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# Leader-Follower Dynamics in Shareholder Activism

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#### **Abstract**

Motivated by the rise of hedge fund activism, we consider a leader blockholder and a follower counterpart who first trade in sequence to build their blocks and then intervene in a firm. With endogenous fundamentals and steering dynamics, the leader ceases to trade in an unpredictable way: she buys or sells to induce the follower to acquire a larger block and thus spend more resources to improve firm value. Key is that the activists have correlated private information—initial blocks, firms' fundamentals, or their own productivity—so that prices either *overreact* or *underreact* to order flows. We link the model's predictions to observables through deriving measures of "abnormal" prices analogous to those documented in empirical studies. The model explains how trades and prices can be used to coordinate non-cooperative attacks, and how block interdependence can be a key factor in the success of multi-activist interventions.

Key words: activism, insider trading, blockholders, hedge funds

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To view the authors' disclosure statements, visit https://www.newyorkfed.org/research/staff\_reports/sr1030.html.

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# 1 Introduction

The past decades have witnessed the rise of activist hedge funds shaking up firms' capital policies, business strategies, and governance structures as a method to unlock value.<sup>1</sup> This phenomenon has coincided with a trend toward more concentrated ownership in U.S. corporations—mostly in the hands of institutional investors—as well as with changes in SEC regulation that permit a non-trivial degree of communication among shareholders. With a smaller number of key players around and an improved ability to exchange information, the strategic complexity of the environment in which activists interact has been reduced. For hedge funds, this is particularly important because their stakes are typically too small to be able to control management, so they usually need the support of fellow activists to influence firms. Not surprisingly, the frequency of activism events featuring multiple hedge funds that engage with the same target has grown considerably (e.g., Becht et al., 2017).

Activism is known to be a very costly endeavor: block acquisition is expensive—which worsens if multiple activists try to build a stake—but so is the actual planning and execution of firms' restructurings.<sup>2</sup> Crucially, these costs reinforce each other: only those who have acquired sufficiently large blocks will have the incentive to spend resources to change firms, because the unlocked value will be applied to more shares. From this perspective, while concentrated ownership is often argued to alleviate the collective-action problem omnipresent in activism, it also triggers strategic considerations regarding block accumulation that are shaped by cost-management motives and the awareness that others can intervene too. As Edmans and Holderness (2017) point out, the theoretical literature has nevertheless focused on settings with an activist building stakes in isolation, or on multiple activists with fixed blocks. Thus, the fundamental question of how investors gear towards an attack, anticipating that other investors have skin in the game too, and can be influenced, is much less understood.

In this paper, we study dynamic strategic interactions among blockholders who actively intervene and trade, a distinctive feature of hedge funds relative to other institutional blockholders (e.g., index funds). From the perspective of microstructure models, we fill a gap by considering a game of influence between traders, which is a natural approach for examining block accumulation towards eventual interventions in firms; regarding activism models, we explore the extent to which ex ante trading complements "voice," rather than acting as an

<sup>&</sup>lt;sup>1</sup>See Brav et al. (2021b) for a comprehensive review. The authors document that almost 900 hedge funds have been involved in more than 4,600 activism events in the U.S. from 1994 to 2018.

<sup>2</sup>Salesforce was a target in December 2022 when its capitalization was around \$130B, and hence a 5% stake approximated \$6.5B. None of the five activists attacking reached 5%—see https://finance.yahoo.co m/news/salesforces-activist-investors-who-are-they-and-what-do-they-want-174655497.html for more details. Away from acquisition costs, Gantchev (2013) finds that activists' campaigns can add up to \$10M, while Albuquerque et al. (2022) structurally estimate activism costs at \$2.43M.

ex post disciplinary force (the so-called "exit" channel); and from an empirical standpoint, we derive novel predictions on market outcomes that can be followed up by empirical work.

In our baseline model, two activists decide how much stake to (de-)accumulate in a market structure à la Kyle (1985), where private information is about initial blocks and firm value is determined by effort choices, as in Back et al. (2018). We add two natural ingredients to this setting. First, there is block interdependence: the initial positions of our activists exhibit correlation—for instance, if positive, because they have similar investment styles. Second, trading is sequential: in the first period, a leader (she) activist acts as the unique informed trader, anticipating that a follower (he) will play that role in the second period. Finally, in the third period, both activists simultaneously exert effort that determines the firm's share (fundamental) value. The model explains how blockholders can coordinate both through the timing of trades and the informational content of prices to manage costs in competitive settings—and how this coordination, as a byproduct, shapes firm values through the channel of activists trying to influence others' likelihood of intervention.

The combination of sequential stake-building and endogenous fundamentals dramatically alters the strategic motive of an 'insider trader' such as our leader. Indeed, Theorem 1 establishes the existence of a novel linear equilibrium in which the leader's orders are nonzero on average—this is in great contrast with decades of microstructure models emanating from the seminal paper of Kyle  $(1985)$ , where trades are unpredictable.<sup>3</sup> Specifically, with positive correlation, the leader sells on average—her order is negative when averaged across all possible block sizes—while the opposite occurs when blocks exhibit negative interdependence. The reason is that the leader distorts her behavior to induce underpricing that the follower is enticed to exploit, ultimately inducing him to build a larger position and exert more effort.

It is intuitive that the leader may want to buy less aggressively, as lower execution prices for the leader translate into lower *quoted* prices for the follower. However, the follower also becomes less optimistic about the leader's block—hence, about her effort—after observing a low order flow, which reduces his incentive to build a block. Not only that: when blocks are negative correlated, the leader buys on average and hence she in fact increases the quoted price faced by the follower; yet, the latter still finds it profitable to buy more shares.

The key to understanding these results is to recognize that the activists' interdependent private information non-trivially shapes the relative inference made by market makers and the follower. When correlation is positive, market makers overreact to large order flows in relative terms: price setters learn about two unknown components of the firm that are positively linked, whereas the follower just learns about one by virtue of his private information—with

<sup>&</sup>lt;sup>3</sup>There, trades respond to the difference between private and public information about the firm's (exogenous) value. We expand on this topic in Section 2.1.

overly responsive prices, only sell orders increase the follower's perceived underpricing. Conversely, if correlation is negative, prices now underreact: large order flows indicative of high effort by the leader are offset by a perception of a small contribution by the follower—with less sensitive prices, only buy orders induce underpricing. While our choice of private initial positions is appropriate given that hedge funds' stakes are typically small—and also relevant given the frequency of so-called "under the threshold campaigns" and the fact that large-cap firms are becoming more frequent targets—our results are not driven by this particular specification.<sup>4</sup> In fact, to demonstrate that what matters is the activists' superior information about each other, we show that identical qualitative results arise if private information is about exogenous components of firms' values, or about the activists' productivity to improve firms; or if the leader trades a second time along with the follower (Section 3.2).

In line with the notion that block size mitigates free riding, our novel equilibrium predicts that larger blockholders acquire more stock than their smaller counterparts, ultimately adding more value in relative terms. But the model also highlights the leader's steering motive as a key force shaping block size in absolute terms, and hence as a central determinant of firm value. When correlation is positive, this motive implies that the leader accumulates fewer shares than if she had acted in isolation: the leader effectively offloads activism costs on the follower in the process of incentivizing him to add more value. By contrast, when correlation is negative, the leader steers the follower by accumulating more shares herself, thus bearing more of the activism costs and developing more skin in the game. Importantly, because the follower does not change his position on average (due to not having manipulation opportunities), all the non-trivial implications for firm values are linked to the leader's behavior: when correlation is positive (negative) the leader overall lowers (increases) firm value relative to the counterfactual world in which blocks do not change on average.

The flexibility of our model enables us to connect its predictions about firm values with the empirical literature that has examined hedge fund activism. First, we show that our leader-follower setup always leads to higher firm values than if a single activist were present, despite the inefficiency that arises under positive interdependence—this is in line with the evidence of Becht et al. (2017) on multiplayer engagements. Second, the paper naturally delivers measures of abnormality analogous to those documented empirically. The idea is to note that if activism opportunities are absent and hence trading is based solely on exploiting informational advantages, trades are expected to be unpredictable: in such "normal times," positions should not change on average. We can then cast our predictions regarding firm

<sup>4</sup>We expand on the importance of smaller blocks on Section 2.2. Campaigns with blocks below 5% were majority in the U.S. in 2021 and the targets had higher market capitalization. https://www.cnbc.com/202 2/01/15/activist-hedge-funds-launched-89-campaigns-in-2021-heres-how-they-fared.html.

value in "price" form: if correlation is positive, prices are predicted to be abnormally low on average (and vice-versa) relative to counterfactual times when activism is not at play.

Section 2.2 argues that increasing levels of block interdependence—in the form of stronger positive correlation or weaker negative correlation—are more likely to manifest as market capitalization grows. In short, institutional blocks (as a fraction) tend to be smaller in higher capitalization stocks, and smaller financial blocks often coexist at the same firms (e.g., Hadlock and Schwartz-Ziv, 2019). Conversely, it is well-known that small stocks draw more disagreement, and that they tend to be disproportionally shorted (e.g., Asquith et al., 2005)—in our model, a mix of activists with a short and a long position is more likely when correlation is negative. From this perspective, our model predicts that the extent of stock appreciation should fall as capitalization grows, which is consistent with buy-andhold abnormal returns documented by Brav et al. (2021b). It also predicts larger abnormal returns if there is disagreement about firms, which is consistent with the findings in Li et al. (2022) that firms featuring traders with large short positions exhibit more stock appreciation. Section 4.2 further elaborates on how our findings can be leveraged in future empirical work.

These findings suggest a fundamental dichotomy linked to such "trading activists:" their ability to overcome collective-action problems may be very effective in smaller firms, but less so in larger ones, purely for strategic reasons—and this issue is important if activists are more likely to cluster in the latter segment due to the costs involved. That said, the fact that the leader acts first matters for this conclusion, so studying factors that favor a sequential structure is key. Section 5.1 in fact shows that there is a sizable region of correlation levels over which both activists are individually better off than if trading simultaneously: coordinating the timing of trades is mutually beneficial. This reflects the benefits of lower acquisition costs in less competitive settings, and it offers a solid foundation for our setup.

To the left of this region, an activist may prefer to trade simultaneously with fellow activist because the latter always provides liquidity when needed. At the other end, increasing levels of positive correlation enhance the leader's ability to influence the follower's trading gains, and so moving first is even more desirable (potentially at the expense of the follower). The bottom line is, a leader is more likely to emerge when there is more *similarity* among fellow activists, in a block-statistical sense. Further, if the interdependence is positive, we show that the benefit of acting as a leader grows when her initial block is larger, because she will expect her fellow activist to place commensurate trades; or when there are multiple small followers, because these can aggressively compete to exploit mispricing; or when an activist's own productivity increases, because her own trades will be increasingly large (Section 5.2).

The paper concludes with two discussions. First, in Section 5.3 we interpret our model and findings from the lens of the so-called "wolf pack" activism, whereby multiple hedge funds attack the same firm in a parallel, seemingly independent, manner after a leader fund acquires a stake. Our model fits many features attributed to this phenomenon: targets are undervalued firms, reflecting a strong sensitivity to underpricing; activists' blocks tend to be similar, and are small to moderate in size; behavior is non-cooperative due to the high costs of acting as a formal group; there are followers who do not disclose positions, and hence necessarily have smaller stakes; and there is strong competition at the moment of trading.

Finally, Section 6 addresses the possibility of other equilibria in which the activists trade against their initial positions to coordinate with each other in terms of creating or destroying value. Despite this being an interesting theoretical possibility, we argue that these equilibria are less suitable as a prediction for activism in practice. Further, we provide conditions under which the equilibrium that we study is the unique prediction within the linear class.

Roadmap We discuss the theoretical literature next—the related empirical literature is discussed throughout our analysis. Section 2 introduces our model, while our main result is in Section 3. Section 4 is devoted to the model's predictions and connection with abnormality measures in practice. Section 5 examines first-mover advantages and wolf packs. Section 6 discusses other equilibria and a refinement result. All proofs are in the Appendix.

Related literature The collective-action problem that arises in activism when ownership is dispersed has been recognized since Berle and Means (1932). The theoretical literature has then focused on how this problem plays out in models of "voice," where a blockholder takes actions that directly affect firm value (e.g., Shleifer and Vishny, 1986, Kahn and Winton (1998) and Maug, 1998); and in models of "exit," where the ex post threat of selling shares can discipline management (e.g., Admati and Pfleiderer, 2009 and Edmans, 2009). Ours is a model of voice, as effort determines firm value; but in some specifications, disposal of shares can happen in equilibrium to induce subsequent activists to govern through voice.

Our research has been influenced by the "program" proposed by Edmans and Holderness (2017) who suggest many areas of research, among them: considering blocks under 5%; that blockholders interact, with their presence affecting the effectiveness of others; that they can act as informed traders, thereby bridging firms' governance with markets' microstructure; and that activists' costs/benefits beyond those related to controlling firms matter (pp. 610– 612). We are not aware of other papers combining these elements in dynamic settings. For instance, in Back et al. (2018), a fully dynamic "activist version" of Kyle (1985), a single trading activist has private information about her initial position and exerts a one-time terminal effort choice. While different activism technologies can have non-trivial implications for market liquidity, equilibrium trading is always unpredictable in their paper; instead, we show that the nature of strategic trading fundamentally changes when other blockholders are present. ('Kyle-type' models are discussed in detail in Section 2.1.) On the other hand, while there are models involving multiple activists, these feature simultaneous moves among them: in Doidge et al. (2021) activists trade non-cooperatively only once to later act as a coalition (in the cooperative-games sense) when exerting effort; in turn, Edmans and Manso (2011) show that competition in trading can make exit more effective; and in Brav et al. (2021a), reputational motives can lead hedge funds to exert effort to attract funding. Thus, none of these papers consider the incentives to induce others to develop skin in the game as a means for controlling private costs or increasing private benefits.

Our model relates to trading models in which strategies of a manipulative nature have real consequences. In Goldstein and Guembel (2008), short-selling is be a profitable strategy for a speculator if it induces a manager to forgo an investment decision; but buy orders are never fruitful there. In Attari et al. (2006), a passive fund may dump shares to insure the value of the remaining block, as activism by a second investor has positive return only when a firm's fundamentals are low. In Khanna and Mathews (2012), a blockholder instead buys shares to counter a speculator's attempt to lower a firm's value. In contrast to these papers, all our players directly influence firm values, and both buying or selling can be optimal.<sup>5</sup>

Finally, we relate to models of belief manipulation employing Gaussian fundamentals and/or shocks in settings other than financial markets, e.g., Holmström  $(1999)$ , Cisternas (2018), Bonatti and Cisternas (2020), Cetemen (2020), and Ekmekci et al. (2020). A key novelty of our model is that noisier signals (here, order flows) can lead to more manipulation, despite beliefs (here, prices) becoming less responsive. This is because the leader's marginal incentives to manipulate beliefs—captured by her terminal block—being endogenous.

### 2 Model

#### 2.1 Setup

A leader activist (she) and a *follower* counterpart (he) hold initial positions of  $X_0^L \in \mathbb{R}$  and  $X_0^F \in \mathbb{R}$  shares in a firm, respectively. In this baseline model, each activist's *block* is their private information, and such stakes are normally distributed with mean  $\mu$ , variance  $\phi$ , and covariance  $\rho \in [-\phi, \phi]$ . In Section 3.2, we explore other forms of private information.

