A Limited Information Approach to the Simultaneous Estimation of Wage and Price Dynamics

Argia M. Sbordone*†
Rutgers University

first draft: May 2003
this revision: April 2004

Abstract

This paper analyzes the dynamics of prices and wages using a limited information approach to estimation. I estimate a two-equation model for the determination of prices and wages derived from an optimization-based dynamic model, where both goods and labor markets are monopolistically competitive, prices and wages can be re-optimized only at random intervals, and, when not re-optimized, can be partially adjusted to the previous period inflation. The estimation procedure is a *two-step* minimum distance estimation, that exploits the restrictions that the model imposes on a time series representation of the data. Specifically, in the first step I estimate an *unrestricted* autoregressive representation of the variables of interest. In the second step, I express the model solution in the form of a *constrained* autoregressive representation of the data, and define the distance between unconstrained and constrained representations as a function of the structural parameters that characterize the joint dynamics of inflation and labor share. This function summarizes the cross-equation restrictions between the model and the time series representations of the data. The parameters of interest are then estimated by minimizing a quadratic function of that distance. I find that the estimated dynamics of prices and wages track actual dynamics quite well, and that the estimated parameters are consistent with the observed length of nominal contracts.

JEL Classification: E32, C32, C52

*Correspondence: Department of Economics, Rutgers University, 75 Hamilton Street, New Brunswick, NJ, 08901-1248; ph.(732)-932-8271; e-mail: sbordone@econ.rutgers.edu.

†I wish to thank for their comments, without implicating them, Jean Boivin, Marc Giannoni, Ken Kuttner, Bruce Preston, Mike Woodford, and seminar participants at the Structural Analysis Division of the Bank of England, the Ente Einaudi in Rome, Barnard College and the Graduate School of Business at Columbia University, and the Research Departments of the Federal Reserve Bank of New York and the Federal Reserve Bank of Chicago.

1. Introduction

This paper is an empirical analysis of the dynamics of wages and prices implied by a model of monopolistic competition in goods and labor markets, with sluggish adjustment of prices and wages. The objective of the paper is to investigate the link between real and nominal variables predicted by an optimization-based model, without specifying the whole structure of a general equilibrium model.

Building on previous work that has shown that inflation fluctuations are fairly consistent with the predictions of an optimizing model of staggered price-setting, if one takes as given the evolution of marginal cost,[1] this paper takes the analysis one step further, endogeneizing the determination of nominal wages, to provide an empirical analysis of the joint dynamics of wages and prices and their interaction with aggregate real variables. Allowing sluggish adjustment of both wages and prices, it also seeks to shed light on whether the source of the inertia that appears to characterize nominal variables rests more on the price or on the wage adjustment mechanism.

The model I analyze is a generalized version of the discrete-time Calvo model for both price and wage setting, of Erceg, Henderson and Levin (2000).[2] Specifically, I assume that monopolistically competitive firms set their prices to maximize the discounted expected value of their future profits, and reset prices only at random intervals. Similarly, monopolistically suppliers of differentiated labor services can re-optimize their wages only at random intervals. On the other hand, I assume that both firms and workers, when not allowed to re-optimize, can adjust their prices to past inflation.

Sluggish price and wage adjustments of this kind, following Calvo modeling, are often introduced in general equilibrium models of business cycle to build in a channel of persistence of monetary policy effects. Estimating the price/wage block within a completely specified general equilibrium model requires further specifications, such as the nature of capital accumulation, the details of fiscal and monetary policy, and the specification of the nature of the shocks. Few papers do so

[1] This is argued by Gali and Gertler (1999), Gali, Gertler and Lopez-Salido (2003) and Sbordone (2002) for the U.S.; Gali, Gertler and Lopez-Salido (2001), Batini, Jackson and Nickell (2002), Gagnon and Khan (2003) for European countries and Canada. The robustness of these estimates has been variously discussed: among the criticisms, see Rudd and Whelan (2002), Kurmann (2002), and Linde' (2002).

[2] This way of modeling the wage and price sector is now widely used in empirical DSGE models; see, for example: Amato-Laubach (2003), Christiano et al. (2001), Altig et al. (2002), Smets and Wouters (2002a,b). A comprehensive exposition of such model can be found in Woodford (2003, ch. 3).

by adopting a full information approach to estimation which uses maximum likelihood methods,[3] others rely on the identification of a single shock and estimate the model parameters by matching theoretical and empirical paths of the impulse response functions to that shock.[4]

The strategy proposed here aims instead at estimating the dynamics of wages and prices implied by this model, without having to specify a whole general equilibrium structure. I compare the equilibrium paths of wages and prices derived from the optimizing model to the paths described by an unrestricted vector autoregression model. Under the null hypothesis that the theoretical model is a correct representation of the stochastic process generating the data, the restrictions that the model solution imposes on the parameters of the time series model should hold exactly. I propose to use these restrictions to construct a two-step distance estimator for the parameters of the structural model.

This approach follows directly from Campbell and Shiller's (1987) analysis, where they suggested to test the present value model of stock prices by testing the restrictions that it imposes on a bivariate time series representation of dividends growth and the price /dividend ratio. The model analyzed here involves as well two present value relationships. When inflation expectations are solved out, price inflation depends upon the present discounted value of expected future deviations of marginal costs from the price level; similarly, when wage expectations are solved out, wage inflation depends upon the present discounted value of expected future deviations of the marginal rate of substitution from the real wage. The joint model then imposes testable restrictions on a multivariate time series representation.

My estimation approach proceeds as follows. I derive the (approximate) equilibrium conditions for price and wage setting from the optimization-based model, and write them in the form of two expectational difference equations in inflation and labor share. I then estimate a multivariate time series model to describe the evolution of all the variables that affect the determination of inflation and labor share. Combining the structural equations and the estimated time series model, I solve for the paths of inflation and labor share as functions of exogenous and predetermined variables. This solution represents a *restricted* autoregressive representation for inflation and labor share, where the parameters are combinations of the structural parameters *and* the parameters of the

[3] For small models, the pioneering work using maximum likelihood estimation is Ireland (1997). Smet and Wouters (2002a,b) have applied that methodology to larger scale models.

[4] For ex., Amato-Laubach (2003), Christiano et al. (2001), and Altig et al (2002).

unrestricted time series process. I can then recover the restrictions imposed by the theoretical model by comparing the coefficients of the restricted and the unrestricted autoregressive representations. These implied restrictions can be interpreted as a measure of the distance between the model and the time series representation: the structural parameters are estimated as those that minimize a quadratic form of this distance.

The estimator I propose is therefore a two-step distance estimator: the first step involves the estimation of the time series model, and the second, taking as given those estimated parameters, minimizes the distance function.

Two important issues are involved in the implementation of the proposed empirical strategy. First, the data need a preliminary transformation so that the stationary variables that define the equilibrium conditions of the model have a measurable counterpart. To handle the presence of a stochastic trend in the time series considered, I use a multivariate approach based on the estimated unrestricted vector autoregression representation: the specification of the VAR is therefore central to both steps of the estimation procedure.

The second issue is the modeling of the marginal rate of substitution, which is the real wage that would prevail in a competitive market, absent wage rigidities; throughout the paper I refer to the marginal rate of substitution as the flexible-wage equilibrium real wage. The expression for this equilibrium wage depends upon the assumptions that one makes about household preferences; without adopting specific functional forms for preferences, I discuss in turn the form that the flexible-wage equilibrium real wage would take under different assumptions.

The rest of the paper is organized as follows. In section 2 I lay out the elements of the optimization model for the determination of the path of price and wage inflation. In section 3 I characterize the two-step estimator, and relate it to similar estimation approaches used in business cycle literature. Section 4 discusses how to model the flexible-wage equilibrium real wage, while section 5 presents the estimation of the time series model and discusses the treatment of the trend. Results are presented and discussed in section 6. Section 7 discusses robustness checks, and section 8 concludes.

2. Wage and price dynamics with backward indexation

As said, the model is based on Erceg et al. (2002), but allows partial indexation of both wages and prices to lagged inflation.[5] Since the basic structure of this model is quite well known, the exposition below is kept to a minimum and targeted to illustrate the meaning of the coefficients to be estimated.

2.1. Staggered price setting with partial indexation

The forward looking model of price setting estimated here generalizes the Calvo model by allowing the firms that can't reset optimally their price to adjust it by the previous period increase in the general level of prices.

Specifically, at any point in time a fraction $(1-\alpha_p)$ of the firms choose a price X_{pt} that maximizes the expected discounted sum of its profits

$$E_t \Sigma_j \alpha_p^j Q_{t,t+j} \left(X_{pt} \Psi_{tj} Y_{t+j}(i) - \mathsf{C}(Y_{t+j}(i)) \right) \tag{2.1}$$

where $Q_{t,t+j}$ is a discount factor between time t and $t+j$, $Y_t(i)$ is the level of output of firm i, $\mathsf{C}(Y_{t,t+j}(i))$ is the total cost of production at $t+j$ of the firm that optimally set prices at t, and

$$\Psi_{tj} = \begin{cases} 1 & j = 0 \\ \Pi_{k=0}^{j-1} \pi_{t+k}^{\varrho_p} & j \geq 1 \end{cases} \tag{2.2}$$

The coefficient $\varrho_p \; \epsilon \; [0,1]$ indicates the degree of indexation to past prices, during the periods in which the firm is not allowed to re-optimize.

