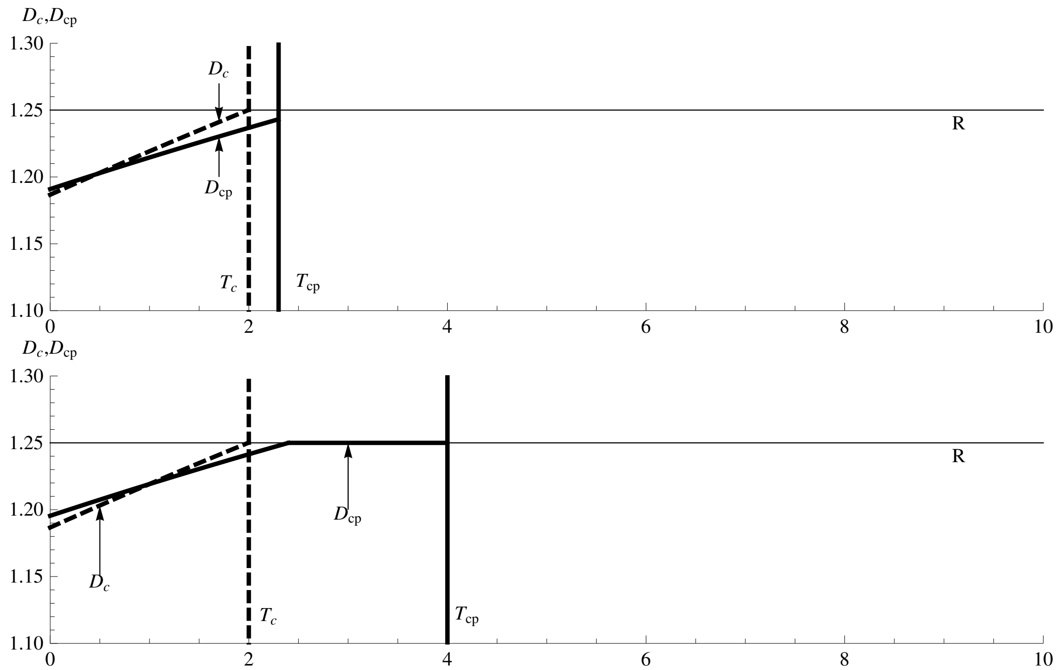

Figure 6: **Central Planer Funding:** The solid line depicts the schedule of face values set by the central planner (D_{cp}), whereas the dashed line depicts face values in the competitive equilibrium (D_c). **Top Panel:** Central planner solution with $\hat{t} < T_{cp}$. **Bottom Panel:** Central planner solution with $\hat{t} > T_{cp}$.

illustrates the maximum funded maturity as a function of λ. The dashed line depicts the benchmark case in which firms cannot adjust their investment decisions. The solid line depicts the equilibrium maximum funded maturity after privately optimal asset-side adjustments by firms. In the benchmark case, all maturities receive financing when λ is sufficiently low. However, once λ crosses a threshold, some long maturities cannot be financed. This captures the rationing of maturities in the benchmark model, as illustrated in Figure 3. The solid line in Figure 7 depicts the full equilibrium with asset-side adjustments. Compared to the benchmark case, in the full equilibrium the maximum funded maturity already drops for lower values of λ, because firms that could get funding at longer maturities find it more attractive to search for second-best projects of potentially shorter maturities. Hence, the endogenous asset-side adjustment by firms precipitates a decrease in the maturities that are funded in equilibrium. Further, the solid line illustrates that, in the full equilibrium, the maximum funded maturity drops substantially when λ increases beyond 0.05. This sharp drop is caused by a switch from an IC-driven equilibrium to an equilibrium driven by the financiers' IR constraint.

The right panel in Figure 7 depicts the percentage change in surplus (aggregate NPV) that arises from firms' asset-side adjustments. Note that, in this example, firms' privately optimal asset side adjustments lead to uniformly lower output than in the benchmark case with exogenous asset side. Moreover, the relative surplus difference between the two cases increases sharply as λ increases. Hence, as a result of privately optimal maturity adjustments, surplus in the full equilibrium with endogenous asset side diverges substantially from the benchmark case in which the asset side is held fixed. As is illustrated in the figure, part of this reduction in surplus comes from a reduction in the total number of projects that is financed (the shaded area in the figure plots the reduction in the number of financed projects or, equivalently, the change in lending that results from endogenizing the asset side). However, a second effect arises, because, in addition to affecting the quantity of investment, firms' asset side adjustments reduce the quality of the average project that is financed. Hence, the overall effect on surplus is driven by both a reduction in quantity of investment and a decrease in the quality of the investment projects undertaken. Relative to the benchmark case with fixed asset side, privately optimal asset side adjustments by firms can thus significantly amplify shocks to the economy.

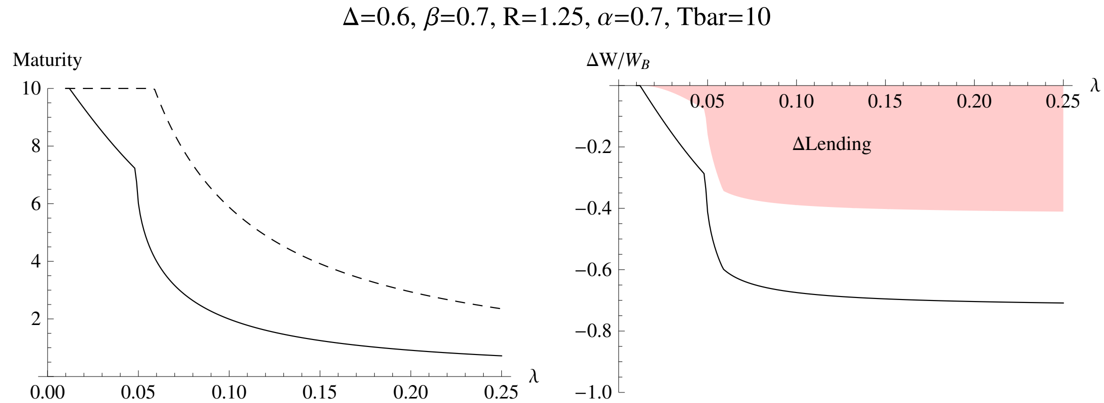

Figure 7: **Equilibrium with (i) exogenous and (ii) endogenous asset side as a function of λ.** The left panel depicts maximum funded maturities in the benchmark model without asset side adjustments (dashed line) and the full model with endogenous asset side (solid line). The right panel depicts the percentage change in total surplus that results from asset side adjustments relative to the case with exogenous asset side $\Delta W/W_B$ (solid line). The shaded area depicts the percentage change in total lending.

5.2 Are firms' privately optimal maturity adjustments efficient?

In the above examples, privately optimal maturity adjustments by firms are undesirable from a social perspective because they lead to an unambiguous decrease in output. More generally, however, whether firms' privately optimal maturity adjustments are socially desirable depends on the degree to which second-best projects are inferior to first best projects, which, in our model, is captured by the parameter α.