The model has three periods. In period 1, the leader acts as a single informed trader in a Kyle (1985) market structure. Specifically, she submits an order for  $\theta^L \in \mathbb{R}$  units of the firm's stock to a competitive market maker who executes it at a public price  $P_1$  after

<sup>&</sup>lt;sup>5</sup>See Yang and Zhu (2021), Boleslavsky et al. (2017), and Ahnert et al. (2020), for models where trading can trigger government interventions, while Chakraborty and Yılmaz (2004), Brunnermeier (2005) and Williams and Skrzypacz (2020) for manipulation in financial markets abstracting from real consequences.

observing the total order flow of the form

$$
\Psi_1 = \theta^L + \sigma Z_1.
$$

In this specification,  $Z_1$  is standard normal random variable independent of the initial positions that captures noise traders, and the volatility  $\sigma > 0$  is a commonly known scalar.

Having observed  $P_1$ , in period 2 the follower replaces the leader as the single informed trader in an identical round of trading: he orders  $\theta^F \in \mathbb{R}$  units from the same market maker who in turn executes the order at a (public) price  $P_2$  after observing the total order flow

$$
\Psi_2 = \theta^F + \sigma Z_2,
$$

where  $Z_2$  is standard normal and independent of  $(X_0^L, X_0^F, Z_1)$ . Finally, in period 3, the activists simultaneously take actions that determine the firm's fundamentals: activist  $i$  exerts effort  $W^i \in \mathbb{R}$  at a cost  $\frac{1}{2}(W^i)^2$ ,  $i \in \{L, F\}$ , resulting in a *true share value* 

$$
W = W^L + W^F.
$$

Turning to payoffs, we use subscript T to capture *terminal* positions (i.e., after the second round of trading), which for each player consists of initial positions plus the amount traded:

$$
X_T^i = X_0^i + \theta^i, \ i \in \{L, F\}.
$$
 (1)

We also let  $(\mathcal{F}_t)_{t=0,1,2}$  denote the public information, which is generated by the prior and the order flows  $(\Psi_t)_{t=1,2}$ . Activist  $i \in \{L, F\}$  then maximizes the value of its holdings net of trading and effort costs:

$$
\sup_{\theta^i, W^i} \mathbb{E}\left[ \left( W^i + W^{-i} \right) X_T^i - P_{t(i)} \theta^i - \frac{1}{2} (W^i)^2 | X_0^i, \mathcal{F}_{t(i)-1} \right]. \tag{2}
$$

Here, the time indices  $t(L) := 1$  and  $t(F) := 2$  link our activists with their corresponding trading periods. Clearly, the optimal effort choice satisfies

$$
W^i = X_T^i, \ i \in \{L, F\}.
$$
\n(3)

Note that a collective-action problem is at play because in this choice, each activist does not internalize the benefit that higher effort has on the value of the other blockholder's total

holdings. Equipped with this, the objective (2) of activist  $i \in \{L, F\}$  then becomes

$$
\sup_{\theta^i} \mathbb{E}\left[ (X_T^i + X_T^{-i}) X_T^i - P_{t(i)} \theta^i - \frac{1}{2} (X_T^i)^2 | X_0^i, \mathcal{F}_{t(i)-1} \right]. \tag{4}
$$

Without loss of generality, throughout the paper we assume  $\mu > 0$ . Also, unless otherwise stated, we will use  $X_0^L > 0$  and  $X_0^F > 0$  to provide intuition: that is, the activists are "long" on the firm and absent any trading they would exert positive effort. But note that the model allows for short positions  $(X_0^i < 0)$  and even negative effort, capturing value destruction.<sup>6</sup>

Interpretation That the game ends after the third period can be rationalized as the firm's value getting revealed after effort is undertaken (which renders subsequent strategic trading unprofitable). Since changes in firms are not immediate, one may then wonder whether not allowing for multiple "pre-revelation" rounds of trading is a limitation. Our belief is that this is not the case: because in practice activists must reveal their *intended plans* when disclosing positions over 5%, valuable information about actions gets revealed well ahead of changes being materialized. Further, since material adjustments to positions or intentions can be disclosed with a delay, trades effectively remain hidden for some time.<sup>7</sup>

Our model is then best interpreted as taking place in such pre-disclosure window when the leader activists are gearing up to quickly finalize their positions and attack. From this perspective, the evidence about such window periods is consistent with our assumptions: disclosing hedge funds tend to trade primarily in the day they cross the 5% threshold—the "trigger date"—or the one after (e.g., Bebchuk et al., 2013 and Collin-Dufresne and Fos, 2015), implying that trades leading to block completion are not spread out over that period, and often happen before the market learns activists' intentions and trades. A key question is how block completion by a leader hedge fund is affected by the common knowledge that subsequent followers will build their stakes too, and the implications for stock prices—we will compare our predictions with the measures of abnormality documented in those windows.

Linear Strategies and Equilibrium Concept As is traditional in the literature following Kyle (1985) we will look for equilibria in linear trading strategies. Our leader conditions on her type  $X_0^L$  and the prior mean  $\mu$  (used by market makers to set the firm's price), while

<sup>&</sup>lt;sup>6</sup>The firm's pre-activism share value has been normalized to zero. Bliss et al.  $(2019)$  for examples of negative activism, and Appel and Fos (2023) for short campaigns run by hedge funds. Refer to https://www. cnbc.com/2019/12/13/reliving-the-carl-icahn-and-bill-ackman-herbalife-feud-on-cnbc.html for a famous case in which investors took opposite positions.

<sup>7</sup>Recently, the traditional disclosure requirement for activists to file a 13D form within 10-day from crossing the 5% threshold has been shortened to 5 business days, while material amendments must be filed within 2 business days: https://www.sec.gov/news/press-release/2023-219.

our follower can, in addition, condition on the first-period price:

$$
\theta^L = \alpha_L X_0^L + \delta_L \mu \quad \text{and} \quad \theta^F = \alpha_F X_0^F + \beta_F P_1 + \delta_F \mu. \tag{5}
$$

A pricing rule is linear if  $P_{t(i)}$  is affine in the current order flow,  $\Psi_{t(i)}$ ,  $i = L, F$ . In equilibrium, (i) the activists' strategies are mutual best-responses when taking as given the pricing rule, and (ii) the latter satisfies  $P_{t(i)} = \mathbb{E}[W^L + W^F | \mathcal{F}_{t(i)}]$  given the activists' strategies.

We will focus on equilibria exhibiting  $\alpha_L > 0$  and  $\alpha_F > 0$ , i.e., market orders with *positive* block sensitivity (PBS). The reason is twofold. First, in this equilibrium larger blockholders acquire more stock, or de-accumulate less, than their smaller counterparts: trading solidifies their a priori relatively stronger willingness to intervene. While the validity of this claim is ultimately an empirical matter, we believe that is appealing when it comes to "positive" activism, as block size is widely seen as a key proxy for willingness to improve firms.

The second reason is that this type of equilibrium conforms with the literature on strategic trading: the activists place a positive weight on their private information, just like in Kyle (1985). An important observation is in order. In Kyle's influential paper, equilibrium trades are based on solely on the extent of mispricing—the difference between private and public information—and hence are expected to be zero conditional on the public information; in the case of our leader, this would amount to  $\mathbb{E}[\theta^L|\mathcal{F}_0] = 0$ , and hence to  $\alpha_L = -\delta_L$ . This form of unpredictability is a pervasive finding in the literature emanating from this paper. Indeed, if fundamentals are exogenous, it arises with any number of traders and degree of correlation in private information (e.g., Foster and Viswanathan, 1996 and Back et al., 2000); with time-invariant non-Gaussian fundamentals (e.g., Back, 1992); with Gaussian fundamentals that evolve (e.g., Caldentey and Stacchetti, 2010); with stochastic volatility (e.g., Collin-Dufresne and Fos, 2016); and so forth. If fundamentals are endogenous, it arises when there are multiple rounds of trading in single-player setups (e.g., Back et al., 2018), or with multiple players in static settings (e.g., Doidge et al., 2021).<sup>8</sup> Our model, which combines endogenous fundamentals, multiple players, and dynamics, will prove fundamentally different.

#### 2.2 Discussing Our Assumptions

Private information Blocks below 5% need not be disclosed, and hence can be an activist's private information.<sup>9</sup> Hedge fund ownership in fact fluctuates around this threshold: Brav et al. (2021b) find that the median stake for this type of fund is 6.6% upon disclo-

<sup>8</sup>See Back et al., 2018 for a discussion of unpredictibility of informed trades in Kyle-type models. The term "inconspicuous insider trading" is used in their paper when referring to trades that are not forecastable.

<sup>&</sup>lt;sup>9</sup>An exception is when a fund holds more than 100 million in shares of publicly traded firms, in which case a form 13F must be filed, even if there is no intention to intervene.

sure, while Collin-Dufresne and Fos (2015) state that, to complete their blocks (e.g., to reach 6.6%), hedge funds purchase around 1% of shares on the day that the threshold is crossed—importantly, these numbers do not include all the (smaller) blocks that are not disclosed. Crucially, the 1%–5% blockholder segment can have substantial power: Lewellen and Lewellen (2022) document that they collectively own around 22% of shares in an average firm compared to the aggregate 20% of blocks above 5%; relatedly, they also show that smaller blockholders are more likely to trade, which gives consistency to our assumptions.

But even if initial blocks are public, our model still is a reasonable approximation when it comes to trading activists. The reason is, as Collin-Dufresne and Fos (2015) state, that activists ultimately have private information about their willingness to intervene: whether this is information about an intensive or extensive margin, any uncertainty regarding how this likelihood is distributed across activists with heterogeneous holdings reduces to uncertainty about an intensive margin of engagement like in our setup. Further, trades that remain hidden for some time can catalyze this process. To make the point, we present two variations of our model that feature public initial blocks (Section 3.2): in the first, the activists have private information about an exogenous component of the firm's value, and in the second they have private information about their costs. As the activists trade, they develop private information about their terminal positions and hence, about their intensive margin for intervention. Our main mechanism is qualitatively identical in these variations.

Payoffs Assuming firm fundamentals that are additive in effort is natural given the wellknown free-rider problem that arises when ownership is dispersed. Our setup then suggests that the leader's ability to take advantage of the follower will only reinforce the collective action problem at play; yet, we will show that improvements can happen. On the other hand, our choice of continuous actions can represent interventions that unlock value to varying degrees, such as intensive margins that are associated with efficient reallocation of resources.<sup>10</sup> That said, many outcomes can have a binary nature: our model can be seen as a linearized version of such settings where the probability of success increases in total effort.

Regarding activism costs beyond block acquisition, the quadratic structure that we employ is convenient because it results in a tractable linear-quadratic-Gaussian structure for a highly non-trivial problem, while keeping with the tradition of Kyle-type models where trading costs are also quadratic. Recently, however, Back et al. (2018) show that moving away from quadratic costs can have non-trivial implications on outcomes such as market liquidity. From this standpoint, the way to read our results is that we can show a robust conceptual departure from the literature without resorting to a mix of different technolo-

 $10$ See Brav et al. (2015) and Brav et al. (2018) in the case of production plants and patents, respectively.

gies. Indeed, at the core of this departure is a simple, albeit fundamental, complementarity: (past) orders and (future) terminal positions across players are strategic complements, as seen in the value of each activist's holdings,  $(X_T^i + X_T^{-i})$  $(T^{i}_{T})X^{i}_{T}$ . The leader's strategic motive will change because the higher the leader's terminal position, the more she benefits from inducing a higher position by the follower.

Block interdependence The statistical relation between initial blocks will turn out to be key in our model, so it is important to discuss how block interdependence manifests in reality. While we are not aware of studies that perfectly fit our exact object of interest interdependence of holdings for blockholders that actively trade and intervene in firms—we can still resort to a combination of (i) economic logic, (ii) descriptive statistics, (iii) empirical work on blockholders to connect correlation in our model with the data.<sup>11</sup>

For homogeneous groups of investors such as hedge funds, economic logic points to blocks exhibiting a baseline level of positive correlation due to these funds' similar investment styles; but repelling effects could be at play too. Importantly, Hadlock and Schwartz-Ziv (2019) document a strong form of positive interdependence for "strategic investors," a category that includes hedge funds and private equity: the likelihood of observing a block from this type of investor increases when a block from the same category is present at a firm. Further, they show that this positive correlation falls as the blocks under consideration grow in size, but it can remain positive and statistically significant.<sup>12</sup> The takeaway is that the degree of block interdependence among strategic investors is naturally linked to block size. One can then explore how this latter variable varies across observable firm characteristics.

Market capitalization is a natural variable, and there is a strong indication that blocks should be smaller as we move from small to large firms, thus favoring positive correlation. The first argument is based on economic logic: activist capital is limited, and the funds needed to acquire a sizable stake grow considerably for large-cap firms (e.g., Brav et al., 2008). The second is based on descriptive statistics: using data from the U.S., Lewellen and Lewellen (2022) indicate a decrease in concentration of institutional ownership when moving from mid- to large-cap firms.<sup>13</sup> On the other hand, the presence of a mix of blockholders with long and short positions is evidence of negative interdependence, which in our model occurs when correlation is negative. As it has been noted, highly shorted stocks tend to come from small-cap firms (e.g., Asquith et al., 2005), which also exhibit more disagreement about their prospects (e.g., Diether et al., 2002).

<sup>&</sup>lt;sup>11</sup>This approach is similar (in spirit) to the "shoe leather empirics" in Edmans and Holderness (2017).

<sup>&</sup>lt;sup>12</sup>See Table C.1 in their Appendix, for blocks above 5% and 10%. This study is not limited to activists. <sup>13</sup>Simple calculations in Table 3 in their paper reveal that (i) the largest, (ii) top 2 and 3 and (iii) the 4-10 blockholders in mid-cap firms have larger fractional holdings on average than their counterparts in large-cap firms. This study encompasses a universe of institutional investors broader than activists hedge funds.

Altogether, the main qualitative conclusion is that stronger interdependence—growing (decreasing) levels of correlation if positive (negative)—are more plausible as market capitalization grows: the average size of long positions falls, thus favoring the case for positive interdependence; conversely, a mix of long and short positions is more likely in small-cap firms, and such a mix is indicative of negative interdependence. In Section 4.2 we will test these ideas against the empirical evidence when discussing our model's predictions.

# 3 Equilibrium Trading

In this section we derive the equilibrium trading strategies for our activists. We note that finding equilibria in environments exhibiting strategic block accumulation and endogenous firm values is in general a difficult task—this issue has been noted before in the literature, and is presumably behind the scarcity of results in the area when it comes to multiplayer analyses.<sup>14</sup> To these features, we are adding interdependent private information and an asymmetry in the timing of moves, which are institutionally relevant but complex to deal with. That being said, we can still obtain valuable insights simply by looking at the firstorder conditions (FOCs) that the activists' trading strategies must satisfy.