The demand for goods of producer i is

$$Y_{t+j}(i) = \left(\frac{X_{pt} \Psi_{tj}}{P_{t+j}} \right)^{-\theta_p} Y_{t+j} \tag{2.3}$$

where $\theta_p > 1$ denotes the Dixit-Stiglitz elasticity of substitution among differentiated goods, and the aggregate price level is

$$P_t = \left[(1-\alpha_p) X_{pt}^{1-\theta_p} + \alpha_p (\pi_{t-1}^{\varrho_p} P_{t-1})^{1-\theta_p} \right]^{\frac{1}{1-\theta_p}} \tag{2.4}$$

The first order condition of this problem can be expressed as

$$E_t \Sigma_j \alpha_p^j Q_{t,t+j} \left\{ Y_{t+j} P_{t+j}^{\theta_p} \Psi_{tj}{}^{1-\theta_p} \left(X_{pt} - \frac{\theta_p}{\theta_p - 1} S_{t+j,t}(i) \Psi_{tj}^{-1} \right) \right\} = 0$$

[5] Full backward indexation was first introduced in Christiano et al. (2001). The generalized model with partial backward indexation is detailed in Woodford (2003), ch.3.

where $S_{t+j,t}(i)$ is nominal marginal cost at $t+j$ of the firms that optimize at time t. Dividing this expression by P_t, and using (2.2), one gets

$$E_t \Sigma_j \alpha_p^j Q_{t,t+j} \left\{ Y_{t+j} P_{t+j}^{\theta_p} \Psi_{tj}{}^{1-\theta_p} \left(x_{pt} - \frac{\theta_p}{\theta_p - 1} s_{t+j}(i) \left(\Pi_{k=1}^{j} \pi_{t+k} \right) \left(\Pi_{k=0}^{j-1} \pi_{t+k}^{\varrho_p} \right)^{-1} \right) \right\} = 0$$

where x_{pt} is the relative price of the firms that optimize at t, and $s_{t+j,t}(i)$ is their real marginal cost at time $t+j$. A log-linearization of this expression around a steady state with zero inflation gives

$$\widehat{x}_{pt} = (1 - \alpha_p \beta) \Sigma_{j=0}^{\infty} (\alpha_p \beta)^j E_t \left(\widehat{s}_{t+j,t} + \Sigma_{k=1}^{j} \widehat{\pi}_{t+k} - \varrho_p \Sigma_{k=0}^{j-1} \widehat{\pi}_{t+k} \right) \tag{2.5}$$

where hat variables are log deviation from steady state values.[6] Under the hypothesis that capital is not reallocated across firms, $s_{t+j,t}$ is in general different from the average marginal cost at time $t+j$, s_{t+j}, so that

$$\widehat{s}_{t+j,t} = \widehat{s}_{t+j} - \theta_p \omega \left(\widehat{x}_{pt} - \left(\Sigma_{k=1}^{j} \widehat{\pi}_{t+k} - \varrho_p \Sigma_{k=0}^{j-1} \widehat{\pi}_{t+k} \right) \right) \tag{2.6}$$

where ω is the output elasticity of real marginal cost for the individual firm.[7] Therefore, substituting (2.6) in (2.5) one obtains

$$(1 + \theta_p \omega) \widehat{x}_{pt} = (1 - \alpha_p \beta) \Sigma_{j=0}^{\infty} (\alpha_p \beta)^j E_t \left(\widehat{s}_{t+j} + (1 + \theta_p \omega) \left(\Sigma_{k=1}^{j} \widehat{\pi}_{t+k} - \varrho_p \Sigma_{k=0}^{j-1} \widehat{\pi}_{t+k} \right) \right) \tag{2.7}$$

Similarly, dividing (2.4) by P_t and log-linearizing:

$$\widehat{x}_{pt} = \frac{\alpha_p}{1 - \alpha_p} \left(\widehat{\pi}_t - \varrho_p \widehat{\pi}_{t-1} \right) \tag{2.8}$$

Finally, combining (2.7) and (2.8) one gets

$$\widehat{\pi}_t - \varrho_p \widehat{\pi}_{t-1} = \frac{(1 - \alpha_p)(1 - \alpha_p \beta)}{\alpha_p (1 + \theta_p \omega)} \Sigma_{j=0}^{\infty} (\alpha_p \beta)^j E_t \left(\widehat{s}_{t+j} + (1 + \theta_p \omega) \left(\Sigma_{k=1}^{j} \widehat{\pi}_{t+k} - \varrho_p \Sigma_{k=0}^{j-1} \widehat{\pi}_{t+k} \right) \right) \tag{2.9}$$

which is equivalently written as[8]

$$\widehat{\pi}_t - \varrho_p \widehat{\pi}_{t-1} = \zeta \widehat{s}_t + \beta \, E_t \left(\widehat{\pi}_{t+1} - \varrho_p \widehat{\pi}_t \right)$$

[6] β is the steady state value of the discount factor, and the index i on variables chosen by the firms that are changing prices is suppressed, since all those firms solve the same optimizing problem.

[7] When the production function takes the Cobb-Douglas form, for example, $\omega = a/(1-a)$, where $(1-a)$ is the output elasticity with respect to labor.

[8] This result is obtained by forwarding (2.9) one period, multiplying it by β, and subtracting the resulting expression from (2.9).

where I set $\zeta = \frac{(1-\alpha_p)(1-\alpha_p\beta)}{\alpha_p(1+\theta_p\omega)}$. This equation describes the evolution of inflation as function of past inflation, expected future inflation and real marginal costs; compared to the standard Calvo model, where $\varrho_p = 0$, this expression contains a backward-looking component that many have argued is a necessary component to fit the inertia of inflation data

$$\widehat{\pi}_t = \frac{\varrho_p}{1+\varrho_p\beta}\widehat{\pi}_{t-1} + \frac{\beta}{1+\varrho_p\beta}E_t\widehat{\pi}_{t+1} + \frac{\zeta}{1+\varrho_p\beta}\widehat{s}_t \tag{2.10}$$

At the other extreme of complete indexation ($\varrho_p = 1$) considered, for ex. in Christiano et al. (2001), the model predicts that the *growth rate* of inflation depends upon real marginal costs and the expected future growth rate of inflation. In this case coefficients on past and future inflation sum to 1, and, for β close to 1, they are approximately the same. For low levels of indexation, instead, the coefficient on past inflation is significantly smaller than the one on future inflation.[9]

2.2. Staggered wage setting with partial indexation

As the firms, households are assumed to set their price (for leisure) in a monopolistically competitive way, analogue to the Calvo price model. Each household (indexed by i) offers a differentiated type of labor services to the firms, and stipulates wage contracts in nominal terms: at the stipulated wage they supply as many hours as are demanded. Unlike Erceg et al. (2000), I allow preferences to be non-separable in consumption and leisure.[10]

Total labor employed by any firm j is an aggregation of individual differentiated hours h_{it}

$$H_t^j = \left[\int_0^1 h_{it}^{(\theta_w-1)/\theta_w} di\right]^{\theta_w/(\theta_w-1)} \tag{2.11}$$

where θ_w is the Dixit-Stiglitz elasticity of substitution among differentiated labor services ($\theta_w > 1$). The wage index is defined as

$$W_t = \left[\int_0^1 W_{it}^{1-\theta_w} di\right]^{1/(1-\theta_w)}$$

The demand function for labor services of household i from firm j is[11]

$$h_{it}^j = (W_{it}/W_t)^{-\theta_w} H_t^j \tag{2.12}$$

[9] A similar equation can be obtained with a slightly different set of assumptions, as, for example, in Gali-Gertler 1999. They assume that part of the optimizing firms behave as 'rule of thumb' price setters, indexing their price to the inflation of the previous period.

[10] Although I do not specify at this point the functional form of preferences, I assume here that they are time separable, and the momentary utility is defined on current values of consumption and leisure.

[11] This demand is obtained by solving firm $j's$ problem of allocating a given wage payment among the differentiated labor services, i.e. the problem of maximizing (2.11) for a given level of total wages to be paid.

which, aggregated across firms, gives the total demand of labor hours h_{it} equal to

$$h_{it} = (W_{it}/W_t)^{-\theta_w} H_t \tag{2.13}$$

where $H_t = \left[\int_0^1 H_t^j dj\right]$.

At each point in time only a fraction $(1 - \alpha_w)$ of the households can set a new wage, which I denote by X_{wt}, independently of the past history of wage changes.[12] The expected time between wage changes is therefore $\frac{1}{1-\alpha_w}$. I also assume, as in Erceg et al. (2000), that households have access to a complete set of state contingent contracts; in this way, although workers that work different amount of time also have different consumption paths, in equilibrium they have the same marginal utility of consumption.

Finally, for wages that are not re-optimized, I allow indexation to previous period inflation : specifically, for $\varrho_w \ \epsilon \ [0,1]$, the wage of a household l that cannot re-optimize at t evolves according to

$$W_t^l = \pi_{t-1}^{\varrho_w} W_{t-1}^l$$

This hypothesis implies that wages reset at time t are expected to grow during the contract period according to

$$X_{wt+j} = X_{wt}\Psi_{tj}, \text{ where } \Psi_{tj} = \begin{bmatrix} 1 & \text{if } j = 0 \\ \prod_{k=0}^{j-1} \pi_{t+k}^{\varrho_w} & \text{if } \ j \geq 1 \end{bmatrix} \tag{2.14}$$

The aggregate wage at any time t is an average of the wage set by the workers that optimize, X_{wt}, and by those who do not:

$$W_t = \left[(1 - \alpha_w)(X_{wt})^{1-\theta_w} + \alpha_w \left(\pi_{t-1}^{\varrho_w} W_{t-1}\right)^{1-\theta_w}\right]^{\frac{1}{1-\theta_w}} \tag{2.15}$$

The wage setting problem is defined as the choice of the wage X_{wt} that maximizes the expected stream of discounted utility from the new wage, defined as the difference between the gain (measured in terms of the marginal utility of consumption) derived from the hours worked at the new wage and the disutility of working the number of hours associated with the new wage. The objective function is then

$$E_t \left\{ \Sigma_{j=0}^{\infty} (\beta\alpha_w)^j \left[\frac{\Lambda_{t+j,t}^c}{P_{t+j}} (X_{wt}\Psi_{tj} h_{t+j,t} - P_{t+j} C_{it+j}) + U(C_{it+j}, h_{t+j,t}) \right] \right\} \tag{2.16}$$

[12] As for the price case, varying α_w between 0 and 1, the model allows various degrees of wage inertia, from perfect wage flexibility ($\alpha_w = 0$) to complete nominal wage rigidity ($\alpha_w \longrightarrow 1$).

where $\Lambda^c_{t+j,t}$ denotes the marginal utility of consumption at $t+j$ of workers that optimize at t, and $h_{t+j,t}$ denotes the hours worked at $t+j$ at the wage set at time t. Given (2.14), the last evolve as

$$h_{t+j,t} = \left(\frac{X_{wt}\Psi_{tj}}{W_{t+j}}\right)^{-\theta_w} H_{t+j} \tag{2.17}$$

The first order condition for this problem can be written as

$$E_t \left\{ \Sigma_{j=0}^{\infty} \ (\beta\alpha_w)^j \left(\frac{X_{wt}\Psi_{tj}}{W_{t+j}}\right)^{-\theta_w} H_{t+j} \left[\frac{X_{wt}\Psi_{tj}}{P_{t+j}} - \frac{\theta_w}{\theta_w - 1} v_{t+j,t}\right]\right\} = 0 \tag{2.18}$$

where $v_{t+j,t}$ is the marginal rate of substitution between consumption and leisure at date $t+j$, when the level of hours is $h_{t+j,t}$. A log-linear approximation of this equation is[13]

$$\widehat{\pi}^w_t - \varrho_w \widehat{\pi}_{t-1} = \gamma\left(\widehat{v}_t - \widehat{\omega}_t\right) + \beta\left(E_t\widehat{\pi}^w_{t+1} - \varrho_w\widehat{\pi}_t\right) \tag{2.19}$$

where $\gamma = \frac{(1-\alpha_w)(1-\beta\alpha_w)}{\alpha_w(1+\theta_w\chi)}$, and the parameter χ reflects the degree of non separability in preferences.[14]

2.3. A complete model

The dynamics of wages and prices is then described by the two log-linearized equilibrium conditions (2.10) and (2.19). Using the fact that the approximations were taken around a point with zero wage and price inflation, $\widehat{\pi}_t = \pi_t \equiv \Delta p_t$, and $\widehat{\pi}^w_t = \pi^w_t \equiv \Delta w_t$. Furthermore, $\widehat{s}_t = w_t - p_t - q_t$ since real wage and labor productivity share the same stochastic trend. Similarly, $\widehat{v}_t - \widehat{\omega}_t = v_t - (w_t - p_t)$, since marginal rate of substitution and real wage also share the same stochastic trend.

