Competitive devaluations: toward a welfare-based approach
TECHNICAL APPENDIX

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1 Introduction

This technical Appendix presents the detailed steps of the analysis. The models are analyzed in the following order:

- Model *with* Intra-Periphery trade and prices set in the *producer*'s currency
- Model *with* Intra-Periphery trade and prices set in the *consumer*'s currency
- Model *without* Intra-Periphery trade and prices set in the *producer*'s currency
- Model *without* Intra-Periphery trade and prices set in the *consumer*'s currency

2 A model with Intra-Periphery trade and prices set in the producer's currency

2.1 Consumer problem

The world is populated by households (consumer-producer units) defined over a continuum of unit mass. Households on the interval $[0, \gamma_A \gamma_P)$ live in country A, households on the interval $[\gamma_A \gamma_P, \gamma_P)$ live in country B and households on the interval $[\gamma_P, 1]$ live in country C, so that the population sizes of country A, B and C are equal to $\gamma_A \gamma_P$, $(1 - \gamma_A) \gamma_P$ and $1 - \gamma_P$ respectively. Each household is assumed to be the sole producer of a brand which is an imperfect substitute to all other available brands. Technology is such that one unit of household labor produces one unit of output.

The objective of household x, living in country j at time t, is defined as:

$$U(x, j, t) = \sum_{s=0}^{\infty} \beta^s \left\{ \ln C_{t+s}^j (x) - \frac{\kappa}{2} \left(Y_{t+s}^j (x) \right)^2 + \chi \ln \left(\frac{M_{t+s}^j (x)}{P_{t+s}^i} \right) \right\}$$

Inside the curly bracket, the first term is the utility from consumption, where $C^j(x)$ is a consumption basket to be defined below; the second term is the disutility from labor effort, where $Y^j(x)$ is the output of the brand produced by household x; the third term is utility from liquidity services, where $M^j(x)$

denotes holdings of country j currency, and P^j is the consumer price index in country j. The discount rate is denoted β, and the other Greek letters denote positive constants.

2.1.1 Consumption allocation

The consumption basket of household x living in country j is defined as the following CES index:

$$C^j(x) = \left[\gamma_P^{\frac{1}{\rho}}\left(C_P^j(x)\right)^{\frac{\rho-1}{\rho}} + (1-\gamma_P)^{\frac{1}{\rho}}\left(C_C^j(x)\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}$$

where ρ is the elasticity of substitution between the types of goods produced in the Center and the Periphery (computers and textiles). In turn, the basket of goods produced in the Periphery is defined as:

$$C_P^j(x) = \left[\gamma_A^{\frac{1}{\psi}}\left(C_A^j(x)\right)^{\frac{\psi-1}{\psi}} + (1-\gamma_A)^{\frac{1}{\psi}}\left(C_B^j(x)\right)^{\frac{\psi-1}{\psi}}\right]^{\frac{\psi}{\psi-1}}$$

where ψ is the elasticity of substitution between the types of goods produced in the Periphery countries (shirts and sweaters).

The sub-indexes across the brands produced in each country, $C_A^j(x)$, $C_B^j(x)$ and $C_C^j(x)$ are given as:

$$\begin{aligned}
C_A^j(x) &= \left[(\gamma_A\gamma_P)^{-\frac{1}{\theta}}\int_0^{\gamma_A\gamma_P}\left(C_A^j(z,x)\right)^{\frac{\theta-1}{\theta}}dz\right]^{\frac{\theta}{\theta-1}} \\
C_B^j(x) &= \left[((1-\gamma_A)\gamma_P)^{-\frac{1}{\theta}}\int_{\gamma_A\gamma_P}^{\gamma_P}\left(C_B^j(z,x)\right)^{\frac{\theta-1}{\theta}}dz\right]^{\frac{\theta}{\theta-1}} \\
C_C^j(x) &= \left[(1-\gamma_P)^{-\frac{1}{\theta}}\int_{\gamma_P}^{1}\left(C_C^j(z,x)\right)^{\frac{\theta-1}{\theta}}dz\right]^{\frac{\theta}{\theta-1}}
\end{aligned}$$

where $C_k^j(z,x)$: consumption by household x in country j of the good produced by household z in country k. The allocation of consumption across the various brands, given the aggregate consumption level $C^j(x)$, is derived using the usual approach. We can then write:

$$C_A^j(z,x) = \left[\frac{P_A^i(z)}{P_A^i}\right]^{-\theta}\left[\frac{P_A^i}{P_P^i}\right]^{-\psi}\left[\frac{P_P^i}{P^i}\right]^{-\rho} C^j(x)$$

$$
\begin{aligned}
C_B^j(z,x) &= \left[\frac{P_B^i(z)}{P_B^i}\right]^{-\theta}\left[\frac{P_B^i}{P_P^i}\right]^{-\psi}\left[\frac{P_P^i}{P^i}\right]^{-\rho} C^j(x) \\
C_C^j(z,x) &= \left[\frac{P_C^i(z)}{P_C^i}\right]^{-\theta}\left[\frac{P_C^i}{P^i}\right]^{-\rho} C^j(x)
\end{aligned}
$$

where $P_k^j(z)$ is the price in country j of the good produced by household z in country k. The utility based price indexes are:

$$
\begin{aligned}
P_A^j &= \left[\frac{1}{\gamma_A\gamma_P}\int_0^{\gamma_A\gamma_P}\left(P_A^j(z)\right)^{1-\theta}dz\right]^{\frac{1}{1-\theta}} \\
P_B^j &= \left[\frac{1}{(1-\gamma_A)\gamma_P}\int_{\gamma_A\gamma_P}^{\gamma_P}\left(P_B^j(z)\right)^{1-\theta}dz\right]^{\frac{1}{1-\theta}} \\
P_C^j &= \left[\frac{1}{1-\gamma_P}\int_{\gamma_P}^{1}\left(P_C^j(z)\right)^{1-\theta}dz\right]^{\frac{1}{1-\theta}} \\
P_P^j &= \left[\gamma_A\left(P_A^j\right)^{1-\psi}+(1-\gamma_A)\left(P_B^j\right)^{1-\psi}\right]^{\frac{1}{1-\psi}} \\
P^j &= \left[\gamma_P\left(P_P^j\right)^{1-\rho}+(1-\gamma_P)\left(P_C^j\right)^{1-\rho}\right]^{\frac{1}{1-\rho}}
\end{aligned}
$$

2.1.2 Demand for brand x

The demand for the brand produced by household x is obtained by integrating the demands presented above across all households worldwide. For brevity, we focus on the demand faced by a household living in country A, the expressions being similar for a household living in country B or C:

$$
\begin{aligned}
Y^A(x) &= \left[\frac{P_A^A(x)}{P_A^A}\right]^{-\theta}\left[\frac{P_A^A}{P_P^A}\right]^{-\psi}\left[\frac{P_P^A}{P^A}\right]^{-\rho}\gamma_A\gamma_P C^A \\
&\quad+\left[\frac{P_A^B(x)}{P_A^B}\right]^{-\theta}\left[\frac{P_A^B}{P_P^B}\right]^{-\psi}\left[\frac{P_P^B}{P^B}\right]^{-\rho}(1-\gamma_A)\gamma_P C^B \\
&\quad+\left[\frac{P_A^C(x)}{P_A^C}\right]^{-\theta}\left[\frac{P_A^C}{P_P^C}\right]^{-\psi}\left[\frac{P_P^C}{P^C}\right]^{-\rho}(1-\gamma_P)C^C
\end{aligned}
$$

where the per-capita consumptions are: $C^A = \frac{1}{\gamma_A\gamma_P}\int_0^{\gamma_A\gamma_P} C^A(q)\,dq$, $C^B = \frac{1}{(1-\gamma_A)\gamma_P}\int_{\gamma_A\gamma_P}^{\gamma_P} C^B(q)\,dq$, $C^C = \frac{1}{1-\gamma_P}\int_{\gamma_P}^{1} C^C(q)\,dq$. The sales revenue for household x living in country A is denoted as $SR^A(x)$ and is given by:

$$\begin{aligned} SR^A(x) &= P_A^A(x)\left[\frac{P_A^A(x)}{P_A^A}\right]^{-\theta}\left[\frac{P_A^A}{P_P^A}\right]^{-\psi}\left[\frac{P_P^A}{P^A}\right]^{-\rho}\gamma_A\gamma_P C^A \\ &+\frac{E^A}{E^B}P_A^B(x)\left[\frac{P_A^B(x)}{P_A^B}\right]^{-\theta}\left[\frac{P_A^B}{P_P^B}\right]^{-\psi}\left[\frac{P_P^B}{P^B}\right]^{-\rho}(1-\gamma_A)\gamma_P C^B \\ &+E^A P_A^C(x)\left[\frac{P_A^C(x)}{P_A^C}\right]^{-\theta}\left[\frac{P_A^C}{P_P^C}\right]^{-\psi}\left[\frac{P_P^C}{P^C}\right]^{-\rho}(1-\gamma_P)C^C \end{aligned}$$

where E^j is the exchange rate for the currency of country j vis-a-vis the Center currency. Note that if the law of one price holds ($P_A^B(x) = P_A^A(x)\cdot E^B/E^A$, $P_A^C(x) = P_A^A(x)/E^A$), we can write:

$$\begin{aligned} Y^A(x) &= \left[\frac{P_A^A(x)}{P_A^A}\right]^{-\theta}\left[\frac{P_A^A}{P_P^A}\right]^{-\psi}\left[\frac{P_P^A}{P^A}\right]^{-\rho}C^w \\ SR^A(x) &= P_A^A(x)Y^A(x) \end{aligned}$$

where $C^w = \gamma_A\gamma_P C^A + (1-\gamma_A)\gamma_P C^B + (1-\gamma_P)C^C$.

2.1.3 Intertemporal optimization

Household x living in country j holds domestic currency, $M^j(x)$, and a nominal bond denominated in the Center's currency, $B^j(x)$. The bond is in zero-net supply worldwide; its nominal yield at time t is denoted i_t. The household budget constraint is therefore:

$$\begin{aligned} &\frac{E_t^j B_{t+1}^j(x)}{P_t^j}+\frac{M_t^j(x)}{P_t^j}+C_t^j(x) \\ =\ &(1+i_t)\frac{E_t^j B_t^j(x)}{P_t^j}+\frac{M_{t-1}^j(x)}{P_t^j}+\frac{SR_t^j(x)}{P_t^j}-\frac{T_t^j(x)}{P_t^j} \end{aligned}$$

where and $T^j(x)$ is a lump sum tax denominated in country j's currency.