### 3.1 Idea of the Construction and Main Result

In a linear-Gaussian equilibrium, prices respond to order flows linearly. Concretely, there are prices  $P_0$ ,  $P_1$  and  $P_2$ , as well as sensitivities—or "price impact" scalars— $\Lambda_1$  and  $\Lambda_2$  satisfying

$$
P_1 = P_0 + \Lambda_1[\Psi_1 - \mathbb{E}[\Psi_1|\mathcal{F}_0]]\tag{6}
$$

$$
P_2 = P_1 + \Lambda_2[\Psi_2 - \mathbb{E}[\Psi_2|\mathcal{F}_1]]. \tag{7}
$$

The exact expressions for these terms are in Appendix A.1. What matters for now is the interpretation: the price  $P_0$  reflects market makers' estimate of the firm's value given candidate equilibrium strategies (5)—namely,  $\theta^L = \alpha_L X_0^L + \delta_L \mu$  and  $\theta^F = \alpha_F X_0^F + \beta_F P_1 + \delta_F \mu$ —before any trading occurs.<sup>15</sup> This price then acts as the "quoted price" in period  $t = 1$ , which gets updated in the direction of the unanticipated order flow  $\Psi_1 - \mathbb{E}[\Psi_1|\mathcal{F}_0]$  from the perspective of market makers. The resulting price  $P_1$  is the execution price at  $t = 1$ —what the leader

<sup>14</sup>Edmans and Holderness (2017) for instance state "Allowing trade to depend on block size may be particularly important in a blockholder trading model (rather than a general informed trading model). Solving for the optimal trading volume is highly complex: while the Kyle (1985) framework allows for trades to derived in closed form, it requires firm value to be normally distributed, but corporate finance models (such as ours) typically feature binary firm value as it substantially improves tractability."

<sup>&</sup>lt;sup>15</sup>Use that  $P_0 = \mathbb{E}[(1 + \alpha_L)X_0^L + \delta_L \mu + (1 + \alpha_F)X_0^F + \beta_F P_1 + \delta_F \mu | \mathcal{F}_0]$  and  $\mathbb{E}[P_1 | \mathcal{F}_0] = P_0$ .

ends up paying—which in turn becomes the price that the follower is quoted at  $t = 2$  before trading. As before, the latter price gets updated using the second-period surprise order flow to determine the execution price  $P_2$  paid by the follower. For reference, price impact at time  $t(i)$  (the time at which activist i trades) is obtained by the classic projection theorem recalling that the firm's true share value is  $X_T^L + X_T^F$ :

$$
\Lambda_{t(i)} = \frac{\text{Cov}(X_T^L + X_T^F, \Psi_{t(i)})}{\text{Var}[\Psi_{t(i)}]}.
$$
\n
$$
(8)
$$

We are now in a position to state our FOCs. Recall that ex post payoffs are given by

$$
\underbrace{(X_T^i+X_T^{-i})X_T^i}_{\text{total value of block}} - \underbrace{P_{t(i)}\theta^i}_{\text{trading costs}} - \underbrace{\frac{1}{2}(X_T^i)^2}_{\text{activation costs}} \ , \ \ i \in \{L, F\}.
$$

Each activist will then decide how much to trade taking as given (i) its counterparty trading strategy and (ii) how the corresponding execution price will "move against them" with a strength  $\Lambda_{t(i)}$ ,  $i = L, F$ . Letting  $\mathbb{E}_i[\cdot]$  denote the expectation operator of activist i at the moment they decide how to trade, and using the functional forms for prices along with  $\Psi_{t(i)} = \theta^i + \sigma Z_{t(i)}$  when a market order of size  $\theta^i$  is placed,  $i \in \{L, F\}$ , these conditions read:

follower: 
$$
0 = -\mathbb{E}_F[P_2] - \theta^F \Lambda_2 + \mathbb{E}_F[X_T^F + X_T^L],
$$
  
leader: 
$$
0 = -\mathbb{E}_L[P_1] - \theta^L \Lambda_1 + \mathbb{E}_L[X_T^F + X_T^L] + X_T^L \frac{\partial \mathbb{E}_L[X_T^F]}{\partial \theta^L}.
$$

Consider the follower's FOC. The first term is the expected cost of the last unit traded. The second term is the cost of moving the price against him after placing a marginally higher order: all the inframarginal units  $\theta^F$  become more expensive according to  $\Lambda_2$ . The last term is the value of a marginally higher block, which the follower values at  $\mathbb{E}_F[X_T^F + X_T^L]$ . (Note that the change in firm value due to a marginally larger terminal block is absent because it cancels out with the change in activism costs, as effort is already at an optimum.)

What is noteworthy about this FOC is that it has the same structure as the one that would arise in a standard static Kyle (1985) setup with an exogenous fundamentals. This has two implications. First, as we will show, the follower will effectively trade in an unpredictable way as in the literature (despite the endogeneity of the fundamentals). Second, inspection of the leader's FOC reveals that any departure from this canonical way of trading by the leader must be driven by the last term, which we refer to as the

value of manipulation: 
$$
X_T^L \frac{\partial \mathbb{E}_L[X_T^F]}{\partial \theta^L}
$$
.

This term is a non-trivial continuation value capturing that the leader's incentives to trade are also influenced by the possibility of inducing the follower to build a larger terminal block, which ultimately maps into more effort. Since  $X_T^F = \theta^F + X_0^F$ , this additional value is achieved by inducing the follower to trade more aggressively. In this regard, the key variable to be influenced is the extent of mispricing from the perspective of the follower—the first and last term in the follower's FOC—which measures his marginal benefit from trading:

$$
mispricing: \quad \mathbb{E}_F[X_T^L + X_T^F] - \mathbb{E}_F[P_2].
$$

Given the Gaussian structure, this wedge is a linear function of the first-period order flow  $\Psi_1$ , so it responds to the leader's trade  $\theta^L$ . Naturally, one channel is the second-period price  $P_2$ : this price is affine in  $P_1$ , which in turn responds to  $\Psi_1$  due to the market makers' learning. But the follower's estimate of the firm's value  $\mathbb{E}_F[X_T^L + X_T^F]$  is also sensitive to  $\Psi_1$ : after seeing the first-period order flow, the follower also updates beliefs about the leader's initial block, and ultimately about the leader's contribution to the firm. The exact form in which this wedge responds to  $\Psi_1$  will be of key importance in our analysis.<sup>16</sup>

The full details for finding PBS equilibria are in Appendix A.1-A.3. Let us just list here the exact steps followed to give a flavor of our construction:

- 1. In addition to computing  $P_0 = \mathbb{E}[X_T^L + X_T^F | \mathcal{F}_0]$  using (5), we also need the activist's private beliefs about each other before any trading happens: due to the correlation, each player's estimate combines their (private) initial blocks and the prior mean  $\mu$ ;
- 2. We then obtain an expression for the leader's optimal order from her FOC using a linear pricing rule as in (6); to compute  $\mathbb{E}_L[X_T^F]$  and  $\frac{\partial \mathbb{E}_L[X_T^F]}{\partial \theta^L}$  in the FOC, we use the leader's estimate of  $X_0^F$  from the previous step and the fact that the leader conjectures that  $\theta^F = \alpha_F X_0^F + \beta_F P_1 + \delta_F \mu$  is being used by the follower;
- 3. Finally, we obtain an expression for the follower's optimal order from his FOC using a linear pricing rule as in (7); this requires computing an interim belief of the follower about the leader's position that updates the one from the first step (absent any trading) with the new information conveyed by the observed order flow  $\Psi_1$ .

The equilibrium condition is that the resulting expressions for the leader's and follower's optimal order must coincide with the strategies (5), when the latter are used as conjectures in the price impact formula (8). The asymmetry in the timing of moves makes the resulting

<sup>&</sup>lt;sup>16</sup>This is the only margin through which the leader can influence the follower. (Price impact  $\Lambda_2$ , while endogenous, is fixed.) Note that the value of manipulation would be absent with exogenous fundamentals.

fixed-point complex not only because the coefficients in the strategies can vary across players, but also because the associated second-order conditions (SOCs) are non-trivially affected:

$$
follower: \quad 0 > 1 - 2\Lambda_2,\tag{9}
$$

$$
leader: \quad 0 > 1 - 2\Lambda_1(1 - \beta_F). \tag{10}
$$

The leader's SOC is non-trivial because of the term  $1 - \beta_F$  accompanying  $\Lambda_1$ . This term reflects how the leader's *effective cost of trading* is determined not only by its impact on prices, but also by the real consequences that trades have through the follower's behavior: if the follower builds his position more aggressively, larger trades are less costly than in a setting with exogenous fundamentals (where price impact is the only disciplining force). This may happen when  $\beta_F$ —the weight that the follower attaches to the first-period price in his strategy—is below but close to 1. We will revisit this topic in greater detail in Section 6.<sup>17</sup>

We are in a position to state our main result, which reads as follows:

**Theorem 1.** Fix  $\sigma > 0$ . There is  $\rho \in (-\phi, 0)$  such that for all  $\rho \in (\rho, \phi]$  a PBS equilibrium exists. In any such equilibrium  $\mathbb{E}[\theta^F|\mathcal{F}_1] = 0$ , while the leader's trades have predictability:

$$
\mathbb{E}[\theta^L|\mathcal{F}_0] \neq 0 \text{ if and only if } \rho \neq 0.
$$

Concretely, in terms of the leader's equilibrium coefficients:

(i) If  $\rho > 0$ , we have  $-\delta_L > \alpha_L > 0$ . Thus,  $\mathbb{E}[\theta^L | \mathcal{F}_0] < 0$ , and the leader sells on average;

(ii) If  $\rho < 0$ , we have  $\alpha_L > -\delta_L > 0$ . Thus,  $\mathbb{E}[\theta^L | \mathcal{F}_0] > 0$ , and the leader buys on average.

It is only when  $\rho = 0$  that  $\alpha_L = -\delta_L$ ; in this case  $\theta^i = \sqrt{\frac{\sigma^2}{\phi^i}}$  $\frac{\sigma^2}{\phi}(X_0^i-\mu),\ i=L,F.$ <sup>18</sup>

Price under/overreaction. When correlation is positive, the leader sells on average. Since the follower behaves neutrally, the leader's behavior reflects a form of downward deviation, and hence that the value of manipulation has negative sign (to be confirmed shortly). This may seem intuitive: by trading less aggressively, the leader can lower  $P_1$ , which is the price that the follower is quoted in the second period. This potentially induces more underpricing, thereby inducing the follower to build a larger block.

The issue is that, by placing a smaller or negative order, the follower also becomes more pessimistic about the leader's contribution to the firm's value, reflected in  $\mathbb{E}_F[X_T^L + X_T^F]$  in

<sup>&</sup>lt;sup>17</sup>The scalar 1 in (9)–(10) reflects a convexity linked to trades affecting firm value via effort choices.

<sup>&</sup>lt;sup>18</sup>We can prove uniqueness of PBS equilibria analytically for  $\rho \in (\rho_0, \phi]$ , where  $\rho_0 \in (\rho, 0)$ . Numerically, uniqueness within the PBS class seems to hold for  $\rho \in (\rho, \rho_0]$ .

the mispricing wedge also falling. To make the matter more stark, consider what happens when  $\rho < 0$ : there, the leader buys on average, therefore expecting to drive  $P_1$  up. As the quoted price moves against the follower, pressure toward a smaller mispricing wedge is created; yet the follower will indeed acquire a larger block (to be confirmed shortly).

At the heart of our main finding is that the activists' private information about each other shapes the relative sensitivity of beliefs of market makers (hence of prices) and that of the follower, which is what matters for the follower's gains from trade. In particular, order flows convey information to market makers about two unknown components of a firm's value, despite carrying the trades of just one activist; by contrast, due to his private information, the follower only updates about one component. When  $\rho > 0$ , therefore, a large first periodorder flow indicates that both components are large: market makers *overreact* to  $\Psi_1$  relative to the follower's updating, which is solely about the leader's contribution to the firm—overly sensitive prices then imply that sell orders amplify the extent of mispricing, and hence are profitable. Conversely, if  $\rho < 0$  market makers underreact to  $\Psi_1$ , as signals that indicate large contributions by the leader are offset by a perception that the follower's contribution will be smaller; by contrast, knowing his own block, the follower becomes relatively more optimistic. With less sensitive prices, only buy orders will generate underpricing, and the follower builds a larger stake despite the quoted price rising.

Only when  $\rho = 0$  are the leader's trades neutral: since blocks are independent, market makers know that the first-period order flow is now a signal solely of the leader's contribution to the firm. With market makers and a follower now learning the same, the mispricing wedge is independent of  $\Psi_1$ , and both activists trade according to  $\sqrt{\frac{\sigma^2}{\phi}}(X_0^i - \mu)$  as is ubiquitous in the literature following Kyle  $(1985)$ .<sup>19</sup> We conclude that, when it comes to strategic trading driven by activism motives, predictability in the leader's trade is a generic property.

Figure 1 illustrates the coefficients in this trading activist's strategy. There, deviations from the horizontal levels  $\pm \alpha^K := \pm \sqrt{\sigma^2/\phi}$  capture the extent of manipulation by the leader: if  $\rho > 0$ , the leader underweighs the importance of her block in her strategy in favor of the prior mean  $\mu$  to generate downward pressure on prices. (The observed ranking with respect to  $\alpha^{K}$  and the decreasing patterns are established in Proposition A.6 in the Appendix.) Further, as  $|\rho|$  grows, the deviation is more acute: because the first-period order flow becomes more informative about the follower's contribution in this case, the mispricing wedge is more responsive, so the value of manipulation grows. The observed asymmetry between positive and negative values of  $\rho$  stems from the effective cost of trading that changes the convexity of the leader's problem depending on the sign of the interdependence, and it is related to

<sup>&</sup>lt;sup>19</sup>In this latter expression, the slope is increasing in  $\sigma$  and  $1/\phi$  because more noise trading or a more precise prior (lower  $\phi$ ) imply less responsive beliefs, and hence a diminished price impact.



Figure 1: Leader's strategy coefficients, along with  $\alpha^K := \sqrt{\sigma^2/\phi}$ . Parameters:  $\mu = \phi = 1, \sigma = .2$ .

our threshold  $\rho < 0$  for existence—we discuss this is more detail in Section 6.

The conclusions made so far are about averages across all possible block sides. Fix a leader's positive block now. Since the value of manipulation is proportional to  $X_T^L$ , larger blockholders deviate more because the extra value supplied by the follower is applied to more shares. If sufficiently large, those leaders still buy when  $\rho > 0$ : but they do so less aggressively so as to not discourage the follower from acquiring a larger block. In addition, those leaders expect their followers to exert more effort, as the follower's trade is not zero from their perspective (i.e.,  $\mathbb{E}_L[\theta^F] \neq \mathbb{E}[\theta^F|\mathcal{F}_0] = 0$ ). Further, since  $X_T^L = (1 + \alpha_L)X_0^L + \delta_L \mu$ and  $\alpha_L > 0$ , larger initial blocks indeed map into more effort in relative terms.<sup>20</sup>

The follower's equilibrium trading Let us confirm our claims regarding the follower's behavior and the value of manipulation. To this end, let  $M_1^F := \mathbb{E}[X_0^F | \mathcal{F}_1]$  and  $\gamma_1^F :=$  $\mathbb{E}[(X_0^F - M_1^F)^2 | \mathcal{F}_1]$  denote the market makers' posterior mean and variance about the follower's position after seeing  $\Psi_1$ , but before the follower trades. Recall that the follower trades according to  $\theta^F = \alpha_F X_0^F + \beta_F P_1 + \delta_F \mu$  in a linear equilibrium.

**Proposition 1.** In any PBS equilibrium, the follower's equilibrium coefficients satisfy:  $\alpha_F =$  $\sqrt{\sigma^2/\gamma_1^F}$ ;  $\delta_F < 0$ ;  $\beta_F < 1$ , with  $sign(\beta_F) = -sign(\rho)$ ; and furthermore,  $\theta^F = \alpha_F(X_0^F - M_1^F)$ .