Recall that T^* and T_B denote the equilibrium funding cutoffs in the full model and the benchmark model without maturity adjustments, respectively. To evaluate the effect of firms' privately optimal maturity adjustments on surplus (aggregate NPV), we can decompose the change in total surplus ΔW into two components, a *direct effect* and a *dilution effect*:

$$\begin{aligned}\Delta W &= \overbrace{\left(\frac{\overline{T}-T_B}{\overline{T}}\right)\left(\frac{T_B}{\overline{T}}\right)NPV\left(\alpha\beta\right)}^{\text{Direct effect}} \\ &\quad \underbrace{-\left(\frac{T_B-T^*}{\overline{T}}\right)\left[NPV\left(\beta\right)-\left(\frac{T^*+T_B-\overline{T}}{\overline{T}}\right)NPV\left(\alpha\beta\right)\right]}_{\text{Dilution effect}}\end{aligned} \tag{14}$$

The direct effect measures the effect of firms' redrawing behavior keeping the funding threshold constant at T_B. If the funding threshold is held fixed, the ability to redraw projects means that firms with initial project maturities on $\left[T_B,\overline{T}\right]$ now have the possibility of finding funding on $[0,T_B]$. This effect is positive whenever second-best projects have positive NPV, i.e., $NPV\left(\alpha\beta\right)>0$. However, as we saw above, the inflow of second-best projects will change the funding threshold T_B to T^* because the dilution of the pool of firms changes financiers' funding decisions (and, in turn, firms' redrawing decisions). Because of this dilution effect, funding is only extended for the range $[0,T^*]$ and all firms on $\left[T^*,\overline{T}\right]$ end up redrawing. The dilution effect thus summarizes the output loss of the maturities $[T^*,T_B]$ now becoming unfunded and redrawing. Since $\frac{T^*+T_B-\overline{T}}{\overline{T}}<1$, the term in square brackets is always (weakly) positive, and thus the dilution effect always leads to a (weak) reduction in surplus.

To see these two effects at work, consider two special cases. First, when $\alpha=1$, redrawn projects are just as good as firms' original projects. In this case, firms' redrawing behavior does not affect the average quality of projects and the cutoff remains unchanged, $T^*=T_B$. Hence, only the direct effect is present: Some firms that were unable to receive financing at their original maturity can now finance an equally attractive positive NPV project of shorter maturity. This results in an unambiguous increase in welfare, because the ability to search for second best projects helps firms circumvent financing frictions for long-term projects without imposing externalities on other firms.

Second, consider the case in which α is such that redrawn projects have zero NPV.[15] In this case, a firm that manages to obtain funding by redrawing its maturity adds nothing to the aggregate NPV produced. Hence, the direct effect is zero, and only the indirect effect is present: Through its redrawing decision, the firm reduces the average quality of projects, which leads to a reduction in T^*, such that some firms that were originally funded are now also forced to redraw. Hence, when second-best projects are zero NPV (or worse than zero NPV), privately optimal redrawing decisions are socially undesirable. For values of α that lie in between these two polar cases both the direct and indirect effects are present.

Finally, consider what happens when we pick $\alpha\beta$ such that total lending in the economy remains fixed before and after redrawing.[16] Since the redrawing of projects increases the density of projects at shorter maturities, for lending to remain fixed it has to be the case that $T^* < T_B$. This, in turn, implies that it has to be the case that $\alpha < 1$, because $\alpha = 1$ implies $T^* = T_B$. But, when $\alpha < 1$, we know that $NPV\left(\hat{\beta}\right) < NPV\left(\beta\right)$, which means that there is an unambiguous welfare loss: When lending remains fixed, the same number of projects are financed but the average project quality is lower than in the benchmark case. Hence, we can conclude that for firm's redrawing decisions to raise surplus, it is a necessary condition that lending increases compared to the benchmark model and that second-best projects have positive NPV.

We illustrate these results in Figure 8, which plots maximum funded maturities (left

[15]The second-best project has zero NPV when $\alpha = \frac{1}{\beta}\frac{1-\Delta R}{(1-\Delta)R}$.

[16]It is straightforward to show that lending in the economy remains fixed when $\frac{T^*}{\overline{T}} + \frac{\overline{T}-T^*}{\overline{T}}\frac{T^*}{\overline{T}} = \frac{T_B}{\overline{T}}$.

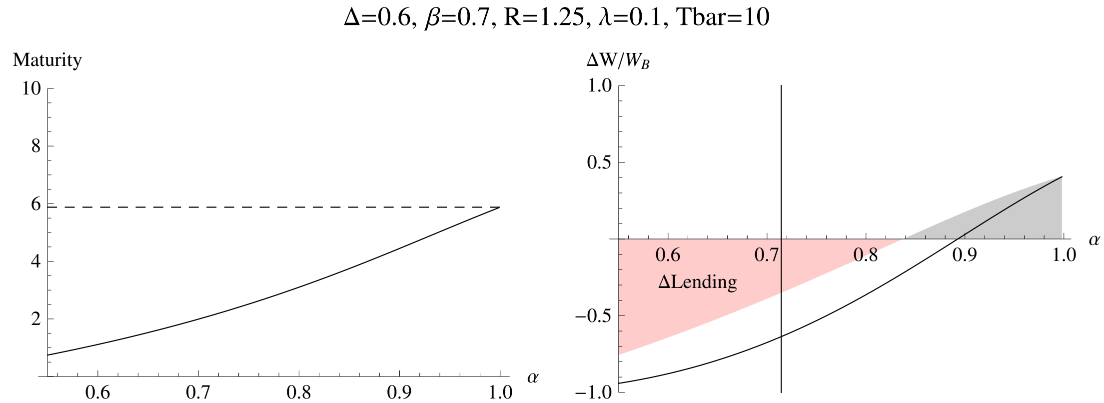

Figure 8: **Equilibrium with (i) exogenous and (ii) endogenous asset side as a function of the dilution parameter α.** The left panel illustrates the equilibrium maximum funded maturity. The dashed line depicts the maximum maturity in the benchmark model with exogenous asset side, which does not depend on the dilution parameter α. The solid line depicts the maximum funded maturity with endogenous asset side, illustrating how dilution through second-best projects reduces the range of financed maturities. The right panel depicts the percentage change in surplus $\Delta W/W_B$ that results from firms' privately optimal asset side adjustments (solid black line). The shaded area depicts the percentage change in total lending. The vertical line depicts the value of α for which $NPV(\alpha\beta) = 0$.

panel) and the percentage change in surplus that results from firms' privately optimal asset side adjustments (right panel) as a function of the dilution parameter α. While the maximum funded maturity is constant in the benchmark case when firms cannot adjust their maturities (dashed line), it decreases as α decreases when firms can adjust their maturities (solid line). When α is lower, the dilutive effect of second-best projects is stronger, leading to a larger reduction in the maximum funded maturity.

In the right panel, the solid line depicts the percentage change in surplus (aggregate NPV) that results from firms' privately optimal redrawing decisions. When there is no or very little quality difference between first-best and second-best projects (i.e., α close to 1), output increases when firms can redraw their project maturity. Thus, for high α, the direct effect outweighs the dilution effect: While firms' maturity adjustments reduce the average quality of funded projects, this negative quality effect is initially outweighed by the increase in the number of projects that can attract financing, which is depicted by the shaded area in the graph. In the figure, this is the case as long as $\alpha > 0.88$. In this region, firms' privately optimal maturity adjustments are also socially desirable

Once $\alpha < 0.88$, privately optimal asset side adjustments by firms reduce surplus. Note that this reduction in surplus occurs even for parameter values in which firms' asset side adjustments lead to an increase in lending. Specifically, for $\alpha \in (0.83, 0.88)$, surplus decreases compared to the benchmark model, even though total lending increases. Hence, in our model an increase in lending is *not* a sufficient condition for an increase in surplus. Finally, the figure illustrates that overall surplus decreases even when second-best projects have positive NPV (which, in the figure, is the case as long as $\alpha \geq 0.714$, as marked by the black vertical line). Hence, while a second-best project may have positive NPV when seen in isolation, because of the dilution, the adoption of positive NPV second-best projects can lead to a decrease in overall surplus. This result illustrates the importance of taking into account cross-firm externalities when evaluating firms' investment choices.

