Online Appendix to: Effect of Constraints on Tiebout Competition: Evidence from a School Finance Reform[*]

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Online Appendix B1: Theoretical Framework

This section constructs a simple model that captures the main features of the Michigan school finance system that prevailed in pre-reform Michigan. The purpose is to analyze, in a simple framework, the impacts of the Michigan school finance reform on public school incentives and responses. Moreover, should one expect the school finance reform to have different impacts in high spending and low spending districts?

In keeping with the literature (Manski (1992), Hoxby (2003a), McMillan (2004), Ferreyra and Liang (2012), Chakrabarti (2013)), the objective of the public school district is to maximize net revenue (rent) which is simply defined as revenue minus costs. School district revenue is given by $p.N$, where p is per pupil revenue and N is the number of students in the public school district. School district cost (C_D) is given by $C_D(N, e) = c_1 + c(N) + C(e)$, where c_1 is a fixed cost and e is school district effort. Both $c(.)$ and $C(.)$ functions are assumed to be increasing and strictly convex in their respective arguments.

The number of students a school district can attract depends on its spending (E) and its effort (e). For simplicity, households are assumed to observe and care for both district spending and effort (Manski (1992), Hoxby (2003), Ferreyra (2007), Chakrabarti (2013)).[1] The net revenue (R) function of the public school district is represented by $R = p.N(E, e) - [c_1 + c(N(E, e)) + C(e)]$, where N is continuous, twice differentiable, additively separable, increasing, and concave function of its arguments.[2]

Under the assumption of "balanced budget" (that is equality of revenue and expenditure), $E = S + F + L$, where S, F, L respectively denote state aid, federal aid, and local revenue. State aid is represented by $s.N$ where s is per pupil state aid and s is exogenously given to the school district based on the state aid formula and the demographic characteristics of the district. Since state aid per pupil is less than per pupil revenue, there exists an $\alpha > 0$ such that $s = p - \alpha$. It is worth noting here

[1] To simplify computations, it is assumed here that number of students depend on spending; note that all results continue to hold if we instead assume it depends on per pupil spending.

[2] Additive separability of the N function in E and e simplifies computations greatly, but all results continue to hold without this assumption.

that while s is assumed to be exogenously given to the school district (to simplify computations), all results continue to hold if s depends on school district effort e – these results are not reported for lack of space, but are available on request.[3] Federal aid F is given to the district based on the federal allocation formula and the school district's demographics.

Local revenue raised by the district is represented by $V(e)$, where V is an increasing and strictly concave function of e.[4,5] In other words, the school districts have local discretion and have the ability to positively affect local revenue through effort. Following from the above discussion,

$$E = (p - \alpha).N(e, E) + F + V(e) \Rightarrow E = E(p, e, V(e), \alpha, F) \qquad (B.1.1)$$

Note that E is expressed as a function of $V(e)$ instead of $E = E(p, e, \alpha, F)$ to highlight the difference in the role of local discretion of school districts before and after the reform; all results continue to hold if under the simpler formulation $E = E(p, e, \alpha, F)$. It follows from (B.1.1) and the discussion above that the net revenue function can be written as:
$R = p.N\big[E(p, e, V(e), .), e\big] - \Big[c_1 + c\Big[N[E(p, e, V(e), .), e]\Big] + C(e)\Big]$. The school district chooses effort to maximize net revenue. There exists a unique effort e^* such that it solves the first order condition:

$$\frac{\delta R(e, .)}{\delta e} = (p - c_N)[N_E(E_e + E_V.V_e) + N_e] - C_e(e) = 0 \qquad (B.1.2)$$

Under strict concavity and convexity of the $N(.)$ and $C(.)$ functions respectively, the net revenue

[3] Before the implementation of the school finance reform, Michigan had a power equalization plan in place under which per pupil state aid for a district was given by: $s = max(0, \$400 + t.(\$102500 - SEV_{pp}))$, where SEV_{pp} is state equalized value per pupil in that district, t is the property tax rate, and the guaranteed tax base in 1994 (just before the reform) was \$102,500. Thus in the pre-program scenario, state aid also depended on local effort (specifically, local tax effort). The assumption of exogeneity of s is made to simplify computations, and also to be more general. A pre-reform scenario is typically (that is, in most other states) characterized by a scenario where s is given (exogenously) to the district.

