Government Guarantees and Financial Stability

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Abstract

We analyze the tradeoff involved in the introduction of government guarantees in a context where both panic and fundamental crises are possible, and banks' and depositors' withdrawal decisions are endogenously determined. On the one hand, government intervention reduces depositors' incentive to run. On the other hand, it induces banks to engage in excessive risk taking. We show that generally a more moderate form of intervention is more efficient than blanket guarantees, even if panic runs still occur. By limiting the size of the intervention, the government increases banks' incentives to behave prudently and thus, contains the likelihood of runs and the costs of intervention. Overall, the optimal level of guarantees depends on the amount of public resources available to finance the scheme.

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JEL classifications: G21, G28

1 Introduction

Government guarantees to financial institutions are common all over the world. They come in different forms, such as deposit insurance provided to depositors who put their money in commercial banks, or implicit guarantees for a bailout provided ex post upon the bank's failure. The recent financial crisis led to renewed interest and debate about the role of government guarantees and their desirability. On the one hand, government guarantees are thought to have a positive role in preventing panic among investors, and hence they help stabilize the financial system. They also help mitigating the negative consequences once a panic had happened. On the other hand, they might create adverse incentives for banks to engage in excessive risk taking. This might even lead to an overall increase in financial fragility.

In this paper, we present a model to analyze the tradeoff involved in the choice of the level of government guarantees. We study an economy as in Diamond and Dybvig (1983) where banks offer deposit contracts to investors, who might face early liquidity needs, and by that provide risk sharing among them. While banks may improve investors' welfare due to the risk sharing they provide, the deposit contracts also expose banks to the risk of a bank run, where many depositors panic and withdraw early out of the self-fulfilling belief that other depositors will do so and the bank will fail. The role of deposit insurance is then to mitigate the panic, by ensuring that depositors will be paid the promised amount in the event of a run, and so avoid the run equilibrium.

In Diamond and Dybvig (1983), deposit insurance is always fully desirable, because it completely prevents the bank-run equilibrium, and so the government never needs to pay anything, and because no moral-hazard problem is generated by the deposit insurance. As real-world events highlighted, however, the situation is often more complex. First, despite the presence of government guarantees, runs are not completely prevented and governments find themselves in cases where they need to pay actual cost to get banks and their investors out of trouble. Second, the presence of government guarantees is often blamed for the excessive risks that banks engage in. Hence, by providing guarantees, the government might even be making the fragility more severe, setting itself to pay excessive amounts to banks and their investors.

Analyzing the desirability of deposit insurance, in light of these considerations, requires a model where the probability of a crisis is determined endogenously and affected by the presence of the deposit insurance. Moreover, the deposit contract offered by the bank, and so the risk that the bank is exposed to, are also determined endogenously and affected by the deposit insurance. Then, one can analyze the overall tradeoff involved in setting the level of guarantees by the government.

To conduct this analysis, we build on the model developed in Goldstein and Pauzner (2005), in which depositors' withdrawal decisions are uniquely determined using the global-game methodology, and so the

probability of a run and how it is affected by the banking contract and by government policy can be determined. Goldstein and Pauzner (2005) study the interaction between the demand deposit contract and the probability of a run. We add a government to this model to study how the government's guarantee policy interacts with the banking contract and the probability of a run.

In the model, there are two periods. Banks raise funds from risk-averse consumers in the form of deposits and invest them in risky projects whose return depends on the fundamental of the economy. Depositors derive utility from consuming both a private and a public good. At the interim date, each depositor receives an imperfect signal regarding the fundamentals and decides when to withdraw based on the information received. In deciding whether to run or not, depositors compare the payoff they would get from going to the bank prematurely and waiting until maturity. These payoffs depend on the fundamentals and the proportion of depositors running.

In this setting, the equilibrium outcome is that runs occur when the fundamentals are below a unique threshold. But, within the range where they occur, they can be classified into *panic-based runs* or *fundamental-based runs*. The former type of run is one that is generated by the self-fulfilling belief of depositors that other depositors will run. The latter type of run happens at the lower part of the run region, where the fundamentals are low enough to make running a dominant strategy for depositors. Overall, the probability of the occurrence of a run (and of both types of run) is uniquely determined and depends on the amount of risk chosen by the bank as represented by the deposit contract offered to depositors.

We first show that the decentralized solution, i.e., without government intervention, is inefficient due to the coordination problem among depositors, leading to panic runs. Moreover, to contain the occurrence of panic runs, banks offer a repayment to depositors that is too low relative to the one that would provide optimal risk sharing. Then, we consider the case in which the government attempts to reduce the probability of runs by guaranteeing (at least part of) the promised repayments through the transfer of resources from the public good to the banking sector. The guarantee scheme used by the government ensures that beyond a certain number of people running on the bank, the government will provide resources that will enable the bank to avoid liquidation, thus eliminating the negative externality a run imposes and so reducing the case for panic to emerge.

Our framework captures the tradeoff generated by the introduction of government guarantees. On the one hand, it reduces depositors' incentives to run and thus the probability of crises. On the other hand, as banks do not fully internalize the costs of the intervention, they are induced to take more risk in the form of a higher repayment to depositors withdrawing prematurely. This in turn increases the government's disbursement and, in case runs are not eliminated, also depositors' incentives to withdraw early. In other words, while the government guarantees reduce (or even eliminate) the probability of panic-based bank runs,

they increase the probability of fundamental-based bank runs by creating incentives for banks to engage in excessive risk taking and offering high payments to depositors. Interestingly, the introduction of government guarantees may even lead to an overall increase in the probability of a run. Hence, in most cases, a more moderate form of intervention is more efficient than blanket guarantees as the limitation in the coverage and scope of the guarantee increases the incentives of banks to behave prudently, thus reducing the probability of runs and the disbursement for the government. We show that the optimal level of guarantees will generally depend on the amount of the government's resources.

We consider ways by which the government can improve its guarantee scheme. One such way is for the government to control the amount of risk taken by the bank, e.g., by limiting the amount it promises to pay to depositors. We show that this scheme can achieve better outcomes, and leads the government to set a high level of insurance eliminating all panic-based runs. However, in practice, the government is likely limited in how much it can control banks' policies. One concern is that setting the deposit rate will limit competition among banks. Another concern is that some forms of risk taking by banks (not modelled here) are not observable to the government. Finally, we consider guarantees schemes where the government prevents both fundamental-based and panic-based runs by ensuring payments even in the long term. Such a policy has the advantage of preventing runs completely and thus limiting the cases in which the government ends up having to pay. However, moral hazard is also very strong here, which might mean that the government will have to bear huge losses in case the bank ends up being insolvent.

Our analysis echoes well the empirical findings and policy observations regarding the downside of deposit insurance. For example, the drawback of deposit insurance that we highlight is consistent with the critique made by Calomiris (1990) that "today's financial intermediaries can maintain higher leverage and attract depositors more easily by offering higher rates of return with virtually no risk of default". Our results are also consistent with the empirical evidence in Demirguc-Kunt and Detragiache (2002), according to which explicit deposit insurance is associated with higher likelihood of crisis. It is also consistent with the conclusion of White (1998) that "... deposit insurance did not substantially reduce aggregate looses from bank failures and may have raised them". Additional empirical evidence is provided by Demirgüç-Kunt and Huizinga (2004) and Ioannidou and Penas (2010). For a survey, see Allen, Carletti, and Leonello (2011).

The analysis in our paper provides a step towards understanding the implications and determination of government guarantees. Due to the complexity of the model, we cannot provide full characterization and we have to focus on particular schemes, but our analysis sheds light on the basic tradeoffs and decisions. The novelty of the paper is to analyze the effects of the introduction of guarantees in the banking sector in a context in which both fundamental and panic crises are possible and both banks' and depositors' decisions are endogenously determined. The paper is linked to other papers that analyze the distortions entailed by deposit

insurance and other forms of guarantees. Boot and Greenbaum (1993) and Cooper and Ross (2002) highlight that public guarantees eliminate runs but at the cost of reducing the incentive of depositors to monitor banks, thus increasing the occurrence of crises and the disbursement for the government. More closely related is Keister (2010), who analyzes the desirability of bailouts in a setting with limited commitment in which banks anticipate that self-fulfilling runs can occur with a certain exogenous probability. The main difference in our paper is that we are able to endogenize the probability with which both fundamental and panic runs can occur, which allows a fuller analysis of the consequences of deposit insurance policy.

The ability to endogenize the probability of a run in our model is achieved by relying on the global-games literature that goes back to Carlsson and van Damme (1993). For an early review, see Morris and Shin (2003). Our paper builds more directly on Goldstein and Pauzner (2005), by deriving a bank-run model where the banking contract is determined endogenously, and the property of global strategic complementarities fails to exist (unlike in most other global-games models). For discussion on the relation of this literature to empirical evidence and the presence of panic vs. fundamental crises in the data, see Goldstein (2012).

The paper proceeds as follows. Section 2 describes the model without government intervention. Section 3 derives the decentralized equilibrium. Section 4 analyzes a guarantee scheme limiting the occurrence of panic runs. Section 5 analyzes two alternative interventions that allows the government to improve upon the guarantee scheme by eliminating either all runs or only the panic ones. Section 6 uses a parametric example to illustrate the properties of the model. Section 7 concludes.

2 The basic model

The basic model is based on Goldstein and Pauzner (2005). There are three dates ($t = 0, 1, 2$) and a continuum $[0, 1]$ of banks and consumers.

Banks raise one unit of funds from consumers in exchange for a deposit contract as specified below, and invest in a risky project. For each unit invested at date 0, the project returns 1 if liquidated at date 1 and a stochastic return $\tilde{R}$ at date 2 given by

$$\tilde{R} = \begin{cases} R > 1 & \text{w. p. } p(\theta) \\ 0 & \text{w. p. } 1 - p(\theta). \end{cases}$$

The variable θ, which represents the state of the economy, is uniformly distributed over $[0, 1]$. We assume that $p(\theta)$ is increasing in θ and that $E_\theta[p(\theta)]R > 1$, which implies that the expected long-run return of the project is superior to the short-run return.

Each consumer is endowed with 1 unit at date 0 and nothing thereafter. At date 0 each consumer deposits its endowment at the bank in exchange for a promised payment c_1 at date 1 or a risky payoff $\tilde{c}_2$ at date 2. Consumers are ex ante identical but can be of two types ex post: each of them has a probability λ of

being early and consuming at date 1, and $1-\lambda$ of being late and consuming at either date. Consumers learn privately their type at date 1.

In addition to the consumption of the good received by the bank, each consumer derives utility also from the provision of a public good g at date 1 and thus its preferences are given by

$$U(c,g) = u(c) + v(g)$$

with $u'(c) > 0$, $v'(g) > 0$, $u''(c) < 0$, $v''(g) < 0$, $u(0) = v(0) = 0$ and relative risk aversion coefficient, $-cu''(c)/u'(c)$, greater than one for any $c \geq 1$.

The state of the economy θ is realized at the beginning of date 1 but is not publicly revealed till date 2. After θ is realized, each consumer receives a private signal x_i of the form

$$x_i = \theta + \varepsilon_i, \tag{1}$$

where ϵ_i are small error terms that are independently and uniformly distributed over $[-\varepsilon, +\varepsilon]$. After the signal is realized, consumers decide whether to withdraw at date 1 or wait till date 2.

