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Fragility of Safe Asset Markets

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Abstract

In March 2020, safe asset markets experienced surprising and unprecedented price crashes. We explain how strategic investor behavior can create such market fragility in a model with investors valuing safety, investors valuing liquidity, and constrained dealers. While safety investors and liquidity investors can interact symbiotically with offsetting trades in times of stress, liquidity investors' strategic interaction harbors the potential for self-fulfilling fragility. When the market is fragile, standard flight-to-safety can have a destabilizing effect and trigger a "dash-for-cash" by liquidity investors. Well-designed policy interventions can reduce market fragility ex ante and restore orderly functioning ex post.

Keywords: safe assets, liquidity shocks, global games, treasury securities, COVID-19

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1 Introduction

In March 2020, U.S. Treasury securities experienced sudden and significant price drops as investors flooded the market with sales in a dash-for-cash unprecedented and counter to historical behavior. Safe asset markets are typically resilient, especially in times of stress when flight-to-safety demand historically more than offsets supply from investors with liquidity needs, and have not historically featured investors preemptively selling out of fear. The March 2020 dash-for-cash was a novel phenomenon and requires a new model that explains how safe asset markets markets can typically be stable and resilient with the well established flight-to-safety but nevertheless harbor the fragility of a dash-for-cash.

We show in a model of regime shifts that a safe asset market functions as expected as long as the market is sufficiently deep. However, under certain conditions the market can break down, with investors rushing to sell and prices falling precipitously, if trade imbalances have to be absorbed by dealers that are subject to balance sheet constraints. Surprisingly, we find that an increase in the demand for safe assets from a standard flight-to-safety can be destabilizing: When the market is relatively fragile, the flight-to-safety among certain investors can trigger the dash-for-cash among other investors. Our model helps understand the unprecedented events in March 2020 and highlights the risks of such events repeating in the future in safe asset markets more broadly.

Our analysis is motivated by the following facts. First, the prices of Treasuries suddenly collapsed in mid-March 2020, in sharp contrast to previous crisis episodes (Panel A of Figure 1). Until the beginning of March, Treasury prices did increase and the S&P 500 decreased with the gradual realization of the severity of the COVID-19 outbreak, consistent with the usual negative correlation between safe and risky assets during a flight-to-safety episode (Nagel, 2016; Adrian, Crump, and Vogt, 2019). However, starting the week of March 9, prices of Treasury notes and bonds rapidly declined together with stock prices as investors moved into cash or ultra-short maturity Treasury bills (i.e. the equivalent of cash). It is well-documented, that dealer balance sheet constraints played an important role in the Treasury price declines (He, Nagel, and Song, 2022; Duffie et al., 2023). Panel B of Figure 1 illustrates how dealer balance sheets were filling up with Treasuries through both the run-up in Treasury prices and their crash, and how the recovery of Treasury prices after March 18 coincided with the receding of dealer balance sheet pressure as emergency purchases by the Federal Reserve ramped up.

However, the existing literature takes as given the Treasury sales in March 2020 and does not address why this episode featured sales so large that they reversed the typical appreciation of safe assets during times of stress and required Fed intervention to "support

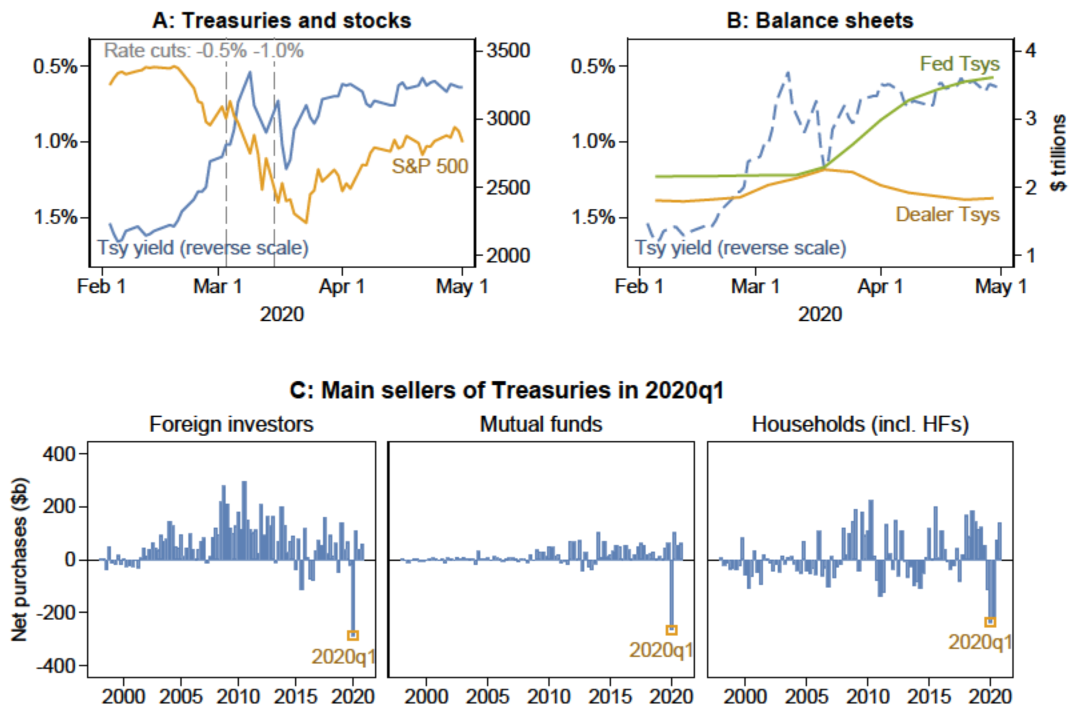

Figure 1: The market for U.S. Treasuries in early 2020. Panel A: market yield on 10-year Treasuries, Federal Reserve rate cuts, and S&P 500 index. Panel B: Treasuries on the balance sheets of the Federal Reserve (outright holdings) and of Primary Dealers (net positions and gross reverse repo). Panel C: net purchases of Treasuries. For details on the data see Appendix A.

the smooth functioning of markets" (FOMC statement on March 15).[1] Panel C of Figure 1 illustrates the historically unprecedented scale of Treasury sales during the dash-for-cash: Foreign investors and mutual funds, the main sellers of Treasuries in 2020q1, each sold roughly $250 billion, an order of magnitude more than in any previous quarter. In the post-2008 period these sales equal three standard deviations for foreign investors and five standard deviations for mutual funds.[2] Sales of such magnitude appear inconsistent with a smooth response to continuously varying underlying fundamentals, suggesting the need for a model of regime shifts.

Crucially, a significant part of these sales appear to have been preemptive, i.e. not due to genuine, immediate liquidity needs. Foreign official agencies (a subset of foreign investors) sold $196 billion of Treasury bonds but reduced their total U.S. Dollar assets by only $48 billion, suggesting that 76% of their sales were preemptive.[3] Among mutual funds, those in the CRSP dataset sold $157 billion of Treasuries but used only $103 billion to satisfy outflows, suggesting that 34% of their sales were preemptive.[4] Consistent with this evidence, the Inter-Agency Working Group for Treasury Market Surveillance (2021) reports that "some Treasury holders appeared to react to the decline in market liquidity by selling securities for precautionary reasons lest conditions worsen further, and these sales only added to the stress on the market."

In sum, investors sold safe assets on an unprecedented scale for what appear to be preemptive reasons. In the language of Haddad, Moreira, and Muir (2021), "selling became viral," with investors selling safe assets — whether Treasuries or investment grade corporate bonds — akin to depositors running on a bank, leading to more severe dislocations for safer and more liquid assets in a reversal of the usual liquidity hierarchy.

In contrast to the existing literature showing how dealer constraints can lead to price dislocations given exogenous sales from investors (e.g. He, Nagel, and Song, 2022), we show how dealer constraints can endogenously induce certain investors to sell, and especially so when there is concurrent flight-to-safety demand from other investors. To do so, we build on and generalize the model of Bernardo and Welch (2004) which is seminal for showing that runs can occur in financial markets but is unable to describe the nuances of

[1]The statement by the Fed's Federal Open Market Committee (FOMC) on March 15 is available at `https://www.federalreserve.gov/newsevents/pressreleases/monetary20200315a.htm`

[2]While hedge funds unwinding the cash-futures basis trade were also net sellers of Treasuries (Barth and Kahn, 2021), panel C of Figure 1 shows that their sales (included in the "household" sector in the Financial Accounts) were more in line with historical experience.

[3]From the international transactions data of the Bureau of Economic Analysis, Table 9.1, International Financial Transactions for Liabilities to Foreign Official Agencies. Reported similarly in Panel B of Table 9 in Vissing-Jørgensen (2021). See Weiss (2022) for additional detail on foreign sales of Treasuries in 2020.

[4]From Table 6 in Vissing-Jørgensen (2021).

runs in safe asset markets.[5] Our model can generate strategic complementarities leading to regime shifts in which continuous changes in fundamentals trigger discontinuous jumps in investor behavior and, therefore, a sudden precipitous drop in equilibrium prices. Using global game techniques, we uniquely link the market outcome to fundamentals including the degree of liquidity risk, the strength of flight-to-safety demand, and the severity of dealer constraints. In "good times," safe asset markets are resilient, with no investors selling for preemptive reasons, but in "bad times," investors that are usually happy to buy and hold safe assets can flood the market with sales for unforced reasons.

Our model captures the two key characteristics of safe assets — safety and liquidity — as well as the central role of constrained dealers to intermediate trade and absorb imbalances. First, safe assets in practice are *safe* in the sense that they will pay par at maturity with very high probability so investors hold them as a store of value, useful for diversification and intertemporal smoothing (e.g. Caballero and Farhi, 2017). In our model, such "safety investors" hold the safe asset in a portfolio together with a risky asset. In times of stress, when fundamentals worsen for the risky asset, these investors rebalance their portfolio to demand more of the safe asset (equivalently, markets reprice the value of safe assets to reflect fundamentals, even in the absence of large trade volume). Such flight-to-safety has been the focus of most existing analyses of safe assets in times of stress.

Second, safe assets in practice are *liquid*, meaning that, typically, they can be easily sold when in need of cash and therefore trade at a convenience yield (e.g. Krishnamurthy and Vissing-Jørgensen, 2012). In our model, there are "liquidity investors" who are subject to liquidity shocks (i.e. immediate consumption needs) and therefore hold the safe asset as liquidity insurance. When hit by the liquidity shock, these investors sell the safe asset in order to consume. Importantly, even in times of stress, not all liquidity investors suffer liquidity shocks. This leaves a group of liquidity investors without genuine liquidity needs who act strategically when deciding whether to sell their assets in the current environment, or whether to hold on and face the risk of a liquidity shock in the near future. An individual investor may sell preemptively if they expect worse market conditions in the future and if their likelihood of having to sell in the future is sufficiently high.

In addition, our model features dealers who buy and sell the safe asset and whose main role is to intermediate over time. Dealers are competitive but subject to balance sheet constraints such as the Supplemental Leverage Ratio rule (SLR), and therefore provide an elastic residual demand for the safe asset. Because dealers' demand in the future is affected by inventory they take on today, they generate an intertemporal link between prices in

[5] We note our generalizations of Bernardo and Welch (2004) and their implications throughout the text and discuss the differences in detail in Appendix C.

different periods. In particular, if dealers absorb a large net supply today then their bids will necessarily be lower in case of additional net supply in the future.

With these ingredients, our model yields two main results. The first result is that the liquidity insurance role of safe assets, together with dealer balance sheet constraints, implies that a safe asset market can be *fragile*, featuring sudden regime changes. For low liquidity risk (low probability of facing a liquidity shock), a strategic liquidity investor never finds it optimal to sell preemptively, irrespective of what other investors are doing; the only equilibrium in this case is for all strategic investors to hold on to the safe asset such that the only investors selling are those with a genuine liquidity need. This regime captures the resiliency that economists typically associate with safe asset markets. For high liquidity risk, the opposite is true: an individual investor finds it dominant to sell preemptively such that the only equilibrium is for all liquidity investors to sell. In this case, the safe asset market is flooded with sales, including by investors who do not actually have liquidity needs — a "market run." The stability of the market is represented by the global game threshold for liquidity risk around which the equilibrium switches from "hold" to "run" with a higher threshold representing a more stable or, equivalently, less fragile market. The severity of dealer balance sheet costs has two effects on the market. First, higher balance sheet costs increase market fragility such that a market run already occurs for lower liquidity risk. Second, higher balance sheet costs increase the magnitude of the price crash conditional on the run occurring.

Our second and key result is that the safety and liquidity roles of safe assets can interact in such a way that a flight-to-safety can *worsen* fragility, making a dash-for-cash more likely. In typical models of fire sales, the crucial friction is slow-moving capital (Duffie, 2010): prices can be depressed because potential buyers cannot enter the market to purchase distressed assets. Thus, in these situations, new buyers entering the market would mitigate fire sales and stabilize asset prices. In contrast, we find that the entry of new capital to purchase safe assets can actually amplify fire sales in a fragile safe asset market.

How can this occur? In principle, safety investors form a natural partnership with liquidity investors as their trades offset during stress episodes. Demand from safety investors absorbs sales from liquidity investors; all else equal, this leads to higher prices for safe assets than would otherwise occur. However, the timing of safety investor demand is key, as it affects the intertemporal tradeoff of strategic liquidity investors. Safety investor demand early on in a stress episode has an ambiguous effect on fragility as it increases prices both contemporaneously (which is destabilizing) and in the future by relaxing dealer balance sheets (which is stabilizing). Additional demand from safety investors today can *induce* liquidity investors to sell today, precisely because the market today has relatively high capacity to absorb sales; but higher prices in the future imply that being forced to sell in the future is less costly, and this is stabilizing.

Which effect dominates — and therefore whether a flight-to-safety prevents or triggers a dash-for-cash — is ambiguous and depends on the inherent fragility of the market. In a relatively stable market, strategic liquidity investors will sell preemptively only if liquidity risk is very high (i.e. only if they are likely to be forced to sell in the future). In that case, the stabilizing effect of the flight-to-safety dominates: the investors weight more their concern about being forced to sell in the future but the flight-to-safety has relaxed dealer balance sheets and increased the price the investors would face in the future. In a relatively fragile market, however, strategic liquidity investors sell preemptively even when liquidity risk is low (i.e. even when they are unlikely to be forced to sell in the future). In this case, the destabilizing effect of the flight-to-safety dominates: the investors put a greater weight on the ability to sell assets at a higher price today even though the fire-sale price tomorrow is also less severe. This means that safety investors have an *amplification effect* on market fragility: When the market is already relatively stable, they stabilize it further (flight-to-safety prevents a dash-for-cash); but if the market is already relatively fragile, they destabilize it even more (flight-to-safety triggers a dash-for-cash). Whether flight-to-safety will trigger a dash-for-cash is unclear unconditionally, but the answer is clear conditional on the degree of market fragility.

It is useful to consider other historical episodes to flesh out two key differences that distinguish the March 2020 events. First, there are standard flight-to-safety episodes, like during the great financial crisis of 2007–2009 (GFC). In these episodes, safe asset prices rally, as we would expect from standard models. Second, there are episodes in which safe asset prices sell off sharply, like during the so-called Taper Tantrum for U.S. Treasuries in 2013 and the LDI crisis for UK gilts in 2022. But in these cases, the fundamentals justified a drop in government bond prices: in the case of the Taper Tantrum, Fed purchases of Treasuries were expected to decline, and in the UK case, the government's fiscal plan pushed up expected long-term rates. In contrast to March 2020, the price declines in these episodes did not occur at a time when one would expect a flight-to-safety. Instead, the precipitous sales reflected more standard amplification mechanisms, such as forced sales by leveraged investors.

Our model helps to understand the differences between standard flight-to-safety episodes like the GFC and the events of March 2020 to understand why the two episodes led to such dramatically different outcomes. First, our model highlights the central role of dealer balance sheet constraints, which are a potentially unintended result of post-GFC regulations, such as the SLR. During the GFC, dealers' activities in Treasury markets were relatively unconstrained and thus investors did not worry about dealers running out of balance sheet space and Treasury prices collapsing. Second, the size of the liquidity shock during the COVID-19 crisis appears to have been much larger than during the GFC. As our analysis shows, very large increases in liquidity risk and flight-to-safety can tilt the system into a region in which investors sell preemptively. Because the GFC did not feature dealers constrained by balance sheet costs, and because the shock to liquidity needs was arguably smaller, the Treasury market remained in the relatively stable region in which flight-to-safety prevents a dash-for-cash, which is why the market behaved as usual despite the tremendous stress in the financial sector. In contrast, in March 2020, the liquidity shock was larger and dealers were more constrained, so much so that the Treasury market suffered a regime change, and flight-to-safety triggered a dash-for-cash. In sum, our analysis suggests that these two episodes did not feature fundamentally different shocks or shocks of different direction, but rather shocks that differed in degree within different regulatory environments.

Our analysis allows us to consider the virtues and costs of various policy interventions, in particular asset purchase facilities and dealer balance sheet regulation. Fragility in our model hinges on the intertemporal considerations of strategic liquidity investors who compare prices today to prices in the near future. In general, there is scope for policy interventions that increase prices both in the present and in the future. However, due to the intertemporal considerations and the coordination effects, the timing of policy interventions is important and announcements can have large effects well before the interventions are executed. We show that an asset purchase facility can have a large effect upon announcement, even if it does not become active until a future date, by shifting strategic investors from the run equilibrium to the hold equilibrium, consistent with the evidence of Haddad, Moreira, and Muir (2021). Similarly, policy interventions that relax dealer balance sheet constraints can be stabilizing. However, because the strategic incentive to sell is caused by fear of low prices in the future, effective policy has to relax balance sheet constraints in the future as well.

The growth of the Treasury market since the GFC has greatly outpaced the capacity of dealers' balance sheets, and that trend is expected to continue (Duffie, 2020). The strategic mechanism in our model will therefore become increasingly relevant unless balance sheet constraints are relaxed. Episodes like March 2020 are likely to become more frequent as dash-for-cash motivations become more pronounced.

After discussing related literature, the rest of the paper proceeds as follows. In Section 2, we present and analyze the baseline model of the strategic interaction among liquidity investors. In Section 3, we consider the behavior of constrained dealers and derive equilibrium prices. In Section 4, we analyze different mechanisms that can generate strategic complementarities among liquidity investors, including the elasticity of dealers' marginal balance sheet costs, investor risk aversion and decentralized trade execution. In Section 5, we use global game techniques to determine the unique equilibrium, and we study how market fragility depends on balance sheet costs, and how a constrained social planner can improve welfare by discouraging preemptive sales. In Section 6, we add safety investors to the model and derive the ambiguous effect of flight-to-safety on market fragility. In Section 7, we discuss the March 2020 policy interventions in light of the theoretical results of our model, and we conclude in Section 8. All proofs are in Appendix B.

Related Literature. The market turmoil in the spring of 2020 has been documented in detail by Vissing-Jørgensen (2021) and He, Nagel, and Song (2022) for Treasuries, and by Haddad, Moreira, and Muir (2021) and Boyarchenko, Kovner, and Shachar (2022) for corporate bonds.[6] In particular, Duffie et al. (2023) show empirically that the typically linear relation between yield volatility and Treasury market liquidity broke down in March 2020 and that the residuals are well explained by the shadow cost of dealer balance sheets.

In a literature that focuses on empirically studying the events, He, Nagel, and Song (2022) stand out as also providing a formal theoretical analysis to understand the implications. Using a model based on Greenwood and Vayanos (2014) but incorporating frictions between dealers and hedge funds, they illustrate how large net sales can generate an "inconvenience yield" for Treasuries. Specifically, He, Nagel, and Song (2022) show that, given large exogenous sales, the presence of regulatory constraints can lead to pricing distortions measured as the spread between Treasuries and overnight-index swap rates, as well as spreads between dealers' reverse repo and repo rates. Importantly, He, Nagel, and Song (2022) take net flows as given and consider in detail the equilibrium pricing consequences. In contrast, our paper shows how, in a strategic environment, the same regulatory constraints can lead to run behavior, thus endogenizing the large net flows. Our focus is on the determinants of large net sales of safe assets during a crisis — the unusual behavior not typically observed — and on the policy implications that can be derived in such a model of regime change.

In contrast to market runs, bank runs have received much greater attention because of the common pool problem inherent with liquidity transformation (e.g., Diamond and

[6] See also D'Amico, Kurakula, and Lee (2020), Fleming et al. (2021), Nozawa and Qiu (2021), Aramonte, Schrimpf, and Shin (2022), and Haughwout, Hyman, and Shachar (2022). For detailed analysis of market liquidity conditions, see Fleming and Ruela (2020), Kargar et al. (2021), O'Hara and Zhou (2021). Ahmed and Rebucci (2022) find sizable estimates of the price impact of foreign officials' sales of Treasuries. The role of mutual funds in particular as large sellers of safe assets has been studied by Falato, Goldstein, and Hortaçsu (2021) and Ma, Xiao, and Zeng (2022). On the role of hedge funds, see e.g. Barth and Kahn (2021).