[4] It is worth pointing out here that in Michigan, due to the presence of the power equalization system in the immediate pre-reform period, an increase in district effort increasing school scores and property value would not directly lead to an increase in total revenue (given tax rate) unlike in a typical Tiebout situation. However, an increase in tax effort or an increase in district effort improving school quality can attract households with higher demand for schooling who, in turn, can vote for a higher property tax rate leading to an increase in both local and total revenue. Thus, an increase in school district effort can still increase local and total revenue in pre-reform Michigan, but through a slightly different mechanism. For the sake of generality (so as to capture both the Tiebout-type system as well as Michigan-type pre-program system with guaranteed tax base and/or matching grants), local revenue is assumed to depend on e, $L = V(e), V_e > 0, V_{ee} < 0$.

[5] While this paper assumes strict concavity or convexity for the various functions (as outlined above), all results go through if at least one of these functions satisfies the strictness assumption.

function is strictly concave and the second order condition is satisfied. Also, note that it follows from the first order condition (B.1.2) that $p - c_N > 0$.

The Michigan school finance reform led to a drastic centralization of school finances, whereby the state set the per pupil expenditure of each district, and the districts virtually lost discretion over local revenue. A key feature of the reform was that the low spending districts saw a marked increase in their per pupil revenue which increased at a higher rate (during the period under consideration here). In contrast, while the high spending districts did not see a decline in their per pupil revenue as they were "held harmless", they saw a marked decline in the rate of growth of per pupil revenue, and their per pupil revenue often grew at a rate lower than the rate of inflation. For simplicity and to avoid messy computations, this is approximated in the model by assuming that the high spending districts faced a decline in per pupil revenue (p) and the low spending districts faced an increase in p. Using comparative statics, this section next investigates the impact of a change in p on school district effort e to understand the incentives faced by the low spending and high spending districts after the reform. An increase in school district effort implies an increase in productivity or efficiency of a district, and vice versa. Such an increase (or decrease) in efficiency should be reflected in the district's decision to allocate its spending between instructional and non-instructional (or support spending). For example, one might expect an increase (decrease) in efficiency to lead a school district to allocate a higher (lower) share of its spending to instructional expenditure which is regarded as more productive and lower (higher) share to non-instruction or support spending.

Proposition 1 *An increase in p can lead to an increase or decrease in equilibrium effort, and vice versa.*

The proof of Proposition 1 is in Online Appendix B2. As can be seen from B.2.1 (in Online Appendix B2), equilibrium effort increases (decreases) with an increase (decrease) in p if and only if b

$$N_E[E_e + E_V V_e] + N_e + (p - c_N)N_{EE}E_p(E_e + E_V V_e) - c_{NN}[N_E E_p[N_E(E_e + E_V V_e) + N_e]] > 0 \qquad (B.1.3)$$

However, as can be seen, the first two terms are positive and the second two terms are negative (as $N_{EE} < 0$ and $c_{NN} > 0$), making the sign of the expression in (B.1.3) ambiguous.

Let's consider the incentives of a school district facing an increase in p. An increase in effort leads to an increase in N both directly (reflected in the second term in (B.1.3)) as well as through an increase in spending (reflected in the first term in (B.1.3)), thus leading to an increase in revenue of the school district. Thus the first two terms induce a district facing an increase in p to respond by increasing e. However, an increase in spending (both through *a direct increase in* p as well as through e) leads to an increase in N (and hence revenue) at a decreasing rate (*that is, marginal revenue declines*) as captured by the third term in (B.1.3). This has a negative impact on e. Moreover an increase in spending (both through *a direct increase in* p as well as through e) increasing N leads to an increase in cost at an increasing rate (*that is, marginal cost increases*) as captured in the last term in (B.1.3). These negatively affect incentives to increase e. Because of these opposing channels, it is not clear whether a school district facing an increase in p will increase effort. The role of local discretion is worth discussing further. In the pre-reform period, an increase in e leads to an increase in property tax revenue (as captured by $E_v.V_e$ in the first term) thus attracting more students and revenue, and consequently inducing the school district to increase effort. The absence of this channel in the post-reform era has a negative impact on effort. However, in the pre-reform period, an increase in spending brought about by the increase in local revenue increases students (and revenue) at a decreasing rate and cost at an increasing rate (third and fourth terms in (B.1.3) respectively), thus discouraging an increase in effort. This force is absent in the post-reform period.