The equilibrium conditions are summarized by the following Euler and money demand equations:

$$\frac{C_{t+1}^j(x)}{C_t^j(x)} = \beta(1+i_{t+1})\frac{P_t^C}{P_{t+1}^C}$$

$$\frac{M_t^j(x)}{P_t^j} = \chi C_t^j(x) \frac{(1+i_{t+1}) E_{t+1}^j}{(1+i_{t+1}) E_{t+1}^j - E_t^j}$$

If prices are flexible, the equilibrium markups are given by

$$\frac{P_{j,t}^j(x)}{P_t^j} = \frac{\theta\kappa}{\theta - 1} C_t^j(x) Y_t^j(x)$$

while, when prices are sticky, goods supply adjusts to meet changes in demand.

2.2 Current account

In equilibrium, all households in a country are identical. We can then drop the x index and interpret the equations in our model as per-capita relations. We abstract from government spending and consider that the seignorage revenue is repaid to the households in a lump-sum transfer. We can then write the following expression for the current account of country j, in per capita terms:

$$\frac{E_t^j B_{t+1}^j}{P_t^j} + C_t^j = (1+i_t) \frac{E_t^j B_t^j}{P_t^j} + \frac{SR_t^j}{P_t^j}$$

The bond being is zero net supply worldwide, we write:

$$\gamma_P \gamma_A B^A + \gamma_P (1-\gamma_A) B^B + (1-\gamma_P) B^C = \gamma_P B^P + (1-\gamma_P) B^C = 0$$

2.3 Symmetric steady state and log linearization

We start by writing the solution of the model in a symmetric steady state where all bond holdings are set to zero: $B^A = B^B = B^P = B^C = 0$. All households worldwide are identical in such a steady state. Each household consumes and produces an amount C_0. The real interest rate is equal to the discount rate ($\beta(1+i) = 1$) and we write:

$$C_0 = \sqrt{\frac{\theta - 1}{\theta} \frac{1}{\kappa}}$$

We solve the model in terms of log-linear approximations around the symmetric steady state. We, denote by X_0 the level of a variable in the initial equilibrium and by X the new level of the variable. Lowercase letters denote log-linear approximations:

$$x \approx \frac{X - X_0}{X_0}$$

We denote long-run variables with upperbars, to distinguish them from short-run variables. For example, c^B and $\bar{c}^B$ are the percentage deviations from the initial steady state for the per capita consumption in country B in the short and the long run respectively. Throughout the paper, we define Periphery-wide and worldwide variables as follows:

$$x^P \equiv \gamma_A x^A + (1 - \gamma_A) x^B \quad (1)$$
$$x^w \equiv \gamma_P x^P + (1 - \gamma_P) x^C \quad (2)$$

The model is solved in for the worldwide effects, and the Center-Periphery, as well as the Intra-Periphery, differences. The solutions for x^w, $x^P - x^C$ and $x^A - x^B$ then allow us to compute the country-specific solution as:

$$\begin{aligned} x^C &= x^w - \gamma_P \left(x^P - x^C \right) \\ x^P &= x^w + (1 - \gamma_P) \left(x^P - x^C \right) \\ x^A &= x^P + (1 - \gamma_A) \left(x^A - x^B \right) \\ x^B &= x^P - \gamma_A \left(x^A - x^B \right) \end{aligned}$$

The initial asset holdings being equal to zero, we define: $\bar{b}^j = B^j / \left(P_0^C C_0 \right)$. We consider that the economy is initially at the symmetric steady state, and analyze the impact of permanent monetary shock in the periphery countries ($\bar{m}^A$ and $\bar{m}^B$).

2.4 The long run

From the money demands and the markups we write:

$$\begin{aligned} \bar{m}^j - \bar{p}^j &= \bar{c}^j \\ \bar{p}_j^j - \bar{p}^j &= \bar{c}^j + \bar{y}^j \end{aligned}$$

for $j = A, B, C$. From the definitions of the price indexes, we can write:

$$\begin{aligned} \bar{p}_P^j &= \gamma_A \bar{p}_A^j + (1 - \gamma_A) \bar{p}_B^j \\ \bar{p}^j &= \gamma_P \bar{p}_P^j + (1 - \gamma_P) \bar{p}_C^j \end{aligned}$$

The output demands imply:

$$
\begin{aligned}
\bar{y}^A &= -\psi\left(\bar{p}_A^A - \bar{p}_P^A\right) + \bar{y}^P,\ \bar{y}^B = -\psi\left(\bar{p}_B^B - \bar{p}_P^B\right) + \bar{y}^P \\
\bar{y}^P &= -\rho\left(\bar{p}_P^C - \bar{p}^C\right) + \bar{c}^w,\ \bar{y}^C = -\rho\left(\bar{p}_C^C - \bar{p}^C\right) + \bar{c}^w
\end{aligned}
$$

The sales revenue and current accounts lead to:

$$
\begin{aligned}
0 &= \frac{1-\beta}{\beta}\bar{b}^A - \bar{c}^A + \bar{p}_A^A - \bar{p}^A + \bar{y}^A \\
0 &= \frac{1-\beta}{\beta}\bar{b}^B - \bar{c}^B + \bar{p}_B^B - \bar{p}^B + \bar{y}^B \\
0 &= \frac{1-\beta}{\beta}\bar{b}^C - \bar{c}^C + \bar{p}_C^C - \bar{p}^C + \bar{y}^C \\
0 &= \gamma_A\gamma_P\bar{b}^A + \gamma_B\gamma_P\bar{b}^B + \gamma_C\bar{b}^C = \gamma_P\bar{b}^P + \gamma_C\bar{b}^C
\end{aligned}
$$

2.4.1 Center-Periphery

We start by computing the results in terms of Center-Periphery differences. The markup equations, output demand and current account are written as:

$$
\begin{aligned}
\left(\bar{p}_P^P - \bar{p}_C^C\right) - \left(\bar{p}^P - \bar{p}^C\right) &= \left(\bar{c}^P - \bar{c}^C\right) + \left(\bar{y}^P - \bar{y}^C\right) \\
\left(\bar{y}^P - \bar{y}^C\right) &= -\rho\left(\bar{p}_P^C - \bar{p}_C^C\right) \\
\left(\bar{c}^P - \bar{c}^C\right) &= \frac{1-\beta}{\beta}\frac{\bar{b}^P}{1-\gamma_P} + \left(\bar{p}_P^P - \bar{p}_C^C\right) - \left(\bar{p}^P - \bar{p}^C\right) \\
&\quad + \left(\bar{y}^P - \bar{y}^C\right)
\end{aligned}
$$

As the law of one price holds, we write:

$$
\bar{p}_P^C + \bar{e}^P = \bar{p}_P^P,\ \bar{p}^C + \bar{e}^P = \bar{p}^P
$$

$$
\begin{aligned}
\left(\bar{p}_P^P - \bar{p}_C^C - \bar{e}^P\right) &= \left(\bar{c}^P - \bar{c}^C\right) + \left(\bar{y}^P - \bar{y}^C\right) \\
\left(\bar{y}^P - \bar{y}^C\right) &= -\rho\left(\bar{p}_P^P - \bar{p}_C^C - \bar{e}^P\right) \\
\left(\bar{c}^P - \bar{c}^C\right) &= \frac{1-\beta}{\beta}\frac{\bar{b}^P}{1-\gamma_P} + \left(\bar{p}_P^P - \bar{p}_C^C - \bar{e}^P\right) + \left(\bar{y}^P - \bar{y}^C\right)
\end{aligned}
$$

which allows us to derive the long run solution as:

$$\begin{aligned}
\bar{c}^P - \bar{c}^C &= \frac{1+\rho}{2\rho}\frac{1-\beta}{\beta}\frac{\bar{b}^P}{1-\gamma_P} \\
\bar{p}_P^P - \bar{p}_C^C - \bar{e}^P &= \frac{1}{2\rho}\frac{1-\beta}{\beta}\frac{\bar{b}^P}{1-\gamma_P} \\
\bar{y}^P - \bar{y}^C &= -\frac{1}{2}\frac{1-\beta}{\beta}\frac{\bar{b}^P}{1-\gamma_P}
\end{aligned}$$

The worldwide effects can be computed as:

$$\bar{c}^w = \bar{y}^w = 0$$

2.4.2 Intra-Periphery

The Intra-Periphery differences can be computed following similar steps as above. This leads to:

$$\begin{aligned}
\bar{c}^A - \bar{c}^B &= \frac{1+\psi}{2\psi}\frac{1-\beta}{\beta}\frac{\bar{b}^A - \bar{b}^P}{1-\gamma_A} \\
\bar{p}_A^A - \bar{p}_B^B - \left(\bar{e}^A - \bar{e}^B\right) &= \frac{1}{2\psi}\frac{1-\beta}{\beta}\frac{\bar{b}^A - \bar{b}^P}{1-\gamma_A} \\
\bar{y}^A - \bar{y}^B &= -\frac{1}{2}\frac{1-\beta}{\beta}\frac{\bar{b}^A - \bar{b}^P}{1-\gamma_A}
\end{aligned}$$

2.5 The short run

In the short run, prices are set in the producer currency. We can then write:

$$\begin{aligned}
p_P^A &= (1-\gamma_A)\left(e^A - e^B\right) \\
p_P^B &= -\gamma_A\left(e^A - e^B\right) \\
p_P^C &= \gamma_A e^A + (1-\gamma_A)\, e^B = e^P \\
p^A &= \gamma_P (1-\gamma_A)\left(e^A - e^B\right) + (1-\gamma_P)\, e^A = e^A - \gamma_P e^P \\
p^B &= -\gamma_P \gamma_A \left(e^A - e^B\right) + (1-\gamma_P)\, e^B = e^B - \gamma_P e^P \\
p^C &= -\gamma_P e^P
\end{aligned}$$

From the Euler equations and the money demands, we write:

$$\begin{aligned}\bar{c}^j - c^j &= \beta di + p^C - \bar{p}^C \\ \bar{m}^j - p^j &= c^j + \frac{\beta}{1-\beta}\left(e^j - \bar{e}^j - \beta di\right)\end{aligned}$$

from which we see that for any pair of countries j and k:

$$\begin{aligned}c^j - c^k &= \bar{c}^j - \bar{c}^k \\ e^j - e^k &= \bar{e}^j - \bar{e}^k\end{aligned}$$

As the purchasing power parity holds at all horizons, the consumption gaps immediately reach their long run values and there is no exchange rate under- or over shooting (the later result uses the long run money demands).