Dybvig, 1983 and Goldstein and Pauzner, 2005). In the case of a market run, there is no common pool threatened by illiquidity. The seminal papers on market runs by Bernardo and Welch (2004) and Morris and Shin (2004) highlight how market frictions can create incentives to front-run other investors by selling assets preemptively. Bernardo and Welch (2004) introduce the intertemporal tradeoff our model relies on, but their model does not feature strategic complementarities and therefore cannot generate regime shifts. Our model with strategic complementarities can generate regime shifts and allows for continuous comparative statics in the analysis of flight-to-safety demand and policy implications. Morris and Shin (2004) consider a static model in which strategic complementarities arise because investors have "stop-loss rules" and will be forced to liquidate if prices fall sufficiently low. The preponderance of sales in March 2020 were from investors subject to liquidity shocks, suggesting that a stop-loss mechanism did not drive preemptive sales during this episode.

There are a variety of other mechanisms in the finance literature generating strategic complementarities. In a non-market setting, papers studying strategic complementarities consider intra-temporal coordination of depositors (Goldstein and Pauzner, 2005), or inter-temporal coordination of creditors with staggered maturity (He and Xiong, 2012) or of mutual fund investors (Zeng, 2017). In a market setting, existing papers generate strategic complementarities with limited resources in defense of a currency peg (Morris and Shin, 1998) or with forced deleveraging due to loss limits (Morris and Shin, 2004). These mechanisms, however, do not apply to the March 2020 event, in which sales were primarily from unleveraged investors.

The literature on safe assets is large; see e.g. Gorton (2017) for an overview. Krishnamurthy and Vissing-Jørgensen (2012) show that Treasuries are valued both for their safety and their liquidity by documenting yield spreads both with respect to assets similarly liquid but not safe and assets similarly safe but not liquid (see also Duffee, 1998, Longstaff, 2004, and Greenwood and Vayanos, 2010, 2014). Our paper focuses on the correlation in times of crisis or market turmoil in which the typical correlation (flight-to-safety) has been otherwise clear (Nagel, 2016; Adrian et al., 2019).[7]

[7] Caballero and Farhi (2017) consider a model where the "specialness" of public debt is its safety during bad aggregate states and where safe assets have "negative beta," as they tend to appreciate in times of aggregate market downturns, providing investors diversification against aggregate macroeconomic risks (see also Maggiori, 2017, Adrian, Crump, and Vogt, 2019, and Brunnermeier, Merkel, and Sannikov, 2022). Acharya and Laarits (2023) show that the convenience yield on Treasuries is high exactly when they provide a good hedge. Safe assets valued for their safety appear in a model of limited participation and risk sharing in Gomes and Michaelides (2007) and through special investors who need safe assets to match liability cash flows in Greenwood and Vayanos (2010). More generally, Treasury bonds have had negative beta over longer horizons in recent decades, rising in price when stock prices fall apart from market turmoil (Baele et al., 2019; Campbell, Sunderam, and Viceira, 2017; Cieslak and Vissing-Jørgensen, 2020).

Safe assets' liquidity is intimately linked to their safety: when payoffs are (nearly) risk free, assets are information-insensitive and thus easily traded "no questions asked" (Gorton and Pennacchi, 1990, Holmström, 2015, Dang, Gorton, and Holmström, 2015). Holmström and Tirole (1998) model the use of safe assets as a store of value and as insurance against liquidity shocks. Safe assets valued for their liquidity appear in Vayanos and Vila (1999) and Rocheteau (2011) as well as in the monetarist literature surveyed by Lagos, Rocheteau, and Wright (2017). The premium for moneyness has been studied empirically, e.g. by Greenwood, Hanson, and Stein (2015), Carlson et al. (2016), and Cipriani and La Spada (2021) (see also Nagel, 2016, and d'Avernas and Vandeweyer, 2021).[8]

The role of dealers and slow-moving capital more generally in short-term price dislocations is introduced, e.g. in Duffie (2010). Fontaine and Garcia (2012) and Hu, Pan, and Wang (2013) show the effects on liquidity in Treasury markets (see also Vayanos and Vila, 2021). Adrian, Boyarchenko, and Shachar (2017) specifically consider the effects of dealer balance sheet constraints on bond market liquidity. Goldberg and Nozawa (2021) show that dealer inventory capacity is a key driver of liquidity in corporate bond markets (see also Bruche and Kuong, 2021).

2 Liquidity Investors and their Strategic Interaction

The model is set in two periods $t = 0, 1$ and has three types of agents and two types of assets, a safe asset and a risky asset. The safe asset, which is the focus of the analysis, has a fundamental value of 1 and is traded among the agents in both periods. Among the agents, there are investors who hold portfolios of the safe asset and the risky asset ("safety investors"), investors who hold the safe asset as protection against liquidity shocks ("liquidity investors"), and dealers who participate in the safe asset market and are subject to balance sheet costs. Safety investors are risk averse and dealers are risk neutral; for expositional clarity, we assume that liquidity investors are risk neutral but we show that assuming them to be risk averse would strengthen our results. All agents have a discount rate of zero and act competitively, and there is a measure one of each type. All asset prices are determined in equilibrium. We defer discussion of the safety investors until Section 6.

[8]Gorton and Ordoñez (2022) study the interaction of public and private provision of safe assets used as store of value and as collateral (see also Holmström and Tirole, 2011, Stein, 2012, Gorton, Lewellen, and Metrick, 2012, Sunderam, 2014 and Krishnamurthy and Vissing-Jørgensen, 2015). Caballero and Krishnamurthy (2008) study flight to quality episodes triggered by uncertainty shocks. He, Krishnamurthy, and Milbradt (2019) study the roles of strategic complementarities and substitutes among investors in determining which asset becomes the safe asset via coordination (see also Farhi and Maggiori, 2017). For recent empirical analysis of safe assets, both current and historical, see Chen et al. (2022) and Choi, Kirpalani, and Perez (2022).

Our model generalizes the model of Bernardo and Welch (2004) along several dimensions in order to appropriately capture safe asset markets; we point out the generalizations and their implications in the following and discuss the differences in detail in Appendix C.

Liquidity investors start out holding one unit of the safe asset and are subject to i.i.d. liquidity shocks, i.e. preference shocks in the style of Diamond and Dybvig (1983), in both periods. If a liquidity investor is hit by the shock, they need to consume immediately and sell their entire holdings of the safe asset. The probability of a liquidity shock at date 0 is $s_0 \in (0,1)$ so, by the law of large numbers, a fraction s_0 of liquidity investors are forced to sell at date 0 at price p_0. Among the remaining fraction $1 - s_0$, each investor has to decide whether to also sell at date 0, receiving p_0 for sure, or to hold on to the safe asset and face liquidity risk at date 1, then with probability $s_1 \in (0,1)$. Investors who hold on to the safe asset at date 0 and then suffer a liquidity shock at date 1 are forced to sell at price p_1. Investors who don't suffer a shock at either date receive a continuation value $v > 1$ akin to a "convenience yield" that reflects the benefit of the safe asset as a liquid store of value for future investment opportunities (e.g., Holmström and Tirole, 1998, 2001).[9] We first analyze the general case with two separate values for liquidity risk, s_0 and s_1, and later specialize to the case $s_0 = s_1$ for expositional clarity.

Examples of real-world liquidity investors we have in mind include foreign official agencies that may face sudden liquidity needs to conduct foreign exchange interventions or mutual funds that may face sudden liquidity needs due to investor withdrawals. Both were among the largest sellers of Treasuries in 2020q1, and their sales were historically unprecedented (Figure 1, Panel C). The consumption good in our model therefore stands in for cash and cash-like instruments, such as bank deposits or short-maturity Treasury bills. Due to its stylized nature, our model cannot not provide a theory of the exact maturity cut-off between short-maturity bills, treated as cash, and longer-maturity notes and bonds, which were not treated as cash during March 2020 and represent the safe asset in our model. While we focus on the strategic interaction among liquidity investors, there are potential additional layers of strategic interaction underlying the liquidity shocks, both in the foreign exchange context (Morris and Shin, 1998) and in the mutual fund context (Chen, Goldstein, and Jiang, 2010).

The only agents with a strategic decision to make are liquidity investors who do not receive a liquidity shock at date 0 and can either voluntarily sell their safe asset at date 0 or hold on to it, risking a liquidity shock at date 1. Let p_0 be the price an investor expects to receive at date 0 and p_1^s be the price the investor expects to receive at date 1 conditional

[9] Joslin, Li, and Song (2021) use a conceptually similar model of liquidity investors and show that its comparative statics match well the empirical features of the Treasury liquidity premium.

on receiving a liquidity shock.[10] Under risk-neutrality, the payoff to selling at date 0 is simply p_0 while the (expected) payoff to selling at date 1 is $s_1 p_1^s + (1 - s_1)\, v$ because, with probability $1 - s_1$, the investor will make it through the stress episode without suffering a liquidity shock. The investor will therefore sell preemptively at date 0 if and only if

$$p_0 > s_1 p_1^s + (1 - s_1)\, v. \tag{1}$$

During normal times, i.e. when few if any investors suffer liquidity shocks, we expect $p_0 \approx p_1^s \approx 1$, such that liquidity investors will generally not sell preemptively because the continuation value is greater than 1. This reflects the fact that safe asset markets function well during normal times, when only investors with genuine liquidity needs sell and can do so easily at prices close to fundamental. Investors without genuine liquidity needs do not consider selling preemptively since they expect to be able to sell at prices close to fundamental if they, in turn, face a liquidity shock at date 1. Condition (1) shows that two things are required in order for investors to sell preemptively at date 0: First, the price at date 1 conditional on a shock has to be considerably lower than the price at date 0. Second, liquidity risk at date 1 has to be sufficiently high.

Formally, we denote by $\lambda \in [0, 1]$ the fraction of strategic liquidity investors who decide to sell at date 0. Then the equilibria of the game among strategic investors are governed by the payoff gain from preemptively selling at date 0 vs. holding on to date 1,

$$\pi(\lambda) = p_0(\lambda) - s_1 p_1^s(\lambda) - (1 - s_1)\, v, \tag{2}$$

where we explicitly account for the fact that prices at date 0 and date 1 depend on strategic sales λ (we will derive the relevant expressions for $p_0(\lambda)$ and $p_1^s(\lambda)$ in Section 3). Under complete information, there are three candidates for Bayesian Nash equilibria:

Hold equilibrium: If the incentive to sell is negative when no other strategic investors sell, that is if $\pi(0) < 0$, then it is a pure-strategy equilibrium for no strategic investors to sell ($\lambda^* = 0$).

Run equilibrium: If the incentive to sell is positive when all other strategic investors sell, that is if $\pi(1) > 0$, then it is a pure-strategy equilibrium for all strategic investors to sell ($\lambda^* = 1$).

[10] The price at date 1 is deterministic because we assume for simplicity that liquidity shocks at date 1 are i.i.d. such that there is no aggregate risk. As we discuss in Section 3 and show in Appendix D, it is straightforward to add aggregate risk to the model such that liquidity shocks at date 1 are correlated, without materially affecting results. We maintain the superscript s to highlight that the relevant variable is conditional on a shock.

Mixed equilibrium: If the incentive to sell is zero when a fraction of strategic investors sell, that is if $\pi(\lambda^*) = 0$ for $\lambda^* \in (0, 1)$, then it is a mixed-strategy equilibrium for all strategic investors to sell with probability λ^*.

As noted above, safe asset markets typically function smoothly — they are considered the most deep and liquid markets in the world. Thus, any empirically realistic model should include the potential for hold equilibria, even as we seek out a candidate run equilibrium. The hold equilibrium exists if

$$p_0(0) < s_1 p_1^s(0) + (1 - s_1)\, v, \tag{3}$$

which holds during normal times when not many investors have liquidity needs (low s_1) and expected prices are not very different between date 0 and date 1 ($p_0 \approx p_1^s$). In the hold equilibrium, the safe asset market features only those investors selling who have a genuine need for liquidity which we consider the typical state for safe asset markets that any desirable model should be able to match. As we show below, our assumption that liquidity investors value the safe asset at a convenience yield $v > 1$ allows for the existence of a pure-strategy hold equilibrium, in contrast to the model of Bernardo and Welch (2004), where $v = 1$ such that a hold equilibrium never exists and, instead, a strictly positive fraction of investors always sells preemptively (see Appendix C). The assumption $v > 1$ is empirically plausible for safe asset markets because they typically function well during normal times with no preemptive sales (i.e. a hold equilibrium normally prevails) and because convenience yields on safe assets have been widely documented in the literature started by Krishnamurthy and Vissing-Jørgensen (2012).

The run equilibrium, in turn, exists if

$$p_0(1) > s_1 p_1^s(1) + (1 - s_1)\, v.$$

In this case, strategic investors prefer to sell early rather than risk having to sell at a worse price in case they suffer a liquidity shock at date 1. Compared to condition (3) for the hold equilibrium, more is needed for a run equilibrium to exist. As noted above, the expected price at date 1 conditional on a shock has to be considerably lower than the price at date 0 and liquidity risk at date 1 has to be sufficiently high. The analysis of our paper shows how frictions can lead prices at date 1 to be lower than at date 0 in times of stress such that a run equilibrium can arise.

The identifying feature of a run equilibrium is the preemptive sales by investors who do not face a genuine liquidity need and who therefore do not "consume" the proceeds of their sales. As noted in the introduction, the detailed analysis of Treasury markets in March 2020 by Vissing-Jørgensen (2021) provides evidence of such preemptive sales: Among the largest sellers, foreign official agencies sold \$196 billion of Treasury bonds but "consumed" only 24% of the proceeds in the form of a \$48 billion reduction in their total U.S. Dollar assets. This evidence, and the viral selling behavior noted by Haddad, Moreira, and Muir (2021) with greater dislocations in the typically safer and more liquid assets, point to a need for a model in which sales reflect strategic, self-fulling decisions.

Finally, there is the potential for both pure-strategy equilibria to exist if the incentive to sell $\pi(\lambda)$ is increasing in the fraction of strategic investors who sell. In such a situation of strategic complementarities, the safe asset market can be truly fragile and break down due to self-fulfilling beliefs. Each individual strategic investor sells early only because they expect other strategic investors to sell early, and the run on the safe asset market could be avoided if beliefs were coordinated instead on the hold equilibrium. As we show below, strategic complementarities can arise (i) if dealers' marginal balance sheet costs are sufficiently elastic, such that inventory taken on at date 0 sufficiently impacts the price at date 1 if there are further sales, (ii) if liquidity investors are risk averse such that a potential price drop between date 0 and date 1 is compounded by an increase in marginal utility, and/or (iii) if trades are sequentially executed, such that an investor selling preemptively at date 0 can expect to front-run some of the other investors selling. These features are in contrast to the model of Bernardo and Welch (2004), where price impact is due to dealer risk aversion, investors are assumed to be risk-neutral throughout, and trades are pooled before execution; therefore strategic complementarities do not arise in their model.

3 Dealers and Equilibrium Safe Asset Prices

A necessary condition for a run equilibrium is $p_1^s < p_0 \leq 1$, which requires frictions such that, in times of stress, the safe asset price can deviate from its fundamental value of 1 and can decline further if there are sustained net sales that continue from date 0 to date 1. The key friction in our model is that dealers with balance sheet costs form the residual demand during times of stress, analogous to dealers with risk aversion in Bernardo and Welch (2004). We further assume that dealers bid competitively à la Bertrand such that their demand is defined by a zero-profit condition, also analogous to Bernardo and Welch (2004) and in the spirit of the literature on market making dealers and limits to arbitrage, including Kyle (1985), Grossman and Miller (1988), and Shleifer and Vishny (1997).

In order to maintain tractability, Bernardo and Welch (2004) assume that dealers are myopic, bidding competitively at date 0 without taking into account date 1; this avoids the complications of a dynamic optimization problem for dealers without materially affecting the strategic interaction of investors at the heart of the model. We show, as a conceptual point, that this friction is not technically necessary and that dealers can be assumed to be forward-looking because, as long as they are competitive and lack commitment, subgame perfection implies that they behave *as if* they were myopic. The intuition is that competition at date 1 implies zero profits in every date 1 subgame, i.e. irrespective of the actions at date 0.

Because we assume that liquidity shocks are i.i.d., there is no aggregate risk in our model such that equilibrium prices are deterministic and the price decline from date 0 to date 1 represents a pure arbitrage. While this may appear unappealing, even considering a time of stress, we show in Appendix D that it is straightforward to add aggregate risk to the model such that liquidity shocks at date 1 are correlated and the price conditional on receiving a shock, $p_1^s \equiv E_0[p_1|\text{shock}]$, is different from the price conditional on not receiving a shock, $p_1^{ns} \equiv E_0[p_1|\text{no shock}]$. These state-contingent prices satisfy $p_1^s < p_0 < p_1^{ns}$ such that the model no longer features a pure arbitrage. However, even in that more general setting, the only date-1 price that is relevant for a strategic investor's decision at date 0 (and thus for the equilibria of our model) is p_1^s such that results do not change materially. We therefore conduct the analysis in the simpler setting without aggregate risk but maintain the superscript s to highlight that the relevant variable is conditional on a shock.

Specifically, we assume that dealers consume at the end of date 1 and are forward looking without commitment, competitive and risk neutral. They value the safe asset at its fundamental value of 1 but face increasing and convex balance sheet costs for any inventory q, given by $C(q)$ with $C(0) = 0$ and $C(1) < 1$. Dealers start out with no inventory at the beginning of date 0 and compete for sales à la Bertrand by quoting prices in each period. We solve for the demand that results from the subgame perfect Nash equilibrium among dealers.

What do our assumptions of competitiveness and of no commitment imply at date 1? First, competition à la Bertrand implies that dealers make zero profit, i.e. that for given inventory q_0 taken on at price p_0 and new supply q_1^s, the price p_1^s has to satisfy

$$\underbrace{(1 - p_0)\, q_0 + (1 - p_1^s)\, q_1^s - C(q_0 + q_1^s)}_{\text{final payoff at date 1}} \;=\; 0. \tag{4}$$

Second, no commitment implies that dealers cannot end up worse from taking on new inventory q_1^s at date 1:

$$\underbrace{(1-p_0)\,q_0 + (1-p_1^s)\,q_1^s - C(q_0+q_1^s)}_{\text{payoff after taking on } q_1^s>0} \geq \underbrace{(1-p_0)\,q_0 - C(q_0)}_{\text{payoff with } q_1^s=0} \tag{5}$$

Because the LHS of (5) is decreasing in p_1^s, competitive bidding implies that the condition holds with equality:

$$(1-p_0)\,q_0 + (1-p_1^s)\,q_1^s - C(q_0+q_1^s) = (1-p_0)\,q_0 - C(q_0) \tag{6}$$

The two conditions (4) and (6) together yield equilibrium prices at date 0 and date 1 summarized in the following proposition. All proofs are in Appendix B.[11]

Proposition 1. *If dealers are forward looking without commitment, competitive and risk neutral, and face strictly convex balance sheet costs $C(q)$ with $C(0)=0$ and $C(1)<1$, then equilibrium safe asset prices satisfy $p_1^s < p_0$ and are given by*

$$p_0(q_0) = 1 - \frac{C(q_0)}{q_0} \quad \textit{and} \quad p_1^s(q_0,q_1^s) = 1 - \frac{C(q_0+q_1^s)-C(q_0)}{q_1^s}. \tag{7}$$

The expressions for equilibrium prices in (7) highlight the effect of existing inventory q_0 on the price at date 1 if there are additional sales q_1^s: for the dealers to take on the additional inventory in that state, the price must be lower compared to date 0, $p_1^s < p_0$, in order to compensate for the additional balance sheet costs $C(q_0+q_1^s)-C(q_0)$ of the additional inventory. The assumption that balance sheet costs are convex has additional implications for the equilibrium prices $p_0(q_0)$ and $p_1^s(q_0,q_1^s)$, as summarized in the following corollary.

Corollary 1. *Equilibrium safe asset prices and are decreasing in investors' asset sales as well as in dealer inventory:*

$$\frac{\partial p_0}{\partial q_0} < 0, \quad \frac{\partial p_1^s}{\partial q_0} < 0, \quad \textit{and} \quad \frac{\partial p_1^s}{\partial q_1^s} < 0.$$

Furthermore, the inventory from date 0 has a greater effect on the price at date 1 then asset sales at date 1:

$$\frac{\partial p_1^s}{\partial q_0} < \frac{\partial p_1^s}{\partial q_1^s}.$$

[11] Note that our framework does not restrict dealer demand at date 1 to be positive. If there is additional demand at date 1 such as from an asset purchase facility discussed in Section 7, we can have dealers sell part of their date-0 inventory at date 1. If the balance sheet costs are symmetric around zero, we can also consider negative dealer demand at date 0 (e.g. if they start with an initial endowment of inventory or if they are able to go short the safe asset).