Equations (2.10) and (2.19) can be then rewritten as

$$\begin{aligned} \Delta p_t &= \frac{\varrho_p}{1+\varrho_p\beta}\Delta p_{t-1} + \frac{\beta}{1+\varrho_p\beta}E_t\Delta p_{t+1} + \frac{\zeta}{1+\varrho_p\beta}\left((w_t - q_t) - p_t\right) + u_{pt} && (2.20)\\ \Delta w_t &= \varrho_w\Delta p_{t-1} + \beta E_t\left(\Delta w_{t+1} - \varrho_w\Delta p_t\right) + \gamma\left(v_t - (w_t - p_t)\right) + u_{wt} && (2.21)\end{aligned}$$

As these equations show, two *gaps* drive the dynamics of prices and wages: the excess of unit labor costs over price (the real marginal cost), and the excess of the 'equilibrium' real wage over the actual wage. The two parameters ζ and γ, defined quite symmetrically, to recall, as $\zeta = \frac{(1-\alpha_p)(1-\alpha_p\beta)}{\alpha_p(1+\theta_p\omega)}$,

[13] See derivation in the Appendix, sect. 9.1.

[14] $\chi = \frac{-\Lambda^c_h H}{\Lambda^c_c C}\eta_c + \eta_h$, where η_c and η_h are, respectively, the elasticity of the marginal rate of substitution with respect to consumption and with respect to hours, evaluated at the steady state. Λ^c_c and Λ^c_h are derivatives of the marginal utility of consumption Λ^c with respect to consumption and with respect to hours. When preferences are separable in consumption and leisure $\Lambda^c_h = 0$.

and $\gamma = \frac{(1-\alpha_w)(1-\beta\alpha_w)}{\alpha_w(1+\theta_w\chi)}$, measure the degree of gradual adjustment of prices and wages to these gaps. These parameters in turn depend upon the parameters that determine the frequency of price and wage adjustments, respectively α_p and α_w; the degree of substitutability between differentiated goods θ_p, and that between differentiated labor services θ_w; the elasticity of firms' marginal costs with respect to their own output ω, and the degree of non separability in households' preferences, χ.

I have included an error term in each equation: these terms may pick up unobservable mark-ups variations, or allow for possible misspecifications induced by the log-linear approximation. I assume that the error terms are mutually uncorrelated, serially uncorrelated: $E(u_{it}u'_{jt-k}) = 0$ for $i, j = p, w$, and $k \neq 0$, and unforecastable, given information set.

These equations show the interdependence of wages and prices, and their dependence upon the evolution of productivity and the other real variables that determine the evolution of the flexible-wage equilibrium real wage. In a fully specified model, this evolution would be described by similar structural relations. Here instead I focus on the restrictions that these equilibrium conditions impose on *any* general model that includes sluggish price and wage adjustment of the form described, independently of the specific form that the other structural relationships may take.

Specifically, I proceed as follows. I assume that the evolution of the variables that determine the path of wages and prices can be summarized by a covariance stationary, m-dimensional process X_t:

$$X_t = \Phi_1 X_{t-1} + ... + \Phi_p X_{t-p} + \varepsilon_t \tag{2.22}$$

(for some lag p to be determined empirically), where $E(\varepsilon_t) = 0$, and $E(\varepsilon_t \varepsilon'_\tau) = \Omega$ for $\tau = t$, and 0 otherwise. This vector includes, in addition to wages and prices, at least labor productivity q and the determinants of the 'flexible-wage equilibrium' real wage v. Letting $Z_t = [X_t \quad X_{t-1} \quad ....X_{t-p+1}]'$, this process can be represented as a first order autoregressive process:

$$Z_t = AZ_{t-1} + Q\varepsilon_t \tag{2.23}$$

where

$$A = \begin{bmatrix} \Phi_1 & \Phi_2 & ... & \Phi_p \\ I & \underline{0} & ... & \underline{0} \end{bmatrix}, \; Q = \begin{bmatrix} I \\ \underline{0} \end{bmatrix}$$

The system of equations (2.20) and (2.21) place a set of restriction on the parameters of the process (2.23). The nature of these restrictions can be recovered as follows: if one considers the joint process of (2.20), (2.21), and (2.23), one can solve for equilibrium processes $\{w_t, p_t\}$, given stochastic processes for $\{v_t, q_t\}$, and initial conditions $\{w_{-1}, p_{-1}\}$. This solution can be expressed as a particular restricted reduced form representation for the vector Z_t

$$Z_t = A^R Z_{t-1} + \widetilde{\varepsilon}_t$$

with $A^R = G(\psi, A)$. ψ is the vector of the structural parameters of interest (defined below), and the function G incorporates the restrictions that the theoretical model imposes on the parameters of the time series representation. The estimation procedure that I present in the next section is based on minimizing the distance between the restricted and the unrestricted representations of the relevant components of vector Z_t (i.e. the relevant elements of matrices A and A^R.)

Before discussing the implementation of the estimation procedure, I will present the model in the form that I solve, which involves the transformation of eqs. (2.20) and (2.21) in equations for price inflation and labor share. I will also derive the specific form of the restrictions that define the distance function used for the estimation of the structural parameters.

In what follows, I'll make use of the following identities

$$\begin{aligned} q_t &= q_{t-1} + \Delta q_t && (2.24) \\ w_t - p_t &= w_{t-1} - p_{t-1} + \Delta w_t - \Delta p_t && (2.25) \end{aligned}$$

and of an expression that defines the theoretical model for the *flexible-wage equilibrium* real wage

$$v_t = q_t + \Xi Z_t \tag{2.26}$$

where the elements of Ξ depend upon assumptions about the long run trend driving the time series, and the specification of the unrestricted representation (2.23). The crucial assumption that delivers (2.26) is that productivity, real wage, output and consumption are all driven by a single stochastic trend, while hours are trend stationary. The specification of the vector X_t, the choice of the lag length p, and the form of the vector of coefficients Ξ will be discussed below.

3. Approach to estimation

Substituting (2.26) in (2.21), and rearranging, we get

$$E_t \Delta w_{t+1} = \frac{1}{\beta} \Delta w_t + \varrho_w \Delta p_t - \frac{\varrho_w}{\beta} \Delta p_{t-1} + \frac{\gamma}{\beta} (w_t - p_t - q_t) - \frac{\gamma}{\beta} \Xi Z_t + \widetilde{u}_{wt} \tag{3.1}$$

Similarly, we can rearrange eq. (2.20) as

$$E_t \Delta p_{t+1} = \frac{1+\varrho_p \beta}{\beta} \Delta p_t - \frac{\varrho_p}{\beta} \Delta p_{t-1} - \frac{\zeta}{\beta} (w_t - p_t - q_t) + \widetilde{u}_{pt} \tag{3.2}$$

where $\widetilde{u}_{wt} = \beta^{-1} u_{wt}$, and $\widetilde{u}_{pt} = (1+\varrho_p \beta)\beta^{-1} u_{pt}$.

It is convenient to rewrite (3.1) as an equation in the labor share variable $s_t \equiv w_t - p_t - q_t$ (which measures the real wage adjusted for productivity, or real unit labor cost), to obtain a two-equation system in inflation and labor share.[15] Subtracting (3.2) and $E_t \Delta q_{t+1}$ from (3.1), we can derive $E_t \Delta s_{t+1} \equiv E_t (\Delta w_{t+1} - \Delta p_{t+1} - \Delta q_{t+1})$ as

$$\begin{aligned} E_t s_{t+1} - s_t &= \frac{1}{\beta} \Delta w_t + \left(\varrho_w - \varrho_p - \frac{1}{\beta} \right) \Delta p_t + \left(\frac{\varrho_p - \varrho_w}{\beta} \right) \Delta p_{t-1} \\ &\quad + \left(\frac{\gamma + \zeta}{\beta} \right) s_t - \frac{\gamma}{\beta} \Xi Z_t - E_t \Delta q_{t+1} + \nu_t \end{aligned} \tag{3.3}$$

where ν_t is a composite error term.

As it will be explained below, productivity growth Δq_t is an element of the vector X_t, so that, by (2.23):

$$E_t \Delta q_{t+1} = e_q' A Z_t \tag{3.4}$$

where the selecting vector e_q' has a 1 in correspondence of productivity growth, and zero elsewhere. Combining the terms in s_t, and using (3.4), eq. (3.3) becomes

$$\begin{aligned} E_t s_{t+1} &= (\varrho_w - \varrho_p) \Delta p_t + \left(\frac{1+\beta+\gamma+\zeta}{\beta} \right) s_t + \left(\frac{\varrho_p - \varrho_w}{\beta} \right) \Delta p_{t-1} - \frac{1}{\beta} s_{t-1} \\ &\quad - \left(\frac{\gamma}{\beta} \Xi - \frac{1}{\beta} e_q' + e_q' A \right) Z_t + \nu_t \end{aligned} \tag{3.5}$$

Defining a vector y_t as

$$y_t = [\pi_t \ s_t \ \pi_{t-1} \ s_{t-1} \ ]' \tag{3.6}$$

and letting $Y_{t+1} = [y_{t+1} \ Z_{t+1}]'$, we can write the system of equations composed of (3.2), (3.5), and (2.23) as

$$E_t Y_{t+1} = M \ Y_t + N \ u_t \tag{3.7}$$

where $u_t = [u_{pt} \ \ u_{wt}]'$, and the square matrix M (of dim. $(4+mp)$) is partitioned as

$$M = \begin{bmatrix} M_{yy} & M_{yZ} \\ 0 & A \end{bmatrix}, \quad N = \begin{bmatrix} N_1 \\ \underline{0} \end{bmatrix}.$$

[15] As will become clear later, this transformation is suggested by the properties of the time series of wage and productivity. The transformed structural equations have therefore the same form of their corresponding unrestricted representation in the process Z_t.