The first thing to note is that since  $\beta_F < 1$ , it follows from the SOCs that  $\Lambda_1 > 0$  in any PBS equilibrium. Likewise  $\Lambda_2 > 0$ . Thus, prices effectively move against our traders and limit their trades. Consider now  $\rho > 0$ . In this case,  $\beta_F < 0$ , so lower first-period prices do lead to more purchases by the follower, as argued. Also, the value of manipulation reads

$$
X_T^L \frac{\partial \mathbb{E}_L[X_T^F]}{\partial \theta^L} = X_T^L \beta_F \frac{\partial P_1}{\partial \Psi_1} = X_T^L \beta_F \Lambda_1,
$$

<sup>&</sup>lt;sup>20</sup>When  $X_0^L < 0$  the logic gets reversed: the leader sells less aggressively to limit block accumulation by the follower; this reduces the follower's effort, which a leader with a negative stake enjoys.

which is negative when the leader is "long," just as we anticipated. Conversely, when correlation is negative,  $\beta_F$  is positive (but less than 1, which is needed for SOCs to hold). In this case, the follower places increasingly large orders after seeing higher quoted prices. The reason is again the differential sensitivity of beliefs between market makers and the follower: since both parties condition on  $\Psi_1$  linearly, their differing sensitivities lead the extent of mispricing to grow in  $\Psi_1$  without bound. Finally, the last part of the proposition confirms that the follower's trades are unpredictable when seen in the "belief space"  $(X_0^F, M_1^F)$ ; the weight  $\sqrt{\sigma^2/\gamma_1^F}$  is just an updated version of  $\sqrt{\sigma^2/\phi}$  when  $\rho = 0.21$ 

### 3.2 Robustness

Before moving to the model's predictions and the connection with empirical work, we briefly discuss variations of our model that deliver a qualitatively identical mechanism.

Leader activist trades twice Because the leader continues to have relevant private information in the second period, she may benefit from trading once again along with the follower. The next figure plots the leader's average trade in the first period of such a model as a function of  $\rho$ , showing that the same distortion from a neutral trade arises.



Figure 2: Leader's expected period-one trade. Parameter values:  $\mu = \phi = 1, \sigma = .8$ .

Observe that the distortion is smaller than in our baseline model. This is due to a competition effect: the follower scales back his trade in response to the presence of the leader, which in turn reduces the value of manipulation for the leader activist in the first period. From this perspective, our choice of model is purely driven by tractability reasons: it delivers the same qualitative insights while permitting much simpler analytic results.

<sup>&</sup>lt;sup>21</sup>See The form of manipulation uncovered is reminiscent of *encouragement effects* in teams, e.g., Bolton and Harris (1999) and Cetemen et al. (2019).

Competition effects (including the retreat effect just mentioned) will be formally studied in Section 5. What matters for our argument right now is that sequentiality is often argued to be the reflection of a desire to escape from competition: hedge funds want to act fast once the 5% threshold is crossed precisely to prevent block acquisition becoming too costly after others jump in. Brav et al. (2008) puts it well: "hedge funds frequently acquire significant stakes in targets within hours of learning that an initial fund has taken a position" (p.1757). Threats from a potentially wider set of investors are possible too.<sup>22</sup>

Two observations are in order. First, leader hedge funds can indeed complete their blocks fast once the 5% threshold is met because there are important costs of ownership above 10%, which means that at most half of a hedge fund's total block is acquired over the 10-day window.<sup>23</sup> Our earlier discussion regarding (i) a median stake of around 6% upon disclosure, (ii) an average purchase of 1% of shares outstanding during the window, and (iii) most of the purchases happening on the trigger date (e.g., Bebchuk et al., 2013) is a demonstration of this. Second, a recent study by Wong (2020) sheds light on the extent of competition at play: he shows that for campaigns involving activist hedge funds who complete their blocks on the trigger date, there is 36% more abnormality in trading by other investors on the same day—a correlation between competition and fast completion.

The bottom line is, examining a potential coordination in the timing of trades is an important topic because it may allow activists to control their costs. We will return to these issues in Section 5, where we also discuss the phenomenon of "wolf-packs."

Other forms of private information The fact that private information is about initial blocks is not essential when it comes to the type of strategic behavior uncovered. What matters is that the activists have interdependent private information: they know more about each other than the rest of the market does.

**Proposition 2.** Suppose that the activists' initial blocks are public, and consider the following variations of our model (in each case, the rest of the assumptions remain unchanged):

- (a) Exogenous components of firm value: the firm's (share) value is  $V^L + V^F + W^L + W^F$ , where  $V^i \sim \mathcal{N}(\mu, \phi)$  is exogenous and is activist i's private information,  $i \in \{L, F\}$ .
- (b) Activist productivity: Activist i's cost of effort is  $\frac{(W^i)^2}{2} \zeta_i W^i$ , where  $\zeta^i \sim \mathcal{N}(\mu, \phi)$  is exogenous and is activist i's private information,  $i \in \{L, F\}$ .

 $^{22}$ Di Maggio et al. (2019) argues that the best clients of brokers handling the order of an activist are much more likely to buy the associated stock during the 10-day window.

<sup>&</sup>lt;sup>23</sup>As an example, the short swing rule or Section  $16(b)$  of the Securities Act gives the issuer the right to ask a hedge fund holding over 10% to return any profits from reversal trades over a 6 month period. Also, insider trader rules that put limitations on trading arise above 10% ownership.

If  $Cov(\xi^L, \xi^F) = \rho \in [-\phi, \phi], \xi \in \{V, \zeta\},$  in both (a) and (b) there is a linear equilibrium with  $\mathbb{E}[\theta^F|\mathcal{F}_1] \equiv 0$ , while  $\mathbb{E}[\theta^L|\mathcal{F}_0] \leq 0$  if and only if  $\rho \geq 0$  (with strict inequality if  $\rho > 0$ ).

Both variations can be seen as capturing "activist expertise" about the target: in the first case, the activists have private information about an exogenous component of firm value (e.g., different divisions of the firm);<sup>24</sup> in the second,  $\zeta$  is a productivity parameter shaping each activist's private costs when unlocking firm value.<sup>25</sup> The main difference with our model is that in these variations, the leader's optimal trading strategy can attach a negative weight to her initial block. This happens when  $\rho > 0$ , and it is driven by an identical logic: through the value of manipulation, leaders with larger blocks benefit more from the follower's effort. While this effect is also present when blocks are private, it is counteracted by the desire to use a block as a source of informational advantage, leading to  $\alpha_L > 0$  for all values of  $\rho$ <sup>26</sup>

Ultimately, it is an empirical question which version is more appropriate for each event. Our view is that elements of all three must be at play in general. When it comes to hedge funds, (i) their rather small blocks, (ii) their sizable purchases at their trigger dates, and (iii) the availability of other financial instruments to circumvent disclosure, suggest that private information about true blocks is a reasonable benchmark case to study first.

Remark 1 (Passive leader). If the leader cannot exert effort but she can trade—thus behaving like a passive fund—the same mechanism ensues, but only sell orders are profitable. Indeed, in this case the follower's (trivial) forecast of the firm's value is independent of the firstperiod order flow, precisely because only the follower contributes to firm value. Since the leader can only affect the price, underpricing can only be created with sell orders. We note that this requires  $\rho \neq 0$ , as the leader's orders are uninformative about firm value otherwise. See the Internet Appendix for the actual construction of a linear equilibrium in this model.

# 4 Predictions and Measures of Abnormality

The predictability of trades is of great importance because it has real consequences: it determines the extent to which initial blocks are expected to change, so it speaks to the question of whether ex ante trading favors or debilitates the so-called costly "voice." Because

 $^{24}$ Brav et al. (2008) and Brav et al. (2021b) argue that firms with more diversified business have a higher probability of being targets. This form of diversification favors the possibility of different activists knowing about different parts of the firm, and even having different views about it (negative correlation case).

<sup>25</sup>Brick et al. (2024) studies how hedge funds' industry experience affect activism.

 $^{26}$ See (A.36) for the leader's equilibrium strategy in each case. In the productivity example, smaller trades are indicative of lower values of  $\zeta$ , and hence of lower firm value. In this proposition we can also determine a negative correlation threshold above which existence of a linear equilibrium is guaranteed, and this is also linked to the effective costs of trading becoming too low as in our theorem.

stock prices simply reflect the market's expectation of firms' true values, our model can link block interdependence, via the implied predictability of trades, with average prices during activism events. To this end, we map block interdependence in practice to our model as in Section 2.2, and then contrast the corresponding predictions on expected firm values with empirical measures of (pre-disclosure) price behavior given the same observables.

#### 4.1 Market Outcomes

We will examine market outcomes averaged across all possible blocks for the leader and follower; to simplify notation, we use  $\mathbb{E}[\cdot]$  to denote  $\mathbb{E}[\cdot|\mathcal{F}_0]$ , which is the relevant expectation. While selection effects can be at play in activism events, our broad average measure is not an unreasonable approximation. On the one hand, one may feel tempted to discard small blockholders based on a (debatable) belief that they are unlikely to play a key role.<sup>27</sup> On the other hand, the largest blockholder in a firm typically is a passive fund; further, the largest blockholders in a firm are less likely to trade (e.g., Lewellen and Lewellen, 2022).

From this perspective, recall that the follower's trades are neutral on average, so  $\mathbb{E}[X_T^F] =$  $\mathbb{E}[X_0^F] = \mu$ . Thus, it is only the leader who ultimately ends up affecting firm value through her trading. It is easy to see then that ex ante firm value and ex ante stock prices read  $\mathbb{E}[W^L + W^F] = \mathbb{E}[P_1] = \mathbb{E}[P_2] = (2 + \alpha_L + \delta_L)\mu$ . We assume  $\mu > 0$  for interpretations.

Proposition 3. In any PBS equilibrium,

- (i) Steering motive and interdependence:  $\mathbb{E}[W^L + W^F] \leq 2\mu$  if and only if  $\rho \geq 0$  (with strict inequality if  $\rho \neq 0$ ). Further, ex ante firm value monotonically decreases with  $\rho$ .
- (ii) Efficacy of multiplayer attacks:  $\mathbb{E}[W^L+W^F] > \mu$  for all  $\rho$  such that a PBS equilibrium exists (i.e.,  $\rho > \rho$ , where  $\rho$  is as in Theorem 1).
- (iii) Effect of market liquidity: Fix  $\rho > 0$ :

(iii.1) Both 
$$
\lim_{\sigma \to +\infty} \mathbb{E}[\theta^L]
$$
 and  $\lim_{\sigma \to +\infty} {\alpha_L - \sqrt{\sigma^2/\phi}}$  exist and take a negative value.  
(iii.2)  $\lim_{\sigma \to 0} \mathbb{E}[\theta^L] = 0$ , while  $\lim_{\sigma \to 0} {\alpha_L - \sqrt{\sigma^2/\phi}} = 0$ .

The first part of the proposition illustrates how the leader's steering motive operates to amplify or mitigate the static free-riding incentives that are inherent to multiplayer engagements. Concretely, absent any trading, ex ante firm value amounts to  $\mathbb{E}[X_0^L + X_0^F] = 2\mu$  due to each activist exerting effort according to their own block. When correlation is positive

 $^{27}$ Brav et al. (2021b) documents an example of a hedge fund owning 0.02 percent of outstanding stock and yet obtaining important concessions.

and the leader sells on average, firm value falls below this benchmark—the leader effectively offloads activism costs on the follower, and the extent of free riding grows. Conversely, when correlation is negative, the leader is inevitably forced to bear more of the activism costs and develop more skin in the game to entice the follower to build his block—remarkably, the steering motive now mitigates the extent of free riding. The last part of (i) simply says that we can analytically show that the inefficiencies grow monotonically as  $\rho$  increases.

Turning to (ii), note that when only one activist is present, the steering motive is trivially absent, so trades are unpredictable; ex ante firm value is then  $\mu$ . This part of the proposition then reads as follows: in a PBS equilibrium, multiplayer engagements always deliver more value than single-player attacks. By always we mean irrespective of the value that  $\rho$  takes, that is, even when the free-riding motive is exacerbated (e.g.,  $\rho > 0$ ). The reason is that, in the presence of a second activist, the only way for the leader to end up lowering firm value is by reversing her initial position on average (i.e.,  $sign(\mathbb{E}[X_0^L]) \neq sign(\mathbb{E}[X_T^L])$ ); but as argued, this is an equilibrium in which the players solidify their positions in relative terms.

Finally, the last part of the proposition explores how the steering motive is affected by the extent of market liquidity as measured by  $\sigma$ , the volatility of order flow—we focus on the case  $\rho > 0$  and limiting values of  $\sigma$  to obtain analytical comparisons. Recall that with positive block interdependence, the leader's motive is driven by her ability to move  $P_1$  downwards. A more liquid market—i.e., a larger  $\sigma$ —means that this ability falls, suggesting that less manipulation is optimal. But the fact that fundamentals are endogenous now kicks in: as the leader trades more aggressively due to her limited ability to move prices, she builds a bigger terminal block. Through this channel, the value of manipulation grows, despite the price's inherent reduced sensitivity. This is what (iii.1) states: there is a non-trivial degree of manipulation in the limit, as measured  $|\alpha_L - \sqrt{\sigma^2/\phi}| \neq 0$ ; and the extent of manipulation is bounded in that the leader sells a finite amount. Conversely, when  $\sigma \searrow 0$  and the market is infinitely illiquid, the leader naturally ceases to trade at all (part (iii.2)).

Figure 3 below confirms that the above findings also hold for intermediate values of  $\sigma$ , and for negative  $\rho$  when possible: the wedge between  $\alpha_L$  and  $-\delta_L$ , and that between  $\alpha_L$  and  $\sqrt{\sigma^2/\phi}$  (both measures of the extent of manipulation), expand as  $\sigma$  grows.

### 4.2 Connection with the Empirical Evidence

The empirical study of activism as an established investment strategy for hedge funds goes back to Brav et al. (2008) at least, who were the first to employ large-scale data to assess the impact of this practice. In discussing multiplayer interactions, they state (pp. 1732-1733):

"It is common for multiple hedge funds to coordinate by cofiling Schedule 13Ds



Figure 3: For each color, (degree of correlation), the continuous curves  $(\alpha_L)$  and their dashed counterparts  $(\delta_L)$  move apart with  $\sigma$  while bounding  $\alpha^K := \sqrt{\sigma^2/\phi}$ . In the limit as  $\sigma \to \infty$ , they become parallel when  $\rho > 0$ . Parameter values:  $\phi = 1$ .

(about 22% of the sample) or acting in tandem without being a formal block. Although some regulators have criticized such informal block behavior as anticompetitive, coordination among hedge funds can benefit shareholders overall by facilitating activism at relatively low individual ownership stakes."

Despite its obvious importance, how exactly this coordination occurs is a highly understudied topic. Any form of coordination must be in each activist's own interest nonetheless. Hence, it is our belief that any approach to the topic must be non-cooperative and must hinge on block accumulation.<sup>28</sup> From this perspective, our model argues that coordinating via the timing of trades—to escape competition effects—coupled with exploiting the informational content of prices—to influence others—is a natural mechanism when it comes to activists (i) sensitive to mispricing opportunities and (ii) who rely on fellow activists to influence firms. By comparing our model's predictions with the evidence on activism, we can then jointly test the plausibility of the mechanism and whether this coordination benefits shareholders.