The bank satisfies consumers' withdrawal demands by liquidating the long term asset. If the liquidation proceeds are not enough to repay the promised c_1 to the depositors withdrawing at date 1, each of them receives a pro-rata share of the liquidation proceeds.

The banking sector is perfectly competitive. Banks make no profits and choose the deposit contract $(c_1, \widetilde{c}_2)$ at date 0 that maximizes depositors' expected utility. As a consequence, the payment $\widetilde{c}_2$ equals the return of the nonliquidated units of the bank's project divided by the number of remaining late depositors.

The timing of the model is as follows. At date 0, each bank chooses the promised payment c_1. At date 1, after receiving the private signal about the state of the fundamentals θ, depositors decide whether to withdraw early or wait till date 2. At date 2, banks' project realizes and the remaining late depositors receive a pro-rata share of the project returns.

3 The decentralized equilibrium

We start by analyzing depositors' withdrawal decisions at date 1 for a given deposit contract promising c_1 to the depositors withdrawing at date 1. As in Diamond and Dybvig (1983), the promised fixed payment must be at least 1 but less than $min\{1/\lambda, R\}$ for the reasons explained below.

Early depositors always withdraw at date 1 to satisfy their consumption needs. Late depositors compare the expected payoffs from going to the bank at date 1 or 2 and withdraw at the date when they expect to obtain the highest utility. Late depositors' expected payoffs depend on the realization of the fundamentals θ as well as on proportion n of depositors withdrawing early. Since the signal x_i provides information on the

expected date 2 repayment and the actions of the other depositors, each late depositor bases his decision on the signal he receives. When the signal is high, a late depositor attributes a high posterior probability to the event that the bank's project yields the positive return R at date 2. Also, upon observing a high signal, he infers that the others have also received a high signal. This lowers his belief about the likelihood of a run and thus his own incentive to withdraw at date 1. Conversely, when the signal is low, a late depositor has a high incentive to withdraw early as he attributes a high likelihood to the possibility that the project's date 2 return will be zero and that the other depositors run.

We assume that there are two regions of extremely bad or extremely good fundamentals, where each late depositor's action is based on the realization of the fundamentals irrespective of his beliefs about the others' behavior. As shown in Goldstein and Pauzner (2005), the existence of these two extreme regions, no matter how small they are, guarantees the uniqueness of the equilibrium in depositors' withdrawal decisions. We start with the lower region.

Lower Dominance Region. When the fundamentals are very bad (θ is very low), the expected utility from waiting until date 2, $p(\theta)u(\frac{1-\lambda c_1}{1-\lambda}R)$, is lower than that from withdrawing at date 1, $u(c_1)$, even if only the early depositors were to withdraw ($n = \lambda$). If, given his signal, a late depositor is sure that this is the case, running is a dominant strategy. We then denote by $\underline{\theta}(c_1)$ the value of θ that solves

$$u(c_1) = p(\theta)u(\frac{1-\lambda c_1}{1-\lambda}R),$$

that is

$$\underline{\theta}(c_1) = p^{-1}\left(\frac{u(c_1)}{u(\frac{1-\lambda c_1}{1-\lambda}R)}\right). \tag{2}$$

We refer to the interval $[0, \underline{\theta}(c_1))$ as the lower dominance region, where runs are only driven by bad fundamentals. Note that $\underline{\theta}(c_1)$ is increasing in c_1 as we have

$$\frac{\partial \underline{\theta}(c_1)}{\partial c_1} = \frac{p^{-1}}{\left[u(\frac{1-\lambda c_1}{1-\lambda}R)\right]^2}\left[u'(c_1)u(\frac{1-\lambda c_1}{1-\lambda}R) + \frac{u(c_1)\lambda R}{1-\lambda}u'(\frac{1-\lambda c_1}{1-\lambda}R)\right] > 0$$

since $u'(c_1) > 0$. Thus, as c_1 increases, the lower dominance region becomes bigger and fundamental runs are more likely to occur.

For the lower dominance region to exist for any $c_1 \geq 1$, there must be feasible values of θ for which all late depositors receive signals that assure them to be in this region. Since the noise contained in the signal x_i is at most ε, each late depositor withdraws at date 1 if he observes $x_i < \underline{\theta}(c_1) - \varepsilon$, that is if $\theta < \underline{\theta}(c_1) - 2\varepsilon$. Given that $\underline{\theta}$ is increasing in c_1, this condition is satisfied for any $c_1 \geq 1$ if $\underline{\theta}(1) > 2\varepsilon$.

Upper Dominance Region. The upper dominance region of θ corresponds to the range $\left(\overline{\theta}, 1\right]$ in which fundamentals are so good that no late depositors withdraw at date 1. Following Goldstein and Pauzner

(2005), we assume that in this region the project is safe, i.e., $p(\theta) = 1$, and that it gives the same return R at dates 1 and 2. Given $c_1 < min\{1/\lambda, R\}$, this ensures that the bank does not need to liquidate more units than the number n of depositors withdrawing at date 1. Then, when a late depositor observes a signal in the interval $(\overline{\theta}, 1]$, he is certain to receive his payment $\frac{1-\lambda c_1}{1-\lambda}R$ at date 2, irrespective of his beliefs on other depositors' action, and thus he has no incentives to run.

Similarly to before, the upper dominance region exists if there are feasible values of θ for which all late depositors receive signals that assure them to be in this range. This is the case if $\overline{\theta} < 1 - 2\varepsilon$.

The Intermediate Region

The two dominance regions are just extreme ranges of fundamentals in which late depositors have a dominant strategy that depends only on the fundamentals θ. In the intermediate range $\left[\underline{\theta}(c_1), \overline{\theta}\right]$ a depositor's decision to withdraw early depends on the realization of θ as well as on his beliefs regarding other late depositors' actions.

To determine late depositors' withdrawal decisions in this region, we calculate their utility differential between withdrawing at date 2 and at date 1 as given by

$$v(\theta, n) = \begin{cases} p(\theta)u\left(\frac{1-nc_1}{1-n}R\right) - u(c_1) & \text{if } \lambda \leq n < n^* \\ \\ 0 - u(\frac{1}{n}) & \text{if } n^* \leq n < 1, \end{cases} \tag{3}$$

where $n^* = 1/c_1 < 1$. The expression for $v(\theta, n)$ states that as long as the bank does not exhaust its resources at date 1, i.e., for $n < n^*$, late depositors waiting till date 2 obtain the residual $\frac{1-nc_1}{1-n}R$ with probability $p(\theta)$ while those withdrawing early obtain c_1. By contrast, for $n \geq n^*$ the bank liquidates its entire project at date 1. Each late depositor receives nothing if he waits till date 2 and the pro-rata share $1/n$ when withdrawing early. The function $v(\theta, n)$ is decreasing in n for $n < n^*$ and increases with n afterwards. However, as the function crosses zero only once for $n < n^*$ and always remains below afterwards, the model exhibits the property of *one-sided strategic complementarity* as in Goldstein and Pauzner (2005) since $v(\theta, n)$ is decreasing in n whenever it is positive. Thus, there exists a unique equilibrium in which a late depositor runs if his signal is below a certain threshold.

Lemma 1 *The model has a unique equilibrium in which late depositors run if they observe a signal below the threshold $x^*(c_1)$ and do not run above. At the limit, as $\varepsilon \to 0$, $x^*(c_1)$ simplifies to*

$$\theta^*(c_1) = p^{-1}\left(\frac{u(c_1)\left[1 - \lambda c_1\right] + c_1 \int_{n=1/c_1}^{1} u(\frac{1}{n})}{c_1 \int_{n=\lambda}^{1/c_1} u\left(\frac{1-nc_1}{1-n}R\right) dn}\right). \tag{4}$$

The threshold $\theta^*(c_1)$ is increasing in c_1.

Proof. See the appendix. ■

The lemma states that a late depositor's action depends uniquely on the signal he receives as it provides information on other depositors' actions. In the range $\left[\underline{\theta}(c_1), \overline{\theta}\right]$ late depositors do not have a dominant strategy and, due to strategic complementarity, each of them withdraw at date 1 in the interval $[\underline{\theta}(c_1), \theta^*(c_1))$ because he expects the others to do the same. Thus, in the intermediate region runs are panic-driven. These occur only if $c_1 > 1$ as otherwise, there would be no coordination problem among depositors and runs would only be driven by fundamentals.

The threshold $\theta^*(c_1)$ increases with the promised payment c_1. The higher c_1 the lower is the payoff $\widetilde{c}_2$ and thus the stronger is the incentive for each late depositors to withdraw early. This implies that the bank's choice of the optimal deposit contract has a direct impact on the probability of occurrence of runs at date 1.

Now that we have characterized the unique equilibrium in depositors' withdrawal decisions, we turn to the bank's choice of deposit contract at date 0. To do this, we focus on the limit case where $\varepsilon \to 0$ and only complete runs occur since all late depositors receive the same signal and take the same action.

At date 0 each bank chooses c_1 to maximize the expected utility of a representative depositor, which is given by

$$\int_0^{\theta^*(c_1)} u\left(1\right) d\theta + \int_{\theta^*(c_1)}^1 \left[\lambda u(c_1) + (1-\lambda) p(\theta) u\left(\frac{1-\lambda c_1}{1-\lambda} R\right)\right] d\theta + v(g). \tag{5}$$

The first term represents depositors' expected utility for $\theta < \theta^*(c_1)$, when a run occurs, the bank liquidates its entire project and each depositor obtains 1. The second term is depositors' expected utility when, for $\theta \geq \theta^*(c_1)$, the bank continues till date 2. The λ early consumers withdraw early and obtain c_1, while the $(1-\lambda)$ late depositors wait and receive the payment $\frac{1-\lambda c_1}{1-\lambda} R$ with probability $p(\theta)$. The last term is the utility that depositors receive from the consumption of the public good g. We have the following result.

Proposition 1 *The optimal deposit contract $c_1^D > 1$ in the decentralized solution solves*

$$\begin{aligned} &\lambda \int_{\theta^*(c_1)}^1 \left[u'(c_1) - p(\theta) R u'\left(\frac{1-\lambda c_1}{1-\lambda} R\right)\right] d\theta + \\ &- \frac{\partial \theta^*(c_1)}{\partial c_1} \left[\lambda u(c_1) + (1-\lambda) p(\theta^*(c_1)) u\left(\frac{1-\lambda c_1}{1-\lambda} R\right) - u(1)\right] = 0. \end{aligned} \tag{6}$$

Proof. See the appendix. ■

The choice of c_1 trades off the positive effect of a higher c_1 on better risk sharing (as represented by the first term in (6)) with the negative effect in terms of a higher probability of panic runs (as represented by the second term in (6)). At the optimum, the bank chooses $c_1^D > 1$ in order to provide some risk sharing although this entails panic runs.