The fact that the indirect effect of additional inventory q_0 reduces the price at date 1 by more than the direct effect of additional sales q_1^s is a crucial ingredient for our results as it implies not only that $p_1^s < p_0$ but also that the date 1 price p_1^s can be sufficiently decreasing in date 0 sales q_0 such that strategic complementarities arise.

Asset sales q_0 and q_1^s are functions of the endogenous strategic sales λ as well as the exogenous liquidity risk s_0 and s_1. Total sales of safe assets at date 0 are sales s_0 by investors with genuine liquidity needs and sales $(1-s_0)\,\lambda$ of the remaining investors that choose to sell preemptively:

$$q_0 = s_0 + (1-s_0)\,\lambda$$

At date 1, only the remaining investors who receive a liquidity shock sell, resulting in

$$q_1^s = s_1\,(1-s_0)\,(1-\lambda)\,.$$

Equilibrium prices $p_0(\lambda, s_0)$ and $p_1^s(\lambda, s_0, s_1)$ therefore ultimately depend on the model primitives λ, s_0 and s_1, and the following corollary shows that we can unambiguously sign all the comparative statics.

Corollary 2. *Equilibrium safe asset prices are decreasing in the endogenous strategic sales and the exogenous degree of liquidity risk:*

$$\frac{\partial p_0}{\partial \lambda} < 0 \quad \textit{and} \quad \frac{\partial p_0}{\partial s_0} < 0$$

as well as

$$\frac{\partial p_1^s}{\partial \lambda} < 0, \quad \frac{\partial p_1^s}{\partial s_0} < 0 \quad \textit{and} \quad \frac{\partial p_1^s}{\partial s_1} < 0.$$

The two key comparative statics in Corollary 2 are the effects of strategic and non-strategic sales at date 0 (λ and s_0) on the price at date 1 if there are more sales:

$$\frac{\partial p_1^s}{\partial \lambda} = \left(\frac{\partial p_1^s}{\partial q_0} - s_1 \frac{\partial p_1^s}{\partial q_1^s}\right)(1-s_0) < 0 \quad \text{and} \quad \frac{\partial p_1^s}{\partial s_0} = \left(\frac{\partial p_1^s}{\partial q_0} - s_1 \frac{\partial p_1^s}{\partial q_1^s}\right)(1-\lambda) < 0$$

Both λ and s_0 increase q_0 (and therefore inventory at date 1) and decrease q_1^s; the indirect effect on p_1^s through inventory is negative while the direct effect on p_1^s through q_1^s is positive, suggesting a potentially ambiguous effect. However, Corollary 2 shows that the inventory effect dominates such that the overall effect is unambiguous: The price at date 1 is decreasing in the primitives λ and s_0 even accounting for the fact that they reduce sales at date 1.

Discussion of Regulatory Constraints. Our modeling of balance sheet costs captures the effects of the Supplementary Leverage Ratio (SLR), an unweighted capital requirement for banks that was introduced as part of the Basel III reforms after the GFC as a backstop to risk-weighted capital regulation and became effective in 2014. Since the largest dealers in the U.S. are part of bank holding companies, the SLR constrains their activity, including in the Treasury market, a potentially unintended consequence of the regulatory reform. Importantly, both the direct holdings of Treasuries and reverse repo positions take up dealers' balance sheet space and are subject to the SLR (for more details, see, e.g. Duffie, 2016). Boyarchenko et al. (2020) show that the constraints pass through to unregulated arbitrageurs who rely on the balance sheet of regulated dealers (see also Du, Hébert, and Li, 2022 and Siriwardane, Sunderam, and Wallen, 2022). The balance sheet costs in our model explicitly capture the regulatory costs of increasing balance sheet size for given equity capital, but also implicitly capture the cost of issuing additional equity to alleviate the regulatory constraints. If, for example, a bank's capital requirement is binding, then expanding assets requires issuing new equity. Equity issuance can be costly due to moral hazard (Jensen and Meckling, 1976) or due to adverse selection (Myers and Majluf, 1984). Convex issuance costs associated with equity issuance would therefore generate convex costs of increasing balance sheet size.

The balance sheet costs matter in markets for safe assets such as Treasuries, as they rely heavily on dealers for intermediating trades. Brain et al. (2019) document that Treasury market trading volume is split roughly evenly between dealer-to-client trades and inter-dealer trades; this suggests that, on average, a trade originating with one investor and ending with another investor passes through two dealers. The effects of balance sheet constraints are also quantitatively meaningful. For example, He, Nagel, and Song (2022) show that Treasury and repo spreads are significantly wider in the post-SLR period. In March 2020, the ability of dealers to provide liquidity in Treasuries was severely impaired as market depth dropped by a factor of more than 10 in the inter-dealer market (Duffie, 2020) while trading volume roughly doubled, reaching historically unprecedented levels (Fleming and Ruela, 2020). Duffie et al. (2023) show strong explanatory power of dealer balance sheet utilization for Treasury market illiquidity after controlling for yield volatility. Furthermore, the SLR constraint was initially not alleviated by the Fed's purchases of Treasuries because they were exchanged for reserves which, though perfectly liquid and safe, are treated the same under the SLR. Only on April 14 did the Fed temporarily exempt both Treasuries and reserves from the SLR rule (announced on April 1). Infante, Favara, and Rezende (2022) document the effect of the SLR and its temporary relaxation on dealers' Treasury market activity. We return to these issues in our discussion of policy implications in Section 7.

For tractability, we model balance sheet constraints as a convex function of *net* dealer demand and abstract from bid-ask spreads. In reality, dealers can rarely net out offsetting trades instantaneously, and so sales or purchases that are not perfectly synchronized at the same dealer will increase balance sheet costs across the financial system, making the role of balance sheet constraints more pronounced. While we model balance sheet costs as convex, in reality the SLR may at times impose hard quantity constraints with effectively infinite costs of expanding balance sheet further (Duffie, 2020). To the extent that regulatory constraints at times become totally binding, our results would be further strengthened. In sum, our modeling decisions bias the analysis toward less significant balance sheet costs. Note that we abstract from the intended benefits of the SLR for the stability of the banking system as these are outside the scope of our model.[12]

4 Strategic Complementarities in the Incentive to Sell

We now return to the liquidity investors' strategic interaction from Section 2 and study if and how strategic complementarities can arise in their incentive to sell preemptively. Given the structure of equilibrium prices derived in Section 3, we start with the case of a general convex balance sheet cost $C(q)$ and show that strategic complementarities can rise for sufficiently elastic marginal costs. We then specialize to a parametric form to sharpen the results and show two features that can strengthen the incentive to sell and can therefore substitute for very elastic marginal balance sheet costs in generating strategic complementarities: (i) investor risk aversion, which is natural for investors demanding liquidity insurance; and (ii) sequential execution of trades, which is appropriate when considering over-the-counter markets.

4.1 General Convex Cost Function

Accounting for the dependence of equilibrium prices p_0 and p_1^s on endogenous strategic sales λ as well as the exogenous liquidity risk s_0 and s_1, the payoff gain π from equation (2) becomes

$$\pi(\lambda, s_0, s_1) = p_0(\lambda, s_0) - s_1 p_1^s(\lambda, s_0, s_1) - (1 - s_1)\, v.$$

[12]The macro-finance literature shows how leverage regulation can have macroprudential benefits, decreasing the probability and severity of crises and fire sales (e.g. Phelan, 2016, Dávila and Korinek, 2017). While the SLR is intended as a "non-risk based backstop measure" (Basel Committee on Banking Supervision, 2014), it's potential to interfere with Treasury market functioning had been anticipated, e.g. by Duffie (2016)

Maintaining the general cost function, we derive the comparative statics for the incentive to sell preemptively at date 0 and provide a sufficient condition for investors to face strategic complementarities, i.e. for $\partial\pi/\partial\lambda > 0$: Liquidity risk s_1 has to be sufficiently high and marginal balance sheet costs have to be sufficiently elastic. Denote the elasticity of marginal costs

$$\eta \equiv \frac{dC'(q)}{dq}\frac{q}{C'(q)} = \frac{qC''(q)}{C'(q)}.$$

Then we have the following result.

Proposition 2. *For strictly convex balance sheet costs $C(q)$ with $C(0) = 0$ and $C(1) < 1$, and with non-negative higher order derivatives:*

- *Strategic complementarities arise for date-1 liquidity risk sufficiently high and/or marginal balance sheet costs sufficiently elastic, i.e. $\partial\pi/\partial\lambda > 0$ uniformly if $s_1 \times \eta > 2$.*
- *Higher date-1 liquidity risk uniformly increases the incentive to sell, i.e. $\partial\pi/\partial s_1 > 0$, while date-0 liquidity risk increases the incentive to sell whenever there are strategic complementarities, i.e. $\partial\pi/\partial s_0 > 0$ if $\partial\pi/\partial\lambda > 0$.*

The intuition for the effect of s_1 is straightforward as higher future liquidity risk s_1 hast two effects: (i) increasing the likelihood of facing p_1^s instead of v and (ii) reducing p_1^s conditional on facing it. Both effects increase the incentive to sell preemptively at date 0 and are therefore destabilizing.

Strategic sales λ and date-0 liquidity risk s_0 affect the payoff gain very similarly. In both cases, the direct effect of lower p_0 is stabilizing but the indirect effect of lower p_1^s through inventory is destabilizing. The intuition is clear: when considering the effect of additional sales at date 0, a strategic investor does not care if the sales originate with more genuine liquidity needs (higher s_0) or with more preemptive sales (higher λ).

The joint condition on s_1 and η guaranteeing strategic complementarities shows that liquidity risk and the elasticity of marginal costs interact. For a given elasticity of marginal costs, future liquidity risk s_1 has to be sufficiently large. In turn, the liquidity risk threshold is lower if marginal balance sheet costs are more elastic. What is the intuition for the sufficient condition for strategic complementarities? Our main object of interest is $\partial\pi/\partial\lambda$ which we can rewrite as

$$\frac{\partial\pi}{\partial\lambda} = (1 - s_0)\left(\frac{\partial p_0}{\partial q_0} + s_1^2\frac{\partial p_1^s}{\partial q_1^s} - s_1\frac{\partial p_1^s}{\partial q_0}\right).$$

For strategic complementarities to arise, we therefore need

$$\underbrace{s_1 \frac{\partial p_1^s}{\partial q_0}}_{<0} < \underbrace{\frac{\partial p_0}{\partial q_0} + s_1^2 \frac{\partial p_1^s}{\partial q_1^s}}_{<0}, \tag{8}$$

that is, we need the indirect inventory effect $\partial p_1^s / \partial q_0$ to dominate (in absolute magnitude) the two direct effects $\partial p_0 / \partial q_0$ and $\partial p_1^s / \partial q_1^s$, and we need s_1 sufficiently large. In Appendix E, we provide graphical intuition for why sufficiently elastic marginal costs guarantee that the inventory effect is large relative to the direct effects. Specifically, the function $C(q)$ has to have weak curvature between 0 and q_0 (for a small direct effect) and strong curvature between q_0 and $q_0 + q_1^s$ (for a large indirect effect). Marginal balance sheet costs $C'(q)$ therefore have to increase relatively slowly for low q and relatively quickly for large q, which translates into sufficiently elastic marginal costs.

4.2 Parametric Cost Functions

We now add structure by considering parametric cost functions of the form $C(q) = cq^n$ with $c > 0$ and $n > 1$ where the elasticity of marginal costs is given by $\eta = n - 1$. We first show analytically that strategic complementarities are not possible for quadratic balance sheet costs ($n = 2$) — which corresponds to the CARA utility in Bernardo and Welch (2004) — but are possible for $n = 3$, as long as future liquidity risk s_1 is sufficiently high. We then show numerically that the same logic applies to $n > 3$, i.e. strategic complementarities arise for s_1 above a threshold $\tilde{s}$. We can derive the threshold $\tilde{s}$ in closed form and show that it is decreasing in n for $n > 2$ such that strategic complementarities are more likely to arise as n increases and marginal balance sheet costs become more elastic.

Proposition 3. *For quadratic balance sheet costs $C(q) = cq^2$ with $c \in (0, 1)$, strategic complementarities cannot arise, i.e. $\partial\pi/\partial\lambda \leq 0$ uniformly. For cubic balance sheet costs $C(q) = cq^3$ with $c \in (0, 1)$, strategic complementarities arise for liquidity risk above a threshold $\tilde{s} = 1 - \sqrt{1/3} \approx 0.42$, i.e. $\partial\pi/\partial\lambda > 0$ uniformly if $s_1 > \tilde{s}$.*

We cannot study strategic complementarities purely analytically for $n > 3$. However, Figure 2 shows that the logic of Proposition 3 extends to $n > 3$ and that the critical value $\tilde{s}$ is decreasing in n such that the range where strategic complementarities arise is larger for higher n. Specifically, Panel A of Figure 2 shows contour plots of $\partial\pi(\lambda, s_0, s_1)/\partial\lambda = 0$, i.e. combinations of s_0 and s_1 where $\partial\pi/\partial\lambda$ switches from negative (strategic substitutes) to positive (strategic complements), for different values of λ as well as different exponents n in the balance sheet cost function $C(q) = cq^n$. The blue lines correspond to the case

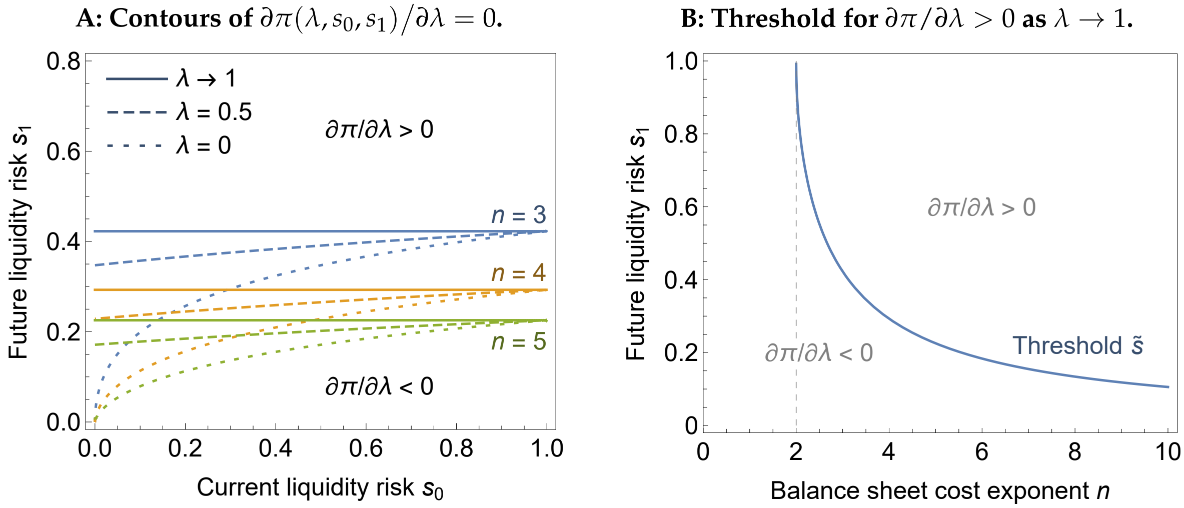

Figure 2: Transition from strategic substitutes to strategic complements. Panel A shows contour plots of $\partial\pi(\lambda, s_0, s_1)/\partial\lambda = 0$ for different values of λ as well as different exponents n in the balance sheet cost function $C(q) = cq^n$ (with $c = 0.25$). Panel B shows the threshold $\tilde{s} = 1 - \sqrt{(n-2)/n}$.

$n = 3$ that we derive analytically in Proposition 3 where $s_1 > \tilde{s}$ is sufficient for strategic complementarities because, in the limit $\lambda \to 1$, the effect of s_0 vanishes as the contour becomes flat. The orange and green lines show that the same applies to the cases $n = 4$ and $n = 5$, where, again, the effect of s_0 vanishes in the limit $\lambda \to 1$ such that s_1 above a threshold $\tilde{s}$ is sufficient for $\partial\pi/\partial\lambda > 0$ uniformly. Across the cases, we see that the threshold $\tilde{s}$ is decreasing in n such that strategic complementarities are more likely to arise as n increases.

In fact, we can analytically derive $\partial\pi/\partial\lambda$ in the limit $\lambda \to 1$ and solve in closed form for the threshold $\tilde{s}$ that is sufficient for strategic complementarities (illustrated in Panel B of Figure 2).

Corollary 3. *For general power balance sheet costs $C(q) = cq^n$ with $c \in (0, 1)$, strategic complementarities arise for liquidity risk above a threshold $\tilde{s} = 1 - \sqrt{(n-2)/n}$ which is decreasing in n.*

In the limit $n \to \infty$, the balance sheet costs turn into a pure capacity constraint, forcing $q_0 + q_1^s \leq 1$ and resulting in strategic complementarities irrespective of the level of liquidity risk as $\tilde{s} \to 0$ (Corollary 3). This is consistent with the effects of occasionally binding constraints documented by Duffie et al. (2023) and the bank executive who stated in March 2020 "We can't bid on anything that adds to the balance sheet right now" (as quoted by Duffie, 2020).

4.3 Investor Risk Aversion

We now consider the case where liquidity investors are risk averse, which would be natural for safe asset investors seeking liquidity insurance and in contrast to the risk neutral equity investors in Bernardo and Welch (2004). We suppose in this section that liquidity investors are risk averse with concave utility $u(x)$ such that the payoff gain π from equation (2) becomes

$$\pi = u(p_0) - s_1 u(p_1^s) - (1 - s_1)\, v.$$

such that marginal utility now appears in the condition $\partial\pi/\partial\lambda > 0$ for strategic complementarities:

$$\frac{\partial \pi}{\partial \lambda} = u'(p_0)\frac{\partial p_0}{\partial \lambda} - s_1 u'(p_1^s)\frac{\partial p_1^s}{\partial \lambda}$$

Risk aversion compounds the effect of a lower price conditional on a shock, $p_1^s < p_0$, since the investor has higher marginal utility if they are forced to sell at date 1 such that the incentive to sell preemptively at date 0 strengthens.

With risk aversion, the sufficient condition for strategic complementarities in Proposition 2 gains an extra factor and becomes $s_1\eta \times \left(1 + \gamma\Delta_p\right) > 2$, where $\gamma = -xu''(p_0)/u'(p_0)$ is the coefficient of relative risk aversion at p_0 and $\Delta_p = (p_0 - p_1^s)/p_0$ is the percent drop in the asset price conditional on a shock. While this conditional price drop is an endogenous object, the extra factor intuitively illustrates how a greater degree of risk aversion and a greater conditional price drop increase the likelihood that strategic complementarities arise. Next, we derive a sufficient condition for strategic complementarities that is not stated in terms of endogenous objects.

For analytical tractability and to highlight the effect of risk aversion on its own, we now use quadratic balance sheet costs $C(q) = cq^2$ which, under risk neutrality, cannot generate strategic complementarities (Proposition 3). Quadratic costs result in simple linear demands from the dealers as the equilibrium price functions of Proposition 1 become

$$p_0(q_0) = 1 - cq_0 \quad \text{and} \quad p_1^s(q_0, q_1^s) = 1 - 2cq_0 - cq_1^s. \tag{9}$$

The linear structure highlights the result from Corollary 1 for general convex balance sheet costs, that the inventory effect $\partial p_1^s/\partial q_0$ is larger than the direct price impact $\partial p_1^s/\partial q_1^s$ — for quadratic cost the inventory effect is twice as large. With CRRA utility and the the price functions (9) linear in λ, we can derive a sufficient condition for strategic complementarities in terms of exogenous parameters.

Proposition 4. *For quadratic balance sheet costs $C(q) = cq^2$ with $c \in (0,1)$ and constant relative risk aversion $u(x) = x^{1-\gamma}/(1-\gamma)$ with $\gamma > 0$, strategic complementarities arise for date-1* *liquidity risk and/or risk aversion sufficiently high, i.e. $\partial\pi/\partial\lambda > 0$ uniformly if $s_1(2-s_1) > (1-cs_1)^\gamma$ which, for given γ, implies a lower bound for s_1 and vice versa.*

Proposition 4 shows that the effect of risk aversion in strengthening the incentive to sell preemptively is sufficiently strong that strategic complementarities can arise even with less elastic marginal balance sheet costs.

4.4 Sequential Trade Execution

We now consider the case where trades are executed sequentially, in contrast to the pooled execution in Bernardo and Welch (2004) but similar to Morris and Shin (2004) and consistent with the decentralized nature of safe asset markets including the Treasury dealer-to-client market (Brain et al., 2019). For analytical tractability we again use quadratic balance sheet costs $C(q) = cq^2$ which, under pooled execution, cannot generate strategic complementarities (Proposition 3) and which result in simple linear equilibrium prices in (9).