6 Conclusion

This paper provides a framework to analyze how financing frictions that arise on the liability side affect firms' investment decisions on the asset side, and vice versa. In our model, financing frictions resulting from limited commitment are more pronounced at longer horizons, leading to less attractive financing terms, or even credit rationing, for long-term investment projects. Firms respond by adjusting their asset-side investments by searching for alternative projects of potentially shorter maturity, even if those projects are second best. Because individual firms' asset-side decisions endogenously determine the magnitude of an asymmetric information friction faced by all firms, an externality arises that leads to inefficient short-termism. In addition, firms' equilibrium asset side adjustments amplify shocks and, while privately optimal, can be socially undesirable. Broadly speaking, our paper highlights the importance of explicitly taking into account the asset side when analyzing the effect of liability side frictions, such as pressure toward short-term financing.

References

Acharya, V. V., D. Gale, and T. Yorulmazer (2011): "Rollover Risk and Market Freezes," *Journal of Finance*, 66(4), 1177–1209.

Bebchuk, L. A., and I. Goldstein (2011): "Self-fulfilling Credit Market Freezes," *Review of Financial Studies*, 24(11), 3519–3555.

Besanko, D., and A. V. Thakor (1987): "Collateral and Rationing: Sorting Equilibria in Monopolistic and Competitive Credit Markets," *International Economic Review*, 28(3), 671–689.

Bester, H. (1985): "Screening vs. Rationing in Credit Markets with Imperfect Information," *American Economic Review*, 75(4), 850–855.

Bigio, S. (2011): "Endogenous Liquidity and the Business Cycle," Working Paper, Columbia University.

Bolton, P., and M. Dewatripont (2005): *Contract Theory*. MIT Press.

Bolton, P., and D. S. Scharfstein (1990): "A Theory of Predation Based on Agency Problems in Financial Contracting," *American Economic Review*, 80(1), 93–106.

Bolton, P., J. Scheinkman, and W. Xiong (2006): "Executive Compensation and Short-Termist Behaviour in Speculative Markets," *Review of Economic Studies*, 73(3), 577–610.

Brunnermeier, M. K., and M. Oehmke (2013): "The Maturity Rat Race," *Journal of Finance*, 68(2), 483–521.

Chen, H., Y. Xu, and J. Yang (2012): "Systematic Risk, Debt Maturity, and the Term Structure of Credit Spreads," Working Paper, MIT Sloan.

Cheng, I.-H., and K. Milbradt (2012): "The Hazards of Debt: Rollover Freezes, Incentives and Bailouts," *Review of Financial Studies*, 25(4), 1070–1110.

DeMeza, D., and D. C. Webb (1987): "Too Much Investment: A Problem of Asymmetric Information," *Quarterly Journal of Economics*, 102(2), 281–292.

Dew-Becker, I. (2011): "Investment and the Cost of Capital in the Cross-Section: The Term Spread Predicts the Duration of Investment," Working Paper, Harvard University.

Dewatripont, M., and E. Maskin (1995): "Credit and Efficiency in Centralized and Decentralized Economies," *Review of Economic Studies*, 62(4), 541–555.

Diamond, D. W. (1991): "Debt Maturity Structure and Liquidity Risk," *The Quarterly Journal of Economics*, 106(3), 709–737.

Diamond, D. W., and P. H. Dybvig (1983): "Bank Runs, Deposit Insurance, and Liquidity," *Journal of Political Economy*, 91(3), 401–419.

Eisfeldt, A. L. (2004): "Endogenous Liquidity in Asset Markets," *Journal of Finance*, 59(1), 1–30.

Freixas, X., and J.-C. Rochet (2008): *Microeconomics of Banking*. MIT Press, 2nd edn.

Froot, K. A., A. F. Perold, and J. C. Stein (1992): "Shareholder Trading Practices and Corporate Investment Horizons," *Journal of Applied Corporate Finance*, 5(2), 42–58.

Harris, M., and A. Raviv (1992): “Frinancial Contracting Theory,” in *Advances in Economic Theory: Sixth World Congress*, ed. by J.-J. Laffont, vol. II. Cambridge University Press.

Hart, O., and J. Moore (1994): “A Theory of Debt Based on the Inalienability of Human Capital,” *Quarterly Journal of Economics*, 109(4), 841–879.

He, Z., and W. Xiong (2012): “Dynamic Debt Runs,” *Review of Financial Studies*, 25(6), 1799–1843.

Jacklin, C. J. (1987): “Demand Deposits, Trading Restrictions and Risk Sharing,” in *Contractual Arrangements for Intertemporal Trade*, ed. by E. C. Prescott, and N. Wallace. University of Minnesota Press, Minneapolis.

Jaffee, D. M., and F. Modigliani (1969): “A Theory and Test of Credit Rationing,” *American Economic Review*, 59(5), 850–872.

Jaffee, D. M., and T. Russell (1976): “Imperfect Information, Uncertainty, and Credit Rationing,” *Quarterly Journal of Economics*, 90(4), 651–666.

Kurlat, P. (2010): “Lemons Markets and the Transmission of Aggregate Shocks,” Working Paper, Stanford University.

Mian, A., and J. Santos (2011): “Liquidity Risk, Maturity Management and the Business Cycle,” Working Paper, Berkeley.

Narayanan, M. P. (1985): “Managerial Incentives for Short-Term Results,” *Journal of Finance*, 40(5), 1469–1484.

Rothschild, M., and J. E. Stiglitz (1976): "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information," *Quarterly Journal of Economics*, 90, 629–649.

Stein, J. C. (1989): "Efficient Capital Markets, Inefficient Firms: A Model of Myopic Corporate Behavior," *Quarterly Journal of Economics*, 104(4), 655–669.

Stiglitz, J. E., and A. Weiss (1981): "Credit Rationing in Markets with Imperfect Information," *American Economic Review*, 71(3), 393–410.

Suarez, J., and O. Sussman (1997): "Endogenous Cycles in a Stiglitz-Weiss Economy," *Journal of Economic Theory*, 76(3), 47–71.

von Thadden, E.-L. (1995): "Long-Term Contracts, Short-Term Investment and Monitoring," *Review of Economic Studies*, 62(4), 557–575.

A Appendix

A.1 Comparative statics of D and $\hat{\beta}$

Differentiating the expressions for D and $\hat{\beta}$, we obtain:

$$\begin{aligned}
D\left(t,\hat{\beta}\right) &= \frac{1}{\hat{\beta}+\left(1-\hat{\beta}\right)\Delta e^{-\lambda t}} \\
\frac{\partial D\left(t,\hat{\beta}\right)}{\partial t} &= \left(1-\hat{\beta}\right)\Delta\lambda e^{-\lambda t} D\left(t,\hat{\beta}\right)^2 > 0 \\
\frac{\partial e^{-\lambda t} D\left(t,\hat{\beta}\right)}{\partial t} &= -\hat{\beta}\lambda e^{-\lambda t} D\left(t,\hat{\beta}\right)^2 < 0 \\
\frac{\partial D\left(t,\hat{\beta}\right)}{\partial\hat{\beta}} &= -\left(1-\Delta e^{-\lambda t}\right) D\left(t,\hat{\beta}\right)^2 < 0 \\
\hat{\beta}'\left(T\right) &= \beta\frac{\overline{T}\left(1-\alpha\right)}{\left(2\overline{T}-T\right)^2} > 0 \\
\hat{\beta}''\left(T\right) &= \beta\frac{2\overline{T}\left(1-\alpha\right)}{\left(2\overline{T}-T\right)^3} > 0
\end{aligned}$$

A.2 Proof of general strategies

A.2.1 Preliminaries

Let the set $\mathcal{I}$ be the set of offered funding so that $t \in \mathcal{I}$ implies that t is funded at some terms $D\left(t,\hat{\beta}_t\right)$. Note that due to competition $D\left(t,\hat{\beta}_t\right)$ is offered if $\hat{\beta}_t$ is the rationally expected quality at t. Further, we know that for an equilibrium that $\mathcal{I}$ has to be break-even, so we assume here that $\mathcal{I}$ is break-even. Furthermore, we make $\mathcal{I}$ as tight as possible, that is, if there is some $\hat{t} \in \mathcal{I}$ that does not accept the funding terms and redraws then we define $\hat{\mathcal{I}}$ excluding $\hat{t}$. Also, we drop all intervals of measure zero from $\mathcal{I}$ (e.g., isolated points).