In the empirical literature, a key measure for evaluating the impact of activism pertains to the time evolution of the abnormal buy-and-hold return in a window around the time of disclosure: stock price appreciation during activism relative to that during "normal times," understood as a period before the aforementioned window in which activism did not take place. Our model delivers analogous measures of abnormality. Specifically, observe that if activism is not at play—and so fundamentals are exogenous from the activists' perspective informed trades should respond to informational advantages only. As argued, this means trades that are neutral, and hence that blocks should not change on average—the average

 $^{28}$ As we argue in Section 5.3 the costs of acting as a formal group—in particular, of using explicit agreements—is very large for activists in the U.S. because block acquisition risks become too costly.

price is then  $2\mu$ . By Proposition 3 (i), however, average prices depart from this benchmark when activism is possible: in absolute terms, prices should be *abnormally low when*  $\rho > 0$ , and vice versa; in relative terms, as  $\rho$  grows, we should expect abnormal returns to fall.

- Performance of multiplayer engagements. Becht et al. (2017) examine hedge fund activism events using data from Asia, Europe and the United States. Of 1,740 events in their sample, a quarter involve multiple activists targeting the same firm. A key finding is that these events perform "strikingly better" (p. 2933) than single-player counterparts: their accumulated total stake as a whole is larger, and so are the abnormal returns observed (p. 2950). These findings are consistent with our robust finding that multiplayer engagements add more value for all  $\rho$  in Proposition 3 (ii).
- Abnormal returns and market capitalization. Brav et al. (2021b) provide comprehensive evidence on hedge fund activism performance in more than 4,600 events between 1994 and 2018 in the United States. One of their exercises is to study abnormal returns across different levels of market capitalization. For windows around both the disclosure and trigger dates, they find that there is substantially more abnormality for small-cap firms, followed by mid-cap, and lastly for the largest firms (even featuring negative abnormality in this latter case). In light of our discussion that positive (negative) block interdependence is likely to grow (fall) with market capitalization (Section 2.2), their finding is consistent with Proposition 3 (i): our measure of abnormality  $\mathbb{E}[W^L +$  $W^F$ ] – 2 $\mu$  falls as  $\rho$  increases (even taking negative values when  $\rho > 0$ ).
- Presence of large negative positions. Li et al. (2022) classify hedge fund attacks based on the presence or absence of investors with large short positions—a negative initial block in our setting—in the target's stock. Their finding is that the presence of large short positions is associated with higher abnormal returns. While in their dataset investors with negative positions need not be actively trying to take actions to undermine value (as it would occur in our model), we offer a qualitatively similar prediction: since observing a mix of one "long" and one "short" activist is more likely in our model when  $\rho < 0$ , prices in this case are predicted to be abnormally higher on average than if both activists are long (which happens with higher probability when  $\rho > 0$ ).<sup>29</sup>

There are a number of ways in which the empirical literature can use our predictions. The first avenue is to examine block interdependence at a more granular level than in Hadlock and Schwartz-Ziv (2019): say, focusing exclusively on hedge funds and splitting firms across

 $^{29}$ Relatedly, Cookson et al. (2022) show that greater disagreement among investors, measured using posts on a social media platform for investors, leads to more informed trading by activists and more short selling.

different levels of market capitalization. This would yield a more conclusive test that the steering motive exacerbates the free-riding effect as firm size grows. The second avenue is to examine block heterogeneity in activist attacks. Concretely, observe that negative correlation in our model need not imply a mix of positive or negative positions: since  $\mu > 0$ , it can also imply that the presence of a moderately large leader activist is necessarily indicative of a smaller follower. Thus, if multiplayer interventions that feature this form of block heterogeneity also exhibit higher abnormal returns, this would be another validation of our model (the effect of block size is examined in the next section). The third avenue is to empirically explore the price manipulation uncovered: this is an interesting possibility because it can occur despite the leader accumulating a long position, and because it can be exacerbated in more liquid markets where prices are less manipulable (Proposition 3 (iii)).

Ultimately, assessing the validity of these claims is important because they would point to a fundamental dichotomy in activism events featuring such "trading blockholders": their ability to overcome collective action problems may be very effective in smaller firms, but less so in larger ones, purely for strategic reasons. Further, this conclusion can be important if we expect groups of activists precisely to conglomerate more frequently around large firms in the future.<sup>30</sup> At this stage, our predictions are best interpreted in relative terms, as small and large in the context of the model is ultimately an empirical question.

### 5 First-Mover Advantages and Wolf Packs

### 5.1 Coordination in the timing of trades

To assess the benefit of acting as a leader, we compare trading strategies and payoffs in our model with those in a one-shot trading game in which both activists trade simultaneously.

**Proposition 4.** In a symmetric PBS equilibrium of a one-shot interaction with simultaneous moves, the activists trade according to  $\theta^{i} = \sqrt{\frac{\sigma^{2}}{2\phi}}$  $\frac{\sigma^2}{2\phi}(X_0^i - \mu)$ ,  $i = L, F$ . Also, there is a region around  $\rho = 0$  in which both traders get a higher ex ante payoff if they move sequentially.<sup>31</sup>

Since the slope  $\sqrt{\sigma^2/2\phi}$  is smaller than  $\sqrt{\sigma^2/\phi}$ , the traders are effectively scaling back due to the presence of a counterparty with market power. It is this competition effect that leads to less steering by the leader in the version in which she trades twice (Section 3.2):

 $30$ Artiga González and Calluzzo (2019) confirms this in campaigns involving hedge funds that are in geographic proximity, and argues that it is consistent with cost-sharing motives.

<sup>31</sup>We can also show that there is a negative threshold level of correlation above which there exists a symmetric PBS equilibrium and is unique. Indeed, the total order flow can become uninformative when  $\rho \ll 0$  due to the two activists' opposing trades reducing price impact, which conflicts with the endogeneity of fundamentals in the players' SOCs.

as the follower partially retreats, the value of manipulation falls. Conversely, looking at the result from the leader's perspective explains why our baseline model is a reasonable approximation: the leader is likely to purchase most of her block when she acts in isolation.

The negative consequences that competition can have for payoffs are reflected in the last part of the proposition: both the leader and the follower can benefit from acting in sequence relative to the simultaneous-move benchmark. In Figure 4 below, the region of interdependence where this mutually advantageous coordination can arise is actually large. To the right of this region, both activists would like to become a leader, and the benefit increases because it is easier to influence market markers' beliefs. Conversely, to the left, acting as a leader is not profitable: the presence of a fellow activist means access to valuable liquidity when needed because activists have opposing needs with high probability.



Figure 4: Leader's and follower's payoffs under sequential vs. simultaneous moves. Between the dashed vertical lines, both players prefer sequential moves. Parameters:  $\mu = \phi = 1, \sigma = .2$ .

These findings offer a strong theoretical underpinning for the notion that hedge funds in general benefit from completing their blocks in less competitive environments, giving validity to the thesis of sequential moves. This is an important observation given the perceived benefits that competition can have on activism via the total amount traded. To illustrate, consider the case  $X_0^L = X_0^F > \mu$ : since  $2\sqrt{\frac{\sigma^2}{2\phi}}(X_0 - \mu) > \sqrt{\frac{\sigma^2}{\phi}}$  $\frac{\sigma^2}{\phi}(X_0 - \mu)$ , the activists' total order when trading simultaneously is larger than in the single-player counterpart, implying a more pronounced impact on a firm's performance. The question is whether we expect it to be in each activist's best interest to act in this way. Our results suggest that this is not sustainable in general because it may not be in line with an activist's individual profit maximization. This demonstrates the importance of examining how blockholders' *private* benefits and costs from interventions can affect governance, as Edmans and Holderness (2017) emphasize.

### 5.2 Other Factors Favoring Leader-Type Behavior

Block size and productivity Our previous result averaged payoffs across all possible blocks to explore the benefits of acting in sequence. The left panel in Figure 5 analyzes the same topic but now conditioning on an activist's block size. Specifically, we plot the expected payoff of a first (top curve) and second (lower curve) mover conditional on a block  $X_0^i$  (horizontal axis), net of the payoff of moving simultaneously with the counterparty; correlation is positive and blocks weakly above average  $(\mu = 1)$ . As blocks grow past a threshold close to the mean, the benefit of acting as a monopolist in any period is increasing in block size—and being a leader is always preferred to being a follower.



Figure 5: Left panel: Expected payoff of  $i = L, F$  conditional on  $X_0^i$  net of simultaneous-move counterpart. Right panel: Ex ante gain for each player when the productive activist moves first. Parameter values:  $\mu = \phi = 1$ ,  $(\rho, \sigma) = (0.5, 1)$  (left) and  $(1, 0.02)$  (right).

We conclude that large blockholders effectively benefit from acting as leaders, and this benefit increases with block size. While the value of manipulation is obviously at play, the novelty is how competition effects play out conditional on block size. Indeed, when correlation is positive, an activist with a larger block expects their counterparty to be larger too, meaning that acquisition costs are expected to be even larger when trading simultaneously both activists then benefit from trading in isolation. Conversely, since small blockholders i.e., those around the mean—do not change their positions too much and do not expect large competitors, acquisition costs are less relevant: the positive effect that competition has on firm value can dominate slightly for them. Small changes in position, however, are inconsistent with the purchases observed around trigger dates (1% of a final 6% block).

Finally, the right panel of Figure 5 explores the question of the optimal sequence for activists who differ in their effort costs: there is an unproductive player with cost  $\frac{1}{2}W^2$ and a productive one with cost  $\frac{1}{2\zeta}W^2$ , where  $\zeta \geq 1$  is publicly known. The curves plot the expected payoff for the productive activist as a leader (top) and the unproductive as a follower (bottom) net of the payoff each would receive if moving in reverse order (payoffs are averaged across all blocks). As  $\zeta$  grows, the productive activist benefits more from being a leader, as expected. Interestingly, for large  $\zeta$ , the unproductive player does not want to lead either: a strong form of mutually advantageous coordination arises in that the activists do not want to change their roles. Indeed, with a more productive player the price is less responsive to order flow surprises; manipulating the price becomes too costly for the unproductive activist, as it would require excessively low trades relative to their block.<sup>32</sup>

Multiple followers Finally, it is natural to explore how our baseline model changes with the number of followers. This relates to the so-called "wolf pack activism" phenomenon, where similar funds simultaneously attack firms, which we discuss shortly.

We will consider the case in which the initial stake of our original follower is split among N individuals: each has an identical initial block  $X_0^F$  which is Gaussian with mean  $\mu/N$  and variance  $\phi/N^2$ , and with  $Cov(X_0^F, X_0^L) = \rho/N$ . This normalization achieves two important goals. First, it keeps fixed the total amount of uncertainty faced by market markers in the second period: otherwise, the leader's incentives may change purely due to a mechanical uncertainty effect. Second, notice that baseline effort—i.e., absent any trading—for any follower is decreasing in  $N$ , since initial positions have a shrinking mean. Put together, these two observations imply that any change in equilibrium outcomes must be due to strategic considerations in the trading game played among the followers.

The firm's value is  $W^L + \sum_{i=1}^{N} W_i$  $i=1$  $W^{F,i}$ , where  $W^j = X^j$  $T<sub>T</sub>$ , is the effort exerted by activist j. Motivated by the notion of similarity attributed to wolf packs, we consider  $\rho > 0$ ; as before, we use  $M_1^F := \mathbb{E}[X_0^F | \mathcal{F}_1]$  and  $\gamma_1^F := \mathbb{E}[(X_0^F - M_1^F)^2 | \mathcal{F}_1]$  to capture the market makers' belief about each follower's individual position after observing  $\Psi_1$  but before the followers trade.

**Proposition 5.** Fix  $\rho \in (0, \phi]$ . In the unique PBS equilibrium, each follower trades via  $\theta^F \,=\, \alpha_F (X_0^F \,-\, M_1^F), \,\, where \,\, \alpha_F \,=\, \sqrt{\frac{\sigma^2}{N \gamma_1^F}}. \,\,\, Also,\,\, \alpha_F \,\,\, is \,\, increasing \,\,\, in \,\, N; \,\, both \,\, \alpha_L \,\,\, and \,\, the \,\, \alpha_L, \,\, and \,\, the \,\, \alpha_L, \,\, and \,\, \alpha_L, \,\, respectively.}$ firm's ex ante value decrease in N; and the leader's ex ante payoff grows  $\sim \sqrt{N}$  for N large. √ If  $\rho = \phi$ , the leader's gain from moving first also grows  $\sim$ √ N for N large.

That the coefficient  $\alpha_F$  in the followers' strategy increases with N reflects strong competitive forces at play. In fact, as N grows, each follower possesses a smaller fraction of the total private information present at  $t = 2$ , which manifests in  $\gamma_1^F$  being proportional to  $1/N^2$ . This implies that any follower's individual contribution to price impact is smaller, incentivizing more aggressive trades. With followers that are more sensitive to mispricing

<sup>&</sup>lt;sup>32</sup>The analogous exercise involving different block sizes would be to fix one activist's stake, and compute the activists' payoffs as in the right panel while varying the other activist's block.

opportunities, the value of manipulation grows, and the leader's coefficient  $\alpha_L$  falls. Since the followers' trades are zero on average, the firm's ex ante value falls with  $N$  too.<sup>33</sup>

The leader's ex ante payoff grows at a rate  $\sqrt{N}$  for N large, despite the follower's trades being zero on average. The reason is the interaction term  $\mathbb{E}[X_T^L N X_T^F]$  capturing the followers' contribution to the value of the leader's total block: as N grows, the leader benefits from an increased block interdependence, now measured in terms of terminal positions that covary more strongly. The last part of the proposition simply says that, with perfect correlation, it is possible to show analytically that the leader's expected payoff net of the simultaneous-move counterpart has the same growth rate—moving first becomes more desirable.

Finally, Figure 6 shows that this competition effect and that of increasing  $\rho$  are in fact complements: when types are more correlated, the leader benefits from having more followers because their increased trading intensity leads to additional firm value that is more in line with the leader's. As the figure suggests, this benefit is likely less important when initial blocks are negatively correlated due to the risk of efforts becoming misaligned.

![](_page_30_Figure_3.jpeg)

Figure 6: Leader's expected payoff as a function of the number of followers, for various levels of covariance. Other parameter values:  $\phi = \mu = \sigma = 1$ .

### 5.3 Wolf Packs

Our model builds on the hypothesis (H1) that the activists involved are strongly sensitive to underpricing. Equipped with this, our mechanism is favored by the following factors:

(H2) Non-cooperative behavior: the activists do not employ formal agreements; rather they maximize their own profits understanding their counterparties' incentives and how trading and the price mechanism can be used to their own advantage;

<sup>&</sup>lt;sup>33</sup>In Edmans and Manso (2011), "voice" is also weakened by the number of traders, but this is due to the free-rider problem worsening among them. To see why this need not be the case here, consider the simpler case in which the leader is absent. Letting  $\hat{X}_0^F$  denote the (fixed) original stake of the follower, it is easy to see that the total volume traded by N followers is  $N\sqrt{\frac{\sigma^2}{N(\phi/N^2)}}$  $\frac{\hat{X}_0^F - \mu}{N}$ , which grows in N if  $\hat{X}_0^F > \mu$ .