We assume that each seller's position in the queue of sequential execution is uniformly distributed such that, for aggregate sales q, the expected position in the queue is $q/2$. Investors therefore expect to sell at expected prices

$$E[p_0(q_0)] = 1 - \frac{1}{2}cq_0 \quad \text{and} \quad E[p_1^s(q_0, q_1^s)] = 1 - 2cq_0 - \frac{1}{2}cq_1^s.$$

The expected prices highlight that sequential execution further strengthens the inventory effect of q_0 on $E\left[p_1^s\right]$ which is now four times as large as the direct price impact of q_1^s. The reason is that an investor selling at date 0 expects to front-run half of total date-0 sales on average while they will have to bear the full inventory effect of date-0 sales once date 1 comes around.

Using these expressions, we can write the payoff gain π from equation (2) explicitly as a function of the endogenous strategic sales λ as well as the exogenous liquidity risk s_0 and s_1:

$$\begin{aligned}\pi(\lambda, s_0, s_1) = {} & \overbrace{1 - \frac{c}{2}\left(s_0 + (1-s_0)\lambda\right)}^{E[p_0]} \\ & - s_1 \underbrace{\left(1 - 2c\left(s_0 + (1-s_0)\lambda\right) - \frac{c}{2}s_1(1-s_0)(1-\lambda)\right)}_{E[p_1^s]} - (1-s_1)v. \quad (10)\end{aligned}$$

The payoff gain (10) is linear in λ and s_0, and quadratic in s_1, which allows us to derive clear comparative statics.

Proposition 5. *For quadratic balance sheet costs $C(q) = cq^2$ with $c \in (0,1)$ and sequential trade execution:*

- *Strategic complementarities arise for date-1 liquidity risk s_1 above a threshold $\widetilde{s} = 2 - \sqrt{3} \approx 0.27$, i.e. $\partial\pi/\partial\lambda > 0$ uniformly if $s_1 > \widetilde{s}$.*
- *Higher date-1 liquidity risk uniformly increases the incentive to sell, i.e. $\partial\pi/\partial s_1 > 0$, while date-0 liquidity risk increases the incentive to sell whenever there are strategic complementarities, i.e. $\partial\pi/\partial s_0 > 0$ if $s_1 > \widetilde{s}$.*
- *Greater dealer balance sheet costs increase the incentive to sell whenever there are strategic complementarities, i.e. $\partial\pi/\partial c > 0$ if $s_1 > \widetilde{s}$.*

Analogous to the effect of risk aversion, Proposition 5 shows that the effect of sequential execution in strengthening the incentive to sell preemptively is sufficiently strong that strategic complementarities can arise even with less elastic marginal balance sheet costs.

5 Equilibrium Analysis

Under complete information, there can be multiple equilibria in the strategic interaction among liquidity investors — a hold equilibrium and a run equilibrium (and a mixed equilibrium), as discussed in Section 2. In this section, we first impose some simplifying assumptions that maintain the key features of the strategic interaction analyzed in Section 4 but simplify the exposition. Then we introduce noise into investors' payoffs to break the common knowledge underpinning the multiplicity and use global game techniques to derive a unique equilibrium.

5.1 Complete Information and Multiple Equilibria

We continue the analysis using the simple linear structure that results from quadratic balance sheet costs and sequential trade execution derived in Section 4.4. Propositions 2 and 5 further show that liquidity risk at the two dates, s_0 and s_1, have very similar effects on the incentive to sell. We also think of the level of liquidity risk as varying at lower frequency than the timing of the liquidity shocks across investors such that, in normal times, both s_0 and s_1 are low while, in times of stress, both s_0 and s_1 are high. To further simplify the exposition we therefore specialize to the case of a single parameter capturing liquidity risk at both dates, $s_0 = s_1 = s$, such that the payoff gain (10) simplifies to

$$\pi(\lambda, s) = \frac{c}{2}\Big((4s-1)\left(s + (1-s)\,\lambda\right) + s^2\,(1-s)\,(1-\lambda)\Big) - (1-s)\,(v-1)\,, \qquad (11)$$

which is linear in strategic sales λ and cubic in liquidity risk s.[13] The following corollary restates Proposition 5 for this case.[14]

Corollary 4. *For quadratic balance sheet costs $C(q) = cq^2$, sequential trade execution, and a single level of liquidity risk $s_0 = s_1 = s$:*

- *Strategic complementarities arise for liquidity risk s above a threshold $\widetilde{s} = 2 - \sqrt{3} \approx 0.27$, i.e. $\partial\pi/\partial\lambda > 0$ uniformly if $s > \widetilde{s}$.*
- *Higher liquidity risk uniformly increases the incentive to sell whenever there are strategic complementarities, i.e. $\partial\pi/\partial s > 0$ if $s > \widetilde{s}$.*
- *Greater dealer balance sheet costs increase the incentive to sell whenever there are strategic complementarities, i.e. $\partial\pi/\partial c > 0$ if $s > \widetilde{s}$.*

With this structure, we can now revisit the possible equilibria under complete information discussed in Section 2 and derive under which conditions each type of equilibrium exists: the hold equilibrium for $\pi(0,s) < 0$, the run equilibrium for $\pi(1,s) > 0$ and a mixed equilibrium if $\pi(\lambda^*,s) = 0$ for some $\lambda^* \in (0,1)$.

The analysis of Section 4, as summarized in Corollary 4 shows that liquidity risk s is a key driver of both the level of the payoff gain $\pi(\lambda,s)$ as well as of its slope $\partial\pi/\partial\lambda$, which we illustrate in Figure 3. For low liquidity risk s, $\pi(\lambda,s)$ is negative for all λ and decreasing (strategic substitutes), and the unique equilibrium is the hold equilibrium ($\lambda^* = 0$). In fact, we have $\pi(0,0) = -(v-1)$ so our assumption that liquidity investors value the safe asset at a convenience yield $v > 1$ guarantees that the hold equilibrium exists for sufficiently low liquidity risk, i.e. the existence of $\underline{s} \in (0,1)$ such that $\pi(0,s) < 0$ for $s < \underline{s}$. As noted above, this is an important difference to the model of Bernardo and Welch (2004), where $v = 1$ such that a hold equilibrium never exists.

For sufficiently high s, $\pi(\lambda,s)$ is positive for all λ and the unique equilibrium is for everyone to sell ($\lambda^* = 1$). We have $\pi(1,1) = 3c/2$ so our assumption that dealers suffer balance sheet costs from holding inventory ($c > 0$) guarantees that the run equilibrium exists for sufficiently high liquidity risk, i.e. the existence of $\bar{s} \in (0,1)$ such that $\pi(1,s) > 0$ for $s > \bar{s}$.

[13] We could instead simplify by considering the case $s_0 = 0$ and $s_1 > 0$ (as in Bernardo and Welch, 2004). However, we want to be able to distinguish between investors with genuine liquidity needs who are forced to sell at date 0, captured by $s_0 > 0$, and investors without genuine liquidity needs who strategically sell at date 0 because they are worried about potential genuine liquidity needs at date 1. We view this as an important distinction for safe asset markets in general and for understanding the events of March 2020 in particular.

[14] We show in Appendix F that the global game analysis goes through with $s_0 \neq s_1$ as long as s_1 is the fundamental that investors receive noisy signals about.

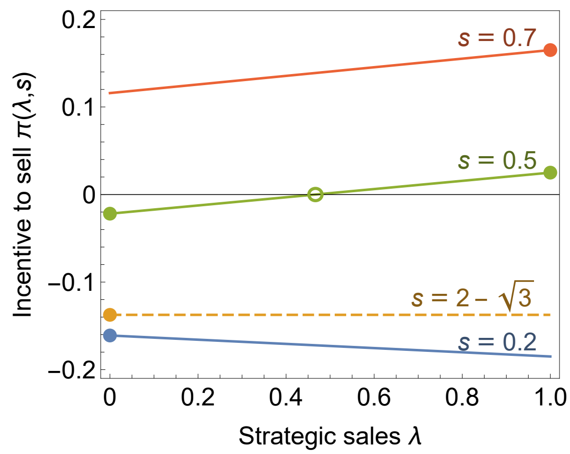

Figure 3: Incentive to sell and equilibria. The figure shows the payoff gain π for different values of liquidity risk s. Circles indicate equilibria of the game under complete information. Parameters: $c = 0.25,\ v = 1.2$.

The only remaining question is whether $\pi(\lambda, s)$ is positive of negative at $s = \tilde{s} \equiv 2 - \sqrt{3}$ where the sign of the slope $\partial\pi/\partial\lambda$ changes. Figure 3 shows strategic complementarities arising at a point where the payoff gain is negative, i.e. the dashed horizontal line is below the horizontal axis which occurs for

$$\pi(\lambda, \tilde{s}) = \frac{c}{2}\tilde{s}^2 - (1 - \tilde{s})(v - 1) < 0. \tag{12}$$

As s increases, the level and slope of $\pi(\lambda, s)$ with respect to λ increase, until it first becomes flat at $s = \tilde{s}$ and then intersects the horizontal axis with positive slope (strategic complements), at which point the game has multiple equilibria (hold, sell and mixed). As noted above, sufficiently elastic marginal balance sheet costs and/or sequential trade execution are key for strategic complementarities, in contrast to the model of Bernardo and Welch (2004), where strategic complementarities do not arise.

In the following, we will focus on the case illustrated in Figure 3 by assuming c and v such that condition (12) holds. What happens if this assumption is not satisfied? In that case, the unique equilibrium is still to hold for sufficiently small s and to sell for sufficiently large s. However, for an intermediate range of s, the unique equilibrium is in mixed strategies since the payoff gain crosses the horizontal axis with negative slope. Since our emphasis is on the potential for fragility, we focus the analysis on the case where multiple pure-strategy equilibria arise in an intermediate range of s and we can have regime shifts. This allows for the use of global game techniques and results in a unique equilibrium for every $s \in [0, 1]$ with the switch from the hold to the run equilibrium at an endogenous threshold. Since the threshold is a continuous function of other model parameters, comparative statics and policy analysis follow naturally.

5.2 Global Game and Unique Equilibrium

We now assume that the liquidity shock probability s is drawn at the beginning of date 0 from a distribution F on $(0,1)$ and that there is imperfect information about s. Specifically, each individual investor i observes an idiosyncratic signal $\widehat{s}_i = s + \sigma_\varepsilon \varepsilon_i$, where the mean-zero signal noise ε_i is i.i.d. across all i with distribution G_ε and $\sigma_\varepsilon > 0$ but arbitrarily small.[15] As a result, a strategic investor faces fundamental uncertainty about the likelihood of a liquidity shock, s, as well as strategic uncertainty about the fraction of other strategic investors who sell preemptively, λ. Making use of standard global game results (e.g. Morris and Shin, 2003), we can derive a unique Bayesian Nash equilibrium for the game among strategic investors.

Proposition 6. *For signal noise $\sigma_\varepsilon \to 0$, the unique Bayesian Nash equilibrium among strategic investors is in switching strategies around a threshold s^* defined by*

$$\int_0^1 \pi(\lambda, s^*)\, d\lambda = 0.$$

For liquidity risk below the threshold, $s < s^*$, all strategic investors hold on to their safe assets and only investors with genuine liquidity needs sell. For liquidity risk above the threshold, $s > s^*$, all strategic investors sell their safe assets and the market suffers a run.

While Appendix B contains the full proof, we provide the following outline for intuition. An investor who receives a signal exactly equal to the switching point has to be indifferent between holding and selling,

$$E\left[\pi(\lambda, s) \,\middle|\, \widehat{s}_i = s^*\right] = 0, \tag{13}$$

where the expectation is with respect to both λ and s. Note from equation (10) that $\pi(\lambda, s)$ is linear in λ and cubic in s. We have $E[s \mid \widehat{s}_i = s^*] = s^*$, and, in the limit $\sigma_\varepsilon \to 0$, we have $E\left[s^2 \mid \widehat{s}_i = s^*\right] \to (s^*)^2$ and $E\left[s^3 \mid \widehat{s}_i = s^*\right] \to (s^*)^3$, so fundamental uncertainty vanishes, and strategic uncertainty in the form of the distribution of λ becomes uniform on $[0,1]$. We therefore have

$$\lim_{\sigma_\varepsilon \to 0} E\left[\pi(\lambda, s) \,\middle|\, \widehat{s}_i = s^*\right] = \int_0^1 \pi(\lambda, s^*)\, d\lambda,$$

[15] Because we focus on the limit of vanishing signal noise, $\sigma_\varepsilon \to 0$, we can treat s as non-random in the exposition except when deriving the global game equilibrium.

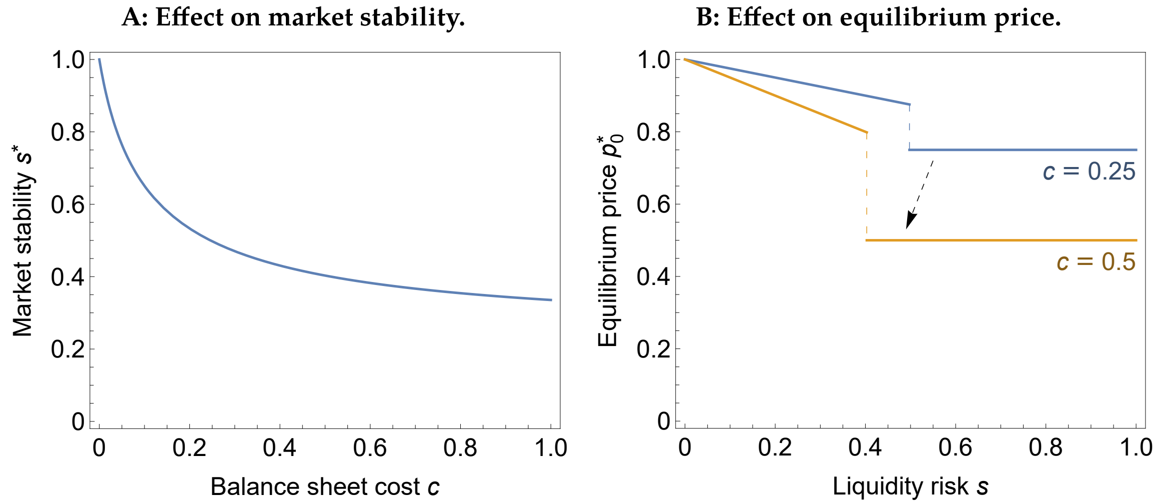

Figure 4: Effect of balance sheet costs on market stability and equilibrium price. Panel A shows market stability measured by the equilibrium threshold s^* as a function of the dealer balance sheet cost c. Panel B shows the equilibrium price at date 0 p_0^* as a function of liquidity risk s for different values of dealer balance sheet cost c. Parameters: $v = 1.2$.

where $\int_0^1 \pi(\lambda, s)\, d\lambda$ is a cubic polynomial in s. We show in the proof of Proposition 6 that $\frac{\partial}{\partial s} \int_0^1 \pi(\lambda, s)\, d\lambda > 0$ with $\int_0^1 \pi(\lambda, 0)\, d\lambda < 0$ and $\int_0^1 \pi(\lambda, 1)\, d\lambda > 0$ so there is a unique threshold s^* that satisfies the indifference condition $\int_0^1 \pi(\lambda, s^*)\, d\lambda = 0$.

The equilibrium switches from hold to sell when liquidity risk s crosses the threshold s^* and a higher threshold implies a larger range of liquidity risk $[0, s^*]$ where the market remains in the hold equilibrium. Given the distribution F of liquidity risk s, the ex-ante probability of the hold equilibrium is therefore $\Pr[s \leq s^*] = F(s^*)$ and the ex-ante probability that the market suffers a run is $\Pr[s > s^*] = 1 - F(s^*)$. The threshold s^* is therefore a well-defined measure of market stability, or $1 - s^*$ a measure of fragility, and we can refer to a market with higher s^* as more stable or, equivalently, less fragile.

Corollary 5. *Market stability s^* is decreasing in dealer balance sheet costs, $\partial s^*/\partial c < 0$ and increasing in liquidity investors' continuation value, $\partial s^*/\partial v > 0$.*

Market stability naturally inherits the properties of the incentive to sell listed in Corollary 4. Consider the effect of dealer balance sheet costs c on market stability s^* illustrated in Figure 4A. If dealers faced no balance sheet costs ($c = 0$), the market would be perfectly stable ($s^* = 1$) and strategic investors would never sell preemptively, even for very high liquidity risk s. However, as balance sheet costs c increase from zero, market stability s^* decreases rapidly and then levels off at higher values of c.

The threshold equilibrium implies that the behavior of strategic liquidity investors and therefore the equilibrium price drops precipitously around the threshold s^*. In particular, total supply at date 0 increases from s to 1 as s crosses the thresholds^*, so the equilibrium price from equation (9) becomes[16]

$$p_0^*(s) = \begin{cases} 1 - cs & \text{for} \quad s < s^*, \\ 1 - c & \text{for} \quad s > s^*. \end{cases} \tag{14}$$

Figure 4B illustrates the equilibrium price p_0^*. When liquidity risk is very low, all strategic investors hold on to their safe assets and only investors who receive a liquidity shock sell — the equilibrium price is therefore steadily decreasing in s, representing the sales of non-strategic investors. However, once liquidity risk crosses the threshold s^*, all strategic investors preemptively sell their safe assets — the market is flooded and the equilibrium price drops precipitously. Figure 4B further illustrates the equilibrium price for two different levels of dealer balance sheet costs c. As balance sheet costs increase, the threshold s^* and therefore market stability decreases (Corollary 5). In addition, the drop in market prices at the discontinuity is much larger for higher balance sheet costs. This is due to the fact that the drop in equation (14) is given by $c\,(1 - s^*)$, where c and s^* interact multiplicatively.

5.3 Welfare and Policy Implications

We now consider the welfare properties of the model and what policies can address the inefficiency. Selling the asset without a genuine liquidity need is generally inefficient because liquidity investors value the safe asset at $v > 1$ when held to maturity. As a result, any runs in our model are inefficient and liquidity investors would be better off if they could coordinate to hold instead. We conduct the welfare analysis including only liquidity investors and dealers because this allows us to focus on the key source of inefficiency in the model. Once we add risk averse safety investors to the model in Section 6, additional distributional effects arise which are not related to the key inefficiency (and do not arise with risk neutral liquidity investors and dealers).

Social welfare is the sum of the payoffs to liquidity investors plus the payoffs to the dealers. The payoff to liquidity investors is

$$q_0 p_0 + q_1 p_1 + (1 - q_0 - q_1)\, v.$$

[16] The expression in (14) represents the zero-noise limit case ($\sigma_\varepsilon \to 0$) where the price drops discontinuously at s^*. For small but positive σ_ε, the price drop would be continuous but very steep in a small neighborhood around s^*.

while the payoff to dealers is

$$q_0\,(1-p_0)+q_1\,(1-p_1)-C(q_0+q_1)\,.$$

Adding the two, we can write ex-post social welfare W as

$$W=v-(q_0+q_1)\,(v-1)-C(q_0+q_1)\,.$$

Social welfare is strictly decreasing in total sales because investors value the safe asset at $v>1$ while dealers value it only at 1 and face balance sheet costs from taking on inventory (the prices paid simply constitute a transfer between investors and dealers). We can decompose total sales into sales that are ex-post necessary and unnecessary, where necessary sales reflect genuine liquidity needs of investors based on liquidity shocks at date 0 *and* date 1:

$$\begin{aligned} q_0+q_1 &= \overbrace{s+(1-s)\,\lambda}^{q_0}+\overbrace{s\,(1-s)\,(1-\lambda)}^{q_1} \\ &= \underbrace{2s-s^2}_{\text{genuine liquidity needs}} + \underbrace{(1-s)^2\,\lambda}_{\text{ex-post unnecessary sales}}\,. \end{aligned}$$

While date-0 and date-1 liquidity risk both create necessary sales, preemptive sales by investors who do not receive a shock at date 0 and would not have received a shock at date 1 are inefficient. Fixing exogenous liquidity risk s, a higher endogenous λ strictly decreases welfare, $dW/d\lambda<0$, and welfare is maximized when the only sales are due to genuine liquidity needs, i.e. when $\lambda=0$.

The threshold equilibrium implies that $q_0+q_1=2s-s^2$ for $s<s^*$ and $q_0+q_1=1$ for $s>s^*$. Ex ante social welfare is therefore given by

$$\begin{aligned} E[W]=v-\int_0^{s^*}\Big[\big(2s-s^2\big)\,(v-1)+C\big(2s-s^2\big)\Big]\,dF(s) \\ -\big(1-F(s^*)\big)\,\big(v-1+C(1)\big)\,. \end{aligned}$$

Analogous to ex-post welfare, a lower incidence of runs, i.e. a higher equilibrium threshold s^*, strictly increases ex-ante welfare, $dE[W]/ds^*>0$, and welfare is maximized when there are no runs, i.e. when the market is perfectly stable with $s^*=1$.