Next, note that the mass of redrawn projects on the set $[0,t]$ is given by

$$F\left(t,\mathcal{I}\right) = \frac{1}{\overline{T}}\left[\int_0^{\overline{T}} \mathbf{1}_{\{s\notin\mathcal{I}\}}\frac{t}{\overline{T}}ds\right]$$

where $\frac{t}{\overline{T}}$ is the probability that a redrawn project is in $[0,t]$. The marginal density contributed by redrawing projects at t is

$$\begin{aligned}
\frac{\partial F}{\partial t} &= \frac{1}{\overline{T}}\int_0^{\overline{T}} \mathbf{1}_{\{s\notin\mathcal{I}\}}\frac{1}{\overline{T}}ds \\
&= \frac{1}{\overline{T}^2}\left|\left|\mathcal{I}^c\right|\right|
\end{aligned}$$

where $\mathcal{I}^c = \left[0,\overline{T}\right]\setminus\mathcal{I}$. We see that $\frac{\partial F}{\partial t}$ is linear in the length $||\mathcal{I}^c|| = \overline{T} - ||\mathcal{I}||$ but independent of t. This is because the redrawing technology redistributes everyone evenly on $\left[0,\overline{T}\right]$ regardless of their original t. Suppose further that there is some probability p with which entrepreneurs who redraw are discovered. Then the average quality on $\mathcal{I}$ is constant and is given by

$$\hat{\beta}\left(T\right) = \beta\frac{1+\alpha p\overline{T}\frac{\partial F}{\partial t}}{1+p\overline{T}\frac{\partial F}{\partial t}}$$

Thus, for any strategy profile $\mathcal{I}$ we know that the average quality on $\mathcal{I}$ is $\hat{\beta}$ and the average quality on $\mathcal{I}^c$ is $\alpha\beta$ and $\hat{\beta}$ is just influenced by the length of $\mathcal{I}$.

Next, suppose the equilibrium offered financial contract $\mathcal{I}$ has funding holes and is based on an average quality $\hat{\beta}$ on the funded region. Define $T \equiv \max \mathcal{I}$. Consider an alternative contract $\mathcal{E}$ which plugs part of a hole from the left by a measure of $\varepsilon = ||\mathcal{E}||$ but lowers T to $T - \delta$ to keep incentives aligned.

Next, note that

$$\frac{D\left(t, \hat{\beta}\right)}{D\left(t, \theta\right)} > 1 \iff \theta > \hat{\beta}$$

and also that

$$\frac{\partial}{\partial t}\left[\frac{D\left(t, \hat{\beta}\right)}{D\left(t, \theta\right)}\right] = -\lambda\Delta\left(\hat{\beta} - \theta\right) e^{-\lambda t} D\left(t, \hat{\beta}\right)^2$$

which is positive for $\theta > \hat{\beta}$ and negative for $\theta < \hat{\beta}$. To summarize, when $\theta > \hat{\beta}$ then the ratio is greater than 1 and increasing in t.

Next, define

$$\begin{aligned}
\pi\left(\theta, t, \hat{\beta}\right) &= \theta\left[R - D\left(t, \hat{\beta}\right)\right] + (1-\theta)\Delta\left[R - e^{-\lambda t} D\left(t, \hat{\beta}\right)\right] \\
&= \left[\theta + (1-\theta)\Delta\right] R - \frac{D\left(t, \hat{\beta}\right)}{D\left(t, \theta\right)}
\end{aligned}$$

which is the expected profit of a type θ of maturity t accepting a contract based on average quality $\hat{\beta}$. Note that $\theta = \{\alpha\beta, \beta\}$ for our purposes and that

$$\pi_t\left(\theta, t, \hat{\beta}\right) = \begin{cases} < 0 & \theta = \beta \\ > 0 & \theta = \alpha\beta \end{cases},$$

that is, the expected profit increases with maturity when the type is $\alpha\beta$ (that is, after redrawing), and decreases with maturity when the type is β (that is, before redrawing).

First, suppose the IC constraint is not binding, i.e. $NVR\left(T, \mathcal{I}\right) < 0$. Then there exists a $\varepsilon > 0$ and $\delta = 0$ that support funding. This is because $\varepsilon > 0 = \delta$ implies more funding is offered (and taken) and thus the average funded quality cannot decline.

A.2.2 Variational approximation

Next, we will use a variational argument to show that we can always find $\delta > 0, \varepsilon > 0$ so that a deviation strategy can be devised. First, note that for providing funding in the hole, the investor is essentially a monopolist. Thus, let $\hat{D} = D\left(t, \hat{x}_t\right)$ be the face value offered in the hole. Here, $\hat{x}$ is the implied quality (which is not equal to the real quality) of the project by mapping the (arbitrary) monopolist face value $\hat{D}$ into the competitive funding terms. It is essentially a more convenient way to write the optimal choice $\hat{D}$.

Next, define

$$cst\left(\mathcal{E}\right) = \int_{\mathcal{I}} \pi\left(\alpha\beta, s, \hat{\beta}\right) ds + \int_{\mathcal{E}} \pi\left(\alpha\beta, s, \hat{x}\right) ds = \pi\left(\beta, T - \delta, \hat{\beta}\right)$$

where $\varepsilon = ||\mathcal{E}||$ and $T - \delta$ defines the new marginal agent given funding at terms $\hat{x}$ on the additional set $\mathcal{E}$. We know that by the monopolist assumption that we must have

$$\pi\left(\beta, t, \hat{x}_t\right) = cst\left(\mathcal{E}\right) \iff \hat{D}\left(t, \mathcal{E}\right) \equiv \frac{1}{\hat{x}_t + (1 - \hat{x}_t)\Delta e^{-\lambda t}} = \frac{\left[\beta + (1-\beta)\Delta\right] R - cst\left(\mathcal{E}\right)}{\beta + (1-\beta)\Delta e^{-\lambda t}}$$

for all $t \in \mathcal{E}$.

Next, define the auxiliary function

$$H(\theta, x, \mathcal{A}) = \int_{\mathcal{A}} \frac{D(t,x)}{D(t,\theta)} dt$$

Then, plugging in $\hat{D}(t, \mathcal{E})$ into the definition of $cst(\mathcal{E})$, we have

$$cst(\mathcal{E}) = \int_{\mathcal{I}} \pi\left(\alpha\beta, s, \hat{\beta}\right) ds + ||\mathcal{E}|| \left[\alpha\beta + (1-\alpha\beta)\Delta\right] R + \left\{cst(\mathcal{E}) - \left[\beta + (1-\beta)\Delta\right] R\right\} H(\alpha\beta, \beta, \mathcal{E})$$

Solving for $cst(\mathcal{E})$, we have

$$cst(\mathcal{E}) = \frac{\int_{\mathcal{I}} \pi\left(\alpha\beta, s, \hat{\beta}\right) ds + ||\mathcal{E}|| \left[\alpha\beta + (1-\alpha\beta)\Delta\right] R - \left[\beta + (1-\beta)\Delta\right] R \cdot H(\alpha\beta, \beta, \mathcal{E})}{1 - H(\alpha\beta, \beta, \mathcal{E})}$$

and we linearly approximate around $\varepsilon = ||\mathcal{E}|| = 0$ to get

$$cst(\varepsilon) \approx cst(0) + \varepsilon \cdot cst'(0)$$

where

$$cst(0) = \int_{\mathcal{I}} \pi\left(\alpha\beta, s, \hat{\beta}\right) ds = \pi\left(\beta, T, \hat{\beta}\right) = \left[\beta + (1-\beta)\Delta\right] R - \frac{D\left(T, \hat{\beta}\right)}{D(T, \beta)}$$

and $cst'(0)$ is bounded.