The output demands imply:

$$\begin{aligned}y^A &= \psi\left(1-\gamma_A\right)\left(e^A - e^B\right) + y^P,\ y^B = -\psi\gamma_A\left(e^A - e^B\right) + y^P \\ y^P &= \rho\left(1-\gamma_P\right)e^P + c^w,\ y^C = -\rho\gamma_P e^P + c^w\end{aligned}$$

The sales revenue and current account equations lead to:

$$\begin{aligned}\bar{b}^A + c^A &= -\left(e^A - \gamma_P e^P\right) + y^A \\ \bar{b}^B + c^B &= -\left(e^B - \gamma_P e^P\right) + y^B \\ \bar{b}^C + c^C &= \gamma_P e^P + y^C\end{aligned}$$

2.5.1 Center-Periphery

We start with the solution in terms of Center-Periphery differences. From the money demands, the output demands and the current accounts, we obtain:

$$\begin{aligned}\left(\bar{m}^P - \bar{m}^C\right) - e^P &= \bar{c}^P - \bar{c}^C \\ y^P - y^C &= \rho e^P \\ \frac{\bar{b}^P}{1-\gamma_P} + \bar{c}^P - \bar{c}^C &= -e^P + \left(y^P - y^C\right)\end{aligned}$$

Combining these expressions with the long run solution, we can establish:

$$\bar{c}^P - \bar{c}^C = \frac{\rho - 1}{\rho}\frac{\left(1-\beta\right)\left(1+\rho\right)}{1+\beta+\rho\left(1-\beta\right)}\left(\bar{m}^P - \bar{m}^C\right)$$

$$
\begin{aligned}
e^P &= \frac{1}{\rho}\frac{1-\beta+\rho(1+\beta)}{1+\beta+\rho(1-\beta)}\left(\bar{m}^P-\bar{m}^C\right) \\
y^P-y^C &= \frac{1-\beta+\rho(1+\beta)}{1+\beta+\rho(1-\beta)}\left(\bar{m}^P-\bar{m}^C\right) \\
\frac{\bar{b}^P}{1-\gamma_P} &= \frac{2\beta(\rho-1)}{1+\beta+\rho(1-\beta)}\left(\bar{m}^P-\bar{m}^C\right)
\end{aligned}
$$

The worldwide effects can be computed as:

$$c^w = y^w = \bar{m}^w$$

2.5.2 Intra-Periphery

Following similar steps as for the Center-Periphery dimension, we establish:

$$
\begin{aligned}
\bar{c}^A-\bar{c}^B &= \frac{\psi-1}{\psi}\frac{(1-\beta)(1+\psi)}{1+\beta+(1-\beta)\psi}\left(\bar{m}^A-\bar{m}^B\right) \\
e^A-e^B &= \frac{1}{\psi}\frac{1-\beta+\psi(1+\beta)}{1+\beta+(1-\beta)\psi}\left(\bar{m}^A-\bar{m}^B\right) \\
y^A-y^B &= \frac{1-\beta+\psi(1+\beta)}{1+\beta+(1-\beta)\psi}\left(\bar{m}^A-\bar{m}^B\right) \\
\frac{\bar{b}^A-\bar{b}^P}{1-\gamma_A} &= \frac{2\beta(\psi-1)}{1+\beta+(1-\beta)\psi}\left(\bar{m}^A-\bar{m}^B\right)
\end{aligned}
$$

2.6 Overall effect

We define the overall effect (net present value, indexed by npv) as $x_{npv} = x + \frac{\beta}{1-\beta}\bar{x}$. The Center-Periphery effects are:

$$
\begin{aligned}
c^P_{npv}-c^C_{npv} &= \frac{\rho-1}{\rho}\frac{1+\rho}{1+\beta+\rho(1-\beta)}\left(\bar{m}^P-\bar{m}^C\right) \\
y^P_{npv}-y^C_{npv} &= \frac{1+\rho}{1+\beta+\rho(1-\beta)}\left(\bar{m}^P-\bar{m}^C\right)
\end{aligned}
$$

The worldwide effects are:

$$c^w_{npv} = y^w_{npv} = \bar{m}^w$$

The Intra-Periphery effects are:

$$c_{npv}^{A} - c_{npv}^{B} = \frac{\psi - 1}{\psi} \frac{1 + \psi}{1 + \beta + (1 - \beta)\psi} \left(\bar{m}^{A} - \bar{m}^{B}\right)$$

$$y_{npv}^{A} - y_{npv}^{B} = \frac{1 + \psi}{1 + \beta + (1 - \beta)\psi} \left(\bar{m}^{A} - \bar{m}^{B}\right)$$

2.7 Welfare

Omitting the direct impact of real balances, the welfare in country j is:

$$u^{j} = c_{npv}^{j} - \frac{\theta - 1}{\theta} y_{npv}^{j}$$

Using our results, the Center-Periphery welfare difference is:

$$u^{P} - u^{C} = \frac{\rho - \theta}{\rho\theta} \left(y_{npv}^{P} - y_{npv}^{C}\right)$$

The worldwide welfare is computed as:

$$u^{w} = \frac{1}{\theta} \bar{m}^{w}$$

And the Intra-Periphery welfare difference is given by:

$$u^{A} - u^{B} = \frac{\psi - \theta}{\psi\theta} \left(y_{npv}^{A} - y_{npv}^{B}\right)$$

2.8 Depreciation and possible responses

We now consider the consequences of a depreciation of the currency of A. There is no policy response in the center ($\bar{m}^{C} = 0$), and country B is faced with three possible reactions:

- - absence of policy reaction (**MST**) ($\bar{m}^{B} = 0$),
- - pegging of the exchange rate vis a vis the center (**PEG**) ($e^{B} = 0$),
- - devaluation to maintain its market share in the center (**DEV**) ($e^{B} = e^{A}$).

2.8.1 Exchange rates and money

As the center does not react, we have: $\bar{m}^C = 0$, $\bar{m}^w = \gamma_P \bar{m}^P = \gamma_P \gamma_A \bar{m}^A + \gamma_P (1 - \gamma_A) \bar{m}^B$. The exchange rates are written as:

$$\begin{aligned} e^P &= \gamma_A e^A + (1-\gamma_A) e^B = \Pi(\rho) \left(\gamma_A \bar{m}^A + (1-\gamma_A) \bar{m}^B \right) \\ e^A - e^B &= \Pi(\psi) \left(\bar{m}^A - \bar{m}^B \right) \end{aligned}$$

where:

$$\Pi(x) = \frac{1}{x} \frac{1 - \beta + x(1+\beta)}{1 + \beta + x(1-\beta)} > 0,\ \Pi(\rho) > \Pi(\psi) \Leftrightarrow \rho < \psi$$

Under the **MST** regime ($\bar{m}^B = 0$), we can show that:

$$\begin{aligned} \bar{m}^A &> 0 \\ e^A &= \left[\gamma_A \Pi(\rho) + (1-\gamma_A) \Pi(\psi) \right] \bar{m}^A > 0 \\ e^B &= \left[\Pi(\rho) - \Pi(\psi) \right] \gamma_A \bar{m}^A > 0 \\ e^A &> e^B \end{aligned}$$

Country B experiences a devaluation that is smaller than for country A. Under the **PEG** regime ($e^B = 0$), remembering that $\Pi(\rho) - \Pi(\psi) > 0$:

$$\begin{aligned} \bar{m}^A &> 0 \\ \bar{m}^B &= -\frac{\Pi(\rho) - \Pi(\psi)}{\gamma_A \Pi(\psi) + (1-\gamma_A) \Pi(\rho)} \gamma_A \bar{m}^A < 0 \\ \bar{m}^P &= \frac{\gamma_A \Pi(\psi)}{\gamma_A \Pi(\psi) + (1-\gamma_A) \Pi(\rho)} \bar{m}^A > 0 \\ e^A &= \frac{\Pi(\rho) \Pi(\psi)}{\gamma_A \Pi(\psi) + (1-\gamma_A) \Pi(\rho)} \bar{m}^A < \Pi(\rho) \bar{m}^A \end{aligned}$$

Country B has to contract its money supply. This contraction is however insufficient to offset the expansion in country A, and there is a Periphery wide monetary expansion. Under the **DEV** regime ($e^B = e^A$):

$$\begin{aligned} \bar{m}^B &= \bar{m}^A > 0 \\ \bar{e}^A &= \bar{e}^B = \frac{1}{\rho} \frac{1 - \beta + \rho(1+\beta)}{1 + \beta + \rho(1-\beta)} \bar{m}^A = \Pi(\rho) \bar{m}^A \end{aligned}$$

and both countries experience the same devaluation as B's policy follows A's.

2.8.2 Current accounts

We use:

$$\begin{aligned}\frac{\bar{b}^P}{1-\gamma_P} &= \bar{b}^P-\bar{b}^C=\frac{2\beta\left(\rho-1\right)}{1+\beta+\rho\left(1-\beta\right)}\bar{m}^P\\ \frac{\bar{b}^A-\bar{b}^P}{1-\gamma_A} &= \bar{b}^A-\bar{b}^B=\frac{2\beta\left(\psi-1\right)}{1+\beta+\left(1-\beta\right)\psi}\left(\bar{m}^A-\bar{m}^B\right)\end{aligned}$$

We can clearly see that the periphery experiences a current account surplus as long as $\rho>1$ (the Marshall-Lerner-Robinson condition). In the short run, a country accumulates assets (or borrows), and consumes (or repays) the interest in the long run. A positive current account corresponds to a positive trade balance in the short run, and a negative trade balance in the long run (expressed in domestic currency).

Under the **MST** regime, we have:

$$\frac{\bar{b}^P}{1-\gamma_P}=\frac{2\beta\left(\rho-1\right)}{1+\beta+\rho\left(1-\beta\right)}\gamma_A\bar{m}^A,\ \bar{b}^A-\bar{b}^B>0$$

The current account effect is more beneficial for country A than for country B. Under the **PEG** regime:

$$\frac{\bar{b}^P}{1-\gamma_P}=\frac{2\beta\left(\rho-1\right)}{1+\beta+\rho\left(1-\beta\right)}\frac{\Pi\left(\psi\right)}{\gamma_A\Pi\left(\psi\right)+\left(1-\gamma_A\right)\Pi\left(\rho\right)}\gamma_A\bar{m}^A,\ \bar{b}^A-\bar{b}^B>0$$

$\bar{b}^P$ and $\bar{b}^A-\bar{b}^B$ are larger than in the **MST** regime. Under the **DEV** regime:

$$\frac{\bar{b}^P}{1-\gamma_P}=\frac{2\beta\left(\rho-1\right)}{1+\beta+\rho\left(1-\beta\right)}\bar{m}^A,\ \bar{b}^A=\bar{b}^B$$

2.8.3 Welfare

We recall the results for the worldwide and difference welfare:

$$\begin{aligned}u^P-u^C &= \frac{\rho-\theta}{\rho\theta}\left(y^P_{npv}-y^C_{npv}\right)\\ u^A-u^B &= \frac{\psi-\theta}{\psi\theta}\left(y^A_{npv}-y^B_{npv}\right)\\ u^w &= \frac{1}{\theta}\gamma_P\bar{m}^P\end{aligned}$$