The decentralized solution is inefficient in various respects. First, banks offer too little risk-sharing to depositors. If runs did not occur, the bank would choose c_1 as the solution to $u'(c_1) = E_\theta[p(\theta)] R u'\left(\frac{1-\lambda c_1}{1-\lambda} R\right)$. When runs are anticipated, the bank chooses a lower level of c_1 in order to limit the likelihood of their

occurrence and thus offers an inefficient risk sharing. Second, runs entail inefficient liquidation. For each unit of project that the bank liquidates, the return R is foregone with probability $p(\theta)$. Thus, liquidation is inefficient whenever a depositor obtains an utility from the liquidated unit which is lower than the utility he would obtain from the same unit if invested till date 2. Formally, this is the case for any $\theta \geq \theta^E$, where θ^E is the solution to

$$u(1) = p(\theta)u(R). \tag{7}$$

Since $c_1^D > 1$, $\theta^E < \underline{\theta}(c_1) < \theta^*(c_1)$. This implies that in the decentralized solution panic runs are always inefficient while fundamental runs are inefficient only in the range $\left(\theta^E, \underline{\theta}(c_1)\right)$.

4 Government intervention at date 1

So far we have showed that the decentralized solution is inefficient because it entails suboptimal risk sharing and inefficient liquidation of the bank's project. In this session we analyze the possibility for the government to improve upon the decentralized solution by transferring some public resources to the banking sector at date 1 in the case that a run occurs. The aim of the intervention is twofold. First, it aims at limiting the coordination problem among depositors and, in turn, the occurrence of runs. Second, it allows depositors to obtain a higher repayment in the case of runs, thus improving upon the decentralized allocation. However, the anticipation of the government intervention introduces a bank moral hazard problem in the choice of the optimal deposit contract. As they do not internalize the cost of the intervention, banks have an incentive to choose an excessively high promised payment c_1 to exploit the public guarantee. This in turn increases the likelihood of runs and thus the fragility of the banking system. Depending on the size of the public good g, the government may then choose to intervene less than what would be required to eliminate runs in order to control the bank moral hazard problem. When this is the case, both fundamental and panic runs still occur even in the presence of government intervention. Even more, given the bank moral hazard problem, runs may be more likely than in the decentralized solution. Despite this, since it allows depositors to obtain a higher payment in case of a run, the government intervention improves the allocation as long as the intervention is not too costly relative to the amount of public resources g available.

To take account of the bank moral hazard problem, we consider a more general form of government intervention than the standard deposit insurance à la Diamond and Dybvig (1983) in which all depositors withdrawing early are guaranteed to receive the promised payment c_1. Specifically, we consider that the government transfers some of the resources g to the banking sector at date 1 after the first $\alpha \geq \lambda$ depositors withdraw. The size of α determines the payoffs that depositors obtain in case of a run and the size of the transfer. The government can either intervene before or after the bank liquidates the entire project.

In the first case, when $\alpha \leq n^* = \frac{1}{c_1} < 1$, the bank is paying c_1 to the withdrawing depositors at the time of the intervention, while, in the second case, when $\alpha > n^*$, it is paying the pro-rata share $\frac{1}{\alpha}$. The government guarantees the same payments as those paid by the bank at the time of the intervention to the $n - \alpha$ withdrawing depositors so that these depositors obtain the same payments as the first α withdrawing depositors. The government transfers to the banking sector the minimum amount needed to guarantee these payments and removes the coordination problem among the $n - \alpha$ late depositors. This means that, when $\alpha \leq n^*$, the bank only liquidates $\frac{1-\alpha c_1}{1-\alpha}$ units of the project for each of the $n - \alpha$ late depositors running and the government transfers $(c_1 - \frac{1-\alpha c_1}{1-\alpha})$ for each of them to the bank. In contrast, when $\alpha > n^*$, the bank has already exhausted its resources and the government transfers to the banking sector $\frac{1}{\alpha}$ for each of the $n > \alpha$ withdrawing depositors.

The government intervention is credible as long as the amount of the public transfer does not exceed the resources g available in the economy. If this is not the case, depositors anticipate that the public guarantee is not feasible and the equilibrium is the same as without intervention. In the rest of the analysis, we restrict our attention to the case where the amount of public good g is sufficient to cover the transfer to the banking sector so that the government intervention is fully credible.

The timing of the model is as follows. At date 0, the government chooses first the optimal value of α and then the bank chooses c_1. At date 1, after receiving the private signal about the state of the fundamentals θ, depositors decide whether to withdraw early or wait till date 2. As before, we solve the model backward starting with depositors' withdrawal decisions. We start characterizing the run thresholds $\underline{\theta}(c_1)$ and $\theta^*(c_1)$ for a generic $\varepsilon > 0$ and we then focus on the limit case $\varepsilon \to 0$.

The lower and the upper dominance regions are the same as in the decentralized equilibrium, and the upper bound of the lower dominance region $\underline{\theta}(c_1)$ is still given by (2). The determination of the intermediate region is more complicated. Late depositors know c_1 and α when making their decisions, and thus they know the amount of bank's resources available at the time of the intervention. This implies that we need to distinguish two cases for the determination of the threshold $\theta^*(c_1)$, depending on whether $\alpha \gtreqless n^*$.

We start with the case where $\alpha \leq n^*$. In this case, the utility differential for a late depositor between withdrawing at date 2 versus date 1 is given by

$$v_1(\theta, n, \alpha) = \begin{cases} p(\theta) u\left(\frac{1-nc_1}{1-n} R\right) - u(c_1) & \text{if } \lambda \leq n < \alpha \\ \\ p(\theta) u\left(\frac{1-\alpha c_1}{1-\alpha} R\right) - u(c_1) & \text{if } \alpha \leq n \leq 1. \end{cases} \tag{8}$$

The utility differential is a piecewise function, which now depends on both n and α. For $\lambda \leq n < \alpha$, the government does not intervene and a late depositor's payoff is the same as in the decentralized solution. He obtains $\frac{1-nc_1}{1-n} R$ with probability $p(\theta)$ when waiting till date 2 and c_1 when withdrawing at date 1. The

date 2 payoff decreases in the number of withdrawing depositors n. For $\alpha \leq n \leq 1$, a late depositor's obtains now the constant repayment of $\frac{1-\alpha c_1}{1-\alpha}R$ with probability $p(\theta)$ if he waits and c_1 if he withdraws prematurely. The reason is that the government intervention limits the liquidation of the bank's asset for $n \geq \alpha$, thus removing the dependence of the date 2 repayment on the number of depositors withdrawing early.

In the case where $\alpha > n^*$, the utility differential is equal to

$$v_2(\theta, n, \alpha) = \begin{cases} p(\theta)u\left(\frac{1-nc_1}{1-n}R\right) - u(c_1) & \text{if } \lambda \leq n < n^* \\ u(0) - u\left(\frac{1}{n}\right) & \text{if } n^* \leq n < \alpha \\ u(0) - u\left(\frac{1}{\alpha}\right) & \text{if } \alpha \leq n \leq 1. \end{cases} \tag{9}$$

The expression for $v_2(\alpha, \theta, n)$ has three intervals depending on the values of n and α. In the first two intervals a late depositor's payoffs are the same as in the decentralized solution since the government does not intervene. In the last interval the government intervenes, thus affecting the payoffs in the case of runs. For $\lambda \leq n < n^*$ the bank does not exhaust its resources at date 1. A late depositor obtains $\frac{1-nc_1}{1-n}R$ with probability $p(\theta)$ when waiting till date 2 and c_1 when withdrawing at date 1. By contrast, for $n \geq n^*$ the bank exhausts its resources at date 1. A late depositor waiting till date 2 always obtains nothing. By contrast, a late depositor withdrawing at date 1 receives $\frac{1}{n}$ for $n^* \leq n < \alpha$ and $\frac{1}{\alpha}$ for $\alpha \leq n \leq 1$ when the government intervenes and uses the public good to guarantee that the remaining $n - \alpha$ depositors withdrawing early obtain a constant payment irrespective of n.

The functions $v_1(\theta, n, \alpha)$ and $v_2(\theta, n, \alpha)$ have different properties. The former is decreasing in n for $\lambda \leq n < \alpha \leq n^*$, and is constant afterwards. The latter is decreasing in n for $n < n^*$, increasing in n for $n^* \leq n < \alpha$ when it is negative, and constant for $\alpha \leq n \leq 1$. Similarly to the function $v(\theta, n)$ in the decentralized economy described in (3), the function $v_2(\theta, n, \alpha)$ crosses zero only once when it is positive and decreasing in n. This implies that the model with the government intervention exhibits the property of *global strategic complementarity* for $\alpha \leq n^*$, while it satisfies *one-sided global strategic complementarity* for $\alpha > \frac{1}{c_1}$. As in the basic model with no government intervention, the model has a unique equilibrium in depositors' withdrawal decisions.

Lemma 2 *The model has a unique equilibrium in which late depositors run if they observe a signal below the thresholds $x_1^*(c_1, \alpha)$ and $x_2^*(c_{1,}\alpha)$ and do not run above. At the limit, as $\epsilon \to 0$, the equilibrium thresholds $x_1^*(c_1, \alpha)$ and $x_2^*(c_1, \alpha)$ simplify to*

$$\theta_1^*(c_1, \alpha) = p^{-1}\left(\frac{(1-\lambda)u(c_1)}{\int_{n=\lambda}^{\alpha} u\left(R\frac{1-nc_1}{1-n}\right) + \int_{n=\alpha}^{1} u\left(R\frac{1-\alpha c_1}{1-\alpha}\right)}\right) \tag{10}$$

if $\alpha \leq n^*$ *and*

$$\theta_2^*(c_1, \alpha) = p^{-1}\left(\frac{(\frac{1}{c_1} - \lambda)u(c_1) + \int_{n=1/c_1}^{\alpha} u\left(\frac{1}{n}\right) + (1-\alpha)u\left(\frac{1}{\alpha}\right)}{\int_{n=\lambda}^{n^*} u\left(R\frac{1-nc_1}{1-n}\right)}\right) \tag{11}$$

when $\alpha > n^*$.

The threshold $\theta_1^*(c_1, \alpha)$ *is increasing both in* c_1 *and* α*, while* $\theta_2^*(c_1, \alpha)$ *is increasing in* c_1 *but decreasing in* α.

Proof. See the appendix. ■

As in the decentralized economy, runs occur when the fundamentals are in the interval $[0, \theta_i^*(c_1, \alpha))$, with $i = 1, 2$. As before, runs are fundamental-driven for $\theta \in [0, \underline{\theta}(c_1))$ and panic-driven for $\theta \in [\underline{\theta}(c_1), \theta_i^*(c_1, \alpha))$. The threshold $\theta_i^*(c_1, \alpha)$ depends now both on the promised date 1 consumption and the size of government intervention as represented by α. A higher c_1 always increases the likelihood of runs, while the government intervention has a different effect depending on the value of α. In particular, a higher α reduces the likelihood of panic runs for $\alpha \leq n^*$, while it increases it for $\alpha > n^*$. The reason is that the government reduces the strategic complementarity between depositors' actions for $\alpha \leq n^*$ as by intervening it limits the liquidation of the bank's asset. By contrast, for $\alpha > n^*$, the bank has already liquidated all its project and the government intervention has the only effect of increasing the repayment of the $n - \alpha$ withdrawing depositors at date 1. The more the government intervenes (i.e., the lower α is), the higher the date 1 repayment in case of a run and thus the stronger depositors' incentives to withdraw early.