Next, the new marginal agent $T - \delta$ is defined by the indifference to redrawing condition

$$\pi\left(\beta, T-\delta, \hat{\beta}\right) = \left[\beta + (1-\beta)\Delta\right] R - \frac{\beta + (1-\beta)\Delta e^{-\lambda[T-\delta]}}{\hat{\beta} + \left(1-\hat{\beta}\right)\Delta e^{-\lambda[T-\delta]}} = cst(\mathcal{E})$$

We rearrange to get

$$\begin{aligned} e^{-\lambda[T-\delta]} &= \frac{\hat{\beta}\left\{\left[\beta + (1-\beta)\Delta\right] R - cst(\mathcal{E})\right\} - \beta}{\Delta\left[(1-\beta) - \left(1-\hat{\beta}\right)\left\{\left[\beta + (1-\beta)\Delta\right] R - cst(\mathcal{E})\right\}\right]} \\ \iff \delta(\mathcal{E}) &= \frac{1}{\lambda} \log\left[\frac{\hat{\beta}\left\{\left[\beta + (1-\beta)\Delta\right] R - cst(\mathcal{E})\right\} - \beta}{\Delta\left[(1-\beta) - \left(1-\hat{\beta}\right)\left\{\left[\beta + (1-\beta)\Delta\right] R - cst(\mathcal{E})\right\}\right]}\right] + T \end{aligned}$$

and we note that $\delta(0) = 0$ by the definition of δ and the fact that $\pi\left(\beta, T, \hat{\beta}\right) = cst(0)$. We write

$$\delta(\varepsilon) \approx \delta(0) + \varepsilon \cdot \delta'(0) = \varepsilon \cdot \delta'(0)$$

where $\delta'(0)$ is bounded.

Next, we will calculate $\tilde{\beta}_t, t \in \mathcal{E}$, the true average quality on $\mathcal{E}$. Note that

$$\tilde{\beta}(\varepsilon) = \hat{\beta}\left(||\mathcal{I}|| + \varepsilon - \delta(\varepsilon)\right) \approx \tilde{\beta}(0) + \varepsilon \cdot \tilde{\beta}'(0)$$

and where

$$\begin{aligned} \tilde{\beta}(0) &= \hat{\beta}(\mathcal{I}) = \hat{\beta} \\ \tilde{\beta}'(0) &= \hat{\beta}'(\mathcal{I})\left[1 - \delta'(0)\right] = \beta \frac{\overline{T}(1-\alpha)}{\left(2\overline{T} - ||\mathcal{I}||\right)}\left[1 - \delta'(0)\right] \end{aligned}$$

Lastly, note that
$$\tilde{\beta}+(1-\tilde{\beta})\,\Delta e^{-\lambda t}\approx\left[\hat{\beta}+\left(1-\hat{\beta}\right)\Delta e^{-\lambda t}\right]+\varepsilon\tilde{\beta}'(0)\left(1-\Delta e^{-\lambda t}\right)$$

A.2.3 The main proof

The proof now hinges on the implementability of this funding. We will use the following repeatedly:

$$\pi\left(\beta,T,\hat{\beta}\right)=cst\,(0)\iff\frac{D\left(T,\hat{\beta}\right)}{D\,(T,\beta)}=[\beta+(1-\beta)\,\Delta]\,R-cst\,(0)$$

by the assumption of indifference at T. For this, we need

$$\begin{aligned} R &\geq D\,(t,\hat{x}_t)\,,\forall t\in\mathcal{E} \\ 1 &\leq \left[\tilde{\beta}+(1-\tilde{\beta})\,\Delta e^{-\lambda t}\right]D\,(t,\hat{x}_t)\,,\forall t\in\mathcal{E} \end{aligned}$$

Let us first concentrate on the first equation. Note that we can write

$$\begin{aligned} R\geq D\,(t,\hat{x}_t) &= \frac{[\beta+(1-\beta)\,\Delta]\,R-cst\,(\mathcal{E})}{\beta+(1-\beta)\,\Delta e^{-\lambda t}} \\ &\approx \frac{[\beta+(1-\beta)\,\Delta]\,R-[cst\,(0)+\varepsilon\cdot cst'\,(0)]}{\beta+(1-\beta)\,\Delta e^{-\lambda t}} \\ &= \left[\frac{D\left(T,\hat{\beta}\right)}{D\,(T,\beta)}-\varepsilon\cdot cst'\,(0)\right]D\,(t,\beta) \end{aligned}$$

We note that $D\,(t,\beta)$ is increasing in t, so we pick $t=T$ to make the condition least likely to hold. However, we know that since $D\left(T,\hat{\beta}\right)<R$ (as we assumed the IC is tight, so the IR is slack except on a measure zero set), we are done.

Second, let us write out

$$\begin{aligned} 1 &\leq \left[\tilde{\beta}+(1-\tilde{\beta})\,\Delta e^{-\lambda t}\right]D\,(t,\hat{x}_t) \\ &= \left[\tilde{\beta}+(1-\tilde{\beta})\,\Delta e^{-\lambda t}\right]\{[\beta+(1-\beta)\,\Delta]\,R-cst\,(\mathcal{E})\}\,D\,(t,\beta) \\ &\approx \left[\frac{1}{D\left(t,\hat{\beta}\right)}+\varepsilon\cdot\tilde{\beta}'\,(0)\left(1-\Delta e^{-\lambda t}\right)\right]\left[\frac{D\left(T,\hat{\beta}\right)}{D\,(T,\beta)}-\varepsilon\cdot cst'\,(0)\right]D\,(t,\beta) \\ &= \frac{D\,(t,\beta)}{D\left(t,\hat{\beta}\right)}\frac{D\left(T,\hat{\beta}\right)}{D\,(T,\beta)}+\varepsilon\left[\tilde{\beta}'\,(0)\left(1-\Delta e^{-\lambda t}\right)\frac{D\left(T,\hat{\beta}\right)}{D\,(T,\beta)}D\,(t,\beta)-\frac{D\,(t,\beta)}{D\left(t,\hat{\beta}\right)}cst'\,(0)\right]+\varepsilon^2... \end{aligned}$$

Ignoring the ε^2 term, we know that
$$\frac{D\,(t,\beta)}{D\left(t,\hat{\beta}\right)}\frac{D\left(T,\hat{\beta}\right)}{D\,(T,\beta)}\geq 1$$

and it only holds with equality for $t=T$ (assuming $\alpha<1$ and thus $\hat{\beta}<\beta$ for $||\mathcal{I}||<\overline{T}$). For any γ such that $T-t\geq\gamma$ (such a γ exists since we know that T is part of an interval that is not of measure zero) we can find a ε small enough such that the above equation holds as

$$\frac{D\,(T-\gamma,\beta)}{D\left(T-\gamma,\hat{\beta}\right)}\frac{D\left(T,\hat{\beta}\right)}{D\,(T,\beta)}\approx 1+\gamma\left(\beta-\hat{\beta}\right)\Delta\lambda e^{-\lambda T}D\left(T,\hat{\beta}\right)D\,(T,\beta)$$

for any $\beta > \hat{\beta}$. Thus, we can always find a δ for a given $\mathcal{E}$ so that the new funding strategy is not making losses.