- (H3) Similarity: the activists hold similar stakes in a statistical sense, in that blocks are not too negatively correlated—this favors the emergence of a leader. For intermediate levels of interdependence, coordinating the timing of trades is mutually beneficial;
- (H4) Moderate stakes: since in practice there is a fixed number of shares, similarity (in the above sense) requires the activists to have small to moderate stakes. Otherwise, the chance that a trade by an activists is satisfied by another fellow activist grows, which undermines the plausibility of sequentiality from the perspective of market makers. Moderate stakes also make the likelihood of trading on a target's stock higher;
- (H5) Multiple small followers. If there is positive interdependence, competition effects associated with the presence of multiple followers make it increasingly profitable for a hypothetical leader to emerge; this effect is reinforced if a leader has a larger block.

Hedge funds are natural candidates to satisfy H1–H5. In particular, these assumptions fit the so-called wolf-pack activism phenomenon, whereby multiple hedge funds of small to moderate size attack a firm in parallel—and in a seemingly non-cooperative manner—after a leader hedge fund has built a stake in the target; see Becht et al. (2017), Brav et al. (2021a), Briggs (2007) and Coffee Jr and Palia (2016) for in-depth treatments of this topic.

The starting point is that hedge funds are the quintessential example of exploitation of mispricing opportunities (H1), and activism as an investment strategy is not the exception: it is argued that hedge funds behave like "value investors" by attacking underpriced firms relative to their potential, as measured by large book-to-market value or a low Tobin's q (Brav et al., 2008; Brav et al., 2021b). In our model, this phenomenon is manifested in the intensive margin of intervention growing in the extent of mispricing.

Regarding H2, there are substantial costs associated with being perceived as a "group" from the standpoint of Section 13(d)(3) of the Securities Exchange Act.<sup>34</sup> The key issue is that an organized set of activists is treated as a single entity with a block equal to the sum of its components. In this situation, there are potential legal fees if the target firm alleges a violation of disclosure requirements (e.g., not disclosing when the aggregate block surpasses 5%), which would be absent if the activists were individually below the 5% threshold and acted non-cooperatively. On the other hand, complying with disclosure rules means that a group necessarily invites undesired competition before achieving a desired block size (in all likelihood above 5%), thereby making block acquisition more costly. Additionally, the target firm may bar the acquisition of more shares by the group members—the identities of which are revealed upon disclosure—which may preclude the success of any engagement.

<sup>34</sup>See for instance, Coffee Jr and Palia (2016), pp. 24–26.

With explicit agreements being risky (and indeed rare in practice, as Becht et al. (2017) argue), activists likely resort to their shared understanding of the economic environment. Two factors have favored a reduction in the strategic complexity of the latter in recent decades: institutional ownership has become more concentrated and changes in SEC regulation imply that activists can communicate in a limited manner without this being characterized as insider trading or trading as a group. In the words of Lewellen and Lewellen  $(2022)$  (pp. 1–2):

"First, Rule  $14a-2(b)(2)$  of the Securities Exchange Act allows an activist to solicit proxies from up to 10 investors without being subject to the usual proxy solicitation rules. Thus, in recent years, an activist could solicit proxies from 37.1% of shares for the value-weighted average firm the holdings of the top 10 institutional shareholders without triggering the stringent filing and informational requirements associated with public proxy contests. Second, shareholders can communicate freely with each other about how they intend to vote and why, and have wide latitude to distribute pre-solicitation material before filing a definitive proxy statement, as long as they do not solicit proxies from each other or coordinate their votes. This communication is easier when ownership is more concentrated and the identity of shareholders is public information. In recent years, a shareholder would need to contact only 5 institutions to reach investors holding 25% of shares and 27 institutions to reach investors holding 50% of shares."

With a smaller number of relevant participants, thinking about others' incentives is easier. Further, with communication, a shared understanding of the environment can be developed. Altogether, these factors pave the way not only for strategic interactions, but also for implicit agreements such as coordinating the timing of trades: immediately attacking after others do, and the common knowledge of it triggering leader behavior in the way that we propose.

Regarding similarity (H3), the niche business models and strategies that hedge funds deploy are a strong suggestion of block similarity in a statistical sense, which is supported by the findings on strategic investors by Hadlock and Schwartz-Ziv (2019) (who in fact argue that positive interdependence is indicative of similar investment styles). Hedge funds' similar trading strategies also suggest similarity in research, hence an overlap in potential targets, further reinforcing their block interdependence. And as we have argued, their stakes are moderately sized, which is consistent with a goal to influence firms and not necessarily exert control (e.g., Brav et al., 2021b). Despite their smaller blocks, this activist category is argued to be the only one within the set of institutional blockholders with a proven record of significantly affecting firms (Brav et al., 2008).

Finally, the evidence on multi-activist engagements traditionally comes from two sources. First, as argued, 13D forms if blocks are larger than 5%, but also from 13F forms if a fund below the threshold is sufficiently large. Second, indirectly, through the analysis of abnormal returns around disclosure events. In this regard, Wong (2020) finds that, in campaigns with a single 13D filer, trades on the trigger date by such leader activists (i.e., before their intentions become public) explain only 25% of the abnormal turnover observed in the data, with the unexplained component averaging 240% of that in normal times; further, he shows that investors who have a prior relationship with the leader in past campaigns are more likely to buy shares. The extent to which such followers operate based on market signals, and whether they contribute to activism expenses, is a matter of debate. Still, these numbers and the anecdotal evidence suggest both a strong incentive by such leaders to move first and a sufficient degree of common knowledge of leader-follower dynamics at play after observing sudden abnormalities. In addition, because many pack members do not disclose, leaders in such attacks are by definition larger than their subsequent followers.

### 6 Other Equilibria and Refinement

We have focused on the case of positive block sensitivity. But equilibria in which at least one of our activists attaches a negative weight to their initial block can also arise—the reason is a coordination motive in value creation/destruction. Suppose that the activists start "long" on the firm (i.e.,  $X_0^L, X_0^F > 0$ ) and that the leader expects the follower to acquire a short position on the firm's value—i.e.,  $\alpha_F < 0$ . In the expectation of a potential negative effort by the follower, the leader may then want to build a negative stake too, as there would be a positive surplus if both players exert negative effort. By the same logic, the follower would choose  $\alpha_F < 0$ , and the expectations are self-fulfilling. Before elaborating on why this type of equilibrium is less appropriate as a prediction for activism events, let us explain what we know about it and how this knowledge connects to our PBS equilibrium.

Concretely, in Section II.A in our Internet Appendix we show that for the  $\rho > 0$  case, if  $\sigma > 0$  is sufficiently large, there is an equilibrium with both  $\alpha_L$  and  $\alpha_F$  taking negative values. (Since  $\rho > 0$  implies that the activists' initial blocks likely have the same sign, this finding is line with the previous logic.) The idea is that when order flow volatility  $\sigma$  is large, it is difficult for the leader to move the price: this facilitates a coordination equilibrium, despite our PBS equilibrium not disappearing.

This brings us to the topic of the lower bound  $\rho < 0$  in Theorem 1, which guarantees the existence of a PBS equilibrium. As argued, in Kyle-type models, price impact is the only force that makes trading costly; but here, there is also the possibility of manipulation. With

positive correlation, more aggressive trading carries the extra cost of lowering the follower's contribution to the firm. By contrast, with negative correlation, trading more aggressively is beneficial in that it encourages the follower to exert effort, a force going against price impact. Thus, through this *effective cost channel*, the leader's problem is more concave when  $\rho > 0$ and more convex when  $\rho < 0$ . This explains why a PBS equilibrium always exists when  $\rho > 0$ , whereas when  $\rho$  becomes sufficiently negative for fixed  $\sigma$ , it may cease to exist: the leader's second-order condition cannot be satisfied by positive  $(\alpha_L, \alpha_F)$  pairs, hence  $\rho < 0$ in our Theorem. As a proof of concept, in our analysis of coordination equilibria in Section II.A of the Internet Appendix, we also show that if  $\rho = -\phi$ , there is no equilibrium in which  $\alpha_F$  and  $\alpha_L$  have the same sign; but one with  $sign(\alpha_L) \neq sign(\alpha_F)$  exists for all  $\sigma > 0$ .

Order flow volatility can then play a dual role: by making manipulation easier, it can make deviations from candidate coordination equilibria more profitable when  $\rho > 0$ ; and by increasing price impact, it can restore concavity in the leader's problem when  $\rho < 0$ . Thus, market illiquidity can refine PBS equilibrium as the unique prediction within the linear class:

**Proposition 6.** Suppose that  $\rho \in (-\phi, \phi)$ . Then for sufficiently small but positive  $\sigma$ , a PBS equilibrium exists and is the unique equilibrium within the linear class.

Coordination equilibria are not unreasonable because they rely on negative firm values, as our model and many others in the microstructure literature allow: after all, it is well-known that acquiring a negative position can be profitable if it triggers a mechanism that lowers a firm's value (Goldstein and Guembel, 2008). When it comes to positive activism, however, it is the feature of revising one's initial choices so radically simply due to the expectation of what others will do that seems stark: such an unwinding before activism occurs means going against the information acquisition and research that in reality leads to the choice of an initial block. Brav et al. (2021b) provide evidence precisely undermining this possibility: hedge funds' average duration of investment in a target is over 530 days, meaning that more than a year and a half passes between disclosure of a position and a major divestiture happen.

# 7 Conclusions

We have proposed a model of interactions among blockholders featuring block accumulation and intervention in firms. Despite its real-world relevance, multiplayer analyses of this kind are a highly understudied topic. The model underscores how coordinating through the price mechanism can be used as a tool to control activism costs in competitive settings, and how such coordination naturally introduces strategic considerations among blockholders: distorting trades to influence others to build skin in the game. We showed that this motive generates non-trivial stock prices that resemble price abnormalities widely documented in the empirical work on hedge fund activism, and also shed light on when and how such a coordination can ameliorate or exacerbate the collective action problem at play.

From a modeling viewpoint, our mechanism is based on the presence of a non-trivial continuation value linked to influencing the gains from trade for other activists—hence, the key force through which the model operates will also be at play under other activism technologies. We would also expect the mechanism to be present, if not reinforced, when there are multiple rounds of trading. Indeed, note that in a fully dynamic environment, the terminal position of any activist now generalizes to a linear aggregate of multiple past trades; each activist would then have the opportunity to influence the subsequent trades of their counterparty at all times (say, by dampening the price if the contemporaneous correlation is positive) with the effect likely compounding if more rounds of trading are left.

Finally, the model has taken the activists' initial positions as exogenous. While there are natural justifications for this choice (e.g., the opportunity to intervene was unanticipated), as well as for the sign of the interdependence (e.g., similar investment styles versus differing views about performance) it is natural explore ways to endogenize this feature. Our Internet Appendix shows that this is possible when enriching our baseline model to incorporate an exogenous component of firm value and a pre-round of trading based on private signals, just like in traditional microstructure models. By varying the degree of correlation of the latter signals and allowing for some interim information revelation about firm value, it is possible to generate early trades—hence "initial" blocks—that exhibit both types of interdependence. Variations of this approach with early rounds of trading are promising if the goal is to develop models where both the roles of leader and follower are determined endogenously; but also to encompass the issue of timing of liquidity as an additional tool to control the costs of block acquisition, as documented by Collin-Dufresne and Fos (2015).

# A Appendix: Proofs

### A.1 Supporting details for learning and pricing

This section derives expressions for beliefs and prices omitted from the main body, which follow from the projection theorem for Gaussian random variables: i.e., if  $(X, Y)$  are jointly Gaussian, then  $\mathbb{E}[X|Y] = \mathbb{E}[X] + \frac{\text{Cov}[X,Y]}{\text{Var}[Y]}(Y - \mathbb{E}[Y])$  while  $\text{Var}[X|Y] = \text{Var}[X] - \frac{\text{Cov}^2[X,Y]}{\text{Var}[Y]}$  $\frac{\mathrm{O}{\mathrm{V}}^{\infty}[X,Y]}{\mathrm{Var}[Y]}$ .

**Lemma A.1.** In any linear equilibrium, beliefs and prices are characterized as follows:

• Prior to trading: Player's private initial beliefs and the initial price are given by

$$
Y_0^i := \mathbb{E}[X_0^{-i}|X_0^i] = \mu + \frac{\rho}{\phi}(X_0^i - \mu), \qquad \nu_0^i := \text{Var}(X_0^{-i}|X_0^i) = \phi - \frac{\rho^2}{\phi}
$$
  
\n
$$
P_0 = \frac{\mu(2 + \alpha_L + \alpha_F + \delta_L + \delta_F)}{1 - \beta_F}.
$$
\n(A.1)

• Period 1: Given  $\Psi_1$  market maker believes  $\begin{pmatrix} X_T^L \\ Y_F^F \end{pmatrix}$  $X^F_0$  $\setminus$  $\sim N$  $\int \int M_1^L$  $M_1^F$  $\setminus$ ,  $\int \gamma_1^L \quad \rho_1$  $\rho_1$   $\gamma_1^F$  $\bigg)$ , where

$$
M_1^L := \mathbb{E}[X_T^L | \mathcal{F}_1] = (1 + \alpha_L) \left[ \mu + \frac{\alpha_L \phi}{\alpha_L^2 \phi + \sigma^2} \left\{ \Psi_1 - \mu (\alpha_L + \delta_L) \right\} \right] + \delta_L \mu \tag{A.2}
$$

$$
M_1^F := \mathbb{E}[X_0^F | \mathcal{F}_1] = \mu + \frac{\alpha_L \rho}{\alpha_L^2 \phi + \sigma^2} \left\{ \Psi_1 - \mu(\alpha_L + \delta_L) \right\}
$$
(A.3)

$$
\gamma_1^L = \frac{\phi \sigma^2 (1 + \alpha_L)^2}{\alpha_L^2 \phi + \sigma^2}, \qquad \gamma_1^F = \frac{\alpha_L^2 [\phi^2 - \rho^2] + \phi \sigma^2}{\alpha_L^2 \phi + \sigma^2}, \qquad \rho_1 = \frac{\rho \sigma^2 (1 + \alpha_L)}{\alpha_L^2 \phi + \sigma^2}.
$$
 (A.4)

The first period price is

$$
P_1 = P_0 + \Lambda_1 \left[ \Psi_1 - (\alpha_L + \delta_L) \mu \right], \ \ with \tag{A.5}
$$

$$
\Lambda_1 := \frac{\alpha_L \phi}{\alpha_L^2 \phi + \sigma^2} \times \frac{1 + \alpha_L + \rho (1 + \alpha_F) / \phi}{1 - \beta_F}.
$$
\n(A.6)

The follower's posterior mean belief about  $X_T^L$ , denoted  $Y_1^F := \mathbb{E}_F[X_T^L | \mathcal{F}_1]$ , is

$$
Y_1^F = (1 + \alpha_L) \left[ Y_0^F + \frac{\alpha_L \nu_0^F}{\alpha_L^2 \nu_0^F + \sigma^2} \left\{ \Psi_1 - (\alpha_L Y_0^F + \delta_L \mu) \right\} \right] + \delta_L \mu. \tag{A.7}
$$

• Period 2: Given  $\Psi_2$ , the market maker's updated beliefs about  $(X_T^L, X_T^F)$  have means

$$
M_T^F := \mathbb{E}[X_T^F | \mathcal{F}_2]
$$
  
=  $(1 + \alpha_F)M_1^F + \beta_F P_1 + \delta_F \mu + \frac{\alpha_F \gamma_1^F (1 + \alpha_F)}{\alpha_F^2 \gamma_1^F + \sigma^2} [\Psi_2 - \alpha_F M_1^F - \beta_F P_1 - \delta_F \mu],$   
(A.8)

$$
M_T^L := \mathbb{E}[X_T^L | \mathcal{F}_2] = M_1^L + \frac{\alpha_F \rho_1}{\alpha_F^2 \gamma_1^F + \sigma^2} [\Psi_2 - \alpha_F M_1^F - \beta_F P_1 - \delta_F \mu]. \tag{A.9}
$$

The second period price is

$$
P_2 = P_1 + \Lambda_2 [\Psi_2 - \alpha_F M_1^F - \beta_F P_1 - \delta_F \mu], \text{ with} \tag{A.10}
$$

$$
\Lambda_2 := \frac{\alpha_F \gamma_1^F}{\alpha_F^2 \gamma_1^F + \sigma^2} \times [1 + \alpha_F + \rho_1/\gamma_1^F].
$$
\n(A.11)

*Proof.* The expressions for  $Y_0^i$  and  $\nu_0^i$  follow immediately from the projection theorem applied to the pair  $(X_0^i, X_0^{-i})$ . Using the conjectured strategies,  $P_0$  satisfies

$$
P_0 = \mathbb{E}[(1 + \alpha_L)X_0^L + \delta_L \mu + (1 + \alpha_F)X_0^F + \beta_F P_1 + \delta_F \mu].
$$
 (A.12)

Using that  $\mathbb{E}[P_1] = P_0$  to eliminate  $P_1$  yields an equation for  $P_0$  with solution (A.1), where the denominator is nonzero due to the leader's second order condition (10).