Along with the overall output results:

$$\begin{aligned} y^P_{npv} - y^C_{npv} &= \frac{1+\rho}{1+\beta+\rho(1-\beta)}\bar{m}^P \\ y^A_{npv} - y^B_{npv} &= \frac{1+\psi}{1+\beta+(1-\beta)\psi}\left(\bar{m}^A - \bar{m}^B\right) \end{aligned}$$

In all regimes, we can write (recalling $\rho < \theta$):

$$\begin{aligned} u^C &= \frac{1}{\theta}\left[1 - \frac{\rho-\theta}{\rho}\frac{1+\rho}{1+\beta+\rho(1-\beta)}\right]\gamma_P\bar{m}^P > 0 \\ u^P &< 0 \Leftrightarrow \gamma_P < \frac{\theta-\rho}{\rho\left(1+\beta\frac{1-\rho}{1+\rho}\right)+\theta-\rho} < 1 \end{aligned}$$

The periphery is adversely affected if it is small and there is little center-periphery substitutability. Focusing on the welfare of country B, we write:

$$\begin{aligned} u^B &= u^P - \gamma_A\left(u^A - u^B\right) \\ &= u^w + (1-\gamma_P)\left(u^P - u^C\right) - \gamma_A\left(u^A - u^B\right) \\ &= \frac{1}{\theta}\gamma_P\bar{m}^P + (1-\gamma_P)\frac{\rho-\theta}{\rho\theta}\frac{1+\rho}{1+\beta+\rho(1-\beta)}\bar{m}^P \\ &\quad -\gamma_A\frac{\psi-\theta}{\psi\theta}\frac{1+\psi}{1+\beta+(1-\beta)\psi}\left(\bar{m}^A - \bar{m}^B\right) \\ &= \frac{1}{\theta}\left(\Psi_A\bar{m}^A + \Psi_B\bar{m}^B\right) \end{aligned}$$

where:

$$\begin{aligned} \Psi_A &= \left\{\left[\gamma_P + (1-\gamma_P)\frac{\rho-\theta}{\rho}\frac{1+\rho}{1+\beta+\rho(1-\beta)}\right]\right. \\ &\quad \left. -\frac{\psi-\theta}{\psi}\frac{1+\psi}{1+\beta+(1-\beta)\psi}\right\}\gamma_A \\ \Psi_B &= \left\{(1-\gamma_A)\left[\gamma_P + (1-\gamma_P)\frac{\rho-\theta}{\rho}\frac{1+\rho}{1+\beta+\rho(1-\beta)}\right]\right. \\ &\quad \left. +\gamma_A\frac{\psi-\theta}{\psi}\frac{1+\psi}{1+\beta+(1-\beta)\psi}\right\} \end{aligned}$$

Therefore, country B prefers an expansionary monetary stance (i.e. **DEV** over **PEG**) if $\Psi_B > 0$:

$$\left[\gamma_P + (1-\gamma_P)\frac{\rho-\theta}{\rho}\frac{1+\rho}{1+\beta+\rho(1-\beta)}\right](1-\gamma_A)+\gamma_A\frac{\psi-\theta}{\psi}\frac{1+\psi}{1+\beta+(1-\beta)\psi} > 0$$

3 A model with Intra-Periphery trade and prices set in the buyer's currency

3.1 Consumer problem

The dynamic model can also be solved for the case where prices are set in the buyer's currency, instead of the seller's currency. Because of deviations from the law of one price, the purchasing power parity does not hold in the short run. Recalling that all households are identical within a country, the Euler and money demand equations are now given by:

$$\frac{C_{t+1}^j}{C_t^j} = \beta(1+i_{t+1})\frac{E_{t+1}^j}{P_{t+1}^j}\frac{P_t^j}{E_t^j}$$

$$\frac{M_t^j}{P_t^j} = \chi C_t^j \frac{(1+i_{t+1})E_{t+1}^j}{(1+i_{t+1})E_{t+1}^j - E_t^j}$$

3.2 Consumption and exchange rate dynamics

Log linear approximations of the Euler, the short run money demand and the long run money demand are given by (recalling that the consumer price indexes are preset in the short run):

$$\begin{aligned}
\bar{c}^j - c^j &= \beta di + \left(\bar{e}^j - e^j\right) - \bar{p}^j \\
\bar{m}^j &= c^j + \frac{\beta}{1-\beta}\left(e^j - \bar{e}^j - \beta di\right) \\
\bar{m}^j - \bar{p}^j &= \bar{c}^j
\end{aligned}$$

We can undertake some algebra on these equations to show that across any pair of countries (recalling that the law of one price holds in the long run):

$$\begin{aligned}
\left(\bar{c}^j - \bar{c}^k\right) &= \left(c^j - c^k\right) - \left(e^j - e^k\right) \\
e^j - e^k &= \bar{e}^j - \bar{e}^k
\end{aligned}$$

The absence of exchange rate under- or overshooting remains valid. This result is a consequence of our assumption of a logarithmic utility for real balances.

3.3 General solution

As prices are flexible in the long run, the long run effects, as functions of $\frac{\bar{b}^P}{1-\gamma_P}$ and $\frac{\bar{b}^A-\bar{b}^P}{1-\gamma_A}$ are identical to the case where prices are set in the producer's currency and the law of one price holds. Of course, the finding that there are no worldwide long run effects is still valid: $\bar{c}^w = \bar{y}^w = 0$.

As there are no relative price effects, the short run output effect exactly reflects the worldwide effect in all countries:

$$y^j = y^w = \bar{m}^w \ \forall j$$

The short run current account can be written as:

$$\begin{aligned}
\bar{b}^A + c^A &= sr^A = c^w + (1-\gamma_A)\gamma_P\left(e^A - e^B\right) + (1-\gamma_P)e^A \\
\bar{b}^B + c^B &= sr^B = c^w - \gamma_A\gamma_P\left(e^A - e^B\right) + (1-\gamma_P)e^B \\
\bar{b}^C + c^C &= sr^C = c^w - \gamma_P e^P
\end{aligned}$$

We start with the Center-Periphery difference. From the current accounts we establish that:

$$\frac{1}{1-\gamma_P}\bar{b}^P = -\left(\bar{c}^P - \bar{c}^C\right)$$

Combining this result with the long run results, we see that $\bar{b}^P = 0$ and there are no long run effects: $\bar{c}^P - \bar{c}^C = \bar{p}^P_P - \bar{p}^C_C - e^P = \bar{y}^P - \bar{y}^C = 0$ given our assumption of logarithmic preferences. As there are no worldwide long run effects, neither the center nor the periphery experience any real effect beyond the short run. The short run effects are then given by:

$$c^P - c^C = e^P = \bar{m}^P - \bar{m}^C$$

and the worldwide effects are given by: $c^w = y^w = \bar{m}^w$.

Similarly, there are no long run effects on the intra-Periphery differences:

$$\frac{\bar{b}^A - \bar{b}^P}{1-\gamma_A} = -\left(\bar{c}^A - \bar{c}^B\right)$$

Combining this with the long run results implies that: $\frac{\bar{b}^A - \bar{b}^P}{1-\gamma_A} = \bar{c}^A - \bar{c}^B = \bar{p}_A^A - \bar{p}_B^B - \left(e^A - e^B\right) = \bar{y}^A - \bar{y}^B = 0$. The only real effects are therefore limited to the short run:

$$c^A - c^B = e^A - e^B = \bar{m}^A - \bar{m}^B$$

3.4 Policy response of country B

From our results, we can see that:

$$e^B = \bar{m}^B - \bar{m}^C = \bar{m}^B$$

The **MST** and **PEG** regime are therefore identical in this model.

3.5 Welfare

The consumption in country B is equal to its nominal balances:

$$c^B = \bar{m}^B$$

It is then straightforward to compute the welfare result for country B as:

$$u^B = \bar{m}^B - \frac{\theta - 1}{\theta}\bar{m}^w$$

The devaluation by country A rises the effort of country B's households, whose consumption isn't affected. Country B has an incentive to abandon its peg to the center, as it gets the entire benefit in terms of consumption, whereas the effort is shared worldwide.

4 A model without Intra-Periphery trade and prices set in the producer's currency

This section analyzes a variant model with no Intra-Periphery trade, and prices set in the producer's currency.

4.1 Consumer problem

The consumption baskets are now different in the Periphery countries. We examine the consumption baskets in all countries.

4.1.1 Center's consumption

In the Center, the consumption basket is unchanged:

$$
\begin{aligned}
C^C(x) &= \left[\gamma_P^{\frac{1}{\rho}}\left(C_P^C(x)\right)^{\frac{\rho-1}{\rho}} + (1-\gamma_P)^{\frac{1}{\rho}}\left(C_C^C(x)\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \\
C_P^C(x) &= \left[\gamma_A^{\frac{1}{\psi}}\left(C_A^C(x)\right)^{\frac{\psi-1}{\psi}} + (1-\gamma_A)^{\frac{1}{\psi}}\left(C_B^C(x)\right)^{\frac{\psi-1}{\psi}}\right]^{\frac{\psi}{\psi-1}}
\end{aligned}
$$

$$
\begin{aligned}
C_A^C(x) &= \left[(\gamma_A\gamma_P)^{-\frac{1}{\theta}}\int_0^{\gamma_A\gamma_P}\left(C_A^C(z,x)\right)^{\frac{\theta-1}{\theta}}dz\right]^{\frac{\theta}{\theta-1}} \\
C_B^C(x) &= \left[((1-\gamma_A)\gamma_P)^{-\frac{1}{\theta}}\int_{\gamma_A\gamma_P}^{\gamma_P}\left(C_B^C(z,x)\right)^{\frac{\theta-1}{\theta}}dz\right]^{\frac{\theta}{\theta-1}} \\
C_C^C(x) &= \left[(1-\gamma_P)^{-\frac{1}{\theta}}\int_{\gamma_P}^{1}\left(C_C^C(z,x)\right)^{\frac{\theta-1}{\theta}}dz\right]^{\frac{\theta}{\theta-1}}
\end{aligned}
$$