The (4×4) block M_{yy} describes the interaction of the structural variables, the $(4 \times mp)$ block M_{yZ} describes the dependence of structural variables upon the exogenous block.[16] If the matrix M has exactly two unstable eigenvalues, the system of equations (3.7) has a unique solution, which can be expressed as an autoregressive form

$$Y_t = GY_{t-1} + F\upsilon_t \tag{3.8}$$

where the matrices G and F depend upon the vector of structural parameters, ψ, and the parameters of the unrestricted VAR process, A, and $\upsilon_t = (u_t', \varepsilon_t')'$. The solution for the endogenous variables π_t and s_t is the upper block of (3.8), and can be expressed as

$$\pi_t \equiv y_{1t} = g^{\pi}(\psi, A)Y_{t-1} + f^{\pi}\upsilon_t = g_y^{\pi} y_{t-1} + g_Z^{\pi} Z_{t-1} + f^{\pi}\upsilon_t \tag{3.9}$$

$$s_t \equiv y_{2t} = g^{s}(\psi, A)Y_{t-1} + f^{s}\upsilon_t = g_y^{s} y_{t-1} + g_Z^{s} Z_{t-1} + f^{s}\upsilon_t \tag{3.10}$$

where g^i and f^i (for $i = \pi, s$) denote the row of the matrices G and F corresponding to variable i. Since both inflation and labor share are elements of the unrestricted process (2.22), they can be expressed as elements of Z_t, with appropriate definitions of selection vectors e_{π}' and e_s' :

$$\pi_t = e_{\pi}' Z_t \qquad \text{and} \qquad s_t = e_s' Z_t \tag{3.11}$$

Similarly, the vector y_{t-1}, which includes lagged inflation and labor share, can be expressed as elements of the vector Z_{t-1}, by way of an appropriate selection matrix $\Upsilon : y_{t-1} = \Upsilon Z_{t-1}$. Using this definition, and substituting (3.11) in (3.9) and (3.10), we get

$$e_{\pi}' Z_t - g_y^{\pi} \Upsilon Z_{t-1} - g_Z^{\pi} Z_{t-1} = f^{\pi}\upsilon_t \tag{3.12}$$

$$e_s' Z_t - g_y^{s} \Upsilon Z_{t-1} - g_Z^{s} Z_{t-1} = f^{s}\upsilon_t \tag{3.13}$$

Finally, projecting both sides of (3.12) and (3.13) onto the information set Z_{t-1}, and observing that, by assumption, $E(\upsilon_t | Z_{t-1}) = 0$, and also $E(Z_t | Z_{t-1}) = AZ_{t-1}$, one obtains

$$\begin{aligned} e_{\pi}' A Z_{t-1} - g_y^{\pi} \Upsilon Z_{t-1} - g_Z^{\pi} Z_{t-1} &= 0 \\ e_s' A Z_{t-1} - g_y^{s} \Upsilon Z_{t-1} - g_Z^{s} Z_{t-1} &= 0 \end{aligned}$$

[16] The matrix N_1 is $\begin{pmatrix} \beta^{-1}(1+\xi\beta) & 0 \\ -\beta^{-1}(1+\xi\beta) & \beta^{-1} \end{pmatrix}$.

Since these equalities must hold for every t, we have that

$$e'_\pi A - g_y^\pi \Upsilon - g_Z^\pi = \underline{0} \tag{3.14}$$

$$e'_s A - g_y^s \Upsilon - g_Z^s = \underline{0} \tag{3.15}$$

Expressions (3.14) and (3.15) form a set of $2 \times mp$ restrictions on the parameters of the unrestricted process (2.23), that must hold if the model is true. The structural parameters can then be estimated as those values that make most likely these restrictions to hold.

The estimation strategy proceeds in two steps. First, I estimate an unrestricted VAR in all the variables of interest, to obtain a consistent estimate $\widehat{A}$ of the autoregressive matrix A. In the second step, taking as given the estimated parameters $\widehat{A}$, and stacking the restrictions (**??**) in a vector function $\digamma(\psi, A) = 0$, I choose the structural parameters ψ to make the empirical value of the function $\digamma$ as close as possible to its theoretical value of zero, namely I choose

$$\widehat{\psi} = \arg\min \ \digamma(\psi, \widehat{A})' W^{-1} \digamma(\psi, \widehat{A}) \tag{3.16}$$

for an appropriate choice of the weighting matrix W.[17]

The proposed estimator can be interpreted as a minimum distance estimators, in application of the approach that Campbell and Shiller (1987) proposed for the empirical evaluation of present-value models. I have in fact interpreted the restrictions that define the function $\digamma$ as measuring the 'distance' between the restricted and the unrestricted representations of the data.[18] This estimator is close in spirit to another distance estimator used in the business cycle literature, based on matching empirical and theoretical impulse response functions to specific structural shocks.[19] That estimator, as the one proposed here, uses an auxiliary VAR model in the first stage to characterize the dynamics of the data; then it minimizes the distance between the dynamic response to identified exogenous shocks estimated in the data and the response predicted by the theoretical model. Unlike

[17] As weighting matrix I use a diagonal matrix with the variance of the estimated parameters A along the diagonal. This choice downweights the parameters estimated with greater uncertainty.

[18] In my previous applications of a similar two-step minimum distance estimation, the objective function had the form of an (unweighted) distance between 'model' and data (Sbordone 2002, 2003).

[19] Rotemberg and Woodford (1997) were the first to propose to estimate the structural parameters of a small monetary model by matching the model's predicted responses to monetary policy shocks to the responses estimated in an identified VAR model. This type of estimator has since then been applied in several monetary models of business cycle, by, among others, Amato and Laubach (2002), Boivin and Giannoni (2003), and Christiano, Eichenbaum and Evans (2001). More recently, it has been applied to match the responses to both technology and monetary shocks by Altig et al. (2002) and Edge et al. (2003).

the estimator based on matching impulse response functions, the one proposed here doesn't rely on further identification restrictions to recover the structural shocks from the VAR innovations. Instead, it exploits the specific restrictions that the VAR specification imposes on the solution of the structural model, and tries to match the dynamic evolution of the endogenous variables implied by the theoretical model with their evolution as described by the data.

Finally, although the distance restrictions are not moments conditions, this estimator is similar to a GMM estimator whose instruments are the variables of the time series representation. However, such an estimator is usually applied to orthogonality conditions that proxy the future values of the endogenous variables, as opposed to solving the expectational equations.[20]

4. Modeling the flexible-wage equilibrium real wage

A crucial step for the implementation of the empirical strategy discussed for this problem is the specification of the flexible-wage equilibrium real wage. Relationship (2.26) expresses the theoretical link between the flexible-wage equilibrium real wage (denoted by v_t) and real variables which are not determined by structural equations (in vector Z_t). The expression for the parameter vector Ξ incorporates therefore the hypotheses about the determinants of the cyclical components of the marginal rate of substitution, together with the hypotheses about the evolution of its trend component.

The real wage v_t is the equilibrium wage that solves the household optimization problem under flexible wages, and is therefore equal to the ratio of the marginal disutility of working Λ_t^h, and the marginal utility of consumption Λ_t^c. If there is no time dependence in the momentary utility function, these marginal utilities depend only upon current values of consumption and hours; with time dependence, for example if one allows habit persistence in consumption, the marginal rate of substitution depends also on past and future expected values of consumption and hours. In the case considered here

$$\widehat{v}_t = \eta_c \widehat{c}_t + \eta_h \widehat{h}_t \tag{4.1}$$

where the coefficients η_i are elasticities. Since 'hat' variables are deviations from steady state of variables transformed by removing a stochastic trend, their natural empirical counterpart are the

[20] An estimator of this kind is used by H. Li (2003) to estimate the NKPC. In single equation models like the NKPC, it can be easily seen that this approach implies focusing on a non-linear transformation of the distance restrictions derived, as here, through the model solution.

cyclical components of the relative series, defined as well as deviations from a stochastic trend. Their derivation is explained in the next section.

5. The time series model

The second crucial step of the empirical methodology that I described is the specification of the *unrestricted* joint dynamics of the variables that appear as endogenous and forcing variables in the structural equations (2.20) and (2.21). These variables are inflation, labor share, labor productivity and, following the discussion of the previous section, consumption and hours of work, which determine the evolution of the flexible-wage equilibrium real wage.

The first order of problems is to choose the appropriate transformation of the data which is consistent with the hypotheses built in the model. The time series of productivity, real wage, consumption and output all contain a unit root, but it appears that the consumption - output ratio, and the ratio of real wage to labor productivity are stationary. Hours, in turn, appear stationary around a deterministic trend. One can then assume that there is only one common stochastic trend to drive the long-run behavior of the series considered.

The hypothesis of a single stochastic trend in the data is consistent with the assumption built into the model that the economy is driven by a single source of non-stationarity.[21] As in the model, I wish to define stationary variables in the sample as deviation from this single stochastic trend. While many stationarity inducing transformations could be applied to the data, my approach is to handle the non stationarity in the same multivariate context that I use for the time series representation, and therefore I use the Beveridge-Nelson (1981) detrending method.