The two run thresholds $\theta_1^*(c_1, \alpha)$ and $\theta_1^*(c_1, \alpha)$ are continuous, that is

$$\theta_1^*(c_1, \alpha) = \theta_2^*(c_1, \alpha)$$

at $\alpha = n^*$. Moreover, when the government decides not to intervene and chooses $\alpha = 1$, we have

$$\theta_2^*(c_1, 1) = \theta^*(c_1) \tag{12}$$

and depositors' withdrawal decisions are the same as in the decentralized economy without intervention. In the opposite case, when the government intervenes as soon as more than λ early depositors withdraw by choosing $\alpha = \lambda$, it holds that

$$\theta_1^*(c_1, \lambda) = \underline{\theta}(c_1)$$

so that only fundamental runs occur.

This discussion highlights one of the main results of our model, namely that the government intervention has an important direct effect on the likelihood and types of runs. By choosing to intervene immediately and set $\alpha = \lambda$, the government can remove all panic runs. By choosing not to intervene and set $\alpha = 1$ the decentralized solution is replicated. For any $\lambda < \alpha < 1$ both fundamental and panic runs occur. When choosing the size of α, the government takes into account the effects of its choice on the likelihood of

runs. Differently from other papers like Keister (2010), the precise size of α matters in our model since the thresholds $\theta_i^*(c_1,\alpha)$ with $i=1,2$ are fully endogenous.

Having characterized depositors' withdrawal decisions at date 1 for given c_1 and α, we now turn to the bank' choice of the optimal deposit contract and the government choice of α at date 0. As in the decentralized solution, we focus on the limit case when $\epsilon \to 0$. We start with the bank's choice of c_1. Given α and anticipating depositors' withdrawal decisions, at date 0 each bank chooses c_1 to maximize

$$\underset{c_1}{Max} \begin{cases} \int_0^{\theta_1^*(c_1,\alpha)} u\left(c_1\right) d\theta + \int_{\theta_1^*(c_1,\alpha)}^1 \left[\lambda u(c_1) + (1-\lambda)p(\theta)u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\right] d\theta + & \\ \int_0^{\theta_1^*(c_1^*,\alpha)} v\left(g-(c_1^*-1)\right) d\theta + \int_{\theta_1^*(c_1^*,\alpha)}^1 v\left(g\right) d\theta & \text{if } \alpha \leq n^* \\ \int_0^{\theta_2^*(c_1,\alpha)} u\left(\frac{1}{\alpha}\right) d\theta + \int_{\theta_2^*(c_1,\alpha)}^1 \left[\lambda u(c_1) + (1-\lambda)p(\theta)u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\right] d\theta + & \\ \int_0^{\theta_2^*(c_1^*,\alpha)} v\left(g-\frac{1-\alpha}{\alpha}\right) d\theta + \int_{\theta_2^*(c_1^*,\alpha)}^1 v\left(g\right) d\theta & \text{if } \alpha > n^*, \end{cases} \tag{13}$$

where c_1^* denotes the equilibrium value of c_1 chosen by all banks. For a given α, each bank chooses the deposit contract c_1 that maximizes depositors' expected utility. This is a piecewise function as it depends on whether $\alpha \lessgtr n^*$. The terms in the two expressions in (13) have a similar meaning in both ranges of α. The first term represents depositors' expected utility when a run occurs for $\theta < \theta_i^*(c_1,\alpha)$. Depositors withdrawing at date 1 receive c_1 if $\alpha \leq n^*$ and a repayment $\frac{1}{\alpha} \in [1,c_1]$ if $\alpha > n^*$. The second term is depositors' expected utility when for $\theta_i^*(c_1,\alpha) \leq \theta \leq 1$ only the λ early consumers withdraw at date 1 and there is no run. The last two terms in each expression represent depositors' expected utility from the public good. For $\theta < \theta_i^*(c_1,\alpha)$ a run occurs and depositors enjoy the public good that remains after the transfer to the banking sector. If $\alpha \leq n^*$ the government uses c_1^*-1 of the public good to guarantee the repayment c_1 to the $1-\alpha$ withdrawing depositors and the utility v is derived from the remaining $g-(c_1^*-1)$ units. If $\alpha > n^*$ the government transfers $\frac{1-\alpha}{\alpha}$ to guarantee the constant payment $\frac{1}{\alpha}$ to the $1-\alpha$ depositors withdrawing after the intervention. Thus, depositors enjoy the utility $v\left(g-\frac{1-\alpha}{\alpha}\right)$ from the remaining units of public good. For $\theta_i^*(c_1,\alpha) \leq \theta \leq 1$ no transfer takes place and depositors obtain utility $v(g)$ from the public good irrespective of the value of α. Banks do not internalize the cost for the provision of the guarantee when choosing the optimal deposit contract. The reason is that each bank is atomistic and takes the equilibrium consumption as given.

The solution to the bank's problem is summarized in the following proposition.

Proposition 2 *The optimal deposit contract $c_1^*(\alpha)$ depends on the size of the government intervention α as follows:*

i) When $\alpha = \lambda$, the optimal deposit contract is $c_1^*(\lambda) \to \overline{c}_{11}(\lambda)$, where $\overline{c}_{11}(\lambda)$ is defined by the following equation

$$\lim_{c_1 \to \overline{c}_{11}(\lambda)} \theta_1^*(c_1^*(\lambda),\lambda) = \underline{\theta}(c_1) = \overline{\theta}$$

and runs occur in the range $\left[0, \overline{\theta}\right]$. *The threshold* $\overline{c}_{11}(\alpha)$ *is decreasing in* α.

ii) When $\alpha = 1$, $c_1^*(1) = c_1^D \geqslant 1/\alpha$ *and runs occur in the range* $[0, \theta_2^*(c_1^*(1), 1))$, *where* $\theta_2^*(c_1^*(1), 1) = \theta^*(c_1) < \overline{\theta}$ *as defined in (12).*

Proof. *See the appendix.* ■

The proposition shows that the bank's choice of c_1 depends crucially on the size of α. When the government intervenes as soon as a run occurs, i.e., when $\alpha = \lambda$, the bank chooses the maximum possible level of c_1. The date 1 promised payment approaches the threshold value $\overline{c}_{11}(\lambda)$ at which runs occur except when θ lies in the upper dominance region $\theta \in \left(\overline{\theta}, 1\right]$. The threshold value $\overline{c}_{11}(\alpha)$ is decreasing in α, suggesting that the optimal choice of $c_1^*(\alpha)$ must be decreasing as well. When the government chooses not to intervene, i.e., when $\alpha = 1$ the bank chooses the same date 1 payment as in the decentralized solution and runs occur with the same likelihood as there.

The proposition suggests two important insights. First, the government intervention introduces a moral hazard problem in the bank's choice of the optimal deposit contract. Since banks do not internalize the cost of the intervention in terms of a lower provision of the public good g, they have an incentive to choose an excessively high c_1 to exploit the guarantee. The optimal choice of c_1 maximizes depositors' expected utility from depositing at the bank, disregarding the effect of a higher c_1 on the utility deriving from the consumption of the public good. The moral hazard problem is maximum when $\alpha = \lambda$ since depositors are guaranteed to obtain c_1 as chosen by the bank in case of a run. The government can limit the moral hazard problem by choosing to intervene less and, at the limit, not to intervene at all. Second, besides the direct effect as analyzed in Lemma 2, the government intervention has also an indirect effect on the likelihood of runs through the bank's choice of c_1. When $\alpha = \lambda$, the government intervention eliminates the coordination problem among depositors and thus only fundamental runs occur. When $\alpha > \lambda$, this problem is not fully eliminated and panic runs remain.

Having characterized the bank's choice of the optimal deposit contract, we now turn to the government's choice of α. Given banks' optimal choice c_1, the government chooses α at date 0 to maximize depositors' total expected utility. This is given by the same two expressions as in (13) evaluated at the bank's optimal choice $c_1^*(\alpha)$. The following result illustrates how the optimal size of government intervention depends on the amount of public resources g available.

Proposition 3 *Define* $\overline{g}$ *as the cutoff value of public resources such that* $\alpha = \lambda$ *is optimal. Then, the government chooses to limit its intervention and set* $\alpha > \lambda$ *for any amount of public resource* $g < \overline{g}$. *As a consequence, panic runs still occur.*

Proof. See the appendix. ■

The proposition shows that when its budget is tight, i.e., when $g < \overline{g}$, the government limits its intervention by choosing $\alpha > \lambda$. As a consequence, panic runs are not fully prevented. The government chooses the optimal size of α to maximize depositors' total expected utility. In doing this, it takes into account the effect that this choice has on banks' and depositors' behavior. In particular, the government anticipates that the intervention entails a moral hazard problem on the side of the banks as they have an incentive to exploit the guarantee and choose an excessively high c_1. The bank's choice influences the likelihood of runs besides the direct effect of α. When the government's budget is tight, the bank moral hazard problem entails large costs. Given the concavity of the function $v(g)$, even a small reduction in the provision of the public good generates a large disutility. Thus, the government finds it optimal to contain the bank moral hazard problem by limiting its intervention and choosing $\alpha > \lambda$. The bank then chooses a lower c_1, which translates ceteris paribus in a lower probability of runs. However, panic runs are not eliminated since for $\alpha > \lambda$ the depositors' coordination problem is not fully prevented.

The consequence of all this is that, although the main objective of the public intervention is to reduce the likelihood of runs relative to the decentralized economy, the government may be unable to do so. Because of the bank moral hazard problem, the government may be forced to intervene less than what is required to eliminate panic runs when it has limited budget. Still, banks may choose a level of c_1 such that runs are more likely with the government intervention than without. Given this, in the next section we analyze two alternative guarantee schemes that may improve upon the government intervention analyzed in this section.

5 Improving upon government intervention

In the previous section we have analyzed a guarantee scheme in which the government chooses to transfer some public resources g to the banking sector at date 1 after the first $\alpha \geq \lambda$ depositors withdraw. The intervention has the objective of reducing the likelihood of runs, thus improving the risk sharing that banks offer to depositors and limiting the inefficient premature liquidation of the banks' project. However, it fails to do so because of the bank moral hazard problem. To contain this, the government intervenes less than what would be requires to eliminate the coordination problem among depositors. As a consequence, the intervention increases depositors' payment in case of a run, but it fails to prevent the occurrence of panic runs and to limit the inefficient liquidation of the banks' projects.

In this section, we analyze two ways to improve upon the government intervention analyzed in the previous section. The first possibility is for the government to control the choice of the bank's deposit contract or, taken it to the extreme, to choose the promised date 1 payment to depositors directly. The second possibility is for the government to offer a guarantee such that all runs −both fundamental and

panic-driven− are prevented. We start with the first case.

5.1 Avoiding the moral hazard problem

Consider now that the government chooses both the size of the intervention α and the deposit contract c_1. Then, as before, the government transfers resources from the public good g to the banking sector after the first $\alpha \geq \lambda$ depositors withdraw. As there is no bank moral hazard problem now, the government always intervenes before the bank has liquidated the entire project and still pays c_1 to the withdrawing depositors. This implies that $\alpha \leq n^*$ and that the government transfers $(n-\alpha)\left(c_1 - \frac{1-\alpha c_1}{1-\alpha}\right)$ to each bank when it intervenes. This allows the bank to pay c_1 to all the $n \geq \alpha$ depositors withdrawing early while liquidating only $\frac{1-\alpha c_1}{1-\alpha} < 1$ resources per depositor.