The consumption allocation for household x living in the Center is given by:

$$
\begin{aligned}
C_A^C(z,x) &= \left[\frac{P_A^C(z)}{P_A^C}\right]^{-\theta}\left[\frac{P_A^C}{P_P^C}\right]^{-\psi}\left[\frac{P_P^C}{P^C}\right]^{-\rho}C^C(x) \\
C_B^C(z,x) &= \left[\frac{P_B^C(z)}{P_B^C}\right]^{-\theta}\left[\frac{P_B^C}{P_P^C}\right]^{-\psi}\left[\frac{P_P^C}{P^C}\right]^{-\rho}C^C(x) \\
C_C^C(z,x) &= \left[\frac{P_C^C(z)}{P_C^C}\right]^{-\theta}\left[\frac{P_C^C}{P^C}\right]^{-\rho}C^C(x)
\end{aligned}
$$

where:

$$
\begin{aligned}
P_A^C &= \left[\frac{1}{\gamma_A\gamma_P}\int_0^{\gamma_A\gamma_P}\left(P_A^C(z)\right)^{1-\theta}dz\right]^{\frac{1}{1-\theta}} \\
P_B^C &= \left[\frac{1}{(1-\gamma_A)\gamma_P}\int_{\gamma_A\gamma_P}^{\gamma_P}\left(P_B^C(z)\right)^{1-\theta}dz\right]^{\frac{1}{1-\theta}} \\
P_C^C &= \left[\frac{1}{1-\gamma_P}\int_{\gamma_P}^{1}\left(P_C^C(z)\right)^{1-\theta}dz\right]^{\frac{1}{1-\theta}} \\
P_P^C &= \left[\gamma_A\left(P_A^C\right)^{1-\psi} + (1-\gamma_A)\left(P_B^C\right)^{1-\psi}\right]^{\frac{1}{1-\psi}}
\end{aligned}
$$

$$P^C = \left[\gamma_P \left(P_P^C\right)^{1-\rho} + (1-\gamma_P)\left(P_C^C\right)^{1-\rho}\right]^{\frac{1}{1-\rho}}$$

4.1.2 Country A's consumption

A household x in country A consumes domestic goods and imports from the Center:

$$C^A(x) = \left[\gamma_P^{\frac{1}{\rho}} \left(C_A^A(x)\right)^{\frac{\rho-1}{\rho}} + (1-\gamma_P)^{\frac{1}{\rho}} \left(C_C^A(x)\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}$$

$$C_A^A(x) = \left[(\gamma_A\gamma_P)^{-\frac{1}{\theta}} \int_0^{\gamma_A\gamma_P} \left(C_A^A(z,x)\right)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}}$$

$$C_C^A(x) = \left[(1-\gamma_P)^{-\frac{1}{\theta}} \int_{\gamma_P}^{1} \left(C_C^A(z,x)\right)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}}$$

$C_A^A(x)$ replaces $C_P^A(x)$, with the same weight. We keep the weight, because consumers value Center and Periphery goods in the same way in all countries. The only specificity is that in country A (and symmetrically in country B). periphery goods are entirely made of domestic goods (in other words, there is a complete home bias within Periphery goods, or equivalently the consumer in country A regards Periphery goods as non-traded).

The consumption allocation is given by are:

$$C_A^A(z,x) = \frac{1}{\gamma_A}\left[\frac{P_A^A(z)}{P_A^A}\right]^{-\theta}\left[\frac{P_A^A}{P^A}\right]^{-\rho} C^A(x)$$

$$C_C^A(z,x) = \left[\frac{P_C^A(z)}{P_C^A}\right]^{-\theta}\left[\frac{P_C^A}{P^A}\right]^{-\rho} C^A(x)$$

where:

$$P_A^A = \left[\frac{1}{\gamma_A\gamma_P}\int_0^{\gamma_A\gamma_P}\left(P_A^A(z)\right)^{1-\theta} dz\right]^{\frac{1}{1-\theta}}$$

$$P_C^A = \left[\frac{1}{1-\gamma_P}\int_{\gamma_P}^{1}\left(P_C^A(z)\right)^{1-\theta} dz\right]^{\frac{1}{1-\theta}}$$

$$P^A = \left[\gamma_P\left(P_A^A\right)^{1-\rho} + (1-\gamma_P)\left(P_C^A\right)^{1-\rho}\right]^{\frac{1}{1-\rho}}$$

4.1.3 Country B's consumption

The consumption basket of household x in country B is given by:

$$C^{B}(x)=\left[\gamma_{P}^{\frac{1}{\rho}}\left(C_{B}^{B}(x)\right)^{\frac{\rho-1}{\rho}}+(1-\gamma_{P})^{\frac{1}{\rho}}\left(C_{C}^{B}(x)\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}$$

$$\begin{aligned}
C_{B}^{B}(x) &= \left[((1-\gamma_{A})\gamma_{P})^{-\frac{1}{\theta}}\int_{\gamma_{A}\gamma_{P}}^{\gamma_{P}}\left(C_{B}^{B}(z,x)\right)^{\frac{\theta-1}{\theta}}dz\right]^{\frac{\theta}{\theta-1}} \\
C_{C}^{B}(x) &= \left[(1-\gamma_{P})^{-\frac{1}{\theta}}\int_{\gamma_{P}}^{1}\left(C_{C}^{B}(z,x)\right)^{\frac{\theta-1}{\theta}}dz\right]^{\frac{\theta}{\theta-1}}
\end{aligned}$$

The consumption allocation is are:

$$\begin{aligned}
C_{B}^{B}(z,x) &= \frac{1}{1-\gamma_{A}}\left[\frac{P_{B}^{B}(z)}{P_{B}^{B}}\right]^{-\theta}\left[\frac{P_{B}^{B}}{P^{B}}\right]^{-\rho}C^{B}(x) \\
C_{C}^{B}(z,x) &= \left[\frac{P_{C}^{B}(z)}{P_{C}^{B}}\right]^{-\theta}\left[\frac{P_{C}^{B}}{P^{B}}\right]^{-\rho}C^{B}(x)
\end{aligned}$$

where:

$$\begin{aligned}
P_{B}^{B} &= \left[\frac{1}{(1-\gamma_{A})\gamma_{P}}\int_{\gamma_{A}\gamma_{P}}^{\gamma_{P}}\left(P_{B}^{B}(z)\right)^{1-\theta}dz\right]^{\frac{1}{1-\theta}} \\
P_{C}^{B} &= \left[\frac{1}{1-\gamma_{P}}\int_{\gamma_{P}}^{1}\left(P_{C}^{B}(z)\right)^{1-\theta}dz\right]^{\frac{1}{1-\theta}} \\
P^{B} &= \left[\gamma_{P}\left(P_{B}^{B}\right)^{1-\rho}+(1-\gamma_{P})\left(P_{C}^{B}\right)^{1-\rho}\right]^{\frac{1}{1-\rho}}
\end{aligned}$$

4.2 Output and sales revenue

The worldwide demand faced by household x in each country is written as:

$$\begin{aligned}
Y^{A}(x) &= \left[\frac{P_{A}^{A}(x)}{P_{A}^{A}}\right]^{-\theta}\left[\frac{P_{A}^{A}}{P^{A}}\right]^{-\rho}\gamma_{P}C^{A} \\
&\quad+\left[\frac{P_{A}^{C}(x)}{P_{A}^{C}}\right]^{-\theta}\left[\frac{P_{A}^{C}}{P_{P}^{C}}\right]^{-\psi}\left[\frac{P_{P}^{C}}{P^{C}}\right]^{-\rho}(1-\gamma_{P})C^{C}
\end{aligned}$$

$$
\begin{aligned}
Y^{B}(x) &= \left[\frac{P_{B}^{B}(x)}{P_{B}^{B}}\right]^{-\theta}\left[\frac{P_{B}^{B}}{P^{B}}\right]^{-\rho}\gamma_{P}C^{B} \\
&+\left[\frac{P_{B}^{C}(x)}{P_{B}^{C}}\right]^{-\theta}\left[\frac{P_{B}^{C}}{P_{P}^{C}}\right]^{-\psi}\left[\frac{P_{P}^{C}}{P^{C}}\right]^{-\rho}(1-\gamma_{P})\,C^{C} \\
Y^{C}(x) &= \left[\frac{P_{C}^{A}(x)}{P_{C}^{A}}\right]^{-\theta}\left[\frac{P_{C}^{A}}{P^{A}}\right]^{-\rho}\gamma_{A}\gamma_{P}C^{A} \\
&+\left[\frac{P_{C}^{B}(x)}{P_{C}^{B}}\right]^{-\theta}\left[\frac{P_{C}^{B}}{P^{B}}\right]^{-\rho}(1-\gamma_{A})\,\gamma_{P}C^{B} \\
&+\left[\frac{P_{C}^{C}(x)}{P_{C}^{C}}\right]^{-\theta}\left[\frac{P_{C}^{C}}{P^{C}}\right]^{-\rho}(1-\gamma_{P})\,C^{C}
\end{aligned}
$$

The sales revenues are:

$$
\begin{aligned}
SR^{A}(x) &= P_{A}^{A}(x)\left[\frac{P_{A}^{A}(x)}{P_{A}^{A}}\right]^{-\theta}\left[\frac{P_{A}^{A}}{P^{A}}\right]^{-\rho}\gamma_{P}C^{A} \\
&+E^{A}P_{A}^{C}(x)\left[\frac{P_{A}^{C}(x)}{P_{A}^{C}}\right]^{-\theta}\left[\frac{P_{A}^{C}}{P_{P}^{C}}\right]^{-\psi}\left[\frac{P_{P}^{C}}{P^{C}}\right]^{-\rho}(1-\gamma_{P})\,C^{C} \\
SR^{B}(x) &= P_{B}^{B}(x)\left[\frac{P_{B}^{B}(x)}{P_{B}^{B}}\right]^{-\theta}\left[\frac{P_{B}^{B}}{P^{B}}\right]^{-\rho}\gamma_{P}C^{B} \\
&+E^{B}P_{B}^{C}(x)\left[\frac{P_{B}^{C}(x)}{P_{B}^{C}}\right]^{-\theta}\left[\frac{P_{B}^{C}}{P_{P}^{C}}\right]^{-\psi}\left[\frac{P_{P}^{C}}{P^{C}}\right]^{-\rho}(1-\gamma_{P})\,C^{C} \\
SR^{C}(x) &= \frac{P_{C}^{A}(x)}{E^{A}}\left[\frac{P_{C}^{A}(x)}{P_{C}^{A}}\right]^{-\theta}\left[\frac{P_{C}^{A}}{P^{A}}\right]^{-\rho}\gamma_{A}\gamma_{P}C^{A} \\
&+\frac{P_{C}^{B}(x)}{E^{B}}\left[\frac{P_{C}^{B}(x)}{P_{C}^{B}}\right]^{-\theta}\left[\frac{P_{C}^{B}}{P^{B}}\right]^{-\rho}(1-\gamma_{A})\,\gamma_{P}C^{B} \\
&+P_{C}^{C}(x)\left[\frac{P_{C}^{C}(x)}{P_{C}^{C}}\right]^{-\theta}\left[\frac{P_{C}^{C}}{P^{C}}\right]^{-\rho}(1-\gamma_{P})\,C^{C}
\end{aligned}
$$