We consider a policy maker who is constrained to using taxes and transfers that cannot be contingent on an investor's type or on the aggregate state of the world. Specifically, we consider a Pigouvian tax $\tau\in(0,1)$ on sales of the safe asset that is rebated, at the end of each period, to all investors who held the asset at the beginning of the period. Under this policy, investors who sell at date t receive $(1-\tau)\,p_t$ and all investors receive $\tau p_t q_t$ at the end of date t. Because the rebates do not depend on the investor's decision whether to sell or not, it cancels out of the payoff gain π which becomes

$$\pi = (1-\tau)\,p_0 - s\,(1-\tau)\,p_1^s - (1-s)\,v.$$

With equilibrium prices $p_1^s < p_0$ (Proposition 1), the incentive to sell preemptively is decreasing in the tax rate τ, $\partial\pi/\partial\tau = -p_0 + sp_1^s < 0$. Furthermore, a sufficiently high tax rate can eliminate runs entirely because $\pi(\tau = 1) = -(1-s)\,v < 0$.

Proposition 7. *Consider a Pigouvian tax τ on sales that is rebated to all investors. Market stability s^* and therefore ex-ante welfare $E[W]$ is increasing in τ. For sufficiently high τ, the market is perfectly stable ($s^* = 1$) and ex-ante welfare is maximized.*

While the Pigouvian tax of Proposition 7 can completely eliminate runs, the tax rate may have to be very high and the policy results in a transfer from investors who receive a liquidity shock and are forced to sell to investors who do not receive a liquidity shock. This transfer does not affect social welfare only because we assume that investors are risk neutral. However, we can reduce the size of this transfer if we allow the social planner to make the tax state contingent. One particularly attractive option would be a tax that only becomes active if total sales are above some threshold, akin to swing pricing in the context of mutual funds (e.g. Jin et al., 2021). Specifically, consider a policy with a tax rate τ such that runs are not completely eliminated ($s^* < 1$) and that only becomes active if sales exceed the threshold s^*.[17] Because equilibrium sales are $q_0 = s$ for $s < s^*$ and fully represent genuine liquidity needs, while preemptive sales only occur for $s > s^*$, such a state-contingent policy would only result in a transfer from investors facing a liquidity shock to investors not facing a shock if $s > s^*$, i.e. in states of the world where the policy has to counter an inefficient run.

In Section 7, we discuss additional policy options such as asset purchases and temporary relaxation of dealer constraints which were implemented during the March 2020 episode and which involve trade-offs outside the scope of our model.

[17] Note that the threshold s^* would still be a function of the tax τ with $\partial s^*/\partial\tau > 0$ although the mapping would be different from the one without state contingency in Proposition 7.

6 Model with Safety Investors

Safe asset markets typically rally in times of stress with investors flocking toward Treasuries in a standard flight-to-safety, instead of selling Treasuries in a dash-for-cash as in March 2020. Accordingly, we want to understand how a market that typically experiences a flight-to-safety without a dash-for-cash could suddenly experience a dash-for-cash during a flight-to-safety episode and whether there is an interaction between the two phenomena.

We now introduce a second type of investors who are risk averse and hold a portfolio of the safe asset and the risky asset. In bad states of the world with negative shocks to the expected payoff of the risky asset, these "safety investors" increase their demand for the safe asset in a standard flight-to-safety, offsetting the flow of sales from liquidity investors or leading to repricing even in the absence of large trade volumes. Examples of real-world safety investors we have in mind include pension funds who face a traditional risk–return tradeoff and were among the largest net buyers of Treasuries in 2020q1 (Financial Accounts Table FU.210).

Safety investors could be active both at date 0 and at date 1. Additional safe asset demand at date 1 directly increases the price at date 1, which reduces the incentive of liquidity investors to sell preemptively at date 0 and therefore has a natural stabilizing effect on the strategic interaction at date 0. In contrast, additional demand at date 0 increases both the price at date 0 as well as the price at date 1 — by reducing dealer inventory — with an ambiguous overall effect on market stability at date 0. To focus attention on this ambiguous effect in the interaction between liquidity investors and safety investors, we restrict attention to the case in which safety investors are active only at date 0. Appendix G discusses the general case.

What if, instead, we modeled flight-to-safety simply as an increase in the marginal investor's valuation? Then a flight-to-safety would be unambiguously destabilizing, as we show in Appendix H. But that is not the typical behavior in safe asset markets where a flight-to-safety typically stabilizes and supports the market. We show that a flight-to-safety that works through dealer inventory can instead produce this stabilizing effect, but can also destabilize the market and trigger a dash-for-cash when markets are fragile.

6.1 Safety Investors' Asset Demand

Safety investors' utility is linear in consumption at date 0 and quadratic in future wealth,

$$u(c_0, w) = c_0 + w - \frac{1}{2}\kappa w^2,$$

where the curvature parameter $\kappa > 0$ commingles risk aversion and intertemporal substitution and we assume $w < 1/\kappa$. In addition to the safe asset with future payoff 1, there is a risky asset with future payoff z distributed according to H_z, where we denote the expected payoff as $\mu_z = \int z \, dH_z(z)$ and the variance as $\sigma_z^2 = \int z^2 \, dH_z(z) - \mu_z^2$.

Given initial wealth w_0, safety investors choose consumption c_0, and a portfolio with holdings q_0^S of the safe asset and q_z of the risky asset, subject to the budget constraint $c_0 + p_0 q_0^S + p_z q_z \leq w_0$, to maximize $E[u(c_0, w)]$, where future wealth is given by $w = q_0^S + z q_z$. After substituting in for c_0 using the budget constraint, we have first-order conditions for q_0^S and q_z given by

$$0 = E\left[1 - \kappa \left(q_0^S + z q_z\right)\right] - p_0 = 1 - \kappa \left(q_0^S + z q_z\right) - p_0,$$

and

$$0 = E\left[z - \kappa \left(q_0^S + z q_z\right) z\right] - p_z = \mu_z - \kappa \left(\mu_z q_0^S + \left(\mu_z^2 + \sigma_z^2\right) q_z\right) - p_z,$$

which are both linear in q_0^S and q_z. Solving, we arrive at safety investors' demand for the safe asset and the risky asset given by

$$q_0^S = \frac{1}{\kappa \sigma_z^2} \left(\sigma_z^2 + \mu_z p_z - \left(\mu_z^2 + \sigma_z^2\right) p_0\right) \quad \text{and} \quad q_z = \frac{1}{\kappa \sigma_z^2} \left(\mu_z p_0 - p_z\right),$$

while their consumption at date 0 is given as the residual $c_0 = w_0 - \left(p_z q_z + p_0 q_0^S\right)$.

To close the model and impose general equilibrium, we assume that safety investors hold the entire supply $Z > 0$ of the risky asset, i.e. $q_z = Z$. In this case, the risky asset price is $p_z = \mu_z p_0 - \kappa \sigma_z^2 Z$ and drops after a negative shock to the risky asset's expected payoff μ_z (as the S&P 500 did in March 2020). Substituting in the equilibrium p_z, safety investors' demand for the safe asset simplifies to

$$q_0^S = a - b p_0, \tag{15}$$

with $a = 1/\kappa - \mu_z Z$ and $b = 1/\kappa$, which is linear in p_0 and has a similar structure to dealers' demand in equation (9). We are interested in shocks to the risky asset's expected payoff μ_z, which enters safety investors' safe asset demand only through the intercept a. A decrease in μ_z is therefore equivalent to an increase in a and implies a flight-to-safety as a level shift in safety investors' demand for the safe asset.[18]

[18] We use a shock to the expected payoff as a tractable way to generate an increase in demand for safe assets within this simple setting. Fluctuations of asset prices are often captured, instead, by variations in required risk premia which would suggest a shock to κ. However, the parameter κ commingles investors' attitudes toward risk and toward intertemporal substitution. A shock only to required risk premia could be captured

6.2 Interaction of Safety Investors and Liquidity Investors

We continue the analysis using the simple linear structure that results from quadratic balance sheet costs and sequential trade execution derived in Section 4.4. Combining the demand from dealers implied by (9), $q_0^D = \frac{1}{c}(1 - p_0)$, with the demand from safety investors, $q_0^S = a - bp_0$, yields total demand $q_0^D + q_0^S$ for safe assets at date 0, which can be rewritten as the equilibrium price at date 0:

$$p_0(q_0) = \frac{1+ac}{1+bc} - \frac{c}{1+bc}\, q_0 \tag{16}$$

An increase in the additional demand from safety investors (higher a) therefore uniformly increases the price at date 0.

At date 1, only dealers buy the safe asset so demand is unchanged from equation (9). However, dealer inventory is no longer the entire date-0 supply q_0 as some of these sales have been absorbed by safety investors. Specifically, at the price $p_0(q_0)$ in (16), dealer inventory is given by $q_0^D = \frac{1}{c}(1 - p_0(q_0)) = (q_0 + b - a) / (1 + bc)$ such that the equilibrium price at date 1 becomes

$$p_1^s(q_0, q_1^s) = 1 - 2c \underbrace{\frac{q_0 + b - a}{1+bc}}_{\text{date-0 inventory}} - cq_1^s.$$

An increase in the additional demand from safety investors at date 0 (higher a) therefore uniformly increases the price at date 1 by decreasing the inventory on dealer balance sheets.

Substituting in the supply $q_0 = s + (1-s)\lambda$ and $q_1^s = s(1-s)(1-\lambda)$ and accounting for the factor $1/2$ due to sequential execution, the payoff gain with safety investors becomes

$$\begin{aligned}\pi(\lambda, s) = {} & \overbrace{\frac{1+ac}{1+bc} - \frac{1}{2}\frac{c}{1+bc}\left(s + (1-s)\lambda\right)}^{E[p_0(\lambda)]} \\ & - s\underbrace{\left(1 - 2c\,\frac{s + (1-s)\lambda + b - a}{1+bc} - \frac{c}{2}\, s(1-s)(1-\lambda)\right)}_{E\left[p_1^s(\lambda)\right]} - (1-s)v.\end{aligned}$$

As anticipated, a flight-to-safety in the form of an increase in safety investor demand

in a model with preferences that separate risk aversion from intertemporal decision (e.g. Epstein and Zin, 1989).

(higher a) has an ambiguous effect on the incentive to sell π. The question is if (or when) flight-to-safety demand is stabilizing (decreases π) or destabilizing (increases π).

Proposition 8. *Flight-to-safety demand increases the incentive to sell preemptively if and only if liquidity risk is low, $\partial\pi/\partial a > 0 \Leftrightarrow s < 1/2$. The effect of additional demand is monotonic in liquidity risk, $\partial^2\pi/(\partial s \partial a) < 0$.*

Where does the ambiguous effect of a on π originate? Similar to strategic sales by liquidity investors, purchases from safety investors have a direct effect and an indirect effect on the payoff gain π. The direct effect of an increase in demand a is an increase in the date-0 price p_0 and therefore an increase in the payoff gain π with a coefficient of $c/(1+bc)$. This effect is destabilizing since a higher price at date 0 incentivizes strategic investors to sell preemptively.

The indirect effect works through relaxing dealer balance sheet constraints, which increases the date-1 price p_1^s and therefore reduces the payoff gain π with a coefficient (in absolute value) of $2s \times c/(1+bc)$, similar to the direct effect except for the factor $2s$. The factor 2 arises because of the larger effect of existing date-0 inventory on dealer demand than of new date-1 inventory (Corollary 1). However, the effect on p_1^s is discounted by the liquidity shock probability s because it is only relevant if the investor actually suffers a liquidity shock at date 1.

Overall, the destabilizing effect of a higher date-0 price dominates the stabilizing effect of a higher date-1 price for low liquidity risk, $s < 1/2$, when the investor is unlikely to face the higher date-1 price. In this case, flight-to-safety increases the incentive to sell preemptively. Vice versa for high liquidity risk, $s > 1/2$, the stabilizing effect dominates such that flight-to-safety decreases the incentive to sell.

The payoff gain with safety investor demand retains the standard global game conditions of Morris and Shin (2003) so, for vanishing signal noise, the unique equilibrium remains in switching strategies around a threshold s^* defined by the indifference condition $\int_0^1 \pi(\lambda, s^*)\, d\lambda = 0$ as in Proposition 6. In particular, recall that π is increasing in s so an exogenous decrease in π leads to a higher threshold s^*, capturing higher market stability. The ambiguous effect of safety investor demand on the payoff gain π from Proposition 8 therefore directly translates into an analogous effect on market stability.

Corollary 6. *Flight-to-safety demand is stabilizing if the market is relatively stable and destabilizing if the market is relatively fragile, $ds^*/da > 0 \Leftrightarrow s^* > 1/2$.*

Figure 5 illustrates the ambiguous effect of flight-to-safety demand on market stability by comparing two markets with different levels of dealer balance sheet cost c. When balance sheet costs are low, the market is relatively stable: the threshold s^* where the price

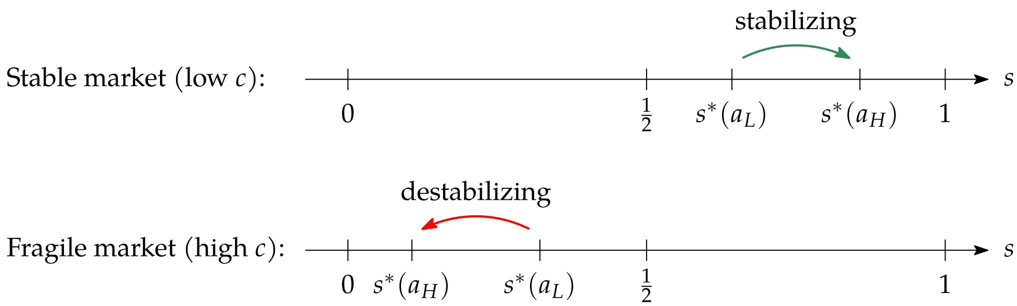

Figure 5: Effect of flight-to-safety on equilibrium market stability. The figure shows the effect of an increase in safety investor demand from a_L to a_H on market stability s^* for different levels of dealer balance sheet cost c.

drops precipitously is above $1/2$. In this case, liquidity investors sell preemptively at date 0 only if liquidity risk s is very high (i.e. only if they are very likely to be forced to sell at date 1). In this environment of high liquidity risk, the stabilizing effect of flight-to-safety demand increasing the price at date 1 dominates and the run threshold s^* is increasing in a, so that runs become less likely as safety demand increases from a_L to a_H.

When balance sheet costs are high, in contrast, the market is relatively fragile with the threshold s^* below $1/2$. In this case, liquidity investors already sell preemptively when liquidity risk s is still low (i.e. when they are unlikely to be forced to sell at date 1). In this environment of low liquidity risk, the destabilizing effect of flight-to-safety demand increasing the price at date 0 dominates and the run threshold s^* is decreasing in a so higher safety demand is destabilizing. In fact, for a given level of liquidity risk that is close to but below the run threshold, an increase in safety investor demand can reduce the threshold sufficiently to tilt the market into the run equilibrium such that flight-to-safety triggers a dash-for-cash.

The interaction of liquidity investors and safety investors therefore results in a feedback effect in market stability. If the market is resilient to begin with (as before the GFC when dealer balance sheet costs were low), then liquidity investors and safety investors interact symbiotically: In times of stress, the additional demand for safe assets from safety investors has a stabilizing effect on the strategic interaction of liquidity investors and attenuates the risk of market breakdown. However, if the market is relatively fragile (as in the post-GFC environment of high dealer balance sheet costs), the relationship reverses: Additional demand from safety investors in times of stress further destabilizes the strategic interaction of liquidity investors, increasing their incentive to sell preemptively and thereby increasing the risk of market breakdown.

6.3 Correlated Liquidity and Safety Shocks

Now suppose the risks faced by liquidity investors and safety investors are correlated. In times of stress, liquidity investors face a higher risk of suffering a liquidity shock (i.e. s is high), and safety investors face a low payoff of the risky asset (i.e. μ_z is low and therefore a is high). To understand the net effect of increases in s and a on the safe asset market, we can derive the equilibrium price at date 0 as a function of s and a.

As before, total supply in the global game equilibrium is s for $s < s^*$ (all strategic investors hold) and 1 for $s > s^*$ (all strategic investors sell). Substituting into equation (16), the equilibrium price becomes

$$p_0^*(s,a) = \begin{cases} \frac{1}{1+bc}\left(1 - c\left(s-a\right)\right) & \text{for} \quad s < s^*(a), \\ \frac{1}{1+bc}\left(1 - c\left(1-a\right)\right) & \text{for} \quad s > s^*(a). \end{cases} \tag{17}$$

Figure 6 illustrates the equilibrium price for combinations of s and a with a contour plot. The figure shows a case in which the market is relatively fragile: The threshold s^* is always below 1/2, so the cliff where the price drops as the equilibrium switches from hold to run is decreasing in (s,a)-space: for liquidity risk s close to s^*, an increase in safety investor demand a can push the market over the cliff and trigger a price crash. In the hold equilibrium (i.e. for $s < s^*$), the expression in equation (17) shows that equal-sized increases in s and a exactly offset each other and leave the price unchanged so the contour lines in Figure 6 have a slope of 1. This implies that whenever safety demand a increases more than 1:1 with liquidity risk s and liquidity risk remains below the threshold s^*, we observe a classic flight-to-safety with p_0^* increasing (i.e. safe assets appreciating). This corresponds to the period from mid-February to early March 2020, where stock prices decreased and Treasury prices increased (Figure 1, Panel A). However, if the balance shifts and the increase in liquidity risk s outweighs the increase in safety demand a, the price p_0^* can decrease and suddenly drop, as s crosses the threshold s^* and the equilibrium shifts to a dash-for-cash. This corresponds to the period in mid-March 2020 when Treasury prices reversed their increase and dropped together with stock prices.

7 Policy Interventions in March 2020

The events of March 2020 triggered an immediate, short-term policy response and have sparked a lively debate about longer-term policy implications. In this section, we study the actual policy interventions through the lens of our model. We highlight the implications

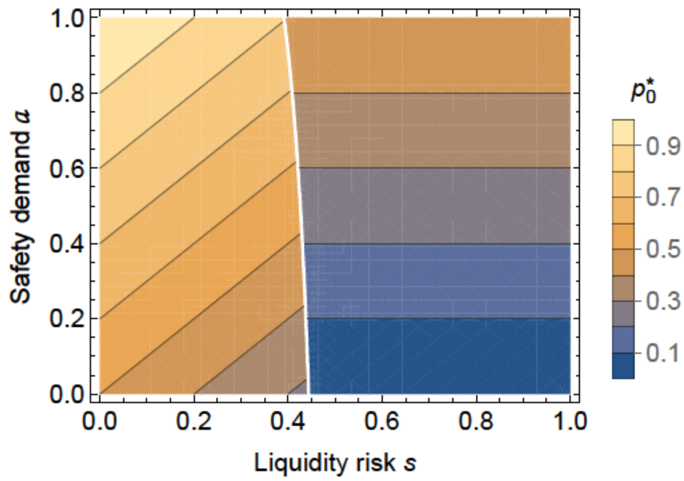

Figure 6: Equilibrium price for combinations of liquidity and safety shocks. The figure shows a contour plot of the equilibrium price p_0^* as a function of liquidity risk s and safety demand a. Parameters: $v = 1.2,\ b = 1, c = 1$.

that follow from the strategic, regime-change nature of our model where policy tools can have a large effects, both expected and unexpected, by switching the equilibrium.

Asset Purchases. Our model provides a clear lens to understand how asset purchases can be used to stabilize safe asset markets, primarily by switching the equilibrium. Consider Figure 7 which shows purchases of Treasuries by the Fed as well as net purchases of foreign official agencies, among the largest sellers of Treasuries in 2020q1 (Figure 1, Panel C). In early March, foreign net purchases started turning moderately negative for two weeks just as Treasury prices peaked, consistent with only non-strategic sales from investors with genuine liquidity needs. Then prices dropped and foreign sales accelerated as the Fed started purchasing Treasuries, consistent with switching to the run equilibrium, in which strategic investors without genuine liquidity needs preemptively sell, potentially amplified by the initial Fed purchases that may have been interpreted as limited in time and scale.[19] Only after the Fed roughly doubled its daily purchases on March 19 and then committed to maintaining them as long as necessary on March 23 did prices recover and foreign sales subside.[20]

[19]For its first purchase of roughly \$40 billion on Friday, March 13, the Fed "brought forward about half of the Treasury purchases previously scheduled for the mid-March to mid-April period into one day" (Federal Reserve Bank of New York, 2021).