Given  $\Psi_1$ , the market maker updates beliefs about  $X_0^F$  (as in (A.3)) and  $X_0^L$  using the projection theorem. Using the leader's conjectured strategy mapping  $X_0^L$  to  $X_T^L$  then yields (A.2). The respective second moments in (A.4) also follow from the projection theorem and applying the leader's conjectured strategy.  $P_1$  is then the fixed point of  $P_1 = \mathbb{E}[X_T^L + X_T^F | \mathcal{F}_1],$ where  $\mathbb{E}[X_{T}^{F}|\mathcal{F}_{1}] = (1 + \alpha_{F})M_{1}^{F} + \beta_{F}P_{1} + \delta_{F}\mu$ , giving (A.5)-(A.6).

Finally, given  $\Psi_2$ , the market maker's updated beliefs in  $(A.8)-(A.9)$  follow from the projection theorem, and (A.10)-(A.11) follow directly from  $P_2 = M_T^F + M_T^L$ .  $\Box$ 

### A.2 Preliminaries for Equilibrium Construction

In this section, we state and prove a proposition, to be used in proving our main results, that characterizes equilibria via a system of equations and inequality conditions derived from the players' first and second order conditions and the pricing equations. The first half of the proposition below provides necessary conditions for equilibrium. The second half of the proposition is a strong converse: it shows that we can focus on the system of equations for the signaling coefficients  $(\alpha_F, \alpha_L)$ ; these coefficients determine price impact and therefore pin down the remaining coefficients.

**Proposition A.1.** The tuple  $(\alpha_F, \beta_F, \delta_F, \alpha_L, \delta_L)$  together with a pricing rule defined by (A.5)-(A.6) and (A.10)-(A.11) characterize an equilibrium only if  $\Lambda_1 \neq 0$ ,  $\Lambda_2 \neq 0$ ,  $\beta_F \neq 1$ ,  $\phi(1 + \alpha_L) + \rho \neq 0$ , and

$$
\alpha_F^2 = \sigma^2 / \gamma_1^F,\tag{A.13}
$$

$$
\beta_F = -\frac{\rho}{\phi(1+\alpha_L) + \rho} \alpha_F,\tag{A.14}
$$

$$
\delta_F = \frac{(\alpha_L + \delta_L)\rho - \alpha_L\phi - (\phi - \rho)}{\phi(1 + \alpha_L) + \rho}\alpha_F,
$$
\n(A.15)

$$
\alpha_L = \frac{\sigma^2}{\phi \alpha_L} - \frac{\rho \alpha_F}{\phi (1 + \alpha_L) + \rho (1 + \alpha_F)},
$$
\n(A.16)

$$
\delta_L = -\frac{\sigma^2}{\phi \alpha_L},\tag{A.17}
$$

$$
0 \ge \sigma^2 - \alpha_L^2 \phi - 2\alpha_L [\rho (1 + \alpha_F) + \phi], \tag{A.18}
$$

$$
0 \ge -\alpha_F[\sigma^2(\phi + \rho(1 + \alpha_L)) + \alpha_L^2(\phi^2 - \rho^2)]. \tag{A.19}
$$

Further, if  $\rho \neq 0$ , one of the following conditions must hold:

$$
\alpha_F = \alpha_{F,1}(\alpha_L) := \sqrt{\frac{\sigma^4 + \alpha_L^2 \sigma^2 \phi}{\sigma^2 \phi + \alpha_L^2 (\phi^2 - \rho^2)}} = \frac{(\rho + \phi + \phi \alpha_L)(\alpha_L^2 \phi - \sigma^2)}{\rho[\sigma^2 - \alpha_L (1 + \alpha_L) \phi]} \text{ or } (A.20)
$$

$$
\alpha_F = \alpha_{F,2}(\alpha_L) := -\sqrt{\frac{\sigma^4 + \alpha_L^2 \sigma^2 \phi}{\sigma^2 \phi + \alpha_L^2 (\phi^2 - \rho^2)}} = \frac{(\rho + \phi + \phi \alpha_L)(\alpha_L^2 \phi - \sigma^2)}{\rho[\sigma^2 - \alpha_L (1 + \alpha_L)\phi]}.
$$
(A.21)

Conversely, suppose  $(\alpha_F, \alpha_L)$  satisfy (A.18) and (A.19), either (A.20) or (A.21), and  $\phi(1 +$  $\alpha_L$ ) +  $\rho \neq 0$ . Then (i)  $(\beta_F, \delta_F, \delta_L)$  are well defined via (A.14), (A.15), and (A.17), with  $\beta_F \neq 1$ ; (ii)  $\Lambda_1 \neq 0$  and  $\Lambda_2 \neq 0$  are well defined via (A.6) and (A.11); and (iii) the associated strategies and pricing rule constitute an equilibrium.

Proof. We first establish necessity, starting with the follower's conditions. The follower's FOC expands as

$$
0 = -\mathbb{E}_F[P_1 + \Lambda_2 \{\Psi_2 - \mathbb{E}[\Psi_2|\mathcal{F}_1]\}|\theta^F] - \Lambda_2 \theta^F + (X_0^F + \theta^F) + Y_1^F
$$
(A.22)

$$
= -P_1 - \Lambda_2(\theta^F - [\alpha_F M_1^F + \beta_F P_1 + \delta_F \mu]) - \Lambda_2 \theta^F + (X_0^F + \theta^F) + Y_1^F,
$$
(A.23)

which we impose at the candidate strategy in  $(5)$ . Now by inverting  $(A.5)$ , we can write  $\Psi_1 = \mu(\alpha_L + \delta_L) + \frac{P_1 - P_0}{\Lambda_1}$ , with  $P_0$  given by (A.1), which we can use to eliminate  $\Psi_1$  in  $M_1^F$ and  $Y_1^F$  (see (A.3) and (A.7)). Recalling that  $Y_0^F$  (appearing in  $Y_1^F$ ) is a linear combination of  $(X_0^F, \mu)$ , the resulting equation is linear in  $(X_0^F, P_1, \mu)$ , and it must be identically zero over  $(X_0^F, P_1, \mu) \in \mathbb{R}^3$ . Hence, the coefficients on each variable  $(X_0^F, P_1, \mu)$  must be zero, delivering three equations. The first of these, from the coefficient on  $X_0^F$ , is

$$
0 = -2\Lambda_2 \alpha_F + (1 + \alpha_F) + \frac{\partial Y_1^F}{\partial X_0^F} = \frac{\tilde{\Lambda}_2}{\gamma_1^F} (\sigma^2 - \alpha_F^2 \gamma_1^F), \tag{A.24}
$$

where  $\tilde{\Lambda}_2 := \frac{\gamma_1^F}{\alpha_F^2 \gamma_1^F + \sigma^2} \times [1 + \alpha_F + \rho_1/\gamma_1^F]$ . The second, from the coefficient on  $P_1$ , is

$$
0 = -1 - \Lambda_2 \left( -\alpha_F \frac{\partial M_1^F}{\partial P_1} \right) - \Lambda_2 \beta_F + \beta_F + \frac{\partial Y_1^F}{\partial P_1}
$$
  
= 
$$
-\frac{\tilde{\Lambda}_2}{\gamma_1^F} \left[ \frac{\rho \sigma^2 (1 - \beta_F)}{\phi (1 + \alpha_L) + \rho (1 + \alpha_F)} + \beta_F \alpha_F \gamma_1^F \right].
$$
 (A.25)

The third, from the coefficient on  $\mu$ , is

$$
0 = -\Lambda_2 \left( -\alpha_F \frac{\partial M_1^F}{\partial \mu} \right) - \Lambda_2 \delta_F + \delta_F + \frac{\partial Y_1^F}{\partial \mu}
$$
  
=  $\frac{\tilde{\Lambda}_2}{\gamma_1^F} \left[ -\sigma^2 + \frac{(2 + \alpha_F + \alpha_L + \delta_F + \delta_L)\rho \sigma^2}{\phi (1 + \alpha_L) + \rho (1 + \alpha_F)} - \alpha_F \delta_F \gamma_1^F \right],$  (A.26)

where the  $\mu$  terms of  $M_1^F$  and  $Y_1^F$  incorporate the elimination of  $\Psi_1$  described above.

We argue that in any linear equilibrium, the right hand sides of  $(A.24)-(A.26)$  are well defined and  $\tilde{\Lambda}_2 \neq 0$ . First,  $\gamma_1^F > 0$  for any (finite)  $\alpha_F$ . Second, (9) implies  $\Lambda_2 \neq 0$ , so  $\tilde{\Lambda}_2$  is well defined and nonzero. Third,  $\Lambda_1 \neq 0$  implies  $\phi(1+\alpha_L) + \rho(1+\alpha_F) \neq 0$  in the denominators in (A.25) and (A.26).

We can now derive (A.13)-(A.15) and (A.19). Since  $\tilde{\Lambda}_2 \neq 0$  is necessary for equilibrium, (A.24) reduces to (A.13). (Note that this implies  $\alpha_F \neq 0$ .) Using this fact to write  $\alpha_F \gamma_1^F =$  $\sigma^2/\alpha_F$ , (A.25) reduces to

$$
0 = -\frac{\tilde{\Lambda}_2}{\gamma_1^F} \left[ \frac{\rho \sigma^2 (1 - \beta_F)}{\phi (1 + \alpha_L) + \rho (1 + \alpha_F)} + \beta_F \frac{\sigma^2}{\alpha_F} \right]
$$
  
= 
$$
-\frac{\tilde{\Lambda}_2 \sigma^2}{\gamma_1^F \alpha_F [\phi (1 + \alpha_L) + \rho (1 + \alpha_F)]} [\rho \alpha_F + \beta_F [\phi (1 + \alpha_L) + \rho]].
$$
 (A.27)

We claim that  $\phi(1+\alpha_L)+\rho\neq 0$  in equilibrium. By way of contradiction, if  $\phi(1+\alpha_L)+\rho=0$ , then (A.27) implies  $\alpha_F = 0$  or  $\rho = 0$ . Equation (A.13) rules out  $\alpha_F = 0$ . And if  $\rho = 0$ , we have  $\alpha_L = -1$ , and thus  $\Lambda_1 = 0$ , violating the leader's SOC. Hence,  $\phi(1 + \alpha_L) + \rho \neq 0$ , and (A.27) reduces to (A.14). Analogous arguments yield (A.15) from (A.26). Lastly, using (A.13) to eliminate  $\alpha_F^2$  terms, the follower's SOC (9) reduces to (A.19).

Next, we derive the leader's identities (A.16)-(A.17) and condition (A.18). For the leader, the following FOC, evaluated at the conjectured strategy, must hold for all  $(X_0^L, \mu) \in \mathbb{R}^2$ :

$$
0 = -\mathbb{E}_L[P_0 + \Lambda_1\{\Psi_1 - \mathbb{E}[\Psi_1]\}|\theta^L] - \theta\Lambda_1 + (X_0^L + \theta^L) + \mathbb{E}_L[X_T^F|\theta^L] + (X_0^L + \theta^L)\frac{\partial \mathbb{E}_L[X_T^F|\theta^L]}{\partial \theta^L}.
$$
\n(A.28)

Setting the coefficients on these variables to 0 and using  $(A.13)$  and  $(A.14)$ , it is straightforward to show that (A.28) reduces to (A.16)-(A.17) where  $\alpha_L \neq 0$  in equilibrium since the leader's SOC implies  $\Lambda_1 \neq 0$ . The leader's SOC is equivalent to (A.18).

To obtain (A.20) or (A.21), first note that the positive and negative values of  $\alpha_F$  solving (A.13) are  $\pm \sqrt{\frac{\sigma^4 + \alpha_L^2 \sigma^2 \phi}{\sigma^2 \phi + \alpha_L^2 \sigma^2 \phi^2}}$  $\frac{\partial^2 \pm \alpha_L \partial^2 \varphi}{\partial^2 \varphi + \alpha_L^2 (\varphi^2 - \rho^2)}$ . Next, solve for  $\alpha_F$  in (A.16) by multiplying through by the denominators on the right hand side and rearrange terms to obtain

$$
\alpha_F \rho [\sigma^2 - \alpha_L (1 + \alpha_L) \phi] = [\phi (1 + \alpha_L) + \rho ](\alpha_L^2 \phi - \sigma^2). \tag{A.29}
$$

We claim that  $\sigma^2 - \alpha_L(1+\alpha_L)\phi \neq 0$  in any solution to (A.29). Indeed, since  $\phi(1+\alpha_L)+\rho \neq 0$ ,  $\sigma^2 - \alpha_L(1 + \alpha_L)\phi = 0$  would imply  $\alpha_L^2 \phi - \sigma^2 = 0$ , but these two equations cannot hold simultaneously. Thus, if  $\rho \neq 0$ , (A.29) implies

$$
\alpha_F = \frac{(\rho + \phi + \phi \alpha_L)(\alpha_L^2 \phi - \sigma^2)}{\rho[\sigma^2 - \alpha_L(1 + \alpha_L)\phi]}.
$$

Since the solutions to (A.13) are  $\alpha_F = \alpha_{F,1}$  and  $\alpha_F = \alpha_{F,2}$ , we obtain (A.20) and (A.21).