I first specify the vector X_t of (2.22) as follows

$$X_t = [\Delta q_t \quad h_t \quad cy_t \quad \pi_t \quad s_t]'$$

where Δq_t is labor productivity growth, h_t is an index of hours, cy_t is the consumption output ratio, s_t is the ratio of real wage to productivity, which is a measure of the share of labor in total output, and inflation is the implicit GDP deflator. Following Beveridge-Nelson, any difference stationary series can be decomposed in a random walk component (the stochastic trend) and a stationary component. In the vector X_t we can identify the stochastic trend with the random walk

[21] This is a stochastic process Θ_t, which I model as a logarithmic random walk. In the model, non-stationary variables such as consumption and real wage, are transformed by dividing through this process.

component of labor productivity. This trend is then defined as the current value of productivity plus all expected future productivity growth. The rationale is that, if productivity growth is expected to be higher than average in the future, then labor productivity today is below trend; viceversa, if productivity growth is expected to be below average, than productivity today is above trend. Formally, letting q_t denote labor productivity, its trend is defined as

$$q_t^T = \lim_{k \to \infty} E_t \left(q_{t+k} - k\mu_q \right) = q_t + \sum_{j=1}^{\infty} E_t \left(\Delta q_{t+j} - \mu_q \right) \tag{5.1}$$

where $\mu_q = E(\Delta q)$. The stationary, or cyclical component, of productivity is then defined as the deviation of the series from its stochastic trend. The assumption of stationary labor share in the VAR in turn implies that the trend in real wage is the same as the trend in productivity, and the stationarity of the consumption-output ratio, together with the stationarity of hours (which correspond to the ratio of output to productivity), imply that consumption shares the same trend as productivity.

Hence the cyclical variables that appear in the theoretical model can be constructed as deviation from their respective trends.[22] From the joint representation of the series in (2.23) the s-step ahead forecast that define the trend are easily computed, for each variable i in vector X, as

$$E_t X_{i,t+s} = e_i' E_t Z_{t+s} = e_i' A^s Z_t \tag{5.2}$$

These forecasts are used to derive the vector of parameters Ξ that expresses the real wage v_t as a function of the exogenous variables in vector Z, as detailed in the Appendix (sect. 9.2). The specification of Ξ completes the specification of the system (3.7) used for the estimation of the structural parameters ψ.

Using (5.2), the trend in productivity defined in (5.1) is

$$q_t^T = q_t + e_q' [I - A]^{-1} A Z_t \tag{5.3}$$

For consumption, since the output-productivity ratio and the consumption-output ratio are stationary, and therefore output, productivity, and consumption share the same stochastic trend, we can

[22] The importance of the transformation to induce stationarity lies in the fact that the theoretical model has implications only for the comovement of the stationary components of real wage, consumption and hours. The specific detrending procedure followed here intends to reflect closely the assumption about the nature of trend which is made in the theoretical model.

write $c_t = (c_t - y_t) + (y_t - q_t) + q_t$, and obtain the following expression for the cyclical component of consumption:

$$c_t^{cyc} = c_t - c_t^T = e'_{cy} Z_t + e'_h Z_t - e'_q [I - A]^{-1} A Z_t \tag{5.4}$$

where I have also used the fact that hours are stationary, so that cyclical hours h_t^{cyc} are simply the appropriate component of vector Z_t.

6. Results

6.1. VAR specification

I use quarterly data for the period 1952:1-2002:1, with periods 1951:2-51:4 used as initial values.[23] Productivity, output, wages and hours are for the non farm business sector of the economy. Nominal wage is hourly compensation, and real wage is nominal wage divided by the implicit GDP deflator for the non farm business sector. Consumption is the aggregate of nondurables and services. All variables are in deviation from the mean, and hours are linearly detrended.[24] I fit a VAR to the vector X_t which includes three lags, so $\Phi(L)$ in (2.22) is:[25]

$$\Phi(L) = I - \Phi_1 L - \Phi_2 L^2 - \Phi_3 L^3 \tag{6.1}$$

Fig. 1 plots the cyclical components of productivity, real wage, and consumption obtained after the removal of the Beveridge-Nelson trend. It also plots the cyclical component of hours, and the series of inflation and labor share. The theoretical model is assumed to describe the dynamic behavior of inflation and labor share and, given the behavior of productivity, the estimated labor share path allows to recover the path of real wages.

6.2. Estimation of structural parameters

The parameter vector, to recall, is

$$\psi = \left(\beta, \varrho_p, \varrho_w, \eta_c, \eta_h, \zeta, \gamma\right)'$$

where β is a discount factor, ϱ_p and ϱ_w are indexation parameters, respectively in the price and wage setting, η_c and η_h are elasticities of the marginal rate of substitution with respect to consumption

[23] The time series are from the FRED database at the Federal Reserve Bank of St. Louis.

[24] I also remove, prior to estimation, a moderate deterministic trend that appears in the consumption-output ratio and the labor share.

[25] The optimal lag length is chosen with the Akaike criterion.

and hours of work, and ζ and γ are measures of the inertia in the price and wage settings. The last two parameters are non linear combinations of other structural parameters which are not separately identified: the frequency of price and wage adjustments, and the structure of technology and preferences. However, calibrating some of these parameters, we can draw some inference on which values of the frequency of price and wage adjustments are consistent with the estimated values of ζ and γ.

Table 1 reports parameter estimates, with standard errors in parentheses[26], and the correlation of the model implied paths of inflation, labor share, and real wage (denoted with superscript m) with their observed counterparts.

TABLE 1
Parameter Estimates - Baseline VAR (52:1-02:1)

β	ϱ_p	ϱ_w	η_c	η_h	ζ	γ
.967	.226	.058	2.41	-.891	.0255	.040
(.007)	(.041)	(.039)	(.537)	(.312)	(.007)	(.014)

Related Statistics

$corr(\pi, \pi^m)$	.905	
$corr(s, s^m)$	.798	$Q = 38.42$
$corr(\omega, \omega^m)$	.908	[p-value: .139]

Most of the estimated parameters are statistically significant. Those of the inflation model are consistent with several of the empirical results in the literature on New Keynesian Phillips curve: in particular, there is role for a backward-looking component in inflation dynamics. However, while the indexation parameter ϱ_p is significantly different from zero, the implied weight on the backward-looking component $(\varrho_p / \left(1 + \beta\varrho_p\right) \simeq .18)$ is quantitatively much smaller than the weight on the forward-looking component $(\beta / \left(1 + \beta\varrho_p\right) \simeq .79)$. The size of the coefficient on the labor share, as it will be explained below, is consistent with other estimates of price inertia in the literature. In the labor share equation, the parameter of wage indexation is much smaller than one, the value

[26] To compute standard errors, I use the empirical distribution of the parameter matrix A to generate N samples A_i $(i = 1, ..N)$, and for each I estimate a vector of structural parameters $\widehat{\psi}_i$. I then compute the sample variance of $\widehat{\psi}_i$,.and report the square roots of the elements on the main diagonal as standard errors. For each estimated vector $\widehat{\psi}_i$ I also compute the value of the distance function $\digamma_i$ and its covariance matrix $\Sigma_\digamma$, to compute a Wald statistic $Q = \digamma(\widehat{\psi})' \Sigma_\digamma \digamma(\widehat{\psi})$, where $\digamma(\widehat{\psi})$ is the value of the distance evaluated at the optimal value of ψ.

imposed in Christiano et al. (2001), and more in the range of that estimated by Smets and Wouters (2002a) for the euro area. Finally, according to the Wald statistic Q, the restrictions that the model imposes on the parameters of A cannot be rejected.

Fig. 2 shows the paths of inflation, labor share, and real wage constructed recursively from the model solution evaluated at the estimated parameters. These paths seem to capture well the underlying dynamics of the actual series. On these accounts, the model of wage and price inflation described seems to fit the data quite well.

Furthermore, the model is able to match the dynamic correlation between inflation and output. As noted in the literature, output leads inflation in the data: the cyclical component of output, variously measured, is positively correlated with future inflation, with the highest value at about three quarters ahead. Purely forward looking NKPC driven by output gap measured as deviation from a deterministic trend are unable to reproduce such result: output gap typically lags inflation in such a model.[27]

Fig. 3 shows the dynamic correlation of cyclical output and inflation implied by the model studied here. Consistently with the estimated time series model, cyclical output is measured as deviation of output from the estimated stochastic trend; the line labeled 'actual' shows the correlation of actual inflation with this measure of output, while the one labeled 'predicted' shows the correlation of the inflation predicted by the model with the same measure of output. As the figure shows, output leads inflation both in the model and in the data, and actual and predicted dynamic correlations peak at about the same time.

6.3. Implied degree of nominal rigidities

Although the parameters that measure the degree of price and wage inertia are significantly different from zero, they do not give a direct estimate of the frequency of price and wage adjustments. In the Calvo model, the frequency of price and wage adjustment is driven by the probability of changing prices or wages at any point in time, measured respectively by α_p and α_w. In order to infer those parameters from the estimated values of ζ and γ, some further hypotheses are needed. From the definition of $\zeta = \frac{(1-\alpha_p)(1-\alpha_p\beta)}{\alpha_p(1+\theta_p\omega)}$ to draw inference on α_p one has to make some assumption about the degree of substitution among differentiated goods θ_p, and the elasticity of real marginal cost to output for the individual firm, ω. On the upper half of table 2 I report the implied degree of

[27] See evidence presented in Sbordone (2001), or Gali-Gertler (1999).

inertia (measured as the average time between price changes, measured in months), under two different assumptions about these two parameters. For the parameter ω I consider two benchmark values, .33 and .54;[28] for θ_p, which is related to the steady state mark-up μ^* by $\mu^* = \theta_p / (\theta_p - 1)$, I consider values that imply a low (20%) and a high steady state mark up (60%), two benchmark values often used in the literature.[29] As the table shows, the average duration of prices ranges from a little more than three quarters to about five quarters, depending on these assumptions.

The bottom part of the table shows the implied degree of wage inertia computed in similar manner. Here the inertia is summarized by $\gamma = \frac{(1-\alpha_w)(1-\beta\alpha_w)}{\alpha_w(1+\theta_w\chi)}$; in order to make inference on α_w some assumption must be made about the value of the parameters θ_w, and therefore about the value of the steady state wage mark-up, and about the degree of non separability between consumption and leisure in preferences, which determines the size of the parameter χ. In the table I consider different values for the steady state mark up, and different degrees of non separability[30] For low degrees of non separability, the average duration of wage contracts is similar to those of prices, while it is shorter for highly non separable preferences.