A.3 Omitted proofs in the main text

Proof: Irrelevance of rollover debt contracts. We now show that a firm with a project of maturity T cannot finance via rollover at an intermediate date t. At the rollover date, the financier updates his belief on whether the financed firm is good or bad. Conditional on a firm having survived until the rollover date t, the probability that the firm is good is given by

$$\check{\beta}(t) \equiv \mathbb{E}[\beta|\text{no default until } t] = \frac{\beta}{\beta + \Delta(1-\beta)e^{-\lambda t}}.$$

This implies that when rolling over its debt from t to T, the maximum a firm can credibly promise to repay is

$$\begin{aligned} X &= \check{\beta}(t) R + \left(1 - \check{\beta}(t)\right) e^{-\lambda(T-t)} R \\ &= \frac{\beta}{\beta + \Delta(1-\beta)e^{-\lambda t}} R + \frac{\Delta(1-\beta)e^{-\lambda t}}{\beta + \Delta(1-\beta)e^{-\lambda t}} e^{-\lambda(T-t)} R \\ &= \frac{R}{\beta + \Delta(1-\beta)e^{-\lambda t}} \left[\beta + \Delta(1-\beta)e^{-\lambda T}\right] \end{aligned}$$

where we used the fact that the maximum discounted face-value that a firm can credibly commit to is R. From 0 to t then the firm will be able to promise a maximal repayment of X if it stays alive until t, which gives the maximum a firm can promise to repay at 0 as

$$\begin{aligned} & X\left[\beta + \Delta(1-\beta)e^{-\lambda t}\right] \\ = & \frac{R}{\beta + \Delta(1-\beta)e^{-\lambda t}} \left[\beta + \Delta(1-\beta)e^{-\lambda T}\right] \left[\beta + \Delta(1-\beta)e^{-\lambda t}\right] \\ = & R\left[\beta + \Delta(1-\beta)e^{-\lambda T}\right] \end{aligned}$$

But this is the same amount the firm can raise by financing directly up to date T. Hence, if maturity T is rationed, a firm with a project of maturity T can also not obtain financing by using rollover finance. ■

Proof of Lemma 1. Note that for any arbitrary competitive funding profile $\mathcal{I}$ that implies an average quality $\hat{\beta}$ we know that $\pi_t\left(\beta, t, \hat{\beta}\right) < 0$ and thus the firm with the highest $t \in \mathcal{I}$ has the highest incentives to redraw. ■

Proof of Proposition 1. When financiers compete by posting funding schedules, we make the following two observations that will guarantee uniqueness. First, we note that a lower cutoff T leads to lower quality on the set $[0, T]$, because $\hat{\beta}'(T) > 0$. Second, suppose there is an equilibrium with $T < T^*$ and $T \in \mathcal{IR} \cap \mathcal{IC}$. By the definition of T^*, we also know that by $T < T^*$ there has to exist a T' with $T < T' \leq T^*$ and $T' \in \mathcal{IR} \cap \mathcal{IC}$. A single financier, by offering funding up to T', will capture the whole market $[0, T']$ as he is able to offer a lower face value than $D\left(t, \hat{\beta}(T)\right)$ and still make a (small) profit. This is because $\hat{\beta}(T') > \hat{\beta}(T)$. By assumption, we have $NVR\left(0, \hat{\beta}(0)\right) < 0 < NVR\left(\overline{T}, \hat{\beta}\left(\overline{T}\right)\right)$ so that by continuity of $NVR\left(T, \hat{\beta}(T)\right)$ we know that it crosses from negative into positive territory at least once. Competition amongst financiers drives the equilibrium funding schedules to extend all the way to $T^* = \max \mathcal{IR} \cap \mathcal{IC}$. ■

Proof of Proposition 2. Rewrite the set $\mathcal{IR} = \left\{T \in \left[0, \overline{T}\right] : 1 \leq f(T) \equiv R\left[\hat{\beta}(T) + \left(1 - \hat{\beta}(T)\right)\Delta e^{-\lambda T}\right]\right\}$. First, note that

$$f''(T) = Re^{-\lambda T}\left[\Delta\lambda^2\left(1 - \hat{\beta}(T)\right) + 2\Delta\lambda\hat{\beta}'(T) + \hat{\beta}''(T)\left(e^{\lambda T} - \Delta\right)\right] > 0$$

since $\hat{\beta}'(T) > 0, \hat{\beta}''(T) > 0$. Then, if $f(0) < 1 \iff \frac{\beta(1+\alpha)}{2} < \frac{1-\Delta R}{(1-\Delta)R} \equiv \underline{\beta}$ and $f\left(\overline{T}\right) < 1 \iff \beta <$

$\frac{2\overline{T}-T}{\overline{T}+\alpha\left(\overline{T}-T\right)}\frac{1-\Delta e^{-\lambda T}R}{\left(1-\Delta e^{-\lambda T}\right)R}$ we know that $f\left(T'\right)<1,\forall T'\in\left(0,T\right)$. Thus, we can rewrite to get

$$\alpha<\overline{\alpha}\left(T\right)\equiv\min\left\{\frac{2}{\beta}\frac{1-\Delta R}{\left(1-\Delta\right)R}-1,\frac{2\overline{T}-T}{\beta\left(\overline{T}-T\right)}\frac{1-\Delta e^{-\lambda T}R}{\left(1-\Delta e^{-\lambda T}\right)R}-\frac{\overline{T}}{\overline{T}-T}\right\}$$

■

For all of the numerical parameters we consider, we find that $\overline{\alpha}\left(T\right)=\frac{2}{\beta}\frac{1-\Delta R}{\left(1-\Delta\right)R}-1$.

A.4 Central planner & Monopolist

Let us first introduce the concept of least-cost-implementation of a funding cutoff T.

Lemma 2 *Consider a funding schedule on $[0,T]$ with the following restrictions*

$$\begin{aligned}(IC):\frac{1}{\overline{T}}\int_0^T\left\{\alpha\beta\left[R-D\left(t\right)\right]+\left(1-\alpha\beta\right)\Delta\left[R-e^{-\lambda t}D\left(t\right)\right]\right\}dt\quad&\leq\quad\beta\left[R-D\left(t\right)\right]+\left(1-\beta\right)\Delta\left[R-e^{-\lambda t}D\left(t\right)\right]\\&\qquad,\forall t\in\left[0,T\right]\\(IR):D\left(t\right)\quad&\leq\quad R,\forall t\in\left[0,T\right]\end{aligned}$$

Then the least-cost-implementation (LCI) is given by

$$D\left(t,\hat{t}\right)=\begin{cases}C\frac{1}{\beta+\left(1-\beta\right)\Delta e^{-\lambda t}} & t<\hat{t}\\R & t>\hat{t}\end{cases}$$

for

$$C=R\left[\beta+\left(1-\beta\right)\Delta e^{-\lambda\hat{t}}\right],\hat{t}<T$$

where $\hat{t}\left(T\right)>0,\hat{t}'\left(T\right)>0$. $\hat{t}\left(T\right)$ might not exist when under no possible contract break-even can be achieved.

The (IC) and (IR) are never binding at the same time except on a measure zero set: The (IC) constraint is binding for all $t\in\left[0,\min\left\{\hat{t},T\right\}\right]$, but the (IR) is slack on this set; whereas on $t\in\left[\min\left\{\hat{t},T\right\},T\right]$ the (IC) constraint is slack and the (IR) constraint is binding. If $\hat{t}>T$ the maximal face-value R is never reached.

Proof. First, suppose that IC and IR are never binding at the same time, and that they are separated by a cutoff $\hat{t}$ so that IC is binding on $\left[0,\hat{t}\right]$ and IR is binding on $\left[\hat{t},T\right]$ if $\hat{t}<T$, or the IR is never binding on $[0,T]$. We note that a binding IC constraint on $\left[0,\hat{t}\right]$ implies that

$$\beta\left[R-D\left(t\right)\right]+\left(1-\beta\right)\Delta\left[R-e^{-\lambda t}D\left(t\right)\right]=\beta\left[R-D\left(t'\right)\right]+\left(1-\beta\right)\Delta\left[R-e^{-\lambda t'}D\left(t'\right)\right]$$

for $t,t'\in\left[0,\hat{t}\right]$.