The problem is similar to the one in Section 4. Anticipating depositors' withdrawal decisions, at date 0 the government chooses the size of the intervention α and the deposit contract c_1 to maximize depositors' total expected utility. Depositors' withdrawal decisions are as in Lemma 2 with the difference that we have to consider only the case where $\alpha < n^*$ and thus only the run threshold $\theta_1^*(c_1, \alpha)$ as defined in (10). Given this, and focusing again on the limit case for $\varepsilon \to 0$ when only complete runs occur (i.e.,$n = 1$) and the size of the transfer is $c_1 - 1$, we can write the government's problem as given by

$$\underset{\alpha, c_1}{Max} \int_0^{\theta_1^*(c_1,\alpha)} u\left(c_1\right) d\theta + \int_{\theta_1^*(c_1,\alpha)}^1 \left[\lambda u(c_1) + (1-\lambda) p(\theta) u\left(\frac{1-\lambda c_1}{1-\lambda} R\right)\right] d\theta \tag{14}$$
$$+ \int_0^{\theta_1^*(c_1,\alpha)} v\left(g - (c_1 - 1)\right) d\theta + \int_{\theta_1^*(c_1,\alpha)}^1 v\left(g\right) d\theta$$

subject to

$$g - (c_1 - 1) \geq 0 \tag{15}$$

The problem is as the one in (13) for $\alpha \leq n^*$ with the important difference that the government internalizes now the cost of the intervention when choosing c_1. This changes the last two terms in (14) representing depositors' utility from the provision of the public good relative to the problem in (13) in that there is now c_1 instead of c_1^*. Also, the government takes into account that the choice of c_1 has an implication on the feasibility of the intervention. In other words, as required by (15), the government internalizes that the transfer $c_1 - 1$ cannot exceed the total amount of public good available. We have the following result.

Proposition 4 *A government that chooses the size of the intervention α and the optimal deposit contract c_1 intervenes after $\alpha = \lambda$ depositors withdraw at date 1 and sets*

$$c_1^G = \min\left\{\widetilde{c}_1^G, g+1\right\},$$

where $\widetilde{c}_1^G \geq 1$ *is the solution to*

$$\int_0^{\underline{\theta}(c_1)} u'(c_1)\, d\theta + \lambda \int_{\underline{\theta}(c_1)}^1 \left[u'(c_1) - p(\theta) R u' \left(\frac{1-\lambda c_1}{1-\lambda} R \right) \right] d\theta +$$

$$-\frac{\partial \underline{\theta}(c_1)}{\partial c_1} \left[v(g) - v\left(g - (c_1 - 1)\right) \right] - \int_0^{\underline{\theta}(c_1)} v'(g - (c_1 - 1)) d\theta = 0 \tag{16}$$

and $\underline{\theta}(c_1)$ *is defined in (2).*

Proof. See the appendix. ■

The proposition shows that the government optimally chooses $\alpha = \lambda$ and a payment c_1^G that, depending on the size of g, is either unconstrained and equal to the solution to (16) or a corner solution. The former trades off a higher payment in case of runs and better risk sharing with a lower provision of g. This solution is attained when g is large enough and/or the function $v(g)$ is not too concave. The latter, which is the solution when g is such that (15) binds, corresponds to the maximum value of $c_1^G = g + 1$ that makes the government intervention feasible and, thus, credible. For either values of c_1^G, only fundamental runs occur. The reason is that by choosing $\alpha = \lambda$, the government eliminates the coordination problem among depositors. Given this, late depositors base their withdrawal decision on the fundamentals and run only if they receive a signal $\theta \leq \underline{\theta}(c_1)$. Still, whenever $c_1^G > 1$, these fundamental runs are not always efficient as they entail an inefficient premature liquidation of the bank's asset for $\underline{\theta}(c_1) > \theta^E$, where θ^E is as in (7). Only in the case when $c_1^G = 1$ and $\underline{\theta}(1) = \theta^E$ runs always entail an efficient liquidation of the bank's project.

The allocation described in Proposition 4 in which the government chooses both the size of the intervention and the optimal deposit contract does not entail any form of moral hazard problem and improves the allocation in which the choice of c_1 is made by the banks in various ways. First, the government intervenes as soon as a run starts. As a consequence, it removes the occurrence of panic runs. Second, depositors always receive c_1 in the case of a run, as the government does not need to limit its intervention to control the bank moral hazard problem. However, despite being clearly beneficial, this intervention may not be possible as it requires that the government has control over the choice of c_1. Such a control may be banned in practice, for example for reasons related to antitrust regulation. When this is the case, the government may offer a different form of guarantee to improve the allocation found in Section 4, which eliminates all runs, both fundamental and panic-driven. Although this intervention may entail an even more severe bank moral hazard problem, it has the advantage of eliminating the effect of c_1 on the likelihood of runs and thus improve on the allocation of Section 4.

5.2 Eliminating runs

Consider now the case where the government uses the public good to guarantee c_1 to all depositors, either at date 1 or date 2, irrespective of whether the bank is solvent at date 2. Under this scheme, runs do not

occur any longer. Waiting till date 2 is a dominant strategy for the late depositors as at date 2 they obtain the payment $\frac{1-\lambda c_1}{1-\lambda}R$ with probability $p(\theta)$ and c_1 with probability $1-p(\theta)$. Anticipating this, each bank's maximization problem at date 0 is given by

$$\underset{c_1}{Max} \int_0^1 \left[\lambda u(c_1) + (1-\lambda)\left(p(\theta)u\left(\frac{1-\lambda c_1}{1-\lambda}R\right) + (1-p(\theta))u\left(c_1\right)\right)\right] d\theta +$$

$$+ \int_0^1 \left[p(\theta)v(g) + (1-p(\theta))v\left(g-(1-\lambda)c_1^*\right)\right] d\theta \tag{17}$$

with c_1^* denoting the equilibrium value of c_1 chosen by all banks. The first term in (17) represents depositors' utility from depositing in the bank. Since there are no runs, the λ early depositors obtain c_1 at date 1, while the $1-\lambda$ late depositors obtain either $\frac{1-\lambda c_1}{1-\lambda}R$ with probability $p(\theta)$ or c_1 with probability $1-p(\theta)$. The second term is the utility from the public good. Each depositor enjoys an utility $v(g)$ with probability $p(\theta)$ and $v\left(g-(1-\lambda)c_1^*\right)$ with probability $1-p(\theta)$ when the government intervenes and transfers $(1-\lambda)c_1^*$ to the bank so that each late depositors obtains c_1^*. As in Section 4, the bank does not internalize the cost of the transfer when choosing c_1. The solution to the bank's problem is summarized in the following proposition.

Proposition 5 *When the government guarantees c_1 to all depositors either at date 1 or 2, each bank chooses c_1^{DI} as given by the solution*

$$\lambda \int_0^1 \left[u'(c_1) - p(\theta)Ru'\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\right] d\theta + (1-\lambda)\int_0^1 (1-p(\theta))u'(c_1)d\theta = 0. \tag{18}$$

Proof. See the appendix. ■

The optimal deposit contract c_1^{DI} trades off the benefit of a higher c_1 in terms of better risk sharing and higher payment at date 2 when the bank is insolvent with the cost in terms of lower payment at date 2 when the bank remains solvent. The proposition highlights that this government guarantee entails again a moral hazard problem since banks do not internalize the cost of the guarantee as represented by a lower provision of the public good. Differently from before though, the bank's choice of c_1 does not affect the likelihood of runs, which are completely eliminated, but only the cost of the intervention $(1-\lambda)c_1^*$. Whether this guarantee represents an improvement relative to the intervention analyzed in Section 4 crucially depends on the probability $1-p(\theta)$ and the size of the public good g, as we show in the next section.

6 A numerical example

In this section we illustrate the properties of the model and in particular the comparison across the various forms of government intervention with the use of a numerical example. We consider the following functional forms for the utility function:

$$u(c) + v(g) = \frac{(c+f)^{1-\sigma}}{1-\sigma} - \frac{(f)^{1-\sigma}}{1-\sigma} + \frac{(g+f)^{1-\sigma}}{1-\sigma} - \frac{(f)^{1-\sigma}}{1-\sigma},$$

and assume $p(\theta) = \theta$, $\sigma = 3$, $R = 5$, $\lambda = 0.3$ and $f = 4$. Regarding the upper dominance region, we assume that $\overline{\theta}$ approaches to 1. We consider different amount of the public good g to show how the optimal government intervention, banks' deposit contract and depositors' withdrawal decisions change depending on the amount of public resources available in the economy. Specifically, we analyze the following cases: $g = 1.55$, $g = 2$ and $g = 2.5$.

The decentralized economy without government intervention and the desirability of the intervention

In this section we report the results relative to the decentralized economy without government intervention (or, equivalently, $\alpha = 1$) and we compare them with those relative to the case in which the government transfers resources from the public good g to the banking sector after the first $\alpha \geq \lambda$ depositors withdraw at date 1.

Insert Table 1

Table 1 shows that there is scope for government intervention. For any possible values of g, the government always chooses to intervene (i.e., $\alpha < 1$), depositors receive a higher repayment in the case of a run and a higher expected utility than in the decentralized solution without intervention. However, the introduction of the guarantee scheme entails a moral hazard problem on the side of the banks as represented by the fact that c_1 is always higher in the solution with intervention than in the one without government intervention. As a consequence, there are more fundamental runs than in the decentralized solution and panic runs are not fully prevented. In order to control for the moral hazard problem and contain its costs, the government is forced to limit the size of the intervention α. Since the costs of the intervention, as represented by a lower provision of the public good, are larger when the public resources g are limited, the government reduces the size of the intervention α as g decreases ($\alpha = 0.95$ when $g = 1.55$ instead of $\alpha = 0.8946$ when $g = 2$ and $\alpha = 0.7$ when $g = 2.5$). This leads to a lower c_1 and, in turn to a lower likelihood of runs as shown in Figure 1.

Insert Figure 1

The example described above shows that, in a context where banks' and depositors' withdrawal decisions are endogenously determined and both fundamental and panic runs are possible, government intervention crucially affects the deposit contract c_1 offered by banks, the probability and the type of runs. On the one hand, the introduction of the guarantee scheme improves depositors' payoff in case of a run. On the other hand, it affects the likelihood of crisis−both panic and fundamental ones−. The effect on the probability of a run is twofold. While the anticipation of the intervention by depositors reduces the coordination problem and thus, their incentive to run, it also increases banks' incentive to exploit the guarantees by offering a higher

payment in case of an early withdrawal, which in turn increases the likelihood of runs. The government chooses the optimal size of the intervention balancing these two effects and optimally allocates resources between the private and the public good. The moral hazard problem on the side of the banks and its negative effect on the likelihood of runs forces the government to limit the size of the intervention, especially when its costs are high (i.e., when g is low). As a consequence, panic runs are not fully prevented.

Improvements upon the government intervention

In this section we turn to analyze the two alternative scheme that improves upon the government intervention by limiting the occurrence of runs−both types or only the panic ones−.

Thus, we present three tables, for the values $g = 1.55$, $g = 2$ and $g = 2.5$.