That preferences should be non separable in consumption and leisure is an implication of the negative sign of the elasticity of the marginal rate of substitution with respect to hours.[31] While most of the business cycle literature adopts separable preference specification, empirical evidence on significant non separability in preferences has been found, most recently, by Basu and Kimball (2000). Moreover, within the class of preferences that are consistent with balanced growth, a negative elasticity of the marginal rate of substitution with respect to hours can be obtained in a generalized indivisible labor model, as shown in King and Rebelo (1999). The interpretation of the large elasticity η_c is more problematic, and requires further investigation. As we will see below, however, a modification in the specification of the time series model reduces its size. Another possibility to be explored, which is left to future research, is that this parameter is overestimated

[28] As mentioned before, in the case of a Cobb-Douglas technology, $\omega = a/(1-a)$, where a is the output elasticity with respect to capital. The two values assumed for ω correspond therefore to an output elasticity with respect to capital of, respectively, .25 and .35.

[29] Values of μ^* above 1.5 are, for example, estimated by Hall (1988) on a large number of U.S. manufaturing industries.

[30] I show in the Appendix (sect. 9.3) that the degree on non separability can be parametrized by calibrating the value of the intertemporal elasticity of substitution in consumption, and the share of labor income in consumption.

[31] This can be shown by expressing the two elasticities of the marginal rate of substitution η_1 and η_4 in terms of the Frish elasticities of consumption and labor supply (see Sbordone 2001).

for an omitted variable problem in the wage equation, as it would be the case if preferences were time dependent.

TABLE 2
Implied degrees of nominal rigidity

	Average time between price changes (months)		
	low mark up (μ^{p*}= 1.2)	high mark up (μ^{p*}= 1.6)	
$\omega = .33$	12.4	15.1	
$\omega = .54$	10.7	13.6	
	Average time between wage changes (months)		
	low wage mark-up ($\mu^{w*} = 1.1$)	mid wage mark-up ($\mu^{w*} = 1.3$)	high wage mark-up ($\mu^{w*} = 1.5$)
low non-sep.	13.4	12.3	16.1
mid non-sep.	8.6	11.4	12.5
high non-sep.	5.8	7.6	8.4

7. Some robustness analysis

The inference presented on the structural parameters relies on the inference in the first step of the procedure: the estimation of the time series model. I made a number of assumptions to model the VAR: the choice of variables has been kept to a minimum, but other variables may be relevant to obtain an optimal forecast of the driving forces of the structural equations. Only one stochastic trend has been modeled in the data, to mimic the trend assumption of the theoretical model; but the data may be consistent with other assumptions about the number of common stochastic trends. Finally, the VAR structure has been modeled as time invariant, while many recent analyses have suggested that changes in policy regime have determined drifts over time in the reduced form representation of the relation between nominal and real variables.[32]

While some of these issues are pursued in separate research[33], here I present alternative estimates

[32] See, for example, Boivin and Giannoni (2003), Cogley and Sargent (2001, 2003).

[33] Cogley-Sbordone (in progress) extend the two-step estimation procedure to the case of a small scale first stage

that address only the first of these issues, namely how sensitive the results presented are to the inclusion of additional variables in the time series model. Specifically, I augment the baseline VAR considered so far with the federal funds rate: although the corresponding equation in the VAR is not meant to represent a policy rule, the introduction of the federal funds rate can be thought of as allowing more directly for the effect of the state of monetary policy on inflation and on the real variables of the system. The drawback of including additional variables in the VAR, though, is an increase in uncertainty when its parameters are not tightly estimated.

A summary of second stage parameter estimates, and implied nominal rigidity is in table 3. The results are qualitatively similar to the previous ones, but the lower estimates of the inertia parameters imply a higher degree of nominal rigidity, especially for prices.

8. Conclusion

In this paper I estimate the joint dynamics of U.S. prices and wages using a partial information approach. I derived the implied price and wage inflations from an optimization-based model of staggered price and wage contracts with random duration, and then implemented a two-step minimum distance estimator of the structural parameters. In the first step, I estimated an unrestricted time series representation for the variables of interest, and derived the restrictions that the model solution imposes on this representation. In the second step, I used these restrictions to define a distance function to be minimized for the estimation of the structural parameters. This methodology allowed me to investigate the dynamics of prices and wages without having to make all the additional assumptions required to close the model and to characterize its entire stochastic structure.

I find that a generalized version of the Calvo mechanism of random intervals between price and wage adjustments fits the data quite well, that there is some backward-looking component in inflation, and that the average duration of both contracts is around a year. The robustness of these results to the specification of the first stage of the proposed estimation procedure is to be further explored.

VAR with drifting parameters.

TABLE 3 Parameter Estimates - Augmented VAR (54:3-02:1)[34]							
	β	ϱ_p	ϱ_w	η_c	η_h	ζ	γ
	.967	.154	.001	2.74	-.71	.018	.033
	.(0027)	(.027)	(.071)	(.581)	(.319)	(.009)	(.034)
Related Statistics							
$corr(\pi, \pi^m)$		.897					
$corr(s, s^m)$		.782				$Q = 36.44$	
$corr(\omega, \omega^m)$		.903				[p-value: .194]	
Average time between price changes (months)							
		low mark up			high mark up		
$\omega = 54$		13.0			16.3		
Average time between wage changes (months)							
		low mark up			high mark up		
low non-sep.		6.63			9.81		
high non-sep.		5.70			8.26		

9. Appendix

9.1. Derivation of (2.19)[35]

Under the hypothesis that there is a single stochastic trend driving long run growth, say Θ_t, with $\gamma_{\Theta_t} = \Theta_t/\Theta_{t-1}$ an i.i.d. process, one can define stationary variables $x_{wt} \equiv \frac{X_{wt}}{W_t}$, $\pi_t^w \equiv \frac{W_t}{W_{t-1}}$, $\widetilde{\omega}_t = \frac{W_t}{\Theta_t P_t}$, and $\widetilde{v}_t = \frac{v_t}{\Theta_t}$. Then, using the fact that $\frac{X_{wt}}{W_{t+j}} = \frac{X_{wt}}{W_t}\frac{W_t}{W_{t+j}}$ and $\frac{X_{wt}}{P_{t+j}} = \frac{X_{wt}}{W_{t+j}}\frac{W_{t+j}}{P_{t+j}}$, eq. (2.18) can be written as

$$E_t\left\{\Sigma_{j=0}^{\infty}\ (\beta\alpha_w)^j \left(x_{wt}\Psi_{tj}\Pi_{k=1}^{j}\left(\pi_{t+k}^{w}\right)^{-1}\right)^{-\theta_w} H_{t+j}\left[x_{wt}\Psi_{tj}\widetilde{\omega}_{t+j}\Pi_{k=1}^{j}\left(\pi_{t+k}^{w}\right)^{-1} - \frac{\theta_w}{\theta_w - 1}\widetilde{v}_{t+j,t}\right]\right\} = 0$$

so that a log-linearization around steady state values $x_w^*, \pi_w^*, \omega^*, v^*$ gives

$$\Sigma_{j=0}^{\infty}\ (\beta\alpha_w)^j\left(\widehat{x}_{wt} + \varrho_w\Sigma_{k=0}^{j-1}\widehat{\pi}_{t+k} - \Sigma_{k=1}^{j}\widehat{\pi}_{t+k}^{w} + \widehat{\omega}_{t+j}\right) = \Sigma_{j=0}^{\infty}(\beta\alpha_w)^j\ E_t\left(\widehat{v}_{t+j,t}\ \right)$$

or

$$\widehat{x}_{wt} = (1-\beta\alpha_w)\ \Sigma_{j=0}^{\infty}(\beta\alpha_w)^j\ E_t\left(\widehat{v}_{t+j,t}\ - \widehat{\omega}_{t+j} - \varrho_w\Sigma_{k=0}^{j-1}\widehat{\pi}_{t+k} + \Sigma_{k=1}^{j}\widehat{\pi}_{t+k}^{w}\right) \qquad (9.1)$$

[34] The shorter sample is due to the federal funds rate data being available only from 54:3.

[35] This derivation follows Sbordone (2001).

To express $\widehat{v}_{t+j,t}$ in terms of the average marginal rate of substitution, write

$$v_{t+j,t} \equiv \frac{\Lambda^h}{\Lambda^c}(c_{t+j,t}, h_{t+j,t}) = \frac{\frac{\Lambda^h}{\Lambda^c}(c_{t+j,t}, h_{t+j,t})}{\frac{\Lambda^h}{\Lambda^c}(c_{t+j}, h_{t+j})} \left(\frac{\Lambda^h}{\Lambda^c}(c_{t+j}, h_{t+j}) \right) \tag{9.2}$$

where $c_t = C_t/\Theta_t$, and Λ^h denotes the marginal disutility of work. A log-linearization of (9.2) gives therefore

$$\widehat{v}_{t+j,t} = \eta_c(\widehat{c}_{t+j,t} - \widehat{c}_{t+j}) + \eta_h \left(\widehat{h}_{t+j,t} - \widehat{h}_{t+j} \right) + \widehat{v}_{t+j} \tag{9.3}$$

where η_x $(x = c, h)$ indicates the elasticity of the marginal rate of substitution between leisure and consumption with respect to x, evaluated at the steady state. By the assumption that changes in consumption occur in a way that maintains the marginal utility of consumption equal across households, $\widehat{c}_{t+j,t}$ and $\widehat{c}_{t+j}$ are respectively function of $\widehat{h}_{t+j,t}$ and $\widehat{h}_{t+j}$. Moreover, from (2.17) we have that

$$\widehat{h}_{t+j,t} - \widehat{h}_{t+j} = -\theta_w \left(\widehat{x}_{wt} + \varrho_w \Sigma_{k=0}^{j-1} \widehat{\pi}_{t+k} - \Sigma_{k=1}^{j} \widehat{\pi}_{t+k}^w \right)$$

Substituting this result in (9.3), we get

$$\widehat{v}_{t+j,t} = -\chi \, \theta_w \left(\widehat{x}_{wt} + \varrho_w \Sigma_{k=0}^{j-1} \widehat{\pi}_{t+k} - \Sigma_{k=1}^{j} \widehat{\pi}_{t+k}^w \right) + \widehat{v}_{t+j} \tag{9.4}$$

where I defined $\chi = \frac{-\Lambda_h^c}{\Lambda_c^c}\eta_c + \eta_h$, and Λ_i^c indicates the derivative of the marginal utility of consumption with respect to argument i.