Next, let's consider the incentives of a district facing a decrease in p. The first two terms in (B.1.3) would dictate a decrease in effort. However a decrease in p (directly operating through a decrease in spending) and a decrease in effort (both directly and through a decrease in spending) decrease N (and hence revenue) at an increasing rate, as can be seen from the third term in (B.1.3) as $N_{EE} < 0$. Moreover, a decrease in p directly operating through spending and a decrease in e (both directly and through spending) decrease N leading to a decrease in cost at a decreasing rate (as follows from the fourth term in (B.1.3)). These last two channels (implying an increase in marginal revenue and a decline in marginal cost) have a positive effect on e. Of note here is that an increase in

e in turn increases N (and hence revenue) both directly and through an increase in spending. Local discretion also has opposing effects (the first two terms versus the last two terms). Thus, once again due to the presence of counteracting effects it is not clear whether a decrease in p would lead to a decrease or increase in school district effort.

To summarize, the theoretical discussion above reveals that a school finance reform has different effects on incentives and responses in high spending and low spending districts. There are a number of mechanisms involved not all of which work in the same direction, and hence the direction of the ultimate effect on school district effort is not clear, both in low spending and high spending districts. Rather, this is more of an empirical question that is addressed in the empirical sections of the paper.

Online Appendix B2: Proofs of results

Proof of Proposition 1. The net revenue function of a public school district is given by $R = p.N\Big[E(p, e, V(e), .), e\Big] - \Bigg[c_1 + c\Big[N[E(p, e, V(e), .), e]\Big] + C(e)\Bigg]$. It follows that there exists a unique effort e^* such that it solves the first order condition:

$$\frac{\delta R(e, .)}{\delta e} = (p - c_N)[N_E(E_e + E_V V_e) + N_e] - C_e(e) = 0$$

Note that

$$\frac{\delta^2 R(e, .)}{\delta e^2} = (p - c_N)[N_E[E_{ee} + E_{VV} V_e^2 + E_V V_{ee}] + N_{EE}[E_e + E_V V_e]^2 + N_{ee}] - c_{NN}[N_E(E_e + E_V.V_e) + N_e]^2 - C_{ee} < 0$$

from the strict concavity of $N(.)$, $E(.)$, $V(.)$ functions and convexity of the cost function in their respective arguments. It follows that the second order condition is satisfied.

Using the first order condition to do comparative statics with respect to p, one obtains:

$$\frac{\delta e}{\delta p} = -\frac{N_E[E_e + E_V V_e] + N_e + (p - c_N) N_{EE} E_p (E_e + E_V V_e) - c_{NN}[N_E E_p [N_E(E_e + E_V V_e) + N_e]]}{(p - c_N)[N_E[E_{ee} + E_{VV} V_e^2 + E_V V_{ee}] + N_{EE}[E_e + E_V V_e]^2 + N_{ee}] - c_{NN}[N_E(E_e + E_V.V_e) + N_e]^2 - C_{ee}} \quad \text{(B.2.1)}$$

The denominator is negative from strict concavity of the rent function. Note that $(p - c_N) > 0$ from the first order condition. The first two terms of the numerator are positive, the third term is negative due to strict concavity of the $N(.)$ function, and the last term is negative due to strict convexity of the cost function. It follows that $\frac{\delta e}{\delta p} \gtrless 0$. ■

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