Second, the (IC) constraint should be binding for any t at which (IR_E) is slack. Suppose that there is an interval that contains t' for which the (IC) constraint is slack and (IR_E) is not tight. Then, we can easily increase the debt face value $D\left(t'\right)$ by a small amount and still have (IC) satisfied for t'. At the same time, we are decreasing the attractiveness of redrawing, i.e., the LHS of the (IC) constraint for all other maturities t, giving us more slack. Finally, such a move, charging a higher face-value for some maturities while not decreasing the face-value anywhere else clearly satisfies the (IR_I) constraint. We can thus conclude that for all t,t' for which $D<R$ we must have the value of the RHS of the (IC) being equal

Thus, we have

$$D\left(t,\hat{t}\right)=\begin{cases}C\frac{1}{\beta+\left(1-\beta\right)\Delta e^{-\lambda t}} & t<\hat{t}\\R & t>\hat{t}\end{cases}$$

Essentially, we are looking for C and $\hat{t}$ for implementing funding on $[0,T]$. As $D\left(t,\hat{t}\right)$ has to be continuous in t, we have

$$C = R\left[\beta + (1-\beta)\,\Delta e^{-\lambda \hat{t}}\right], \hat{t} < T$$

On $t \in \left[0,\hat{t}\right]$, the RHS of (IC) is constant. Thus, let us define a function G that evaluated at $t=0$

$$\begin{aligned} G\left(T,\hat{t}\right) &= \frac{1}{\overline{T}}\int_0^T \left(\alpha\beta\left[R - D\left(t,\hat{t}\right)\right] + (1-\alpha\beta)\,\Delta\left[R - e^{-\lambda t}D\left(t,\hat{t}\right)\right]\right) dt \\ &\quad - \left(\beta\left[R - D\left(0,\hat{t}\right)\right] + (1-\beta)\,\Delta\left[R - D\left(0,\hat{t}\right)\right]\right) \end{aligned}$$

The IC constraint is slack if $G<0$, and violated if $G>0$. First, we note that $\hat{t} = 0 < T$ implies $D(t) = R$, which results in

$$G(T,0) = \frac{1}{\overline{T}}\int_0^T (1-\alpha\beta)\,\Delta R\left(1 - e^{-\lambda t}\right) dt > 0$$

and thus IC is violated. Next, let us take the derivative w.r.t. $\hat{t}$ while conjecturing that $\hat{t} < T$. Note that

$$\partial_{\hat{t}} D\left(t,\hat{t}\right) = \begin{cases} \frac{-\lambda R(1-\beta)\Delta e^{-\lambda \hat{t}}}{\beta + (1-\beta)\Delta e^{-\lambda t}} & t < \hat{t} \\ 0 & t > \hat{t} \end{cases}$$

and we have

$$\begin{aligned} \partial_{\hat{t}} G\left(T,\hat{t}\right) &= -\frac{1}{\overline{T}}\int_0^T \left(\alpha\beta + (1-\alpha\beta)\,\Delta e^{-\lambda t}\right)\partial_{\hat{t}} D\left(t,\hat{t}\right) dt \\ &\quad + \left(\beta + (1-\beta)\,\Delta\right)\partial_{\hat{t}} D\left(0,\hat{t}\right) \\ &= \lambda R\,(1-\beta)\,\Delta e^{-\lambda \hat{t}}\left[\frac{1}{\overline{T}}\int_0^{\min\left\{\hat{t},T\right\}} \left(\frac{\alpha\beta + (1-\alpha\beta)\,\Delta e^{-\lambda t}}{\beta + (1-\beta)\,\Delta e^{-\lambda t}}\right) dt\right] \\ &\quad -\lambda R\,(1-\beta)\,\Delta e^{-\lambda \hat{t}} \\ &< 0 \end{aligned}$$

Thus, a higher $\hat{t}$ for a given T relaxes the IC constraint on $\left[0,\hat{t}\right]$ by lowering the face-value required, which affects the non-redrawing profit more than the expected redrawing profit. Therefore, there exists at most one $\hat{t}$ that characterizes the least-cost funding policy that fulfills both the IC and IR constraints. If it exists, let us define $\hat{t}(T)$ as the solution to $G\left(T,\hat{t}(T)\right) = 0$. If no solution exists, we set $\hat{t}(T) = \infty$, but in this case we need to adjust C even further down. However, for $\hat{t} = \infty$, there is a loss generated at each maturity as $D(t,\infty) = R\frac{\beta}{\beta+(1-\beta)\Delta e^{-\lambda t}} < R$. Thus, if $\beta R < 1$, even when funding from the original cross-section with repayment probability $\beta + (1-\beta)\,\Delta e^{-\lambda t}$, there is a loss to the financier for all t. Consequently, $\hat{t} = \infty$ or no solution for $\hat{t}$ indicates that the financiers will never break even and thus will never enter the market.

Next, note that

$$\partial_T G\left(T,\hat{t}\right) = \frac{1}{\overline{T}}\left(\alpha\beta\left[R - D\left(T,\hat{t}\right)\right] + (1-\alpha\beta)\,\Delta\left[R - e^{-\lambda T}D\left(T,\hat{t}\right)\right]\right) > 0$$

as $D(T) \leq R$. Thus, we have

$$\hat{t}'(T) = \frac{\partial \hat{t}}{\partial T} = -\frac{\frac{\partial G}{\partial T}}{\frac{\partial G}{\partial \hat{t}}} > 0$$

so that as the funding horizon T extends, the required face-value weakly decreases for a given $\hat{t}$. If no $\hat{t}$ solves the equation, then it is never profitable undertake funding for either the monopolist or the central planner, both who have to at least break even. ■

First, note the density of projects on $[0,T_{cp}]$ if T_{cp} is the funding cutoff is $\frac{1}{\overline{T}} + \frac{\overline{T}-T}{\overline{T}}\frac{1}{\overline{T}} = \frac{2\overline{T}-T}{\overline{T}^2}$. Then, the central planner program $\mathcal{P}$ is to pick a sequence of face-values $D_{cp}(t)$ and a funding cut-off T_{cp} to maximize overall output which is funding mass times average NPV,

$$\max_{T,\{D(t)\}} \frac{2\overline{T}-T}{\overline{T}^2}\int_0^T \left[\hat{\beta}(T)R+\left(1-\hat{\beta}(T)\right)\Delta R\right]dt$$

subject to the following constraints:

$$\begin{aligned}
(BE): \frac{2\overline{T}-T}{\overline{T}^2}\int_0^T \left\{D(t)\left[\hat{\beta}(T)+\left(1-\hat{\beta}(T)\right)\Delta e^{-\lambda t}\right]-1\right\}dt &\geq 0\\
(IC): \frac{1}{\overline{\overline{T}}}\int_0^T \left\{\alpha\beta\left[R-D(t)\right]+(1-\alpha\beta)\Delta\left[R-e^{-\lambda t}D(t)\right]\right\}dt &\leq \beta\left[R-D(t)\right]+(1-\beta)\Delta\left[R-e^{-\lambda t}D(t)\right]\\
&\quad ,\forall t\in[0,T]\\
(IR): D(t) &\leq R
\end{aligned}$$

Proposition 4 *The central planner uses an LCI schedule for any T and will pick the highest funding cutoff T_{cp} that exactly breaks even.*

Proof. First, we observe that the objective function is increasing in T as

$$\frac{\partial}{\partial T}\frac{2\overline{T}-T}{\overline{T}^2}\int_0^T \left[\hat{\beta}(T)R+\left(1-\hat{\beta}(T)\right)\Delta R\right]dt = \frac{2\left(\overline{T}-T\right)}{\overline{T}^2}R\left[\Delta+\hat{\beta}(T)(1-\Delta)\right]+\frac{2\overline{T}-T}{\overline{T}^2}T\hat{\beta}'(T)(1-\Delta)>0$$

and thus the central planner will pick the highest T that is consistent with the constraints.