In (2.15), dividing both sides by W_t, and log-linearizing, one obtains

$$\widehat{x}_{wt} = \frac{\alpha_w}{1 - \alpha_w} \left(\widehat{\pi}_t^w - \varrho_w \widehat{\pi}_{t-1} \right) \tag{9.5}$$

Substituting (9.5) and (9.4) into (9.1) one obtains that

$$\left(\widehat{\pi}_t^w - \varrho_w \widehat{\pi}_{t-1} \right) = \gamma \, \Sigma_{j=0}^{\infty} (\beta\alpha_w)^j \, E_t \left(\widehat{v}_{t+j} - \widehat{\omega}_{t+j} + (1 + \chi \, \theta_w) \left(\Sigma_{k=1}^{j} \widehat{\pi}_{t+k}^w - \varrho_w \Sigma_{k=0}^{j-1} \widehat{\pi}_{t+k} \right) \right) \tag{9.6}$$

where $\gamma = \frac{(1-\alpha_w)(1-\beta\alpha_w)}{\alpha_w(1+\theta_w\chi)}$.

Finally, forwarding (9.6) one period, premultiplying it by $\beta\alpha_w$, and subtracting the resulting expression from (9.6), one obtains the wage equation (2.19) in the text.

9.2. Empirical implementation

To compute the solution, I cast the model in the following canonical form

$$Y_{t+1} = MY_t + \Psi u_{t+1} + \Pi \eta_{yt+1} \tag{9.7}$$

where $\eta_{y,t+1} = y_{t+1} - E_t y_{t+1}$ are expectational errors.

The definitions of the vector Y_t and of the matrix M are as in the text, and the matrices Ψ and Π are :

$$\Psi = \begin{bmatrix} N_1 & 0 \\ \underline{0} & 0 \\ \underline{0} & Q \end{bmatrix}, \text{ and } \Pi = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \underline{0} & \underline{0} \end{bmatrix}$$

Furthermore,

$$M_{yy} = \begin{bmatrix} \frac{1+\varrho_p\beta}{\beta} & -\frac{\zeta}{\beta} & -\frac{\varrho_p}{\beta} & 0 \\ \varrho_w - \varrho_p & \frac{1+\beta+\gamma+\zeta}{\beta} & \frac{\varrho_p-\varrho_w}{\beta} & -\frac{1}{\beta} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad M_{yZ} = \begin{bmatrix} \underline{0} \\ -\left(\frac{\gamma}{\beta}\Xi - \frac{1}{\beta}e_q' + e_q' A\right) \\ \underline{0} \\ \underline{0} \end{bmatrix}$$

As indicated in the text, the vector Ξ depends on the chosen specification of preferences and on the assumptions about the trend removal.

Since $v_t = v_t^T + v_t^{cyc} = q_t^T + v_t^{cyc}$, from the definition of the trend in productivity (5.3), we have that

$$v_t = q_t + e_q'[I-A]^{-1}A + \eta_c c_t^{cyc} + \eta_h h_t^{cyc}$$

the vector Ξ is therefore defined as

$$\begin{aligned} \Xi &= e_q'[I-A]^{-1}A + \eta_c\left(e_{cy}' + e_h' - e_q'[I-A]^{-1}A\right) + \eta_h e_h' \\ &= (1-\eta_c)\, e_q'[I-A]^{-1}A + \left[\eta_c\left(e_{cy}' + e_h'\right) + \eta_h e_h'\right]. \end{aligned}$$

The parameters of interest in this expression are the elasticities η_c and η_h, which are estimated together with the adjustment parameters of the wage and price equations, .

9.3. Inference on wage rigidity

To translate the estimate of the 'inertia' parameter γ into an estimate of the degree of wage rigidity, I need to parametrize χ, which is

$$\chi = \frac{-\Lambda_h^c H}{\Lambda_c^c C}\eta_c + \eta_h \tag{9.8}$$

I first consider a slight transformation of this expression[36] (9.8)

$$\chi = \frac{-\Lambda_h^c \Lambda^c}{\Lambda_c^c \Lambda^h}\left(\frac{\Lambda^h H}{\Lambda^c C}\right)\eta_c + \eta_h \tag{9.9}$$

[36] A more detailed discussion of this parametrization is in Sbordone (2001)

and then write the expression for η_c as

$$\eta_c = -\frac{\Lambda_c^c C}{\Lambda^c} + \frac{\Lambda_h^c C}{\Lambda^h} = \sigma + \frac{\Lambda_h^c C}{\Lambda^h} = \sigma + \frac{\Lambda_h^c}{\Lambda_c^c}\left(\frac{\Lambda_c^c C}{\Lambda^c}\right)\frac{\Lambda^c}{\Lambda^h} = \sigma\left(1 - \frac{\Lambda_h^c}{\Lambda_c^c}\frac{\Lambda^c}{\Lambda^h}\right) \tag{9.10}$$

where, with conventional notation, I indicate with σ the inverse of the intertemporal elasticity of substitution in consumption. Expression (9.10) implies that

$$\frac{\Lambda_h^c}{\Lambda_c^c}\frac{\Lambda^c}{\Lambda^h} = \frac{\sigma - \eta_c}{\sigma}$$

Substituting this result in (9.9), I obtain

$$\chi = \left(\frac{\sigma - \eta_c}{\sigma} * \tau\right)\eta_c + \eta_h$$

Therefore, given the estimated η_c and η_h, one can determine the value of χ for any value that one wishes to assign to σ, and to the ratio wH/C, which I have denoted by τ. The computations in table 2 are based on three different assumptions about the value of the intertemporal elasticity of substitution in consumption (corresponding to $\sigma = 4$, 5, or 10), and the value of $\tau = 1$. Every value of σ implies in turn a different degree of non-separability in preferences.

References

[1] Altig, D., L.J. Christiano, M. Eichenbaum, and J. Linde' (2002) "Technology Shocks and Aggregate Fluctuations", unpublished.

[2] Amato, J.D. and T. Laubach (2003) "Estimation and control of an optimization-based model with sticky prices and wages", *Journal of Economic Dynamic and Control* 27: 1181-1215.

[3] Basu, S. and M.S. Kimball (2000) "Long-Run Labor Supply and the Elasticity of Intertemporal Substitution for Consumption" unpublished, University of Michigan.

[4] Batini, N., Jackson, B., Nickell, S., (2002) "An Open Economy New Keynesian Phillips Curve for the U.K.", unpublished.

[5] Beveridge, S. and C. R. Nelson (1981) "A New Approach to Decomposition of Economic Time Series into Permanent and Transitory Components with Particular Attention to Measurement of the 'Business Cycle"', *Journal of Monetary Economics.* 7:. *151-174.*

[6] Boivin, J. and M. Giannoni (2003) "Has Monetary Policy Become More Effective?", NBER working paper n.9459.

[7] Calvo, G. A. (1983) "Staggered Prices in a Utility -Maximizing Framework", *Journal of Monetary Economics* 12: 983-998.

[8] Campbell, J.Y. and R.J. Shiller (1987) "Cointegration and Tests of Present Value Model", *Journal of Political Economy* 95: 1062-1088.

[9] Christiano,L.,M. Eichenbaum and C. Evans (2001) "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy", unpublished.

[10] Cogley, T. and T. J. Sargent (2001) "Evolving Post-World War II U.S. Inflation Dynamics" *NBER Macroeconomics Annual* , vol. 16

[11] ———- (2003) "Drifts and Volatilities: Monetary Policies and Outcomes in the Post WWII U.S., unpublished.

[12] Cogley, T. and A.M. Sbordone "A Search for a Structural Phillips Curve", in progress.

[13] Edge,R., T. Laubach, and J. Williams (2003), "Productivity Slowdowns and Speedups: A Dynamic General Equilibrium Approach", unpublished, Board of Governors.

[14] Erceg, C. J., D. W. Henderson, and A. T. Levin (2000) "Optimal Monetary Policy with Staggered Wage and Price Contracts" *Journal of Monetary Economics*, 46: 281-313.

[15] Gagnon, E. and H. Khan (2003) "New Phillips curve under alternative production technologies for Canada, the United States and the Euro area", unpublished.

[16] Gali, J., and M. Gertler (1999), "Inflation Dynamics: A Structural Econometric Analysis", *Journal of Monetary Economics* 44: 195-222.

[17] Gali, J., Gertler, M., Lopez-Salido, J.D. (2001) "European Inflation Dynamics". *European Economic Review* 45: 1237-1270.

[18] Gali, J., Gertler, M., Lopez-Salido, J.D. (2003) "Robustness of the Estimates of the Hybrid New Keynesian Phillips Curve", unpublished.

[19] Ireland, P. (1997) "A small, structural, quarterly model for monetary policy evaluation.", *Carnegie-Rochester Conference Series on Public Policy* 47: 83-108.

[20] Hall, R. (1988) "The Relation Between Price and Marginal Cost in U.S. Industry", Journal of Political Economy 96: 921-947.

[21] King, R. J., and S. T. Rebelo (1999), "Resuscitating Real Business Cycles", in J. B. Taylor and M. Woodford, eds., *Handbook of Macroeconomics,* vol. 1B, Elsevier.

[22] Kurmann, A. (2002). "Quantifying the Uncertainty about a Forward-Looking New Keynesian Pricing Model", unpublished.

[23] Li, H. (2003) "Testing Alternative Theories of Aggregate Supply", unpublished, Princeton University.

[24] Linde' J. (2002) "Estimating New-Keynesian Phillips Curves: A Full Information Maximum Likelihood Approach", Sveriges Riksbank Working Paper Series No. 129.

[25] Rotemberg, J. and M. Woodford (1997) "An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy", *NBER Macroeconomics Annual*, vol. 12: 297-346.

[26] Rotemberg, J. and M. Woodford (1999) "The Cyclical Behavior of Prices and Costs", in J.B. Taylor and M. Woodford, eds., *Handbook of Macroeconomics*, vol. 1B, Elsevier.

[27] Rudd, J. and K. Wheelan (2002) "New Tests of the New-Keynesian Phillips Curve", unpublished, Board of Governors.

[28] Sbordone, A.M. (2001) "An optimizing model of U.S. wage and price dynamics", Rutgers University, working paper series 2001-10.

[29] Sbordone, A.M. (2002) "Prices and unit labor costs: a new test of price stickiness", *Journal of Monetary Economics* 49: 265-292.

[30] Sbordone, A.M. (2003) "Inflation dynamics and real marginal costs", unpublished, Rutgers University.

[31] Smets, F. and R. Wouters (2002a) " An Estimated Stochastic Dynamic General Equilibrium Model of the Euro Area" ECB working paper series, n.171

[32] Smets, F. and R. Wouters (2002b) "Sources of Business Cycle Fluctuations in the U.S.: A Bayesian DSGE Approach", unpublished.