Note that when the law of one price holds:

$$
\begin{aligned}
Y^{A}(x) &= \left[\frac{P_{A}^{A}(x)}{P_{A}^{A}}\right]^{-\theta}\left\{\left[\frac{P_{A}^{A}}{P^{A}}\right]^{-\rho}\gamma_{P}C^{A}+\left[\frac{P_{A}^{C}}{P_{P}^{C}}\right]^{-\psi}\left[\frac{P_{P}^{C}}{P^{C}}\right]^{-\rho}(1-\gamma_{P})\,C^{C}\right\} \\
Y^{B}(x) &= \left[\frac{P_{B}^{B}(x)}{P_{B}^{B}}\right]^{-\theta}\left\{\left[\frac{P_{B}^{B}}{P^{B}}\right]^{-\rho}\gamma_{P}C^{B}+\left[\frac{P_{B}^{C}}{P_{P}^{C}}\right]^{-\psi}\left[\frac{P_{P}^{C}}{P^{C}}\right]^{-\rho}(1-\gamma_{P})\,C^{C}\right\}
\end{aligned}
$$

$$Y^{C}(x) = \left[\frac{P_{C}^{C}(x)}{P_{C}^{C}}\right]^{-\theta} \left\{ \begin{array}{c} \left[\frac{P_{C}^{A}}{P^{A}}\right]^{-\rho} \gamma_{A}\gamma_{P}C^{A} + \left[\frac{P_{C}^{B}}{P^{B}}\right]^{-\rho} (1-\gamma_{A})\gamma_{P}C^{B} \\ + \left[\frac{P_{C}^{C}}{P^{C}}\right]^{-\rho} (1-\gamma_{P}) C^{C} \end{array} \right\}$$

$$\begin{aligned} SR^{A}(x) &= P_{A}^{A}(x) Y^{A}(x) \\ SR^{B}(x) &= P_{B}^{B}(x) Y^{B}(x) \\ SR^{C}(x) &= P_{C}^{C}(x) Y^{C}(x) \end{aligned}$$

4.3 Optimization

The Euler equations are similar as before (recalling that all households are identical within a country):

$$\frac{C_{t+1}^{j}}{C_{t}^{j}} = \beta(1+i_{t+1}) \frac{E_{t+1}^{j}}{P_{t+1}^{j}} \frac{P_{t}^{j}}{E_{t}^{j}}$$

Note that it isn't necessarily true that $P^{j} = E^{j}P^{C}$, because the Center price index includes goods from the other periphery country. The money demands are:

$$\frac{M_{t}^{j}}{P_{t}^{j}} = \chi C_{t}^{j} \frac{(1+i_{t+1}) E_{t+1}^{j}}{(1+i_{t+1}) E_{t+1}^{j} - E_{t}^{j}}$$

The optimal markups and current accounts are:

$$\begin{aligned} \frac{P_{jt}^{j}}{P_{t}^{j}} &= \frac{\theta\kappa}{\theta-1} C_{t}^{j} Y_{t}^{j} \\ \frac{E_{t}^{j} B_{t+1}^{j}}{P_{t}^{j}} + C_{t}^{j} &= (1+i_{t}) \frac{E_{t}^{j} B_{t}^{j}}{P_{t}^{j}} + \frac{SR_{t}^{j}}{P_{t}^{j}} \end{aligned}$$

In the symmetric steady state where: $B^{A} = B^{B} = B^{P} = B^{C} = 0$, everyone in the world produces and consumes C_{0} given by: $C_{0} = \sqrt{\frac{\theta-1}{\theta\kappa}}$.

4.4 The long run

We take log linear approximations of the main equations of the model. From the money demands, the markup and the current accounts we write

$$\bar{m}^{j} - \bar{p}^{j} = \bar{c}^{j}$$

$$\begin{aligned}
\bar{p}_j^j - \bar{p}^j &= \bar{c}^j + \bar{y}^j \\
\bar{c}^j &= \frac{1-\beta}{\beta}\bar{b}^j + \bar{y}^j + \bar{p}_j^j - \bar{p}^j
\end{aligned}$$

The output demands are now given by:

$$\begin{aligned}
\bar{y}^A &= \left(\gamma_P \bar{c}^A + (1-\gamma_P)\,\bar{c}^C\right) - \psi\,(1-\gamma_P)\left(\bar{p}_A^C - \bar{p}_P^C\right) \\
&\quad -\rho\left[\gamma_P\left(\bar{p}_A^A - \bar{p}^A\right) + (1-\gamma_P)\left(\bar{p}_P^C - \bar{p}^C\right)\right] \\
\bar{y}^B &= \left(\gamma_P \bar{c}^B + (1-\gamma_P)\,\bar{c}^C\right) - \psi\,(1-\gamma_P)\left(\bar{p}_B^C - \bar{p}_P^C\right) \\
&\quad -\rho\left[\gamma_P\left(\bar{p}_B^B - \bar{p}^B\right) + (1-\gamma_P)\left(\bar{p}_P^C - \bar{p}^C\right)\right] \\
\bar{y}^C &= \bar{c}^w - \rho\left[\gamma_A\gamma_P\left(\bar{p}_C^A - \bar{p}^A\right) + (1-\gamma_A)\,\gamma_P\left(\bar{p}_C^B - \bar{p}^B\right) + (1-\gamma_P)\left(\bar{p}_C^C - \bar{p}^C\right)\right] \\
\bar{y}^P &= \gamma_A \bar{y}^A + (1-\gamma_A)\,\bar{y}^B \\
&= \bar{c}^w - \rho\left[\gamma_P\left(\gamma_A \bar{p}_A^A + (1-\gamma_A)\,\bar{p}_B^B - \bar{p}^P\right) + (1-\gamma_P)\left(\bar{p}_P^C - \bar{p}^C\right)\right]
\end{aligned}$$

Therefore:

$$\begin{aligned}
\bar{y}^P - \bar{y}^C &= -\rho\left[\gamma_P\left(\gamma_A\left(\bar{p}_A^A - \bar{p}_C^A\right) + (1-\gamma_A)\left(\bar{p}_B^B - \bar{p}_C^B\right)\right) + (1-\gamma_P)\left(\bar{p}_P^C - \bar{p}_C^C\right)\right] \\
\bar{y}^A - \bar{y}^B &= \gamma_P\left(\bar{c}^A - \bar{c}^B\right) - \rho\gamma_P\left[\left(\bar{p}_A^A - \bar{p}^A\right) - \left(\bar{p}_B^B - \bar{p}^B\right)\right] \\
&\quad -\psi\,(1-\gamma_P)\left(\bar{p}_A^C - \bar{p}_B^C\right)
\end{aligned}$$

4.4.1 Center-Periphery

We define the following prices:

$$\begin{aligned}
\bar{p}_P^P &\equiv \gamma_A \bar{p}_A^A + (1-\gamma_A)\,\bar{p}_B^B \\
\bar{p}^P &\equiv \gamma_A \bar{p}^A + (1-\gamma_A)\,\bar{p}^B
\end{aligned}$$

As the law of one price holds for any trade flow, we can write:

$$\begin{aligned}
\bar{p}_P^C &= \gamma_A \bar{p}_A^C + (1-\gamma_A)\,\bar{p}_B^C = \bar{p}_P^P - \bar{e}^P \\
\bar{p}^P &= \bar{e}^P + \bar{p}^C
\end{aligned}$$

Therefore, the purchasing power parity holds between the Center and the Periphery. The relative output demand is then:

$$\bar{y}^P - \bar{y}^C = -\rho\left(\bar{p}_P^P - \bar{p}_C^C - \bar{e}^P\right)$$

Which we combine with the markup and current account equations:

$$\begin{aligned}\bar{p}_P^P - \bar{p}_C^C - \bar{e}^P &= \left(\bar{c}^P - \bar{c}^C\right) + \left(\bar{y}^P - \bar{y}^C\right) \\ \left(\bar{c}^P - \bar{c}^C\right) &= \frac{1-\beta}{\beta}\frac{\bar{b}^P}{1-\gamma_P} + \left(\bar{y}^P - \bar{y}^C\right) + \bar{p}_P^P - \bar{p}_C^C - \bar{e}^P\end{aligned}$$

From these equations, we derive:

$$\begin{aligned}\bar{c}^P - \bar{c}^C &= \frac{1+\rho}{2\rho}\frac{1-\beta}{\beta}\frac{\bar{b}^P}{1-\gamma_P} \\ \bar{p}_P^P - \bar{p}_C^C - \bar{e}^P &= \frac{1}{2\rho}\frac{1-\beta}{\beta}\frac{\bar{b}^P}{1-\gamma_P} \\ \bar{y}^P - \bar{y}^C &= -\frac{1}{2}\frac{1-\beta}{\beta}\frac{\bar{b}^P}{1-\gamma_P}\end{aligned}$$

The worldwide effects are:

$$\bar{c}^w = \bar{y}^w = 0$$

The absence of Intra-Periphery trade therefore has no implication on the worldwide variables and the Center-Periphery differences.

4.4.2 Intra-Periphery

There is no direct law of one price between the two Periphery countries. Note however that:

$$\begin{aligned}\bar{p}_C^A &= \bar{p}_C^C + \bar{e}^A \Rightarrow \bar{p}^A = \gamma_P \bar{p}_A^A + (1-\gamma_P)\left(\bar{p}_C^C + \bar{e}^A\right) \\ \bar{p}_C^B &= \bar{p}_C^C + \bar{e}^B \Rightarrow \bar{p}^B = \gamma_P \bar{p}_B^B + (1-\gamma_P)\left(\bar{p}_C^C + \bar{e}^B\right)\end{aligned}$$

$$\Rightarrow \left(\bar{p}_A^A - \bar{p}_B^B\right) - \left(\bar{p}^A - \bar{p}^B\right) = (1-\gamma_P)\left(\left(\bar{p}_A^A - \bar{p}_B^B\right) - \left(\bar{e}^A - \bar{e}^B\right)\right)$$

Furthermore:

$$\bar{p}_A^C = \bar{p}_A^A - \bar{e}^A, \, \bar{p}_B^C = \bar{p}_B^B - \bar{e}^B$$

$$\Rightarrow \bar{p}_A^C - \bar{p}_B^C = \left(\bar{p}_A^A - \bar{p}_B^B\right) - \left(\bar{e}^A - \bar{e}^B\right)$$

We now take the markup, output demands and current account equations to write:

$$\left(\bar{p}_A^A - \bar{p}_B^B\right) - \left(\bar{p}^A - \bar{p}^B\right) = \left(\bar{c}^A - \bar{c}^B\right) + \left(\bar{y}^A - \bar{y}^B\right)$$

$$
\begin{aligned}
\bar{c}^A - \bar{c}^B &= \frac{1-\beta}{\beta}\frac{\bar{b}^A - \bar{b}^P}{1-\gamma_A} + \left(\bar{y}^A - \bar{y}^B\right) + \left(\bar{p}_A^A - \bar{p}_B^B\right) - \left(\bar{p}^A - \bar{p}^B\right) \\
\bar{y}^A - \bar{y}^B &= \gamma_P\left(\bar{c}^A - \bar{c}^B\right) - \rho\gamma_P\left[\left(\bar{p}_A^A - \bar{p}_B^B\right) - \left(\bar{p}^A - \bar{p}^B\right)\right] \\
&\quad -\psi\left(1-\gamma_P\right)\left(\left(\bar{p}_A^A - \bar{p}_B^B\right) - \left(\bar{e}^A - \bar{e}^B\right)\right)
\end{aligned}
$$