Insert Table 2,3 and 4

Comparing Tables 1 with 2, 3 and 4 shows that the scheme in which the government chooses both the size of the intervention α and the deposit contract c_1 represents, for any value of g, the best allocation as illustrated in Figure 2. By choosing $\alpha = \lambda$, the government removes all panic runs while leaving only the fundamental ones. Independently of the amount of resources g, the government offers a higher level of c_1 than in the solution in which the choice of the deposit contract is made by the banks (i.e., $c_1^G > c_1^\alpha$). Nevertheless, when g is not too small (i.e., except in the case where $g = 1.55$) there are fewer runs than in the solution in which the government cannot control c_1.

When the choice of the deposit contract is made by the banks, the desirability of the different forms of intervention depends crucially on the amount of resources g available in the economy. The scheme in which the government chooses the size of the intervention α is the best one for $g = 1.55$ and $g = 2$. However, for $g = 2.5$ the deposit insurance scheme promising at least c_1 to all depositors and eliminating all types of runs dominates. This is due to the fact that, when the amount of resources g is large, the government is less concerned about limiting the moral hazard problem on the side of the bank. Thus, the government chooses a lower α and, in turn banks set a higher c_1 and more runs occur. By contrast, the deposit insurance scheme guaranteeing c_1 removes all runs and, even if it entails a greater moral hazard problem ($c_1^{DI} > c_1^\alpha$), it ensures a higher depositors' total expected utility.

Insert Figure 2

7 Concluding Remarks

In this paper we have developed a simple model where both panic and fundamental runs are possible and both banks' and depositors' decisions are endogenously determined. We have shown that government intervention is desirable as it reduces the inefficiency of the decentralized economy arising from the coordination problem among consumers. However, the introduction of government's guarantees generates a tradeoff. On the one hand, by limiting the premature liquidation of banks' asset, government intervention reduces depositors' incentives to run and thus the probability of crises. On the other hand, as banks do not fully internalize the costs of the intervention, it induces them to take excessive risk in the form of a higher repayment offered to consumers withdrawing early. In some cases, the moral hazard problem on the side of the banks offsets the benefits of the introduction of the guarantee scheme leading to inefficient outcomes in which the system is more fragile than in the case without intervention and the disbursement for the government is large. To account for the moral hazard problem, we have considered a more general form of government intervention than the standard deposit insurance à la Diamond and Dybvig in which the government chooses the optimal size of the intervention. By limiting the level of guarantees, the government reduces banks' incentives to exploit the guarantees but panic runs still occur as the intervention does not fully remove the coordination problem among depositors.

We consider ways for the government to improve upon its guarantees by preventing the occurrence of runs-either both types or only the panic ones-. One possibility is for the government to control the choice of the deposit contract set by the banks. While it allows the government to set a high level of insurance and, thus to completely prevent the occurrence of panic runs, this solution may not always be feasible as it requires the government to control the amount of risk taken by the banks. A second possibility is a scheme that ensures the payment even in the long run with the consequence that both fundamental-based and panic-based runs are prevented. In this case, the moral hazard problem can be still severe, which means that the government will have to bear huge losses in the case banks are insolvent. We have shown that the optimal level of guarantees depends on the amount of resources available to finance the scheme. In economies in which the government has a tight budget, the consequences of the moral hazard problem are severe. In this case, the most efficient form of intervention is a more moderate form of intervention which limits moral hazard by leaving panic runs. On the contrary, when government has a large amount of resources to transfer to the banking sector, blanket guarantees, which removes all types of runs, are more efficient than other more moderate form of intervention.

The paper offers an ideal framework to evaluate the implications of government guarantees, as the likelihood and types of runs and the deposit contract are completely endogenous and affected by the presence of

the deposit insurance. The analysis sheds light on the tradeoff generated by the introduction of government guarantees and offers insights for future research. One potentially interesting extension would be the analysis of the feedback effect between government guarantees and financial stability. When the introduction of a guarantee scheme entails an actual disbursement for the government, it can threaten the solvency of the country and thus undermines the credibility of the guarantees themselves. The threat of sovereign default represents a new source of risk that has been completely overlooked in the literature on government intervention so far, but, as the recent European sovereign crisis has shown, it is a very relevant drawback of the massive government intervention which took place during the crisis.

Another possible extension would be removing the assumption of full commitment so that the government only intervenes if it is ex post optimal. The credibility of the intervention will then be conditional on its ex post optimality which endogenously depends on the features of the guarantee introduced. There is a growing literature analyzing different form of interventions in a context of limited commitment (e.g., Ennis and Keister, 2009 and 2010 and Cooper and Kempf, 2011), but all those contributions consider an exogenous probability of runs.

A Proofs

Proof of Lemma 1: The proof follows Goldstein and Pauzner (2005). The arguments in their proof establish that there is a unique equilibrium in which depositors run if and only if the signal they receive is below a common signal $x^*(c_1)$. The number n of depositors withdrawing at date 1 is equal to the probability of receiving a signal x_i below $x^*(c_1)$ and, given that the posterior distribution of θ is uniform over the interval $[x^*(c_1)-\epsilon, x^*(c_1)+\epsilon]$, it is given by:

$$n(\theta, x^*(c_1)) = \begin{cases} 1 & \text{if } \theta \leq x^*(c_1)-\varepsilon \\ \lambda + (1-\lambda)\left(\frac{x^*(c_1)-\theta+\varepsilon}{2\varepsilon}\right) & \text{if } x^*(c_1)-\varepsilon \leq \theta \leq x^*(c_1)+\varepsilon \\ \lambda & \text{if } \theta \geq x^*(c_1)+\varepsilon \end{cases} \tag{19}$$

The posterior distribution of n is uniform over the range $[\lambda, 1]$. When θ is below $x^*(c_1)-\epsilon$, all patient depositors receive a signal below $x^*(c_1)$ and run. When θ is above $x^*(c_1)+\epsilon$, all late depositors wait until date 2 and only the λ early consumers withdraw early. In the intermediate interval, when θ is between $x^*(c_1)-\epsilon$ and $x^*(c_1)+\epsilon$, there is a partial run as some of the late depositors run. The proportion of late consumers withdrawing decreases linearly with θ as fewer agents observe a signal below the threshold.

Having characterized the proportion of agents withdrawing for any possible value of the fundamentals θ, we can now compute the threshold signal $x^*(c_1)$. A patient depositor who receives the signal $x^*(c_1)$ must be indifferent between withdrawing at date 1 and at date 2. The threshold $x^*(c_1)$ can be then found as the solution to

$$f(\theta, c_1) = \int_{n=\lambda}^{\frac{1}{c_1}} \left[p(\theta(n)) u\left(\frac{1-nc_1}{1-n} R\right) - u(c_1) \right] + \int_{n=\frac{1}{c_1}}^{1} \left[u(0) - u\left(\frac{1}{n}\right) \right] = 0, \tag{20}$$

where, from (19), $\theta(n) = x^*(c_1) + \epsilon - 2\epsilon\frac{(n-\lambda)}{1-\lambda}$. Equation (20) follows from (3) and requires that a late depositor's expected utility when he withdraws at date 1 is equal to that when he waits till date 2. Note that at the limit, when $\epsilon \to 0$, $\theta(n) \to x^*(c_1)$, and we denote it as $\theta^*(c_1)$. Solving (20) with respect to $\theta^*(c_1)$ gives the threshold as in the proposition.

To prove that $\theta^*(c_1)$ is increasing in c_1, we use the implicit function theorem and obtain

$$\frac{\partial\theta^*(c_1)}{\partial c_1} = -\frac{\frac{\partial f(\theta^*,c_1)}{\partial c_1}}{\frac{\partial f(\theta^*,c_1)}{\partial\theta^*}}.$$

It is easy to see that $\frac{\partial f(\theta^*,c_1)}{\partial\theta} > 0$. Thus, the sign of $\frac{\partial\theta^*(c_1)}{\partial c_1}$ is given by the opposite sign of $\frac{\partial f(\theta^*,c_1)}{\partial c_1}$, where

$$\frac{\partial f(\theta^*,c_1)}{\partial c_1} = -\frac{1}{c_1^2}\left[p(\theta^*)u(\frac{1-\frac{1}{c_1}c_1}{1-\frac{1}{c_1}}R) - u(c_1)\right] + \frac{1}{c_1^2}\left[0 - u(c_1)\right] - \int_{n=\lambda}^{\frac{1}{c_1}}\left[p(\theta^*)\left(\frac{nR}{1-n}\right)u'(\frac{1-nc_1}{1-n}R) + u'(c_1)\right] =$$

$$= -\int_{n=\lambda}^{\frac{1}{c_1}}\left[p(\theta^*)\left(\frac{nR}{1-n}\right)u'(\frac{1-nc_1}{1-n}R) + u'(c_1)\right] < 0.$$

This implies $\frac{\partial\theta^*(c_1)}{\partial c_1} > 0$. □

Proof of Proposition 1: Differentiating (5) with respect to c_1 gives the optimal deposit contract c_1^D as the solution to (6).

To show that $c_1^D > 1$, we evaluate (6) at $c_1 = 1$. From (4), at $c_1 = 1$ the threshold $\theta^*(c_1)$ simplifies to

$$\theta^*(1) = p^{-1}\frac{(1-\lambda)u(1)}{(1-\lambda)u(R)},$$

and, from (2), it is then

$$\theta^*(1) = \underline{\theta}(1).$$

Thus, when $c_1 = 1$, (6) can be rewritten as follows:

$$\lambda\int_{\underline{\theta}(1)}^{1}\left[u'(1) - p(\theta)Ru'(R)\right] - \frac{\partial\underline{\theta}(c_1)}{c_1}|_{c_1=1}(1-\lambda)\left[p(\underline{\theta}(1)u(R) - u(1))\right]$$

The second term is equal to zero because of the definition of $\underline{\theta}(c_1)$ in (2), and thus the expression simplifies to

$$\lambda\int_{\underline{\theta}(1)}^{1}\left[u'(1) - p(\theta)Ru'(R)\right].$$

Since the relative risk aversion coefficient is bigger than 1, it holds

$$1\cdot u'(1) > Ru'(R),$$

so that $\lambda\int_{\underline{\theta}(1)}^{1}\left[u'(1) - p(\theta)Ru'(R)\right] > 0$ and thus $c_1^D > 1$. □

Proof of Lemma 2: The proof is analogous to the one of Lemma **1** with the difference that now we have to compute two equilibrium thresholds depending on the value of α relatively to n^*. In both cases (i.e. $\alpha \lessgtr n^*$), a patient depositor who receives the signal $x^*(c_1)$ must be indifferent between withdrawing at date 1 and at date 2.

We start from the case $\alpha \leqslant n^*$. The threshold $x_1^*(c_1, \alpha)$ can be then found as the solution to

$$f_1(\theta, c_1, \alpha) = \int_{n=\lambda}^{\alpha} \left[p(\theta(n)) u(\frac{1-nc_1}{1-n} R) - u(c_1) \right] + \int_{n=\alpha}^{1} \left[p(\theta(n)) u(\frac{1-\alpha c_1}{1-\alpha} R) - u(c_1) \right] = 0, \tag{21}$$

where, similarly to (19), $\theta(n) = x_1^*(c_1, \alpha) + \epsilon - 2\epsilon \frac{(n-\lambda)}{1-\lambda}$. Equation (21) follows from (8) and requires that a late depositor's expected utility when he withdraws at date 1 is equal to that when he waits till date 2. At the limit, when $\epsilon \to 0$, $\theta(n) \to x_1^*(c_1, \alpha)$, and we denote it as $\theta_1^*(c_1, \alpha)$. Solving (21) with respect to $\theta_1^*(c_1, \alpha)$ gives the threshold as in the proposition.