[33] Sims, C.A (2001) "Comments on papers by Jordi Gali and by Stefania Albanesi, V.V. Chari and Lawrence J. Christiano", unpublished, Princeton University.

[34] Woodford, M. (2003), *Interest and Prices: Foundations of a Theory of Monetary Policy*, forthcoming , Princeton University Press.

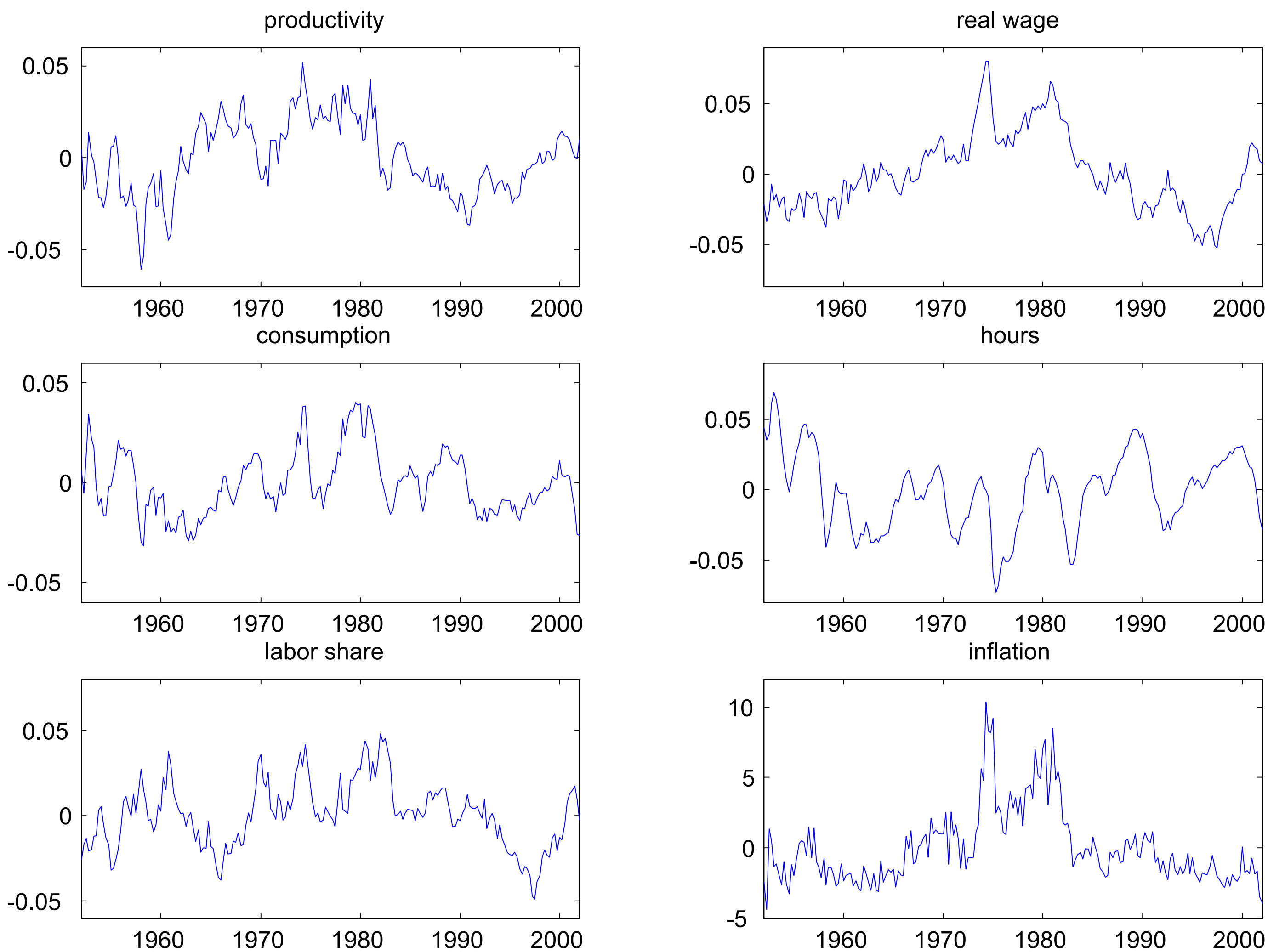

Figure 9.1: Cyclical Components

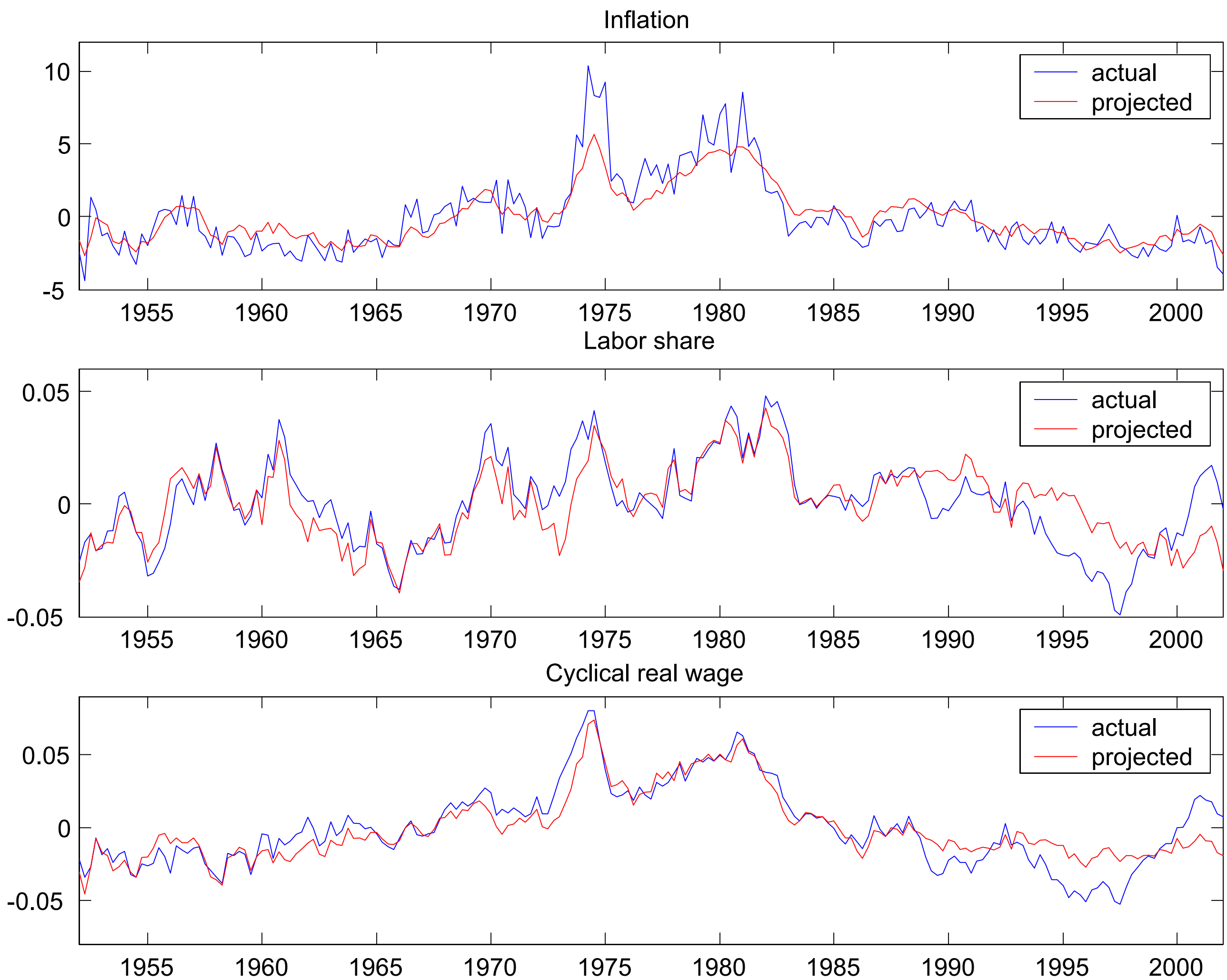

Figure 9.2: Inflation, Labor share, and Cyclical real wage

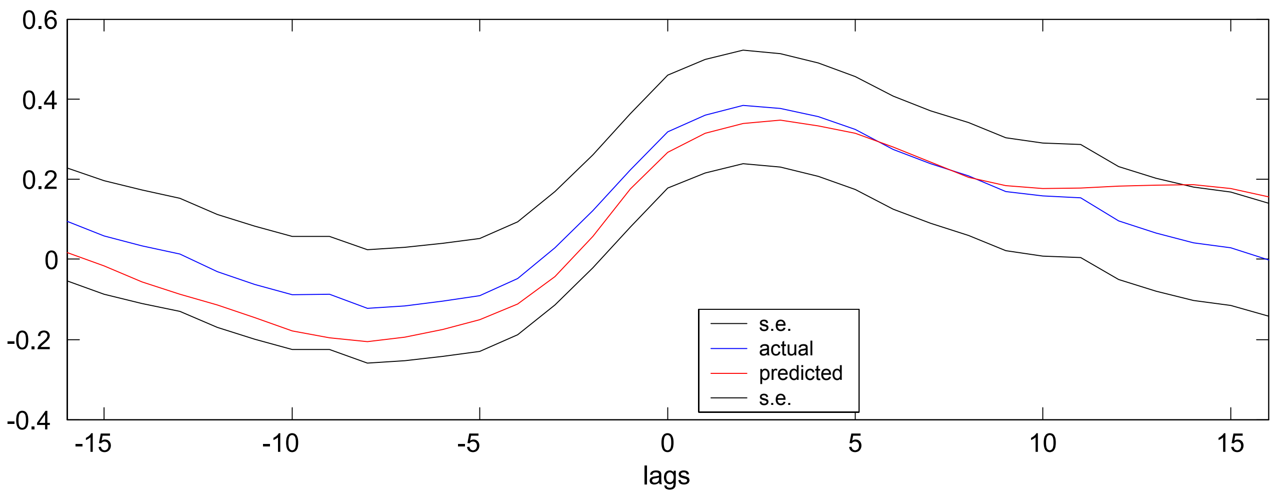

Figure 9.3: Dynamic crosscorrelations: output(t)-inflation(t+k)