[20]The statement by the Fed's Federal Open Market Committee (FOMC) on March 23 reads "The Federal Reserve will continue to purchase Treasury securities and agency mortgage-backed securities in the amounts needed to support smooth market functioning and effective transmission of monetary policy to broader financial conditions." Available at https://www.federalreserve.gov/newsevents/

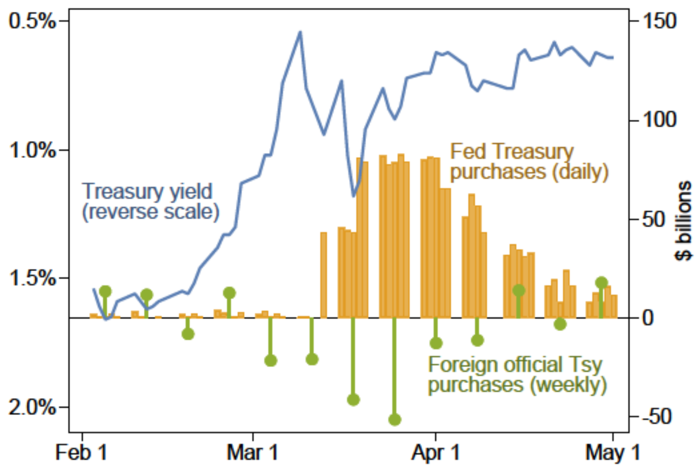

Figure 7: Federal Reserve purchases and foreign official sales of Treasuries. The figure shows the market yield on 10-year Treasuries as well as Federal Reserve Treasury purchases and foreign official agencies' net Treasury purchases inferred from changes in custody holdings. For details on the data see Appendix A.

Our model helps us interpret this behavior, which is consistent with the market switching from the run equilibrium to the hold equilibrium once investors were confident that they would not face worse prices in the future. Suppose the policy maker announces at date 0 an asset purchase facility that will purchase a quantity q_1^F of the safe asset at date 1. This leaves demand at date 0 unchanged but adds to dealer demand at date 1, such that the payoff gain becomes

$$\pi(\lambda, s) = 1 - \frac{c}{2}\left(s + (1-s)\,\lambda\right) \\ - s\underbrace{\left(cq_1^F + 1 - 2c\left(s + (1-s)\,\lambda\right) - \frac{c}{2}\,s\,(1-s)\,(1-\lambda)\right)}_{p_1^s \text{ affected by } q_1^F} - (1-s)\,v.$$

The payoff gain is uniformly decreasing in the size of the facility q_1^F, and this stabilizing effect, given by $\partial\pi/\partial q_1^F = -sc$, is larger (in absolute value) for both higher degrees of liquidity risk s and higher dealer balance sheet costs c.

Figure 8 illustrates the effects of the purchase facility. Panel A shows the effect of the facility size q_1^F on market stability at date 0 as measured by the equilibrium threshold s^*. Consistent with the stabilizing effect of q_1^F being increasing in liquidity risk s, we see that market stability is increasing and convex in q_1^F until s^* reaches 1 and the market is perfectly stable. Panel B of Figure 8 shows the announcement effect of a facility on the date-0 price

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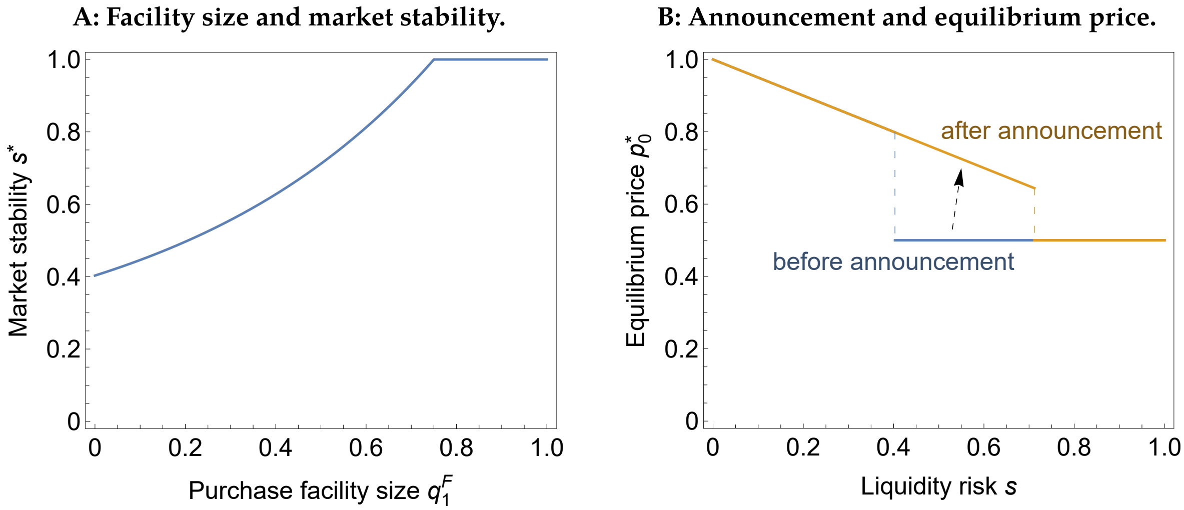

Figure 8: Effects of date-1 purchase facility announced at date 0. Panel A shows the effect of the facility size q_1^F on date-0 market stability as measured by the equilibrium threshold s^*. Panel B shows the effect of the announcement of a facility with $q_1^F = 0.5$ on the equilibrium price at date 0. Parameters: $v = 1.2,\ c = 0.5$.

p_0^*. Upon announcement, the equilibrium threshold s^* increases from the value without a facility, s^*_{pre}, to the value with a facility, $s^*_{\text{post}} > s^*_{\text{pre}}$. For intermediate levels of liquidity risk, $s \in \left[s^*_{\text{pre}}, s^*_{\text{post}}\right]$, the announcement leads to a switch from the run equilibrium to the hold equilibrium and therefore a discrete jump in the date-0 price.

Our theoretical results suggest that what matters for stabilizing a fragile market is the announcement more than the purchases directly. However, this result should be interpreted with some care especially when there is little time between the announcement and execution of purchases as was the case for the Treasury market in March 2020 (Vissing-Jørgensen, 2021). In this case, the purchases can be interpreted as falling into period 0 or into period 1 with potentially opposite effects as official sector purchases in period 0 can be destabilizing and trigger strategic sales in the same way that purchases from safety investors can (Section 6).

The corporate bond market provides a clean illustration of the announcement effects implied by our model. While corporate bonds are not considered as safe (or liquid) as Treasuries, highly rated ones are on a spectrum of relative safety slightly below agency MBS (He and Song, 2022) and also feature flight-to-safety (Baele et al., 2019). Haddad, Moreira, and Muir (2021) document that, in March 2020, prices of corporate bonds suffered a crash similar to that in Treasuries. Surprisingly, the dislocations were *worse* for bonds considered safer, which is consistent with safe asset fragility as shown in our model.

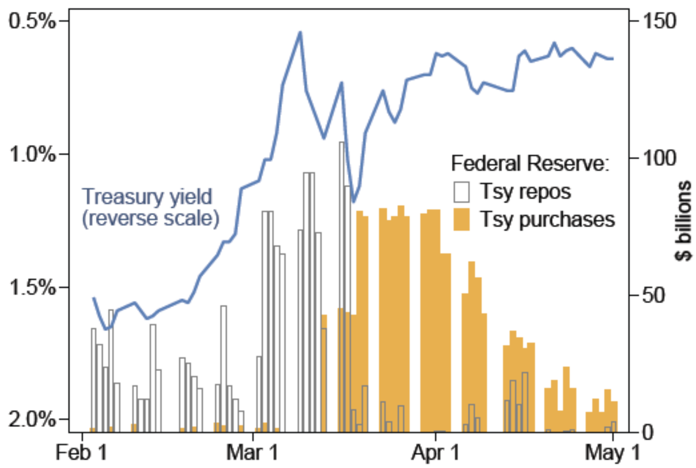

Figure 9: Federal Reserve interventions in Treasury markets. The figure shows the market yield on 10-year Treasuries and two types of Federal Reserve interventions: Treasury repos (lending against Treasury securities) and outright Treasury purchases. For details on the data see Appendix A.

Further, Haddad, Moreira, and Muir (2021) show in detail that the Fed's purchase facilities for corporate bonds had large positive effect on prices *at the time they were announced* in March even though purchases would not start until June (see also Boyarchenko, Kovner, and Shachar, 2022).

Dealer Constraints. In response to widespread claims at the time that tight regulatory constraints impeded dealers' ability to intermediate trades, the Fed pursued a number of interventions designed to alleviate dealer balance sheet constraints. Our results show that interventions to alleviate constraints only in the short-run could backfire.

Figure 9 shows in more detail the sequence of Fed interventions aimed at the Treasury market in the spring of 2020. Going into March, the Fed was conducting limited Treasury repo operations (lending against Treasuries) as part of its regular monetary policy implementation and was actually shrinking the offering size of these operations.[21] As conditions deteriorated starting March 9, repo offering sizes were increased to over $1 trillion by March 12 but, as shown in Figure 9, take-up by dealers was only moderate at around $100 billion and the liquidity provision through repos was not effective against the drop in Treasury prices. This is consistent with the SLR being the binding constraint on dealers, as the SLR is not relaxed by funding a Treasury position with a loan from the Fed (Duffie,

[21] On February 4 the New York Fed's Open Market Trading Desk decreased term repo operation offering size from $35 billion to $30 billion and again on February 13 from $30 billion to $25 billion, concurrent with a reduction in overnight repo from $120 billion to $100 billion (Federal Reserve Bank of New York, 2021).

2020).[22] Figure 9 shows that the recovery in Treasury prices in mid-March coincided with a switch by the Fed — from lending against Treasuries to purchasing them outright — and that the Fed was able to scale back purchases once Treasury holdings were exempted from the SLR on April 14 (announced on April 1).

How can we understand the effects of these policies? Dealer balance sheet costs play a crucial role in the strategic interaction of liquidity investors, since dealer inventory is the key link between the price at date 0 and the price at date 1. When considering only the interaction of liquidity investors, higher dealer balance sheet costs result in a more fragile safe asset market (i.e. a market that is more prone to runs and sudden price crashes; Figure 4B). Also taking into account the effect of additional demand from safety investors, an increase in dealer balance sheet costs can tip the market from a relatively stable region in which flight-to-safety has a stabilizing effect to a relatively fragile region in which flight-to-safety has a destabilizing effect (Figure 5). However, a policy that aims to relax dealer balance sheet constraints in times of stress has to be designed with care due to the subtleties of the strategic interaction. For example, if the policy relaxes dealer constraints only at date 0 (or relatively more at date 0), then it can increase the incentive to sell preemptively at date 0. If the market is in a run equilibrium, such a policy will appear to not have an effect, and if it is in the hold equilibrium then a short-run relaxation of constraints can precipitate a run. In addition, policy has to target the constraint that is actually binding.

If the SLR had been relaxed earlier, including for Treasury repos, the switch from the hold to the run equilibrium and the resulting market collapse could potentially have been avoided. Of course, the SLR and other post-GFC regulatory constraints were introduced for good reason. However, the fact that these constraints interfered with intermediation in safe assets during times of stress seems like an unintended consequence. For future stress episodes, a temporary relaxation of the SLR would therefore improve market stability in our model. Indeed, relaxing the SLR in times of stress would be a way to mitigate the fragility emphasized in our paper while maintaining the stabilizing macroprudential consequences for which the regulation was designed.

8 Conclusion

We focus on three key features of safe asset markets: investors who value the assets' safety, investors who value the assets' liquidity, and dealers who face balance sheet constraints. Combining these features, we show that safe asset markets, which are typically resilient,

[22]See Infante, Favara, and Rezende (2022) for a detailed study of the effect of the SLR on dealer's activity in the Treasury market.

can be fragile in that they are susceptible to sudden price crashes due to coordination effects among investors valuing liquidity that are amplified by investors valuing safety.

Our model helps us understand the unprecedented events in the U.S. Treasury market at the onset of the COVID-19 pandemic in March 2020 as a "perfect storm" of the three features: First, financial regulation in the wake of the GFC had significantly tightened dealer balance sheet constraints, increasing the inherent fragility of the market. Second, the pandemic threatened a global economic slowdown, leading to a powerful flight-to-safety demand, further destabilizing the market. Third, lockdowns created unprecedented liquidity needs among consumers and official agencies. The result, according to our model, was a market run in a market that is typically deep and resilient, featuring indiscriminate sales by liquidity investors, including those without genuine liquidity needs who feared having to sell at even worse conditions in the future.

The issues of dealer balance sheet constraints is almost surely only going to get worse over time as the federal deficit grows and Treasury supply increases. So long as dealers' balance sheet capacity grows more slowly than the stock of Treasuries, the market relying on dealer balance sheet capacity will have insufficient ability to intermediate trades (Duffie, 2020). Our model implies that this will exacerbate preemptive selling and increase the frequency of dash-for-cash episodes.

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Appendices

A Data

Treasury yield: The market yield on U.S. Treasury securities at 10-year constant maturity, daily frequency, from the Federal Reserve's H.15 via FRED series `DGS10`.

S&P 500: The S&P 500 index, daily frequency, from Standard & Poors via FRED series `SP500`.

Fed holdings of Treasuries: Federal Reserve outright holdings of Treasury notes and bonds (both nominal and TIPS), weekly frequency as of Wednesday, from the Federal Reserve's H.4.1 via FRED series `WSHONBNL` and `WSHONBIIL`.

Dealer net positions of Treasuries: Primary Dealers' net position in Treasuries (both nominal and TIPS) from the New York Fed's Primary Dealer statistics available at https://www.newyorkfed.org/markets/counterparties/primary-dealers-statistics.

Dealer reverse repo against Treasuries: Primary Dealers' gross reverse repurchase agreements against Treasuries (both nominal and TIPS), including other financing activity and securities borrowed, from the New York Fed's Primary Dealer statistics available at https://www.newyorkfed.org/markets/counterparties/primary-dealers-statistics.

Net purchases of Treasuries: Net purchases of Treasuries (all types), quarterly frequency (not seasonally adjusted), from the Federal Reserve's Financial Accounts Table FU.210 available in the CSV files at https://www.federalreserve.gov/releases/z1. The label "foreign investors" refers to the sector "rest of the world" in the original table.

Fed Treasury Purchases: Federal Reserve Treasury purchases (all types), daily frequency, from the New York Fed's Treasury securities operations, available at https://www.newyorkfed.org/markets/desk-operations/treasury-securities.

Foreign Official Treasury Purchases: Net Treasury purchases inferred from changes in Treasury securities held in custody for foreign officials and international accounts, weekly frequency as of Wednesday, from the Federal Reserve's H.4.1 via FRED series `WMTSECL1`.

Fed Treasury Repos: Federal Reserve Treasury repurchase agreements (overnight and term) in temporary open market operations, daily frequency, from the New York Fed via FRED series `RPTSYD`.

B Proofs

Proof of Proposition 1. Solving condition (6) for p_1^s implies an equilibrium price at date 1 given by

$$p_1^s(q_0, q_1^s) = 1 - \frac{C\left(q_0 + q_1^s\right) - C(q_0)}{q_1^s}.$$

In turn, combining conditions (4) and (6) implies an equilibrium price at date 0 given by

$$p_0(q_0) = 1 - \frac{C(q_0)}{q_0}.$$

We have $p_1^s < p_0$ if and only if

$$\frac{C\left(q_0 + q_1^s\right) - C(q_0)}{q_1^s} > \frac{C(q_0)}{q_0},$$

which can be rearranged as

$$\frac{C\left(q_0 + q_1^s\right)}{q_0 + q_1^s} > \frac{C(q_0)}{q_0},$$

which holds if C is strictly convex.

Given our focus on an equilibrium with $p_1^s < p_0$, let us explicitly consider why $p_1^s = p_0$ cannot be an equilibrium under our assumptions of competition and no commitment. In an equilibrium with $p_1^s = p_0 \equiv p$, the zero-profit condition at date 1 (4) becomes $(1-p)\left(q_0 + q_1^s\right) = C\left(q_0 + q_1^s\right)$, so that the total profit from buying assets at a discount to fundamental value exactly equals the total balance sheet costs. In this proposed equilibrium, convexity of balance sheet costs $C(q)$ requires

$$(1-p)\, q_0 > C(q_0) \quad \text{and} \quad (1-p)\, q_1^s < C(q_0 + q_1^s) - C(q_0). \tag{18}$$

The first inequality in (18) implies that dealers would earn positive profits on the date-0 inventory q_0 and then give away those profits on the additional date-1 inventory. But an optimizing dealer would never take on the additional inventory q_1^s at the same price as before if it reduces profits so $p_1^s = p_0$ cannot be an equilibrium. $\square$

Proof of Corollary 1. Convexity of $C(q)$ and $C(0) = 0$ imply that $C''(q) > 0$ and that $C'(q) > C(q)/q$ for $q > 0$. The fact that both prices are decreasing in both quantities follows:

$$
\begin{aligned}
\frac{\partial p_0}{\partial q_0} &= \frac{1}{q_0}\left(\frac{C(q_0)}{q_0} - C'(q_0)\right) < 0 \\
\frac{\partial p_1^s}{\partial q_0} &= -\frac{C'\left(q_0 + q_1^s\right) - C'(q_0)}{q_1^s} < 0 \\
\frac{\partial p_1^s}{\partial q_1^s} &= \frac{1}{q_1^s}\left(\frac{C\left(q_0 + q_1^s\right) - C(q_0)}{q_1^s} - C'(q_0 + q_1^s)\right) < 0
\end{aligned}
$$

To show that the inventory effect through q_0 is greater than the direct effect through q_1^s, i.e. $\partial p_1^s/\partial q_0 < \partial p_1^s/\partial q_1^s$, we need

$$
\begin{aligned}
-\frac{C'\left(q_0 + q_1^s\right) - C'(q_0)}{q_1^s} &< -\frac{C'\left(q_0 + q_1^s\right)}{q_1^s} + \frac{C\left(q_0 + q_1^s\right) - C(q_0)}{\left(q_1^s\right)^2} \\
\Leftrightarrow \quad C'(q_0) &< \frac{C\left(q_0 + q_1^s\right) - C(q_0)}{q_1^s},
\end{aligned}
$$

which holds because the properties of C imply

$$
C'(q_0) < \frac{C\left(q_0 + q_1^s\right) - C(q_0)}{q_1^s} < C'(q_0 + q_1^s).
$$

$\square$

Proof of Corollary 2. The comparative statics are given by

$$
\begin{aligned}
\frac{\partial p_0}{\partial \lambda} &= \frac{\partial p_0}{\partial q_0}(1 - s_0) < 0 \\
\frac{\partial p_0}{\partial s_0} &= \frac{\partial p_0}{\partial q_0}(1 - \lambda) < 0 \\
\frac{\partial p_1^s}{\partial \lambda} &= \left(\frac{\partial p_1^s}{\partial q_0} - s_1\frac{\partial p_1^s}{\partial q_1^s}\right)(1 - s_0) < 0 \\
\frac{\partial p_1^s}{\partial s_0} &= \left(\frac{\partial p_1^s}{\partial q_0} - s_1\frac{\partial p_1^s}{\partial q_1^s}\right)(1 - \lambda) < 0 \\
\frac{\partial p_1^s}{\partial s_1} &= \frac{\partial p_1^s}{\partial q_1^s}(1 - s_0)(1 - \lambda) < 0
\end{aligned}
$$

and the signs of the first, second and last follow directly from Corollary 1.