We now consider the case $\alpha > n^*$. Using the same arguments as above, the threshold $x_2^*(c_1, \alpha)$ is the solution to

$$f_2(\theta, c_1, \alpha) = \int_{n=\lambda}^{\frac{1}{c_1}} \left[p(\theta(n)) u(\frac{1-nc_1}{1-n} R) - u(c_1) \right] + \int_{n=\frac{1}{c_1}}^{\alpha} \left[u(0) - u(\frac{1}{n}) \right] + \int_{n=\alpha}^{1} \left[u(0) - u(\frac{1}{\alpha}) \right] =$$

$$= \int_{n=\lambda}^{\frac{1}{c_1}} \left[p(\theta(n)) u(\frac{1-nc_1}{1-n} R) - u(c_1) \right] - \int_{n=\frac{1}{c_1}}^{\alpha} u(\frac{1}{n}) - \int_{n=\alpha}^{1} u(\frac{1}{\alpha}) = 0. \tag{22}$$

Thus, at the limit when $\epsilon \to 0$, we denote as $\theta_2^*(c_1, \alpha)$ the solution to (22).

To prove that $\theta_1^*(c_1, \alpha)$ and $\theta_2^*(c_1, \alpha)$ are increasing in c_1, we use the implicit function theorem and obtain

$$\frac{\partial \theta_i^*(c_1, \alpha)}{\partial c_1} = - \frac{\frac{\partial f_i(\theta_i^*, c_1, \alpha)}{\partial c_1}}{\frac{\partial f_i(\theta_i^*, c_1, \alpha)}{\partial \theta_i^*}}$$

with $i = 1, 2$.

It is easy to see that $\frac{\partial f_i(\theta_i^*, c_1, \alpha)}{\partial \theta_i^*} > 0$ for any $i = 1, 2$. Thus, the sign of $\frac{\partial \theta_i^*(c_1, \alpha)}{\partial c_1}$ is given by the opposite sign of $\frac{\partial f_i(\theta_i^*, c_1, \alpha)}{\partial c_1}$.

For $\theta_1^*(c_1, \alpha)$, we have

$$\frac{\partial f_1(\theta_1^*, c_1, \alpha)}{\partial c_1} = - \int_{n=\lambda}^{\alpha} p(\theta_1^*) \left(\frac{nR}{1-n} \right) u'(\frac{1-nc_1}{1-n} R) +$$

$$- \int_{n=\alpha}^{1} p(\theta_1^*) \left(\frac{\alpha R}{1-\alpha} \right) u'(\frac{1-\alpha c_1}{1-\alpha} R) - \int_{n=\lambda}^{1} u'(c_1) < 0.$$

This implies $\frac{\partial \theta_1^*(c_1, \alpha)}{\partial c_1} > 0$.

For $\theta_2^*(c_1, \alpha)$, we have

$$\frac{\partial f_2(\theta_2^*, c_1, \alpha)}{\partial c_1} = - \int_{n=\lambda}^{\frac{1}{c_1}} \left[p(\theta_2^*) \left(\frac{nR}{1-n} \right) u'(R \frac{1-nc_1}{1-n}) + u'(c_1) \right] < 0.$$

This implies $\frac{\partial \theta_2^*(c_1, \alpha)}{\partial c_1} > 0$.

To prove that $\theta_1^*(c_1, \alpha)$ is increasing in α while $\theta_2^*(c_1, \alpha)$ is decreasing in α, we use again the implicit function theorem and obtain

$$\frac{\partial \theta_i^*(c_1, \alpha)}{\partial \alpha} = - \frac{\frac{\partial f_i(\theta_i^*, c_1, \alpha)}{\partial \alpha}}{\frac{\partial f_i(\theta_i^*, c_1, \alpha)}{\partial \theta_i^*}}$$

with $i = 1, 2$.

Being $\frac{\partial f_i(\theta_i^*, c_1, \alpha)}{\partial \theta_i^*} > 0$ for any $i = 1, 2$, the sign of $\frac{\partial \theta_i^*(c_1,\alpha)}{\partial \alpha}$ is given by the opposite sign of $\frac{\partial f_i(\theta_i^*, c_1, \alpha)}{\partial \alpha}$.

For $\theta_1^*(c_1, \alpha)$, we have

$$\frac{\partial f_1(\theta_1^*, c_1, \alpha)}{\partial \alpha} = -\int_{n=\alpha}^{1} p(\theta_1^*) \frac{(c_1 - 1)}{(1-\alpha)^2} R u'(\frac{1 - \alpha c_1}{1 - \alpha} R) < 0.$$

This implies $\frac{\partial \theta_1^*(c_1,\alpha)}{\partial \alpha} > 0$.

For $\theta_2^*(c_1, \alpha)$, we have

$$\frac{\partial f_2(\theta_2^*, c_1, \alpha)}{\partial \alpha} = -u(\frac{1}{\alpha}) + u(\frac{1}{\alpha}) + \frac{1}{\alpha^2}\int_{n=\alpha}^{1} u'(\frac{1}{\alpha}) = \frac{1}{\alpha^2}\int_{n=\alpha}^{1} u'(\frac{1}{\alpha}) > 0.$$

This implies $\frac{\partial \theta_2^*(c_1,\alpha)}{\partial \alpha} < 0$.

Thus, the lemma follows.□

Proof of Proposition 2: Denote as $\overline{c}_{11}$ and $\overline{c}_{12}$ the maximum level of consumption that the bank can choose. The two upper bounds are defined by the following equations, respectively

$$\lim_{c_1 \to \overline{c}_{11}} \theta_1^*(c_1, \alpha) = \overline{\theta} \tag{23}$$

and

$$\lim_{c_1 \to \overline{c}_{12}} \theta_2^*(c_1, \alpha) = \overline{\theta}. \tag{24}$$

As $\theta_1^*(c_1, \alpha)$ is increasing in c_1 and α, $\overline{c}_{11}$ is decreasing in α. On the contrary, given that $\theta_2^*(c_1, \alpha)$ is increasing in c_1 but decreasing in α, $\overline{c}_{12}$ is increasing in α.

Consider first the case $\alpha \leq n^*$. Differentiating **(13)** with the respect to c_1 gives

$$\begin{aligned} &\int_0^{\theta_1^*(c_1,\alpha)} u'(c_1)\, d\theta + \lambda \int_{\theta_1^*(c_1,\alpha)}^{1} \left[u'(c_1) - Rp(\theta) u'\left(\frac{1 - \lambda c_1}{1 - \lambda} R \right) \right] d\theta + \\ &- \frac{\partial \theta_1^*(c_1, \alpha)}{\partial c_1} (1 - \lambda) \left[p(\theta_1^*(c_1, \alpha)) u\left(\frac{1 - \lambda c_1}{1 - \lambda} R \right) - u(c_1) \right] = 0. \end{aligned} \tag{25}$$

When $\alpha = \lambda$, (**25**) simplifies to

$$\int_0^{\underline{\theta}(c_1)} u'(c_1)\, d\theta + \lambda \int_{\underline{\theta}(c_1)}^{1} \left[u'(c_1) - Rp(\theta) u'\left(\frac{1 - \lambda c_1}{1 - \lambda} R \right) \right] d\theta.$$

Evaluating the expression above at $c_1 \to \overline{c}_{11}(\lambda)$ and given that $\overline{\theta} \to 1$, it simplifies to

$$\int_0^{\overline{\theta}} u'(c_1)\, d\theta > 0.$$

Thus, banks optimally choose $c_1^* \to \overline{c}_{11}$ and runs occur in the range $\left[0, \overline{\theta}\right]$.

Now we turn to the interval $\alpha > n^*$. Differentiating **(13)** with the respect to c_1 gives

$$\begin{aligned} &\lambda \int_{\theta_2^*(c_1,\alpha)}^{1} \left[u'(c_1) - Rp(\theta) u'\left(\frac{1 - \lambda c_1}{1 - \lambda} R \right) \right] d\theta + \\ &- \frac{\partial \theta_2^*(c_1, \alpha)}{\partial c_1} \left[(1 - \lambda)\, p(\theta_2^*(c_1, \alpha)) u\left(\frac{1 - \lambda c_1}{1 - \lambda} R \right) + \lambda u(c_1) - u\left(\frac{1}{\alpha} \right) \right] = 0. \end{aligned} \tag{26}$$

At $\alpha = 1$, bank's problem is the same as the problem of a bank in the decentralized economy without government intervention. As shown in Proposition **1,** $c_{1D} \equiv c_1^* > 1$. It is easy to show that in this case, $c_1^* < \overline{c}_{12}(1)$ as (**26**), evaluated at the limit for $c_1 \to \overline{c}_{12}(1)$ and $\overline{\theta} \to 1$, simplifies to

$$\lim_{c_1 \to \overline{c}_{12}(1)} - \frac{\partial \theta_2^*(c_1, \alpha)}{\partial c_1} \left[(1-\lambda)\, p(\theta_2^*(c_1, \alpha)) u \left(\frac{1-\lambda c_1}{1-\lambda} R \right) + \lambda u(c_1) - u\left(\frac{1}{\alpha}\right) \right]$$

and it is negative as $\frac{\partial \theta_2^*(c_1,\alpha)}{\partial c_1} > 0$ and $\lim_{c_1 \to \overline{c}_{12}(1)} p(\theta_2^*(c_1, \alpha)) = p(\overline{\theta}) = 1$ and $\overline{c}_{12} > \frac{1}{\alpha}$. Thus, the proposition follows.□

Proof of Proposition 3: Denote as $\overline{g}$ the amount of public good for which the government optimally chooses $\alpha^* = \lambda$. From the proof of Proposition 2, when $\alpha = \lambda$ it is the case that banks choose a corner solution $c_1^* \to \overline{c}_{11}(\lambda)$ and $\theta_1^*(\overline{c}_{11}(\lambda)) \to \overline{\theta}$. In this case, depositors' expected utility is given by

$$\int_0^{\overline{\theta}} u(\overline{c}_{11}(\alpha)) d\theta + \int_{\overline{\theta}}^1 \left[\lambda u(\overline{c}_{11}(\alpha)) + (1-\lambda) u \left(\frac{1-\lambda \overline{c}_{11}(\alpha)}{1-\lambda} R \right) \right] d\theta + \int_0^{\overline{\theta}} v(g - \overline{c}_{11}(\alpha) + 1) d\theta + \int_{\overline{\theta}}^1 v(g) d\theta. \quad (27)$$

Differentiating (27) with respect to α and considering the limit where $\overline{\theta}$ approaches to 1, $\alpha^* = \lambda$ is the solution to

$$\int_0^{\overline{\theta}} u'(\overline{c}_{11}(\alpha)) d\theta - \int_0^{\overline{\theta}} v'(\overline{g} - \overline{c}_{11}(\alpha) + 1) d\theta = 0. \quad (28)$$

Consider now $g < \overline{g}$. Given the concavity of $v'(.)$, a decrease in the amount of public good available in the economy implies that the term $\int_0^{\overline{\theta}} v'(\overline{g} - \overline{c}_{11}(\alpha) + 1) d\theta$ in (28) becomes larger. Given that both $u''(.) < 0$ and $\frac{\partial \overline{c}_{11}(\alpha)}{\partial \alpha} < 0$ from the proof of Proposition **2,** it must be that $\alpha > \lambda$ for (28) to be satisfied with equality when $g < \overline{g}$. Thus, the proposition follows.□

Proof of Proposition 4: Differentiating (14) with respect to α gives

$$-\frac{\partial \theta_1^*(c_1, \alpha)}{\partial \alpha} \left[p(\theta_1^*(c_1, \alpha)) u(\frac{1-\lambda c_1}{1-\lambda} R) - u(c_1) \right] (1-\lambda) +$$

$$-\frac{\partial \theta_1^*(c_1, \alpha)}{\partial \alpha} \left[v(g) - v(g - c_1 + 1) \right] < 0$$

as $\frac{\partial \theta_1^*(c_1,\alpha)}{\partial \alpha} > 0$. This implies that the government chooses $\alpha = \lambda$. Consider now the choice of c_1.