The system can be solved as:

$$
\begin{aligned}
\bar{c}^A - \bar{c}^B &= \frac{1 + (\psi + \rho\gamma_P)}{2\left[\psi + (\rho-1)\gamma_P\right]}\frac{1-\beta}{\beta}\frac{\bar{b}^A - \bar{b}^P}{1-\gamma_A} \\
\left(\bar{p}_A^A - \bar{p}_B^B\right) - \left(\bar{e}^A - \bar{e}^B\right) &= \frac{1}{1-\gamma_P}\frac{1+\gamma_P}{2\left[\psi + (\rho-1)\gamma_P\right]}\frac{1-\beta}{\beta}\frac{\bar{b}^A - \bar{b}^P}{1-\gamma_A} \\
\bar{y}^A - \bar{y}^B &= -\frac{1}{2}\frac{1-\beta}{\beta}\frac{\bar{b}^A - \bar{b}^P}{1-\gamma_A}
\end{aligned}
$$

Note that the purchasing power parity does not hold across the two Periphery countries:

$$
\begin{aligned}
\left(\bar{e}^A - \bar{e}^B\right) - \left(\bar{p}^A - \bar{p}^B\right) &= -\gamma_P\left[\left(\bar{p}_A^A - \bar{p}_B^B\right) - \left(\bar{e}^A - \bar{e}^B\right)\right] \\
&= -\frac{\gamma_P}{1-\gamma_P}\frac{1+\gamma_P}{2\left[\psi + (\rho-1)\gamma_P\right]}\frac{1-\beta}{\beta}\frac{\bar{b}^A - \bar{b}^P}{1-\gamma_A}
\end{aligned}
$$

The country with the largest current account surplus experiences a real appreciation of its currency in the long run, vis-a-vis the other Periphery country.

4.5 The short run

We start be linearizing the Euler equations as:

$$
\begin{aligned}
\bar{c}^A - c^A &= \left(\bar{e}^A - e^A\right) - \left(\bar{p}^A - p^A\right) + \beta di \\
\bar{c}^B - c^B &= \left(\bar{e}^B - e^B\right) - \left(\bar{p}^B - p^B\right) + \beta di \\
\bar{c}^C - c^C &= -\left(\bar{p}^C - p^C\right) + \beta di
\end{aligned}
$$

Turning to the short and long run money demands, we have:

$$
\begin{aligned}
\bar{m}^A - p^A &= c^A - \frac{\beta}{1-\beta}\beta di - \frac{\beta}{1-\beta}\left(\bar{e}^A - e^A\right) \\
\bar{m}^B - p^B &= c^B - \frac{\beta}{1-\beta}\beta di - \frac{\beta}{1-\beta}\left(\bar{e}^B - e^B\right) \\
\bar{m}^C - p^C &= c^C - \frac{\beta}{1-\beta}\beta di
\end{aligned}
$$

$$\bar{m}^A - \bar{p}^A = \bar{c}^A,\ \bar{m}^B - \bar{p}^B = \bar{c}^B,\ \bar{m}^C - \bar{p}^C = \bar{c}^C$$

Prices being set in the producer's currency, we write:

$$\begin{aligned} p^A &= (1-\gamma_P)\, e^A,\ p^B = (1-\gamma_P)\, e^B \\ p^P &= (1-\gamma_P)\, e^P \\ p^C &= -\gamma_P e^P \end{aligned}$$

4.5.1 Center-Periphery

We start with the Center-Periphery dimension. Recall that: $\bar{p}^P = \bar{e}^P + \bar{p}^C$, and we can show that: $p^P = e^P + p^C$: the purchasing power parity holds between the Center and the Periphery. The Euler equations therefore imply:

$$c^P - c^C = \bar{c}^P - \bar{c}^C$$

We can show that there is no overshooting of the exchange rate

$$\bar{e}^P = e^P$$

The output demands, current accounts and money demands are:

$$\begin{aligned} y^P - y^C &= \rho e^P \\ \frac{\bar{b}^P}{1-\gamma_P} &= -\left(\bar{c}^P - \bar{c}^C\right) + y^P - y^C - e^P \\ \left(\bar{m}^P - \bar{m}^C\right) - e^P &= \bar{c}^P - \bar{c}^C \end{aligned}$$

We can derive the solution as:

$$\begin{aligned} \bar{c}^P - \bar{c}^C &= \frac{\rho-1}{\rho}\frac{(1-\beta)(1+\rho)}{1+\beta+\rho(1-\beta)}\left(\bar{m}^P - \bar{m}^C\right) \\ e^P &= \frac{1}{\rho}\frac{1-\beta+\rho(1+\beta)}{1+\beta+\rho(1-\beta)}\left(\bar{m}^P - \bar{m}^C\right) \\ y^P - y^C &= \frac{1-\beta+\rho(1+\beta)}{1+\beta+\rho(1-\beta)}\left(\bar{m}^P - \bar{m}^C\right) \\ \frac{\bar{b}^P}{1-\gamma_P} &= \frac{2\beta(\rho-1)}{1+\beta+\rho(1-\beta)}\left(\bar{m}^P - \bar{m}^C\right) \end{aligned}$$

The worldwide solution is:

$$c^w = y^w = \bar{m}^w$$

The worldwide results, along with the Center-Periphery dimension, are unaffected by the absence of Intra-Periphery trade.

4.5.2 Intra-Periphery

The Euler and money demands imply:

$$\begin{aligned}\left(\bar{c}^A-\bar{c}^B\right)-\left(c^A-c^B\right) &= \left(\bar{e}^A-\bar{e}^B\right)-\left(e^A-e^B\right)-\left(\bar{p}^A-\bar{p}^B\right)+\left(p^A-p^B\right)\\ \left(\bar{m}^A-\bar{m}^B\right)-\left(p^A-p^B\right) &= \left(c^A-c^B\right)-\frac{\beta}{1-\beta}\left(\left(\bar{e}^A-\bar{e}^B\right)-\left(e^A-e^B\right)\right)\\ \left(\bar{m}^A-\bar{m}^B\right)-\left(\bar{p}^A-\bar{p}^B\right) &= \left(\bar{c}^A-\bar{c}^B\right)\end{aligned}$$

Combining these equations, we show that there is no exchange-rate overshooting:

$$e^A-e^B=\bar{e}^A-\bar{e}^B$$

Furthermore: $p^A-p^B=(1-\gamma_P)\left(e^A-e^B\right)$. Recall from the long run that:

$$\bar{p}^A-\bar{p}^B=\gamma_P\left(\bar{p}_A^A-\bar{p}_B^B\right)+(1-\gamma_P)\left(\bar{e}^A-\bar{e}^B\right)$$

Plugging these results in the Euler equations, we derive:

$$\left(\bar{c}^A-\bar{c}^B\right)-\left(c^A-c^B\right)=-\gamma_P\left(\bar{p}_A^A-\bar{p}_B^B\right)$$

The output demand, current account are written as:

$$\begin{aligned}y^A-y^B &= \gamma_P\left(c^A-c^B\right)+(\psi+\rho\gamma_P)(1-\gamma_P)\left(e^A-e^B\right)\\ c^A-c^B &= -\frac{\bar{b}^A-\bar{b}^P}{1-\gamma_A}+\left(y^A-y^B\right)-(1-\gamma_P)\left(e^A-e^B\right)\end{aligned}$$

From the long run results, we write:

$$\begin{aligned}\left(c^A-c^B\right) &= \left(\bar{c}^A-\bar{c}^B\right)+\gamma_P\left(\bar{p}_A^A-\bar{p}_B^B\right)\\ &= \frac{1}{1-\gamma_P}\frac{1+\gamma_P+(1-\gamma_P)\left[\psi+(\rho-1)\gamma_P\right]}{2\left[\psi+(\rho-1)\gamma_P\right]}\frac{1-\beta}{\beta}\frac{\bar{b}^A-\bar{b}^P}{1-\gamma_A}\\ &\quad +\gamma_P\left(e^A-e^B\right)\end{aligned}$$

After some algebra, this implies:

$$\begin{aligned}c^A-c^B &= \frac{1}{\psi+(\rho-1)\gamma_P}\cdot\\ &\quad \frac{2\beta\gamma_P\left[\psi+(\rho-1)\gamma_P\right]+(\psi+\rho\gamma_P-1)\,\Omega\,(1-\beta)}{2\beta+\Omega\,(1-\beta)}\left(\bar{m}^A-\bar{m}^B\right)\end{aligned}$$

$$\begin{aligned} e^A - e^B &= \frac{1}{\psi + (\rho - 1)\gamma_P} \frac{2\beta\left[\psi + (\rho - 1)\gamma_P\right] + \Omega(1-\beta)}{2\beta + \Omega(1-\beta)} \left(\bar{m}^A - \bar{m}^B\right) \\ \frac{\bar{b}^A - \bar{b}^P}{1 - \gamma_A} &= \frac{2\beta(1-\gamma_P)\left[(\psi + (\rho - 1)\gamma_P) - 1\right]}{2\beta + \Omega(1-\beta)} \left(\bar{m}^A - \bar{m}^B\right) \end{aligned}$$

where:

$$\Omega = 1 + \gamma_P + (1 - \gamma_P)(\psi + (\rho - 1)\gamma_P) > 0$$

Assuming $\psi + (\rho - 1)\gamma_P > 0$, which is reasonable.

We can see that the purchasing power parity does not hold at the Intra-Periphery level:

$$\left(e^A - e^B\right) - \left(p^A - p^B\right) = \gamma_P \left(e^A - e^B\right)$$

A relative expansion by country A ($\bar{m}^A - \bar{m}^B > 0$) results in a nominal and real depreciation of its currency vis-a-vis country B. If the parameters are such that the current account effect is stronger for A ($\bar{b}^A - \bar{b}^P > 0$), the real depreciation remains in the long run.