The only non-obvious comparative statics are the effects of λ and s_0 on p_1^s. Note that

$\partial p_1^s / \partial q_1^s < 0$ implies

$$\frac{\partial p_1^s}{\partial q_1^s} < s_1 \frac{\partial p_1^s}{\partial q_1^s},$$

and we also have

$$\frac{\partial p_1^s}{\partial q_0} < \frac{\partial p_1^s}{\partial q_1^s}.$$

Combining the two implies

$$\frac{\partial p_1^s}{\partial q_0} < s_1 \frac{\partial p_1^s}{\partial q_1^s}$$

and therefore $\partial p_1^s / \partial \lambda < 0$ and $\partial p_1^s / \partial s_0 < 0$. ☐

Proof of Proposition 2. We want to determine when $\partial \pi / \partial \lambda > 0$, i.e. when $\partial p_0 / \partial \lambda > s_1 \partial p_1^s / \partial \lambda$. In words, the initial price impact of more sellers, $\partial p_0 / \partial \lambda$, is negative, but we want to show that the future price impact of more sellers, weighted by liquidity risk, $s_1 \partial p_1^s / \partial \lambda$, is even more negative. Substituting in the expressions from the proofs above, using $C(q_0 + q_1^s) - C(q_0) = \int_0^{q_1^s} C'(q_0 + x)\, dx$ as well as $q_1^s = (1 - q_0)\, s_1$, and rearranging, the condition becomes

$$C'(q_0 + q_1^s) - C'(q_0) - s_1 \left(C'(q_0 + q_1^s) - \frac{1}{q_1^s} \int_0^{q_1^s} C'(q_0 + x)\, dx \right)$$
$$> \frac{1 - q_0}{q_0} \left(C'(q_0) - \frac{C(q_0)}{q_0} \right). \tag{19}$$

We provide a sufficient condition for (19) by deriving a lower bound on the LHS and an upper bound on the RHS:

- The upper bound on the RHS is simply

$$\frac{1 - q_0}{q_0} \left(C'(q_0) - \frac{C(q_0)}{q_0} \right) < \frac{1 - q_0}{q_0}\, C'(q_0) \tag{20}$$

- For the the lower bound on the LHS of (19), we proceed in two steps. First, convexity implies

$$C'(q_0 + q_1^s) - \frac{1}{q_1^s} \int_0^{q_1^s} C'(q_0 + x)\, dx > 0$$

and therefore $s_1 < 1$ implies

$$\begin{aligned} C'(q_0+q_1^s) - C'(q_0) - s_1\left(C'(q_0+q_1^s) - \frac{1}{q_1^s}\int_0^{q_1^s} C'(q_0+x)\,dx\right) \\ > C'(q_0+q_1^s) - C'(q_0) - \left(C'(q_0+q_1^s) - \frac{1}{q_1^s}\int_0^{q_1^s} C'(q_0+x)\,dx\right) \\ = \frac{1}{q_1^s}\int_0^{q_1^s} C'(q_0+x)\,dx - C'(q_0). \end{aligned}$$

Second, since C has non-negative higher order derivatives, a Taylor expansion implies $C'(q_0+x) \geq C'(q_0) + xC''(q_0)$ and therefore

$$\begin{aligned} \frac{1}{q_1^s}\int_0^{q_1^s} C'(q_0+x)\,dx - C'(q_0) &\geq \frac{1}{q_1^s}\int_0^{q_1^s} \left(C'(q_0) + xC''(q_0)\right)dx - C'(q_0) \\ &= \frac{1}{2}q_1^s C''(q_0). \end{aligned} \tag{21}$$

Using the two bounds (20) and (21), a sufficient condition for (19) is therefore

$$\frac{1}{2}q_1^s C''(q_0) > \frac{1-q_0}{q_0}\,C'(q_0),$$

which we can rewrite, using $q_1^s = (1-q_0)\,s_1$, as

$$\frac{1}{2}s_1 C''(q_0) > \frac{1}{q_0}\,C'(q_0).$$

For a general cost function C, a sufficient condition for strategic complementarities is therefore

$$s_1 \times \frac{qC''(q)}{C'(q)} > 2 \quad \text{for all} \quad q$$

Turning to the comparative statics for s_0 and s_1, note that future liquidity risk s_1 enters the payoff gain π directly and indirectly:

$$\frac{\partial \pi}{\partial s_1} = (v - p_1^s) - s_1\frac{\partial p_1^s}{\partial s_1} > 0$$

The effect is unambiguously positive as $v > p_1^s$ and $\partial p_1^s/\partial s_1 < 0$ from Corollary 2. Current liquidity risk s_0 affects π analogous to strategic sales λ:

$$\frac{\partial \pi}{\partial s_0} = \frac{\partial p_0}{\partial s_0} - s_1\frac{\partial p_1^s}{\partial s_0}$$

Substituting in for the partial derivatives of p_0 and p_1^s (see the expressions in the proof of Corollary 2), we therefore have $\partial\pi/\partial s_0 > 0$ if $\partial\pi/\partial\lambda > 0$. $\square$

Proof of Proposition 3. Quadratic balance sheet costs $C(q) = cq^2$ imply that the equilibrium prices from Proposition 1 are given by

$$p_0 = 1 - cq_0 \quad \text{and} \quad p_1^s = 1 - 2cq_0 - cq_1^s.$$

Substituting into the payoff gain and differentiating, we have

$$\frac{\partial\pi}{\partial\lambda} = c\,(1 - s_0)\,(s_1\,(2 - s_1) - 1)\,,$$

which is positive if and only if $s_1\,(2 - s_1) > 1$ but that cannot happen since $s_1\,(2 - s_1)$ has a maximum of 1 at $s_1 = 1$.

Cubic balance sheet costs $C(q) = cq^3$ imply that the equilibrium prices from Proposition 1 are given by

$$p_0 = 1 - cq_0^2 \quad \text{and} \quad p_1^s = 1 - c\left(3q_0^2 + 3q_0q_1^s + (q_1^s)^2\right)$$

Substituting into the payoff gain and differentiating, we have

$$\frac{\partial\pi}{\partial\lambda} = c\,(1 - s_0)\left((3 - 2s_1)\,s_1^2 - 2\,(s_0 + (1 - s_0)\,\lambda)\,(1 - s_1)^3\right),$$

the sign of which still depends on s_0 and λ. Note, however, that the second derivative is

$$\frac{\partial^2\pi}{\partial\lambda^2} = c\,(1 - s_0)^2\left(\left(2s_1^2 - 6s_1 + 6\right)s_1 - 2\right),$$

which is negative for $s_1 \in [0, 1]$. Therefore $\partial\pi/\partial\lambda > 0$ at $\lambda = 1$ is sufficient for $\partial\pi/\partial\lambda > 0$ uniformly. We have

$$\left.\frac{\partial\pi}{\partial\lambda}\right|_{\lambda=1} = c\,(1 - s_0)\left(-3s_1^2 + 6s_1 - 2\right),$$

which is positive for $s_1 > 1 - \sqrt{3}/3 \approx 0.42$. $\square$

Proof of Corollary 3. General power balance sheet costs $C(q) = cq^n$ imply that the equilibrium prices from Proposition 1 are given by

$$p_0 = 1 - cq_0^{n-1} \quad \text{and} \quad p_1^s = 1 - c\frac{\left(q_0 + q_1^s\right)^n - q_0^n}{q_1^s}$$

Substituting into the payoff gain and differentiating, we have

$$\frac{\partial \pi}{\partial \lambda} = c\left(1-s_0\right)\left(-\left(n-1\right)q_0^{n-2} - s_1\left(n\frac{\left(1-s_1\right)\left(q_0+q_1^s\right)^{n-1} - q_0^{n-1}}{q_1^s} + s_1\frac{\left(q_0+q_1^s\right)^n - q_0^n}{\left(q_1^s\right)^2}\right)\right),$$

with a limit given by

$$\lim_{\lambda\to 1}\frac{\partial \pi}{\partial \lambda} = c\left(1-s_0\right)\left(n-1\right)\left(\frac{n}{2}\left(2-s_1\right)s_1 - 1\right).$$

Solving $\lim_{\lambda\to 1}\partial\pi/\partial\lambda = 0$ for s_1 yields the threshold $\widetilde{s} = 1-\sqrt{(n-2)/n}$. $\square$

Proof of Proposition 4. With the linear price impacts $\partial p_0/\partial\lambda = -c\left(1-s_0\right)$ and $\partial p_1^s/\partial\lambda = -2c\left(1-s_0\right) + cs_1\left(1-s_0\right)$, the derivative of the payoff gain can be written as

$$\frac{\partial \pi}{\partial \lambda} = c\left(1-s_0\right)\left(s_1\left(2-s_1\right)u'(p_1^s) - u'(p_0)\right),$$

such that $\partial\pi/\partial\lambda > 0$ if and only if

$$s_1\left(2-s_1\right) > \frac{u'(p_0)}{u'(p_1^s)}.$$

With constant relative risk aversion $u(x) = x^{1-\gamma}/(1-\gamma)$, the condition simplifies to

$$s_1\left(2-s_1\right) > \left(\frac{p_1^s}{p_0}\right)^{\gamma}. \tag{22}$$

The linear price functions (9) imply that the ratio p_1^s/p_0 is decreasing in λ and s_0 such that condition (22) is hardest to satisfy for $\lambda = 0$ and $s_0 = 0$ where $p_1^s/p_0 = 1-cs_1$. A sufficient condition for $\partial\pi/\partial\lambda > 0$ uniformly is therefore

$$s_1\left(2-s_1\right) > \left(1-cs_1\right)^{\gamma}. \tag{23}$$

Note that $c, s_1 \in (0,1)$ imply $1-cs_1 < 1$ and therefore

$$\frac{d}{ds_1}\left(s_1\left(2-s_1\right) - \left(1-cs_1\right)^{\gamma}\right) = 2\left(1-s_1\right) + c\gamma\left(1-cs_1\right)^{\gamma-1} > 0,$$

and

$$\frac{d}{d\gamma}\left(s_1\left(2-s_1\right) - \left(1-cs_1\right)^{\gamma}\right) = -\left(1-cs_1\right)^{\gamma}\ln\left(1-cs_1\right) > 0,$$

such that condition (23) requires a sufficiently high s_1 and/or γ. $\square$

Proof of Proposition 5. Differentiating the payoff gain π in (10) with respect to λ yields

$$\frac{\partial \pi}{\partial \lambda} = \frac{c}{2}\left(1-s_0\right)\left(s_1\left(4-s_1\right)-1\right), \tag{24}$$

with $c > 0$, $s_0, s_1 \in (0,1)$, $v > 1$, and $\lambda \in [0,1]$, which imply the following comparative statics:

- We have $\partial\pi/\partial\lambda > 0$ if and only if $s_1\left(4-s_1\right)-1 > 0$ which has one root in the unit interval given by $\widetilde{s} = 2-\sqrt{3}$.
- Differentiating π with respect to s_1, we have

  $$\frac{\partial \pi}{\partial s_1} = \left(v - p_1^s\right) + s_1 \frac{c}{2}\left(1-s_0\right)\left(1-\lambda\right),$$

  which is positive.
- Differentiating π with respect to s_0, we have

  $$\frac{\partial \pi}{\partial s_0} = \frac{c}{2}\left(1-\lambda\right)\left(s_1\left(4-s_1\right)-1\right),$$

  which is analogous to $\partial\pi/\partial\lambda$ above and therefore positive for $s_1 > \widetilde{s}$.
- Differentiating π with respect to c, we have

  $$\frac{\partial \pi}{\partial c} = \frac{1}{2}\left(s_1^2\left(1-s_0\right)\left(1-\lambda\right) + \left(4s_1-1\right)\left(s_0+\left(1-s_0\right)\lambda\right)\right)$$

  which is positive for s_1 sufficiently large. The threshold is lower than $\widetilde{s}$ as

  $$\left.\frac{\partial \pi}{\partial c}\right|_{s_1=\widetilde{s}} = \frac{1}{2}\left(7-4\sqrt{3}\right) \approx 0.036,$$

  so $\partial\pi/\partial c$ is positive for $s_1 > \widetilde{s}$. $\square$

Proof of Corollary 4. The proof is analogous to the proof of Proposition 5 after setting $s_0 = s_1 = s$ in the payoff gain (10). Differentiating π with respect to λ yields

$$\frac{\partial \pi}{\partial \lambda} = \frac{c}{2}\left(1-s\right)\left(s\left(4-s\right)-1\right),$$

with $c > 0$, $s \in (0,1)$, $v > 1$, and $\lambda \in [0,1]$, which imply the following comparative statics:

- We have $\partial\pi/\partial\lambda > 0$ if and only if $s(4-s)-1 > 0$ which has one root in the unit interval given by $\widetilde{s} \equiv 2-\sqrt{3}$.

- Differentiating π with respect to s, we have

$$\frac{\partial\pi}{\partial s} = \frac{c}{2}\left(\left(10-3s\right)\left(1-\lambda\right)s + \left(5\lambda-1\right)\right) + v - 1,$$

  which is positive unless λ is small. For $s > \frac{1}{3}\left(5-\sqrt{22}\right) \approx 0.103$, it is positive for all λ and therefore also for $s > \widetilde{s}$.

- Differentiating π with respect to c, we have

$$\frac{\partial\pi}{\partial c} = \frac{1}{2}s^2 + \frac{1}{2}\left(s\left(4-s\right)-1\right)\left(s+\left(1-s\right)\lambda\right),$$

  which is positive for $s > \widetilde{s}$. □

Proof of Proposition 6. In order to apply the standard global game result that there is a unique equilibrium and that it is in switching strategies, we have to show that the payoff gain $\pi(\lambda,s)$ satisfies certain properties (Morris and Shin, 2003). Corollary 4 establishes State Monotonicity and Action Monotonicity, that is $\pi(\lambda,s)$ is increasing in s and increasing in λ for $s > \widetilde{s}$, which is satisfied if there are multiple equilibria of the complete-information game. The payoff gain satisfies Strict Laplacian State Monotonicity since we have

$$\int_0^1 \pi(\lambda,s)\,d\lambda = \frac{c}{2}\left(s^2 + s\left(s\left(4-s\right)-1\right) + \frac{1}{2}\left(1-s\right)\left(s\left(4-s\right)-1\right)\right) - \left(1-s\right)\left(v-1\right), \tag{25}$$

which satisfies

$$\int_0^1 \pi(\lambda,0)\,d\lambda = -\left(v-1\right) - \frac{c}{4} < 0,$$

and

$$\int_0^1 \pi(\lambda,1)\,d\lambda = \frac{3c}{2} > 0,$$

as well as

$$\frac{\partial}{\partial s}\int_0^1 \pi(\lambda,s)\,d\lambda = \frac{c}{2}\left(2s + \frac{1}{2}\left(\left(s\left(4-s\right)-1\right) + \left(1+s\right)\left(4-2s\right)\right)\right) + \left(v-1\right) > 0,$$

for $s > \widetilde{s}$ and therefore a unique $s^* \in (\widetilde{s}, 1)$ solves $\int_0^1 \pi(\lambda, s^*)\, d\lambda = 0$. Finally, $\pi(\lambda, s)$ satisfies Uniform Limit Dominance since we have

$$\pi(\lambda, 0) = -(v-1) - \frac{c}{2}\lambda < 0,$$

and

$$\pi(\lambda, 1) = \frac{3c}{2} > 0.$$

Under these properties, Morris and Shin (2003) show that, in the limit $\sigma_\varepsilon \to 0$, the global game has a unique equilibrium and that the equilibrium is in switching strategies around a threshold s^* defined by the indifference condition $\int_0^1 \pi(\lambda, s^*)\, d\lambda = 0$ where the distribution of λ conditional on signal $\widehat{s}_i = s^*$ is uniform on $[0, 1]$. □

Proof of Corollary 5. From the equilibrium condition $\int_0^1 \pi(\lambda, s^*)\, d\lambda = 0$, implicit differentiation using (25) yields

$$\frac{ds^*}{dc} = -\frac{\frac{1}{2}\left((s^*)^2 + s^*(s^*(4-s^*)-1) + \frac{1}{2}(1-s^*)(s^*(4-s^*)-1)\right)}{\frac{c}{2}\left(2s^* + \frac{1}{2}((s^*(4-s^*)-1) + (1+s^*)(4-2s^*))\right) + (v-1)} < 0$$

and

$$\frac{ds^*}{dv} = \frac{1-s^*}{\frac{c}{2}\left(2s^* + \frac{1}{2}((s^*(4-s^*)-1) + (1+s^*)(4-2s^*))\right) + (v-1)} > 0$$

as stated in the corollary. □

Proof of Proposition 7. The payoff gain under the Pigouvian tax is given by

$$\begin{aligned}\pi(\lambda, s) = (1-\tau)&\left(\frac{c}{2}\left((4s-1)(s+(1-s)\lambda) + s^2(1-s)(1-\lambda)\right)\right) \\ &- (1-s)(v-(1-\tau))\end{aligned}$$

and the other properties of π relevant to the global game are unaffected. Analogous to the proof of Corollary 5, implicit differentiation of the equilibrium condition $\int_0^1 \pi(\lambda, s^*)\, d\lambda = 0$ where

$$\begin{aligned}\int_0^1 \pi(\lambda, s)\, d\lambda = (1-\tau)\frac{c}{2}&\left(s^2 + s(s(4-s)-1) + \frac{1}{2}(1-s)(s(4-s)-1)\right) \\ &- (1-s)(v-(1-\tau))\end{aligned}$$

yields

$$\frac{ds^*}{d\tau} = -\frac{-\frac{c}{2}\left(s^2 + s\left(s\left(4-s\right)-1\right) + \frac{1}{2}\left(1-s\right)\left(s\left(4-s\right)-1\right)\right) - \left(1-s\right)}{\frac{c}{2}\left(2s^* + \frac{1}{2}\left(\left(s^*\left(4-s^*\right)-1\right) + \left(1+s^*\right)\left(4-2s^*\right)\right)\right) + \left(v - \left(1-\tau\right)\right)} > 0,$$

such that market stability s^* and therefore ex-ante welfare $E[W]$ is increasing in τ. Furthermore, for $\tau = 1$, we have

$$\int_0^1 \pi(\lambda, s)\, d\lambda = -\left(1-s\right) v < 0$$

such that a sufficiently high tax rate can eliminate runs entirely. ☐

Proof of Proposition 8. We can rewrite the payoff gain with additional demand as

$$\begin{aligned}\pi(\lambda, s) = &\frac{c}{2}s^2 + \frac{c\left(a-b\right)}{1+bc}\left(1-2s\right) - \left(1-s\right)\left(v-1\right) \\ &+ \frac{c}{2}\left(\frac{1}{1+bc}\left(4s-1\right) - s^2\right)\left(s + \left(1-s\right)\lambda\right),\end{aligned}$$

and differentiate with respect to a to obtain

$$\frac{\partial \pi}{\partial a} = \frac{c}{1+bc}\left(1-2s\right),$$

and

$$\frac{\partial^2 \pi}{\partial s \partial a} = -\frac{2c}{1+bc}.$$

We therefore have $\partial\pi/\partial a > 0$ if and only if $s < 1/2$ as well as $\partial^2\pi/(\partial s\partial a) < 0$. ☐

Proof of Corollary 6. The global game threshold is defined by $\int_0^1 \pi(\lambda, s^*)\, d\lambda = 0$ and implicit differentiation yields

$$\frac{ds^*}{da} = -\frac{\int_0^1 \frac{\partial}{\partial a}\pi(\lambda, s^*)\, d\lambda}{\int_0^1 \frac{\partial}{\partial s^*}\pi(\lambda, s^*)\, d\lambda},$$

and therefore $ds^*/da > 0$ if and only if $s^* > 1/2$. ☐

C Comparison to Bernardo and Welch (2004)

Our model deviates from Bernardo and Welch (2004), hereafter BW, in a few important details which allow us to apply global game methods in a model of regime shifts and to derive a state-contingent interaction between flight-to-safety and dash-for-cash.

The first key difference between the two models is as follows: BW is intended as a model of stock price crashes and has only one (risky) asset which dealers and investors both value at its expected value μ. In our model, dealers value the (safe) asset at its par value of 1 while liquidity investors value the asset at a convenience yield $v > 1$. Such a difference in valuations is natural when thinking of a safe asset that conveys specific benefits to certain investors. We have $\pi(\lambda = 0, s = 0) = -(v-1)$ so $v > 1$ guarantees that the hold equilibrium $\lambda^* = 0$ exists for sufficiently low liquidity risk, i.e. the existence of $\underline{s} \in (0,1)$ such that $\pi(0,s) < 0$ for $s < \underline{s}$. In contrast, BW do not have a hold equilibrium, only a mixed or a run equilibrium. The possibility of a hold equilibrium is both empirically plausible and technically important: Empirically plausible because we do not think that investors routinely sell assets preemptively during normal times as the BW model implies; technically important because it provides the second pure-strategy equilibrium that is necessary for a true model of regime shifts.

The second key difference is that our model allows for strategic complementarities which are another necessary ingredient for a model of regime shifts as it allows for multiplicity of equilibria under complete information. We show that strategic complementarities can arise (i) if dealers' marginal balance sheet costs are sufficiently elastic, such that inventory taken on at date 0 sufficiently impacts the price at date 1 if there are further sales, (ii) if liquidity investors are risk averse such that a potential price drop between date 0 and date 1 is compounded by an increase in marginal utility, and/or (iii) if trades are sequentially executed, such that an investor selling preemptively at date 0 can expect to front-run some of the other investors selling. These features are in contrast to the model of Bernardo and Welch (2004), where price impact is due to dealer risk aversion, investors are assumed to be risk-neutral throughout, and trades are pooled before execution; therefore strategic complementarities do not arise in their model.