Suppose first that (15) is not binding. For $\alpha = \lambda$, the threshold $\theta_1^*(c_1, \alpha)$ is equal to $\underline{\theta}(c_1)$.Then, the first order condition with respect to c_1 is equal to

$$\begin{aligned} &\int_0^{\underline{\theta}(c_1)} u'(c_1)\, d\theta + \lambda \int_{\underline{\theta}(c_1)}^1 \left[u'(c_1) - p(\theta) R u'(c_{2\lambda}) \right] d\theta + \\ &-\frac{\partial \underline{\theta}(c_1)}{\partial c_1} (1-\lambda) \left[p(\underline{\theta}(c_1))\, u(c_{2\lambda}) - u(c_1) \right] - \frac{\partial \underline{\theta}(c_1)}{\partial c_1} \left[v(g) - v(g - (c_1 - 1)) \right] + \\ &-\int_0^{\underline{\theta}(c_1)} v'(g - (c_1 - 1)) d\theta = 0. \end{aligned} \quad (29)$$

From the definition of $\underline{\theta}(c_1)$ in (2), the term $\frac{\partial \underline{\theta}(c_1)}{\partial c_1} (1-\lambda) \left[p(\underline{\theta}(c_1))\, u(c_{2\lambda}) - u(c_1) \right]$ is equal to zero and the expression above simplifies to (16) as in the proposition.

To see when the constraint (15) is binding, we substitute $c_1 = g + 1$ in (16) and obtain

$$\int_0^{\underline{\theta}(g+1)} u'(g+1)d\theta + \lambda \int_{\underline{\theta}(g+1)}^1 \left[u'(g+1) - p(\theta)Ru'\left(\frac{1-\lambda(g+1)}{1-\lambda}R\right)\right] d\theta +$$
$$-\frac{\partial \underline{\theta}(c_1)}{\partial c_1}|_{c_1=g+1} v(g) - \int_0^{\underline{\theta}(g+1)} v'(0)d\theta. \tag{30}$$

Denote as $\overline{g}^G$ the amount of public good for which (30) is equal to zero. Given the concavity of the function (14), (30) is negative for $g > \overline{g}^G$ and positive for $g < \overline{g}^G$. This implies that the constraint (15) is not binding for $g > \overline{g}^G$, while it is for $g < \overline{g}^G$.

The proposition follows. □

Proof of Proposition 5: Differentiating (17) with respect to c_1 gives c_1^{DI} as the solution to (18) and the proposition follows. □

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TABLE 1

g		α	c_1 c_2	$E\left[u\left(c_1,c_2\right)\right]$ $E[v(g)]$	$[SW\left(c_1,c_2,g\right)]$	$\frac{\underline{\theta}}{\theta^*}$
$g=1.55$	*Decentralized economy*	1	1.0076 4.98372	0.0139512 0.0150176	0.0289377	0.451436 0.463162
	Economy with transfers	0.95	1.05133 4.89001	0.01409 0.0148694	0.0289594	0.467607 0.486561
$g=2$	*Decentralized economy*	1	1.0076 4.98372	0.0139202 0.0173611	0.0312813	0.451436 0.463162
	Economy with transfers	0.8946	1.09951 3.37316	0.0142449 0.0170879	0.0313328	0.485305 0.578313
$g=2.5$	*Decentralized economy*	1	1.0076 4.98372	0.0139202 0.0194157	0.333359	0.451436 0.463162
	Economy with transfers	0.7	1.2936 4.10101	0.0148707 0.0186018	0.0334725	0.555976 0.710063

$TABLE\ 2 : (g = 1.55)$

	α	$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$	$\begin{bmatrix} E\left[u\left(c_1, c_2\right)\right] \\ E[v(g)] \end{bmatrix}$	$[SW\left(c_1, c_2, g\right)]$	$\begin{bmatrix} \underline{\theta} \\ \theta^* \end{bmatrix}$
Limiting	λ	1.20096	0.0147178	0.0290863	0.522304
moral hazard		4.56937	0.0143685		*None*
Eliminating	*None*	2.17671	0.0185618	0.0262741	*None*
all runs		2.47848	0.00771226		*None*

$TABLE\ 3 : (g = 2)$

	α	$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$	$\begin{bmatrix} E\left[u\left(c_1, c_2\right)\right] \\ E[v(g)] \end{bmatrix}$	$\left[SW\left(c_1, c_2, g\right)\right]$	$\begin{bmatrix} \underline{\theta} \\ \theta^* \end{bmatrix}$
Limiting	λ	1.32918	0.0152042	0.0316213	0.56893
moral hazard		4.29462	0.0164171		*None*
Eliminating	*None*	2.17671	0.0185618	0.0303906	*None*
all runs		2.47848	0.0118288		*None*

$TABLE\ 4 : (g = 2.5)$

	α	$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$	$\begin{bmatrix} E\left[u\left(c_1, c_2\right)\right] \\ E[v(g)] \end{bmatrix}$	$[SW\left(c_1, c_2, g\right)]$	$\begin{bmatrix} \underline{\theta} \\ \theta^* \end{bmatrix}$
Limiting	λ	1.47692	0.0157435	0.0339448	0.623291
moral hazard		3.97802	0.0182013		*None*
Eliminating	*None*	2.17671	0.0185618	0.0337992	*None*
all runs		2.47848	0.0152374		*None*

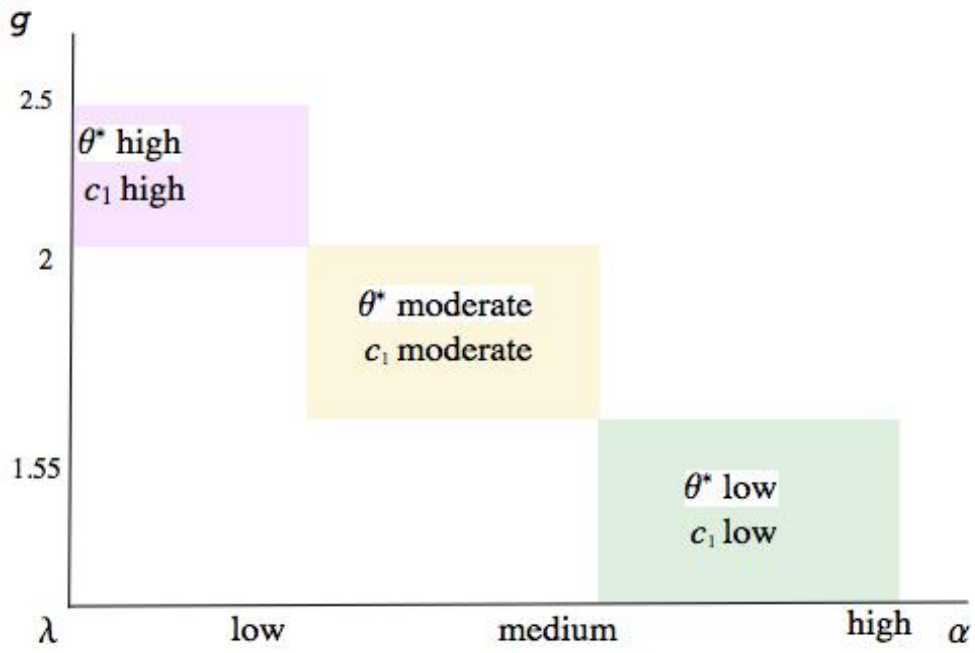

Figure 1: The figure shows how the optimal size of intervention α varies with the amount of public resources g available in the economy and its effects on the likelihood of runs and optimal deposit contract. When g is small, the government chooses a high α in order to limit the moral hazard problem on the side of the banks. As a consequence, banks choose a low c_1 and in turn the probability of runs is small. As g increases, the government chooses a lower α and, banks a more generous deposit contract c_1. In turn, the higher c_1 implies that runs are more likely.

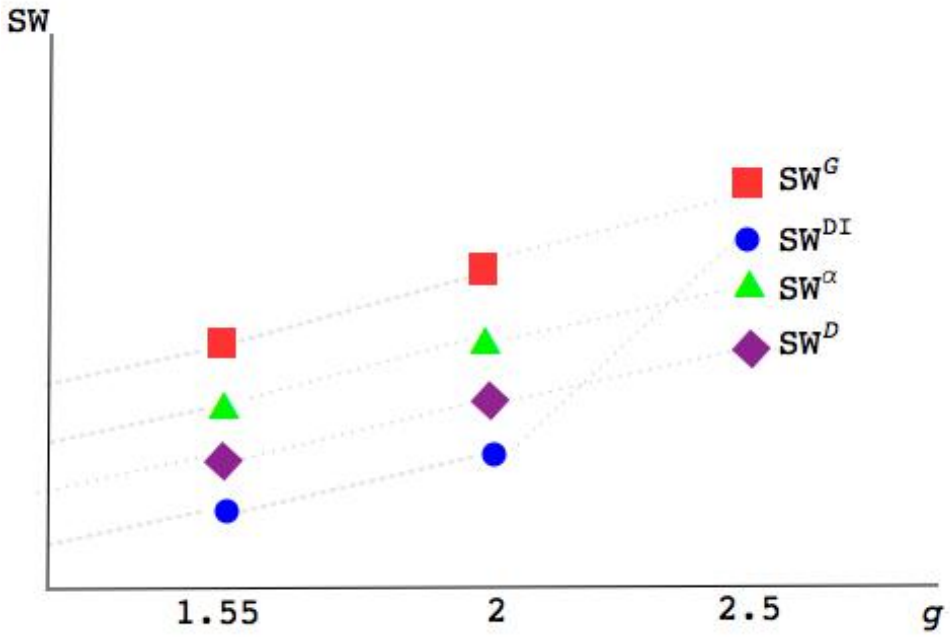

Figure 2: The figure shows the comparison of social welfare across the various forms of intervention for different values of g. The scheme in which the government chooses both the size of the intervention and the deposit contract (denoted as G) is the optimal one for any possible value of g. Comparing the interventions in which the government cannot control the choice of c_1, the scheme in which the government chooses the size of the intervention α (denoted as α) represents an improvement upon the decentralized solution (denoted as D) for any possible value of g. However, it is preferable to the intervention in which all runs are prevented (denoted as DI) only when g is not too high. When $g = 2.5$, the guarantee scheme eliminating all runs represents the best form of intervention if the governement cannot control the choice of the deposit contract c_1.