4.6 Welfare

The worldwide and Center-Periphery welfares are the same with or without Intra-Periphery trade:

$$\begin{aligned} u^w &= \frac{1}{\theta}\bar{m}^w \\ u^P - u^C &= \frac{\rho - \theta}{\rho\theta} \frac{1 + \rho}{1 + \beta + \rho(1-\beta)} \left(\bar{m}^P - \bar{m}^C\right) \end{aligned}$$

Setting $\bar{m}^C = 0$ we see:

$$u^P < 0 \Leftrightarrow \gamma_P < \frac{\theta - \rho}{\rho\left(1 + \beta\frac{1-\rho}{1+\rho}\right) + \theta - \rho} < 1$$

Turning to the Intra-Periphery dimensions, we can show that:

$$\begin{aligned} u^A - u^B = &\frac{1}{\psi + (\rho - 1)\gamma_P} \frac{\Omega}{2\beta + \Omega(1-\beta)} \cdot \\ &\frac{\psi + (\rho - 1)\gamma_P - \theta(1 - \gamma_P)}{\theta} \left(\bar{m}^A - \bar{m}^B\right) \end{aligned}$$

Therefore, focusing on the effect of $\bar{m}^B$ on u^B, we write:

$$\begin{aligned} u^B &= \left\{ (1-\gamma_A) \left[\gamma_P + (1-\gamma_P) \frac{\rho-\theta}{\rho} \frac{1+\rho}{1+\beta+\rho(1-\beta)} \right] \right. \\ &\quad \left. + \gamma_A \frac{\psi + (\rho-1)\gamma_P - \theta(1-\gamma_P)}{\psi + (\rho-1)\gamma_P} \frac{\Omega}{2\beta + \Omega(1-\beta)} \right\} \frac{\bar{m}^B}{\theta} \end{aligned}$$

which we can compare with the corresponding equation in the case with Intra-Periphery trade:

$$\begin{aligned} u^B &= \left\{ (1-\gamma_A) \left[\gamma_P + (1-\gamma_P) \frac{\rho-\theta}{\rho} \frac{1+\rho}{1+\beta+\rho(1-\beta)} \right] \right. \\ &\quad \left. + \gamma_A \frac{\psi-\theta}{\psi} \frac{1+\psi}{1+\beta+(1-\beta)\psi} \right\} \frac{\bar{m}^B}{\theta} \end{aligned}$$

For comparison, focus on the case where $\rho = 1$. The equation with no Intra-Periphery trade becomes:

$$\begin{aligned} u^B &= \left\{ (1-\gamma_A) \left[\gamma_P + (1-\gamma_P) \frac{\rho-\theta}{\rho} \frac{1+\rho}{1+\beta+\rho(1-\beta)} \right] \right. \\ &\quad \left. + \gamma_A \frac{\psi - \theta(1-\gamma_P)}{\psi} \frac{1+\gamma_P+(1-\gamma_P)\psi}{2\beta + [1+\gamma_P+(1-\gamma_P)\psi](1-\beta)} \right\} \frac{\bar{m}^B}{\theta} \end{aligned}$$

There is more incentive to devalue without Intra-Periphery trade if:

$$[\psi - \theta(1-\gamma_P)] \frac{1+\gamma_P+(1-\gamma_P)\psi}{2\beta + [1+\gamma_P+(1-\gamma_P)\psi](1-\beta)} > [\psi - \theta] \frac{1+\psi}{1+\beta+(1-\beta)\psi}$$

Which is a complex expression. Note however that if $\psi = 0$, it can be rewritten as:

$$\beta < \frac{1+\gamma_P}{1-\gamma_P}$$

which is true. If $\psi = \theta$, it becomes:

$$\gamma_P \frac{1+\gamma_P+(1-\gamma_P)\theta}{2\beta + [1+\gamma_P+(1-\gamma_P)\theta](1-\beta)} > 0$$

which is true. If $\psi \to \infty$, converges to:

$$\frac{1}{1-\beta} > \frac{1}{1-\beta}$$

Therefore, the incentive to devalue is always stronger in the absence of Intra-Periphery trade (a numerical analysis with $\rho \neq 1$ confirms this point).

5 A model without Intra-Periphery trade and prices set in the buyer's currency

The last model is the case with no Intra-Periphery trade, and prices set in the buyer's currency.

5.1 The long run

In the long run, prices are adjusted and the solution is the same as in the model where prices are set in the producer's currency:

$$\bar{c}^w = \bar{y}^w = 0$$

$$\begin{aligned}
\bar{c}^P - \bar{c}^C &= \frac{1+\rho}{2\rho}\frac{1-\beta}{\beta}\frac{\bar{b}^P}{1-\gamma_P} \\
\bar{p}^P_P - \bar{p}^C_C - \bar{e}^P &= \frac{1}{2\rho}\frac{1-\beta}{\beta}\frac{\bar{b}^P}{1-\gamma_P} \\
\bar{y}^P - \bar{y}^C &= -\frac{1}{2}\frac{1-\beta}{\beta}\frac{\bar{b}^P}{1-\gamma_P}
\end{aligned}$$

$$\begin{aligned}
\bar{c}^A - \bar{c}^B &= \frac{1+(\psi+\rho\gamma_P)}{2\left[\psi+(\rho-1)\gamma_P\right]}\frac{1-\beta}{\beta}\frac{\bar{b}^A-\bar{b}^P}{1-\gamma_A} \\
\left(\bar{p}^A_A - \bar{p}^B_B\right) - \left(\bar{e}^A - \bar{e}^B\right) &= \frac{1}{1-\gamma_P}\frac{1+\gamma_P}{2\left[\psi+(\rho-1)\gamma_P\right]}\frac{1-\beta}{\beta}\frac{\bar{b}^A-\bar{b}^P}{1-\gamma_A} \\
\bar{y}^A - \bar{y}^B &= -\frac{1}{2}\frac{1-\beta}{\beta}\frac{\bar{b}^A-\bar{b}^P}{1-\gamma_A}
\end{aligned}$$

5.2 The short run

Turning to the short run, we recall that all consumer prices are set. The Euler equations are then written as:

$$\begin{aligned}
\bar{c}^A - c^A &= \left(\bar{e}^A - e^A\right) - \bar{p}^A + \beta di \\
\bar{c}^B - c^B &= \left(\bar{e}^B - e^B\right) - \bar{p}^B + \beta di \\
\bar{c}^C - c^C &= -\bar{p}^C + \beta di
\end{aligned}$$

Turning to the short and long run money demands, we have:

$$\begin{aligned}\bar{m}^A &= c^A - \frac{\beta}{1-\beta}\beta di - \frac{\beta}{1-\beta}\left(\bar{e}^A - e^A\right) \\ \bar{m}^B &= c^B - \frac{\beta}{1-\beta}\beta di - \frac{\beta}{1-\beta}\left(\bar{e}^B - e^B\right) \\ \bar{m}^C &= c^C - \frac{\beta}{1-\beta}\beta di\end{aligned}$$

$$\bar{m}^A - \bar{p}^A = \bar{c}^A,\ \bar{m}^B - \bar{p}^B = \bar{c}^B,\ \bar{m}^C - \bar{p}^C = \bar{c}^C$$

The output demands are:

$$\begin{aligned}y^A &= \gamma_P c^A + (1-\gamma_P)\, c^C \\ y^B &= \gamma_P c^B + (1-\gamma_P)\, c^C \\ y^C &= c^w\end{aligned}$$

Whereas the current account equations are given by:

$$\begin{aligned}\bar{b}^A + c^A &= sr^A = \gamma_P c^A + (1-\gamma_P)\, c^C + (1-\gamma_P)\, e^A \\ \bar{b}^B + c^B &= sr^B = \gamma_P c^B + (1-\gamma_P)\, c^C + (1-\gamma_P)\, e^B \\ \bar{b}^C + c^C &= sr^C = c^w - \gamma_P e^P\end{aligned}$$

5.2.1 Center-Periphery

From the Euler equations and the money demands, we can show that:

$$\begin{aligned}\left(\bar{c}^P - \bar{c}^C\right) &= \left(c^P - c^C\right) - e^P \\ e^P &= \bar{e}^P\end{aligned}$$

The current account equations imply:

$$\frac{\bar{b}^P}{1-\gamma_P} = -\left(\bar{c}^P - \bar{c}^C\right)$$

which along with the long run results leads to $\bar{b}^P = 0$. There are therefore no long run differences between the Center and the Periphery. From the money demands, we easily show that:

$$c^P - c^C = e^P = \bar{m}^P - \bar{m}^C$$

In addition $y^P = y^C$. The worldwide results are:

$$c^w = y^w = \bar{m}^w$$

The Intra-Periphery trade is again irrelevant for the worldwide and Center-Periphery results.

5.2.2 Intra-Periphery

We now turn to the Intra-Periphery dimension. From the Euler equations and the money demands, we show that:

$$\begin{aligned} e^A - e^B &= \bar{e}^A - \bar{e}^B \\ \left(\bar{c}^A - \bar{c}^B\right) &= \left(c^A - c^B\right) - \left(\bar{p}^A - \bar{p}^B\right) \end{aligned}$$

From the long run money demand, we see that:

$$c^A - c^B = \bar{m}^A - \bar{m}^B$$

From the output demand and the current account equations, we write:

$$\begin{aligned} y^A - y^B &= \gamma_P \left(c^A - c^B\right) \\ \frac{\bar{b}^A - \bar{b}^P}{1 - \gamma_A} + \left(c^A - c^B\right) &= \gamma_P \left(c^A - c^B\right) + \left(1 - \gamma_P\right) \left(e^A - e^B\right) \end{aligned}$$

The next step is to combine this with the long run solution, using the fact that

$$\left(e^A - e^B\right) - \left(\bar{p}^A - \bar{p}^B\right) = -\frac{\gamma_P}{1 - \gamma_P} \frac{1 + \gamma_P}{2\left[\psi + (\rho - 1)\gamma_P\right]} \frac{1 - \beta}{\beta} \frac{\bar{b}^A - \bar{b}^P}{1 - \gamma_A}$$

to derive:

$$\begin{aligned} \frac{\bar{b}^A - \bar{b}^P}{1 - \gamma_A} &= -\left(1 - \gamma_P\right)\left[\left(\bar{c}^A - \bar{c}^B\right) + \left(\bar{p}^A - \bar{p}^B\right) - \left(e^A - e^B\right)\right] \\ &= -\frac{1 + \gamma_P + \left(1 - \gamma_P\right)\left(\psi + (\rho - 1)\gamma_P\right)}{2\left[\psi + (\rho - 1)\gamma_P\right]} \frac{1 - \beta}{\beta} \frac{\bar{b}^A - \bar{b}^P}{1 - \gamma_A} \end{aligned}$$

which implies that there are no long run effects at the Intra-Periphery level either:

$$\bar{b}^A - \bar{b}^P = 0$$

$$\Rightarrow e^A - e^B = \bar{m}^A - \bar{m}^C$$

We can then show that the short run solution is:

$$\begin{aligned}
c^A &= \bar{m}^A,\ c^B = \bar{m}^B,\ c^C = \bar{m}^C \\
y^A &= \gamma_P \bar{m}^A + (1-\gamma_P)\,\bar{m}^C \\
y^B &= \gamma_P \bar{m}^B + (1-\gamma_P)\,\bar{m}^C \\
y^C &= \bar{m}^w
\end{aligned}$$

Consumption and output in country B are not affected by the monetary shocks in country A. There is no beggar-thy-neighbor effect, because of the absence of any direct link between country A and country B.