Second, the central planner will use the LCI schedule as it extracts the maximum amount from low maturities that are then used for cross-subsidization. ■

The monopolist's problem is more complicated. Because the monopolist maximizes profit (instead of surplus), he may choose not to cross subsidize, or, at least, limit cross-subsidization relative to the planner. The monopolist's program $\mathcal{M}$ is given by picking a sequence of face values $D_m(t)$ and a funding cut-off T_m that solve

$$\begin{aligned}
\max_{T,D(t)} \frac{2\overline{T}-T}{\overline{T}^2}\int_0^T \left\{D(t)\left[\hat{\beta}(T)+\left(1-\hat{\beta}(T)\right)\Delta e^{-\lambda t}\right]-1\right\}dt &\\
s.t. &\\
(IC): \frac{1}{\overline{\overline{T}}}\int_0^T \left\{\alpha\beta\left[R-D(t)\right]+(1-\alpha\beta)\Delta\left[R-e^{-\lambda t}D(t)\right]\right\}dt &\leq \beta\left[R-D(t)\right]+(1-\beta)\Delta\left[R-e^{-\lambda t}D(t)\right]\\
&\quad ,\forall t\in[0,T]\\
(IR): D(t) &\leq R
\end{aligned}$$

The monopolist will also use the LCI schedule given above. However, picking the cutoff T_m is not as straightforward as in the central planer case. The reason becomes apparent once we differentiate the objective function—with the optimal schedule $D\left(t,\hat{t}(T)\right)$ plugged in—with respect to T:

$$\begin{aligned}
\frac{\partial(\cdot)}{\partial T} = & \frac{2\overline{T}-T}{\overline{T}^2}\left\{D\left(T,\hat{t}(T)\right)\left[\hat{\beta}(T)+\left(1-\hat{\beta}(T)\right)\Delta e^{-\lambda T}\right]-1\right\}\\
&+\frac{2\overline{T}-T}{\overline{T}^2}\int_0^T D\left(t,\hat{t}(T)\right)\hat{\beta}'(T)\left(1-\Delta e^{-\lambda t}\right)dt\\
&+\frac{2\overline{T}-T}{\overline{T}^2}\int_0^T \frac{\partial D\left(t,\hat{t}\right)}{\partial \hat{t}}\hat{t}'(T)\left[\hat{\beta}(T)+\left(1-\hat{\beta}(T)\right)\Delta e^{-\lambda t}\right]dt\\
&-\frac{1}{\overline{T}^2}\int_0^T \left\{D\left(t,\hat{t}(T)\right)\left[\hat{\beta}(T)+\left(1-\hat{\beta}(T)\right)\Delta e^{-\lambda t}\right]-1\right\}dt
\end{aligned}$$

The first term represents the net present value of funding $\frac{2\overline{T}-T}{\overline{T}^2}$ more projects of average quality $\hat{\beta}(T)$ at terms $D\left(T,\hat{t}(T)\right)$. This is the *direct effect*. The second term is the marginal benefit of increasing by $\hat{\beta}'(T)$ the average quality of funded projects as a result of increasing the funding threshold T. This effect leads to an added profit on all funded projects on $[0,T]$ and is unambiguously positive. This is the *dilution effect*. The third term is the infra-marginal loss that results from having to adjust the funding terms for infra-marginal firms to satisfy the IC and IR constraints. A higher funding threshold tightens the IC constraint, and thus requires lowering the face-value schedule. Formally, this is represented by $\hat{t}'(T) > 0$ and $\frac{\partial D\left(t,\hat{t}\right)}{\partial \hat{t}} < 0$, and therefore leads to an unambiguous loss. This is the *incentive effect*. The fourth term captures the effect that changes in the density of projects (brought about by changes in the funding threshold T) have on the monopolist's profit. This is the *density effect*. At each funded maturity, the monopolist has an expected profit of

$$D\left(t,\hat{t}\right)\left[\hat{\beta}+\left(1-\hat{\beta}\right)\Delta e^{-\lambda t}\right]-1=\begin{cases} C(T)\frac{\hat{\beta}+\left(1-\hat{\beta}\right)\Delta e^{-\lambda t}}{\beta+(1-\beta)\Delta e^{-\lambda t}}-1, & t<\hat{t} \\ R\left[\hat{\beta}+\left(1-\hat{\beta}\right)\Delta e^{-\lambda t}\right]-1, & t\geq\hat{t} \end{cases}$$

which, by inspection, is decreasing in t. Thus, the monopolist makes the most profit on short maturity projects. Increasing the funding threshold T uniformly reduces the density of projects on $[0,T]$ and thus hurts the monopolist's profits by reducing the density of the most profitable projects. This is the *density effect*. Both the *density* and *dilution effect* come from our technological assumptions on the endogenous asset side choice (i.e., the firm's redrawing technology), whereas the *incentive effect* is a contractual effect that arises as a result of the asymmetric information friction in the model.

To gain a better understanding of the trade-offs facing the monopolist, consider a situation in which the direct effect is zero. To this end, we fix a funding threshold T_m s.t.

$$D\left(T_m,\hat{t}(T_m)\right)\left[\hat{\beta}(T_m)+\left(1-\hat{\beta}(T_m)\right)\Delta e^{-\lambda T_m}\right]=1,$$

such that the monopolist just breaks even on the last funded project. Because the expected profit is decreasing w.r.t. t, we know that the *density effect* will be negative. Hence, the threshold T_m is only an equilibrium if the *dilution effect* outweighs both the *density* and the *incentive effects*. The *dilution effect* is more pronounced, that is the larger $\hat{\beta}'$ or the smaller α. However, α also effects the choice of $\hat{t}$ and thus the *incentive effect*. This can be easily seen by the observation that

$$\partial_\alpha G\left(T,\hat{t}\right)=\frac{1}{\overline{T}}\beta\int_0^T\left\{\left[R-D\left(t,\hat{t}\right)\right]-\Delta\left[R-e^{-\lambda t}D\left(t,\hat{t}\right)\right]\right\}dt$$

has an ambiguous sign (the integrand is positive for $t=0$ and is negative at $t=\hat{t}$). Thus, $\frac{\partial\hat{t}}{\partial\alpha}=-\frac{\partial G}{\partial\alpha}/\frac{\partial G}{\partial\hat{t}}$ has an ambiguous sign as well. Overall, this implies that, depending on the relative sizes of the different effects, it is possible that the monopolist cross subsidizes and funds some loss-making maturities in order to extract more from shorter maturities. However, it is also possible that the monopolist finds it optimal not to cross subsidize and picks a funding cutoff where the marginal project is still strictly profitable.

Parameters, Variables & Functions			
Variable	Interpretation	Value	Eq.
$\left[0, \overline{T}\right]$	Range of initial project maturities	$[0, 10]$	
β	Initial population share of good projects	0.7	
λ	Default intensity of bad projects	0.1	
$\Delta \in \left(\frac{1}{2}, \frac{1}{R}\right)$	Probability that bad project gets off the ground	0.6	
α	Decrease in probability of good project if switching	0.7	
$R \in (1, 2)$	Discounted payoff of good projects	1.25	
NPV_G	NPV good project	0.25	(1)
NPV_B	NPV bad project	-0.25	(2)
$D\left(t, \beta\right)$	Discounted face value at maturity t and pool quality β		(5)
$\hat{\beta}\left(T\right)$	Share of good projects on no redrawing interval $[0, T]$		(6)
$\alpha\beta$	Share of good projects on redrawing interval $\left[T, \overline{T}\right]$	0.49	
$\mathcal{IC}$	Set of incentive-compatible cutoffs given $D\left(t, \beta\right)$		(10)
$\mathcal{IR}$	Set of funded cutoffs strategies: $T \in \mathcal{IR}$ means $[0, T]$ funded		(11)

Table 1: Summary of parameters and variables of the model.