A third difference is that BW only consider liquidity risk at date 1, i.e. assume $s_0 = 0$ and $s_1 = s$ and therefore cannot distinguish between sales due to genuine liquidity needs and preemptive sales for strategic reasons. As a result of these differences, the unique equilibrium in the BW model features preemptive sales $\lambda^*(s)$ that are continuously increasing in the degree of liquidity risk s from $\lambda^*(0) = 0$ to $\lambda^*(\bar{s}) = 1$ at some $\bar{s} \leq 1$. Whenever there is positive liquidity risk $s > 0$, the BW model predicts that some investors sell preemptively and that the date 0 price is continuously decreasing in s until $\bar{s}$ (and then constant). In contrast, the unique global game equilibrium in our model features a discrete regime shift. As long as liquidity risk is sufficiently low, only investors with genuine liquidity needs sell and no investors sell preemptively; once liquidity risk exceeds the global game threshold, liquidity investors that do not have current liquidity needs suddenly sell preemptively. In our model, preemptive sales therefore jump discontinuously from $\lambda^*(s) = 0$ for s below the switching point s^* to $\lambda^*(s) = 1$ for s above s^* and the date 0 price drops precipitously as s crosses the threshold.

Finally, our model features safety investors whose demand for the asset has an ambiguous effect on liquidity investors' strategic sales. In the BW model, market depth at date 0 is destabilizing and market depth at date 1 is stabilizing but the net effect of more market depth at both dates does not vary with the degree of liquidity risk — it is either uniformly positive or uniformly negative. In contrast, our modeling of safety investors — who increase market depth both at date 0 and at date 1 — combined with our regime shift model of liquidity investors generates one of our key results: more market depth at both dates is stabilizing if the market is relatively stable (high s^*) but destabilizing if the market is relatively unstable (low s^*).

D Model with Aggregate Risk

Suppose, that investors only face liquidity risk at date 1 if the aggregate environment is stressed (or continues to be stressed) which happens with probability α. With probability $1-\alpha$, the aggregate environment is "back to normal" at date 1, such that investors are not at risk of liquidity shocks anymore. With this simple binomial aggregate state, liquidity shocks at date 1 are correlated and the price conditional on receiving a shock, $p_1^s \equiv E_0[p_1|\text{shock}]$, is different from the price conditional on not receiving a shock, $p_1^{ns} \equiv E_0[p_1|\text{no shock}]$.

Specifically, sales at date 0 are unchanged as $q_0 = s_0 + (1-s_0)\,\lambda$ while sales at date 1 now depend on the aggregate state with $q_1^s = s_1\,(1-s_0)\,(1-\lambda)$, as in the main text, and $q_1^{ns} = 0$. The equilibrium prices at date 0 is similarly unchanged from Proposition 1 as $p_0(q_0) = 1 - C(q_0)/q_0$ but the price at date 1 depends on the aggregate state. In the bad state, the price is $p_1^s(q_0, q_1^s) = 1 - \big(C(q_0 + q_1^s) - C(q_0)\big)/q_1^s$, as derived in the main text. As we show below, all the properties derived for the date-1 price in the main text are really only required for p_1^s and therefore do not imply any restrictions on p_1^{ns}, the price conditional on *not* receiving a liquidity shock; in the good state, where dealers don't have to take on additional inventory since $q_1^{ns} = 0$, we can therefore assume, e.g. that no-arbitrage holds such that the price equals fundamentals as $p_1^{ns} = 1$. Then the state-contingent prices satisfy $p_1^s < p_0 < p_1^{ns}$ such that the model no longer features a pure arbitrage. Because the prices p_0 and p_1^s are unchanged from the main text such that Proposition 1 and its Corollaries 1 and 2 continue to hold.

Turning to the strategic interaction among liquidity investors, the payoff gain becomes

$$\pi = p_0 - \alpha s_1 p_1^s - (1 - \alpha s_1)\, v.$$

Crucially, the only date-1 price that is relevant for a strategic investor's decision at date 0 is p_1^s and the payoff gain with aggregate risk is almost identical to the one in the main text in (2), except for replacing s_1 by αs_1. In principle, we could therefore redefine $\widehat{s}_1 \equiv \alpha s_1$.

However, note that s_1 also enters π indirectly through the effect of q_1^s on p_1^s and that α does not appear in q_1^s (because it is conditional on the bad state). For the sufficient condition in Proposition 2, this does not matter such that we can follow the same steps as in the proof in Appendix B and arrive at the sufficient condition $\alpha s_1 \times \eta > 2$ which simply replaces s_1 by αs_1. For the tighter sufficient conditions in Propositions 3 to 5, s_1 appears in the relevant expressions on its own and as αs_1 but the monotonicity of the expressions with respect to s_1 remains unchanged such that any threshold for the redefined $\widehat{s}_1$ is similar to the corresponding threshold for s_1 in the model without aggregate risk. As a result, the analysis remains essentially unchanged except for replacing statements of the form "for sufficiently high date-1 liquidity risk s_1" by statements of the form "for sufficiently high risk α of a stressed environment at date 1 and sufficiently high liquidity risk s_1 conditional on the stressed environment."

To verify this and gauge the quantitative effect, consider the threshold for strategic complementarities in Proposition 5 which yields Corollary 4 and is therefore the basis of our global game analysis. With aggregate risk, the payoff gain (10) becomes

$$\begin{aligned}
\pi(\lambda, s_0, s_1) &= 1 - \frac{c}{2}\left(s_0 + (1 - s_0)\,\lambda\right) - \alpha s_1 \Big(1 - 2c\left(s_0 + (1 - s_0)\,\lambda\right) - \frac{c}{2} s_1 \left(1 - s_0\right)\left(1 - \lambda\right)\Big) \\
&\quad - \left(1 - \alpha s_1\right) v \\
&= \frac{c}{2}\Big(\left(4\alpha s_1 - 1\right)\left(s_0 + (1 - s_0)\,\lambda\right) + \alpha s_1^2 \left(1 - s_0\right)\left(1 - \lambda\right)\Big) - \left(1 - \alpha s_1\right)\left(v - 1\right)
\end{aligned}$$

Differentiating this payoff gain with respect to λ yields

$$\frac{\partial \pi}{\partial \lambda} = \frac{c}{2}\left(1 - s_0\right)\left(\alpha s_1 \left(4 - s_1\right) - 1\right).$$

Compared to the analogous expression (24) for the case without aggregate risk, one of

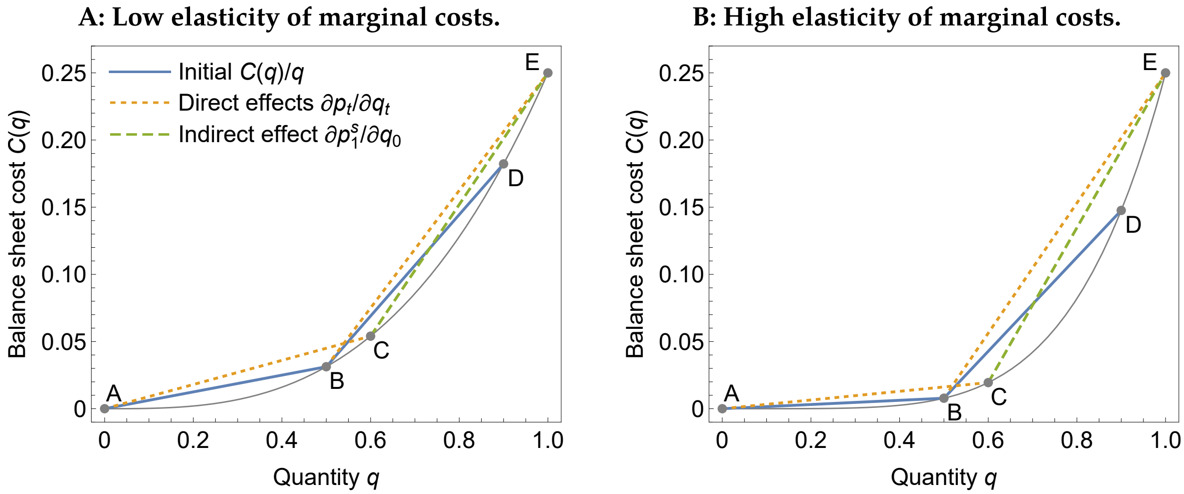

Figure 10: Elasticity of marginal costs and strategic complementarities. The figure shows the balance sheet cost $C(q)$ (gray curve) and average marginal costs $C(q)/q$ between different points (colored lines) for balance sheet cost function $C(q) = cq^n$. Panel A shows low elasticity of marginal costs for $n = 3$. Panel B shows high elasticity of marginal costs for $n = 5$. Other parameters: $c = 0.25$, $q_0 = 0.5$, $q_1^s = 0.4$ and $\delta = 0.1$.

the two occurrences of s_1 is replaced by αs_1. With aggregate risk, we have strategic complementarities, $\partial\pi/\partial\lambda > 0$, if and only if $\alpha s_1 (4 - s_1) - 1 > 0$ which implies a threshold $2\alpha - \sqrt{\alpha(4\alpha - 1)}$ for the redefined $\widehat{s}_1$. This threshold is greater than the corresponding threshold $2 - \sqrt{3}$ for s_1 in the case without aggregate risk but the difference is decreasing in α and small even for intermediate levels of α. For example, with equal probability of the environment remaining stressed or returning back to normal ($\alpha = 0.5$), the threshold is $2\alpha - \sqrt{\alpha(4\alpha - 1)} \approx 0.29$ compared to $2 - \sqrt{3} \approx 0.27$.

E Graphical Intuition for Elasticity of Marginal Costs

Considering the price functions in Proposition 1, we see that the direct effects in condition (8) correspond to changes in the average slope of the cost function C. For example, increasing q_0 by δ reduces p_0 by

$$\frac{C(q_0 + \delta)}{q_0 + \delta} - \frac{C(q_0)}{q_0},$$

which is the change in the average slope $C(q)/q$ when extending the interval from $[0, q_0]$ to $[0, q_0 + \delta]$. This is illustrated in Figure 10 by going from the slope of the blue line between points A and B, which equals $C(q_0)/q_0$, to the slope of the orange line between points A and C, which equals $C(q_0+\delta)/(q_0+\delta)$. The direct effect $\partial p_1^s/\partial q_1^s$ works the same way but with the origin shifted from $(0,0)$ to $\big(q_0, C(q_0)\big)$. Increasing q_1^s by δ reduces p_1^s by

$$\frac{C(q_0+q_1^s+\delta)}{q_1^s+\delta}-\frac{C(q_0+q_1^s)}{q_1^s},$$

which is the change in the average slope when extending the interval from $\left[q_0, q_0+q_1^s\right]$ to $\left[q_0, q_0+q_1^s+\delta\right]$. This is illustrated in Figure 10 by going from the slope of the blue line between points B and D to the slope of the orange line between points B and E.

In contrast, while the indirect inventory effect $\partial p_1^s/\partial q_0$ also starts with the average slope on $\left[q_0, q_0+q_1^s\right]$, an increase of q_0 by δ shifts both the beginning and the end of the interval and results in the average slope on $\left[q_0+\delta, q_0+q_1^s+\delta\right]$, i.e. it reduces p_1^s by

$$\frac{C(q_0+q_1^s+\delta)-C(q_0+\delta)}{q_1^s}-\frac{C(q_0+q_1^s)-C(q_0)}{q_1^s}.$$

This is illustrated in Figure 10 by going from the slope of the blue line between points B and D to the slope of the green line between points C and E. By convexity, this effect is bigger than the direct effect $\partial p_1^s/\partial q_1^s$ (as used in the proof of Corollary 1).

Condition (8) therefore requires two things: First, the difference between the indirect effect $\partial p_1^s/\partial q_0$ and the direct effect $\partial p_1^s/\partial q_1^s$ (in absolute magnitude) has to be sufficiently large; in Figure 10 this means that the slope of the line from C to E has to be sufficiently greater than the slope of the line from B to E, i.e. the function $C(q)$ has to have strong curvature between q_0 and $q_0+q_1^s$. Second, the direct effect $\partial p_0/\partial q_0$ has to be sufficiently small; in Figure 10 this means that the slope of the line from A to C has to be not much greater than the slope of the line from A to B, i.e. the function $C(q)$ has to have weak curvature between 0 and q_0.

Taken together, these two requirement translate into sufficiently elastic marginal balance sheet costs, i.e. marginal costs $C'(q)$ have to increase relatively slowly for low q and relatively quickly for large q. The two panels of Figure 10 illustrate this using the parametric cost function $C(q)=cq^n$ that we will analyze further in the following section and that have elasticity of marginal costs given by $\eta=n-1$. Panel A shows the case of $n=3$ such that marginal balance sheet costs have relatively low elasticity $\eta=2$; here the indirect inventory effect (green line) is not much stronger than the direct effects (orange lines). Panel B shows the case of $n=5$ such that marginal balance sheet costs are more elastic with $\eta=4$ and the indirect inventory effect becomes stronger.

F Global Game Analysis with $s_0 \neq s_1$

The payoff gain (10) with $s_0 \neq s_1$ is

$$\pi(\lambda, s_0, s_1) = \frac{c}{2}\left(s_1^2\left(1-s_0\right)\left(1-\lambda\right) + \left(4s_1-1\right)\left(s_0+\left(1-s_0\right)\lambda\right)\right) - \left(1-s_1\right)\left(v-1\right)$$

A sell equilibrium $\pi(1, s_0, s_1) \geq 0$ exists for sufficiently high s_1:

$$\begin{aligned} \pi(1, s_0, s_1) &= \frac{c}{2}\left(4s_1-1\right) - \left(1-s_1\right)\left(v-1\right) > 0 \\ \Leftrightarrow \quad s_1 &> \frac{\frac{c}{2}+\left(v-1\right)}{2c+\left(v-1\right)} \end{aligned}$$

The condition for a hold equilibrium $\pi(0, s_0, s_1) \leq 0$ depends on s_0:

$$\pi(0, s_0, s_1) = \frac{c}{2}\left(s_1^2\left(1-s_0\right) + \left(4s_1-1\right)s_0\right) - \left(1-s_1\right)\left(v-1\right) < 0$$

However, a hold equilibrium exists for sufficiently low s_1 because, for $s_1 = 0$, we have a hold equilibrium irrespective of s_0:

$$\pi(0, s_0, 0) = -\frac{c}{2}s_0 - \left(v-1\right) < 0$$

Together with the properties derived in Proposition 5, the payoff gain $\pi(\lambda, s_0, s_1)$ with s_1 as the fundamental state variable therefore satisfies the global game properties of Morris and Shin (2003). Specifically, Proposition 5 establishes State Monotonicity and Action Monotonicity, i.e. $\pi(\lambda, s_0, s_1)$ is increasing in s_1 and increasing in λ for $s_1 > \widetilde{s}$, which is satisfied if there are multiple equilibria of the complete-information game. The payoff gain satisfies Strict Laplacian State Monotonicity since we have

$$\int_0^1 \pi(\lambda, s_0, s_1)\, d\lambda = \frac{c}{4}\left(s_1^2\left(1-s_0\right) + \left(4s_1-1\right)\left(1+s_0\right)\right) - \left(1-s_1\right)\left(v-1\right),$$

which satisfies

$$\int_0^1 \pi(\lambda, s_0, 0)\, d\lambda = -\frac{c}{4}\left(1+s_0\right) - \left(v-1\right) < 0,$$

and

$$\int_0^1 \pi(\lambda, s_0, 1)\, d\lambda = \frac{c}{2}\left(2+s_0\right) > 0,$$

as well as

$$\frac{\partial}{\partial s_1}\int_0^1 \pi(\lambda, s_0, s_1)\, d\lambda = \frac{c}{4}\left(2s_1\left(1-s_0\right) + 4\left(1+s_0\right)\right) + \left(v-1\right) > 0,$$

and therefore a unique $s_1^* \in (\widetilde{s}, 1)$ solves $\int_0^1 \pi(\lambda, s_0, s_1^*)\, d\lambda = 0$. Finally, $\pi(\lambda, s_0, s_1)$ satisfies Uniform Limit Dominance since we have

$$\pi(\lambda, s_0, 0) = -(v-1) - \frac{c}{2}\left(s_0 + (1-s_0)\lambda\right) < 0,$$

and

$$\pi(\lambda, s_0, 1) = \frac{c}{2}\left((1-s_0)(1-\lambda) + 3\left(s_0 + (1-s_0)\lambda\right)\right) > 0.$$

Under these properties, Morris and Shin (2003) show that, in the limit $\sigma_\varepsilon \to 0$, the global game has a unique equilibrium and that the equilibrium is in switching strategies around a threshold s_1^* defined by the indifference condition $\int_0^1 \pi(\lambda, s_0, s_1^*)\, d\lambda = 0$ where the distribution of λ conditional on signal $\widehat{s}_{1i} = s_1^*$ is uniform on $[0,1]$. This establishes that the analog of Proposition 6 holds for the case $s_0 \neq s_1$.

The indifference condition $\int_0^1 \pi(\lambda, s_0, s_1^*)\, d\lambda = 0$ which defines the threshold s_1^* is given by

$$\frac{c}{4}\left(\left(s_1^*\right)^2(1-s_0) + (4s_1^*-1)(1+s_0)\right) - (1-s_1^*)(v-1) = 0.$$

Implicit differentiation shows same comparative statics as in Corollary 5:

$$\frac{ds_1^*}{dc} = -\frac{\frac{1}{4}\left(\left(s_1^*\right)^2(1-s_0) + (4s_1^*-1)(1+s_0)\right)}{\frac{c}{4}\left(2s_1^*(1-s_0) + 4(1+s_0)\right) + (v-1)} < 0 \quad \text{for} \quad s > \widetilde{s}$$

and

$$\frac{ds_1^*}{dv} = -\frac{-(1-s_1^*)}{\frac{c}{4}\left(2s_1^*(1-s_0) + 4(1+s_0)\right) + (v-1)} > 0$$

G Model with Safety Investors Active at Both Dates

Suppose we have additional demand $q_0^S = a_0 - b_0 p_0$ at date 0 and $q_1^S = a_1 - b_1 p_1$ at date 1. Things are unchanged at date 0 with expected price

$$E[p_0(\lambda)] = \frac{1 + a_0 c}{1 + b_0 c} - \frac{1}{2}\frac{c}{1 + b_0 c}\left(s + (1-s)\lambda\right).$$

At date 1, dealers demand $q_1^D = \frac{1}{c}(1 - p_1) - 2q_0^D$ with inventory q_0^D as in the main text. With additional demand q_1^S, total demand at date 1 can be written as

$$p_1^s(q_1^s) = \frac{1 + a_1 c - 2cq_0^D}{1 + b_1 c} - \frac{c}{1 + b_1 c} q_1^s.$$

With total supply $q_1^s = s(1-s)(1-\lambda)$ and substituting in dealer inventory

$$q_0^D = \frac{s + (1-s)\lambda + b_0 - a_0}{1 + b_0 c},$$

we have an expected price

$$E[p_1^s(\lambda)] = \frac{1 + a_1 c - 2c\left(\frac{s+(1-s)\lambda}{1+b_0 c} - \frac{a_0 - b_0}{1+b_0 c}\right)}{1 + b_1 c} - \frac{1}{2}\frac{c}{1+b_1 c} s(1-s)(1-\lambda).$$

Collecting terms, we have expected prices given by

$$\begin{aligned}
E[p_0(\lambda)] &= \frac{1 + \left(a_0 - \frac{1}{2}s\right)c}{1 + b_0 c} - \frac{1}{2}\frac{c}{1+b_0 c}(1-s)\lambda, \\
E[p_1^s(\lambda)] &= \frac{1 + \left(a_1 - \frac{1}{2}s(1-s)\right)c}{1 + b_1 c} + \frac{2c(a_0 - b_0 - s)}{(1+b_1 c)(1+b_0 c)} \\
&\quad - \frac{1}{1+b_1 c}\left(\frac{2c}{1+b_0 c} - \frac{c}{2}s\right)(1-s)\lambda
\end{aligned}$$

As before, a_0 has twice the effect on p_1^s as on p_0 but p_1^s is discounted by s, so for a_0 to be stabilizing, we need $s > 1/2$. In contrast, a_1 only affects p_1^s, so it is always stabilizing.

H Flight-to-Safety as Increase in Dealer Valuation

Suppose that dealers value the safe asset at $v_D \in [1, v)$. In this case, the equilibrium prices from Proposition 1 would be

$$p_0 = v_D - \frac{C(q_0)}{q_0} \quad \text{and} \quad p_1^s = v_D - \frac{C(q_0 + q_1^s) - C(q_0)}{q_1^s}.$$

We can consider an increase in v_D as a temporary shock to dealers' fundamental valuation of safe assets, perhaps reflecting decreases in funding costs or relaxation of collateral constraints.

Let π_1 denote the payoff gain when $v_D = 1$ and let π_D denote the payoff gain for a given $v_D > 1$. Then we have a simple relation between π_D and π_1 given by

$$\pi_D = \pi_1 + (1-s)(v_D - 1),$$

which directly implies that the payoff gain is increasing in the dealers' valuation v_D.

In other words, a positive shock to dealers' valuation strictly increases the incentive for liquidity investors to sell preemptively. The intuition is straightforward. The increase in dealers' valuation increases the price in both periods by the same amount, and since liquidity investors only weight the future price by the liquidity risk s, an equal increase in both prices strictly increases the incentive to sell.