For the sufficiency half of the proposition, take  $(\alpha_F, \alpha_L)$  as in the statement. Clearly, either  $\alpha_F = \alpha_{F,1}$  or  $\alpha_F = \alpha_{F,2}$  implies (A.13). Now given  $\phi(1 + \alpha_L) + \rho \neq 0$ , we can multiply through (A.20) or (A.21) by  $\rho[\sigma^2 - \alpha_L(1 + \alpha_L)\phi]$  to recover (A.29). To recover (A.16) from (A.29), simply note that (A.18) can be rewritten as  $\sigma^2 + \alpha_L^2 \phi - 2\alpha_L[\rho(1+\alpha_F) + \phi(1+\alpha_L)] \leq 0$ , which implies  $\alpha_L \neq 0$  and  $\phi(1 + \alpha_L) + \rho(1 + \alpha_F) \neq 0$ . Given that  $\phi(1 + \alpha_L) + \rho \neq 0$  by supposition,  $(\beta_F, \delta_F)$  are well defined by (A.14)-(A.15). Further,  $\phi(1+\alpha_L) + \rho(1+\alpha_F) \neq 0$ implies that  $1 \neq -\frac{\rho \alpha_F}{\phi(1+\alpha_L)+\rho} = \beta_F$ . This establishes (i). It follows that  $\Lambda_1$  and  $\Lambda_2$  are well defined by  $(A.6)$  and  $(A.11)$ , respectively. Moreover, by construction,  $(A.18)$ - $(A.19)$  imply  $(10)-(9)$ , so  $\Lambda_1 \neq 0$  and  $\Lambda_2 \neq 0$ , establishing (ii).

For part (iii) of the sufficiency claim, observe that since the players' best responses problems are quadratic, it suffices to check first and second order conditions. Given that the inequalities  $\Lambda_1 \neq 0$ ,  $\Lambda_2 \neq 0$ ,  $\beta_F \neq 1$ ,  $\phi(1 + \alpha_L) + \rho \neq 0$  are satisfied, the equations  $(A.13)-(A.17)$  imply the FOCs  $(A.22)$  and  $(A.28)$  by construction, and as noted for part (ii), the SOCs (10) and (9) are satisfied.  $\Box$ 

### A.3 Proof of Theorem 1

To prove that  $\mathbb{E}[\theta^F|\mathcal{F}_1]=0$ , we simply use the fact that, by Proposition 1,  $\theta^F=\alpha_F(X_0^F-\theta_F^F)$  $M_1^F$ ), where  $M_1^F$  was defined as  $\mathbb{E}[X_0^F|\mathcal{F}_1]$ , and take expectations conditional on  $\mathcal{F}_1$ .<sup>35</sup>

The rest of the proof is divided into four components as follows. First, we first address  $\rho = 0$ , in which case the unique linear equilibrium can be characterized in closed form (Proposition A.2). Second, we consider  $\rho \in (0, \phi]$ , for which we establish existence of a PBS equilibrium and uniqueness within the PBS class (Proposition A.3). Third, we show that for all  $|\rho| > 0$  sufficiently small (allowing for positive or negative  $\rho$ ), there exists a unique

<sup>&</sup>lt;sup>35</sup>There is no circularity in our argument since the proof of this part of Proposition 1 does not rely on the current theorem.

equilibrium within the whole linear class, and it is a PBS equilibrium (Proposition A.4). For both positive and negative  $\rho$  we prove the inequalities stated in the proposition. Fourth, we show that a PBS equilibrium fails to exist if  $\rho$  is sufficiently low (Proposition A.5), and we construct  $\rho \in (-\phi, 0)$  presented in the proposition and  $\rho_0$  mentioned in footnote 18. Recall that  $\alpha^K := \sqrt{\frac{\sigma^2}{\phi}}$ .

**Proposition A.2.** For  $\rho = 0$ , there is a unique linear equilibrium: for  $i \in \{L, F\}$ , trader i trades  $\theta^i = \alpha^K (X_0^i - \mu)$ , and  $\mathbb{E}[\theta^L | \mathcal{F}_0] = 0$ .

*Proof.* For  $\rho = 0$ , (A.19) becomes  $-\alpha_F[\sigma^2 \phi + \alpha_L^2] \leq 0$ . The only solution to (A.13) satisfying this is  $\alpha_F = \sqrt{\frac{\sigma^2}{\gamma_1^F}} = \alpha^K$  (as  $\rho = 0$  implies  $\gamma_1^F = \phi$ ). Equation (A.16) then yields  $\alpha_L =$  $\pm \alpha^{K}$ . Of these, only  $\alpha_{L} = \alpha^{K}$  satisfies (A.18). Given  $(\alpha_{F}, \alpha_{L}) = (\alpha^{K}, \alpha^{K}), (\beta_{F}, \delta_{F}, \delta_{L}) =$  $(0, -\alpha^{K}, -\alpha^{K})$  is the unique solution to (A.14), (A.15), and (A.17). These strategies and the pricing rule in  $(A.5)$  and  $(A.10)$  satisfy the first and second order conditions, so they constitute an equilibrium. Moreover,  $\mathbb{E}[\theta^L|\mathcal{F}_0] = \mathbb{E}[\alpha^K(X_0^L - \mu)|\mathcal{F}_0] = \alpha^L(\mu - \mu) = 0.$  $\Box$ 

In the next two propositions, note that the ranking of  $\alpha_L$  and  $-\delta_L$  determines the sign of  $\mathbb{E}[\theta^L|\mathcal{F}_0] = (\alpha_L + \delta_L)\mu$ .

**Proposition A.3.** If  $\rho \in (0, \phi]$ , there is a unique PBS equilibrium, and  $0 < \alpha_L < \alpha^K < -\delta_L$ .

*Proof.* By Proposition A.1, (A.20) is a necessary condition for  $(\alpha_F, \alpha_L)$  to be part of PBS equilibrium. Let  $L(\alpha_L)$  and  $R(\alpha_L)$  denote the left and right sides of (A.20). Define  $\hat{\alpha}$  :=  $-\frac{\phi+\sqrt{\phi^2+4\sigma^2\phi}}{2\phi} > 0$  to be the positive root of the denominator on the right side of (A.20). Note that  $\alpha^K > \hat{\alpha}$ .

L is positive and strictly increasing in  $\alpha_L$  for  $\alpha_L \geq 0$ . Meanwhile, R is continuous on  $[0, \hat{\alpha}) \cup (\hat{\alpha}, +\infty)$  and satisfies  $R(\hat{\alpha}-) = -\infty$ ,  $R(\hat{\alpha}+) = +\infty$ , and  $R(\alpha^{K}) = 0$ . Further, for  $\alpha_L \in [0, \hat{\alpha}) \cup (\hat{\alpha}, +\infty),$ 

$$
R'(\alpha_L) = -\phi \frac{(\alpha_L^2 \phi - \sigma^2)^2 + (\rho + \phi)(\alpha_L^2 + \sigma^2) + 2\alpha_L^3 \phi^2}{\rho[\sigma^2 - \alpha_L(1 + \alpha_L)\phi]^2},
$$

which is unambiguously strictly negative when  $\rho > 0$ . Thus, R is strictly decreasing on  $(\hat{\alpha}, +\infty)$ , so there exists a solution to  $(A.20)$  on  $(\hat{\alpha}, \alpha^K)$  and this is the only solution on  $(\hat{\alpha}, +\infty)$ . Since  $L(0) > 0$ , while  $R(0) = -(\rho + \phi)/\rho < 0 < L(0)$  (given  $\rho > 0$ ), there is no solution on  $[0, \hat{\alpha})$ , so this solution is the unique among  $\alpha_L \geq 0$ . And by (A.17),  $\alpha_L < \alpha^K$ implies  $\alpha^K < -\delta_L$  (and  $\delta_L < 0$ ).

Given a unique candidate for PBS equilibrium, we now verify SOCs. For the leader, note that since  $\alpha_L, \alpha_F > 0$ , (A.18) is bounded above by  $\sigma^2 - \alpha_L^2 \phi - \alpha_L \phi$ , which is negative since  $\alpha_L > \hat{\alpha}$ . For the follower, (A.19) holds by inspection for  $\rho > 0$  since  $\alpha_L > 0$  and  $\alpha_F > 0$ .  $\Box$  Next, we turn to  $|\rho| > 0$  close to 0.

**Proposition A.4.** If  $|\rho| >$  is sufficiently small, there exists a unique linear equilibrium, and it is a PBS equilibrium. If  $\rho > 0$ ,  $\alpha_L < \alpha^K < -\delta_L$ , and if  $\rho < 0$ ,  $\alpha_L > \alpha^K > -\delta_L > 0$ .

*Proof.* Assume throughout that  $\rho \neq 0$ . Let us call any pair  $(\alpha_L, \alpha_F)$  satisfying (A.20) or (A.21) a candidate signaling pair. We construct two candidate signaling pairs  $(\alpha_L^*, \alpha_F^*)$  and  $(\alpha_L^{\flat}, \alpha_F^{\flat})$ . We then show that for small  $|\rho|$ , there are no other candidate signaling pairs satisfying the leader's second order condition, and of these two pairs, only  $(\alpha_L^*, \alpha_F^*)$  satisfies the follower's SOC. We then invoke the converse part of Proposition A.1 to establish existence of a unique equilibrium based on  $(\alpha_L^*, \alpha_F^*)$ .

We claim that if  $\rho < 0$ , there exists  $\alpha_L^* \in (\alpha^K, \infty)$  solving  $(A.20)$  and  $\alpha_L^{\flat} \in (\hat{\alpha}, \alpha^K)$  solving (A.21). Analogous arguments for the case  $\rho > 0$  establish the existence of  $\alpha_L^* \in (\hat{\alpha}, \alpha^K)$  and  $\alpha_L^{\flat} \in (\alpha^K, \infty)$ ; we omit this case for brevity. In either case, we will ultimately show that  $\alpha_L^*$  is the unique equilibrium value of  $\alpha_L$  for small  $|\rho|$ . As before, let  $R(\alpha_L)$  denote the right hand side common to (A.20) and (A.21). Note that R is continuous on  $(\hat{\alpha}, \infty)$ , and it has the properties  $\lim_{\alpha_L \to +\infty} R(\alpha_L) = +\infty$ ,  $\lim_{\alpha_L \downarrow \hat{\alpha}} R(\alpha_L) = -\infty$ , and  $R(\alpha^K) = 0$ . The left hand side of (A.20) is strictly positive and bounded, so by the intermediate value theorem (IVT), there exists a solution  $\alpha_L^* \in (\alpha^K, \infty)$  to (A.20). Similarly, the left hand side of (A.21) is strictly negative and bounded, so by the IVT, there exists a solution  $\alpha_L^{\flat} \in (\hat{\alpha}, \alpha^K)$  to (A.21).

Define  $\alpha_F^* := \alpha_{F,1}(\alpha_L^*)$  and define  $\alpha_F^{\flat} = \alpha_{F,2}(\alpha_L^{\flat})$ . By definition, both  $(\alpha_L^*, \alpha_F^*)$  and  $(\alpha_L^{\flat}, \alpha_F^{\flat})$  are candidate signaling pairs.

To assess other candidate signaling pairs, we derive a polynomial equation such that  $(\alpha_L, \alpha_F)$  is a candidate signaling pair only if  $\alpha_L$  is a root of this equation. By squaring either  $(A.20)$  or  $(A.21)$ , we obtain a necessary condition

$$
\frac{\sigma^4 + \alpha_L^2 \sigma^2 \phi}{\sigma^2 \phi + \alpha_L^2 (-(\rho)^2 + (\phi)^2)} = \left( \frac{(\rho + \phi + \phi \alpha_L)(\alpha_L^2 \phi - \sigma^2)}{\rho [\sigma^2 - \alpha_L (1 + \alpha_L) \phi]} \right)^2, \tag{A.30}
$$

and by cross multiplying, an eighth-degree polynomial equation

$$
0 = Q(\alpha_L; \rho) = \sum_{i=0}^{8} A_i \alpha_L^i, \quad \text{where}
$$
\n
$$
A_8 = -\phi^4(\phi^2 - \rho^2),
$$
\n
$$
A_6 = \phi^2(\rho^2 - \phi^2)[\rho^2 + 2\rho\phi + \phi(-\sigma^2 + \phi)],
$$
\n
$$
A_7 = -2(\phi - \rho)\phi^3(\rho + \phi)^2,
$$
\n
$$
A_8 = 2\sigma^2\phi^2[-2\rho^3 - \rho^2\phi + \rho\phi^2 + \phi^3],
$$
\n
$$
A_9 = \sigma^2\phi[-2\rho^4 - 4\rho^3\phi + 2\rho\phi^3 + \phi^3(\sigma^2 + \phi)],
$$
\n
$$
A_9 = 2\sigma^4\phi[\rho^3 + \rho^2\phi + \rho\phi^2 + \phi^3],
$$
\n
$$
A_1 = -2\sigma^6\phi[\rho^2 + \phi\phi + \phi^2],
$$
\n
$$
A_0 = \sigma^6[\rho^2(\sigma^2 - \phi) - 2\rho\phi^2 - \phi^3].
$$
\n(M. 31)

Being an eighth-degree polynomial,  $Q(\cdot;\rho)$  has exactly eight complex roots, counting multiplicity; two of these are  $\alpha_L^*$  and  $\alpha_L^{\flat}$ .

We now show that of all candidate signaling pairs, when  $|\rho|$  is sufficiently small, only  $(\alpha_L^*, \alpha_F^*)$  satisfies both activists' SOCs. To that end, it is useful to approximate all of the roots of  $(A.31)$  for small  $|\rho|$ . We will make use of a standard result on the continuous dependence of the (complex) roots of a polynomial on its coefficients:

**Lemma A.2** (Uherka and Sergott (1977)). Let  $p(x) = x^n + \sum_{k=1}^n a_k x^{n-k}$  and  $p^*(x) =$  $x^n + \sum_{k=1}^n a_i^* x^{n-k}$  be two nth degree polynomials. Suppose  $\lambda^*$  is a root of  $p^*$  with multiplicity m and  $\epsilon > 0$ . Then for  $|a_i - a_i^*|$  sufficiently small  $(i = 1, ..., n)$ , p has at least m roots within  $\epsilon$  of  $\lambda^*$ .

For a proof, see Uherka and Sergott (1977) or the references therein.

We apply this lemma to the polynomial Q indexed by  $\rho$ . (While Lemma A.2 assumes a leading coefficient of 1, we can divide through our polynomial  $Q(\cdot;\rho)$  in  $(A.31)$  by  $A_8$ , which is bounded away from 0 provided that  $|\rho| < |\phi|$ , allowing us to apply the lemma.) In the limit as  $\rho \to 0$ ,

$$
Q(\alpha_L; 0) = -(1 + \alpha_L)^2 \phi^3 (\sigma^2 - \alpha_L^2 \phi)^2 (\sigma^2 + \alpha_L^2 \phi).
$$

By inspection,  $Q(\cdot; 0)$  is nonpositive and has double roots at  $-1$  and  $\pm \alpha^{K}$ , and it has complex roots at  $\pm \alpha^{K}i$ .

Lemma A.2 then has two important implications about candidate signaling pairs. We state the first one as a corollary.

### Corollary A.1. As  $\rho \to 0$ ,  $\alpha_L^* \to \alpha^K$ ,  $\alpha_L^{\flat} \to \alpha^K$ ,  $\alpha_F^* \to \alpha^K$ , and  $\alpha_F^{\flat} \to -\alpha^K$ .

The limits of  $\alpha_L^*$  and  $\alpha_L^{\flat}$   $\alpha_L^*$ ,  $\alpha_L^{\flat} \ge 0$ , so they can only converge to  $\alpha^K$  (among the roots of  $Q(\cdot; 0)$ ); the corresponding limits of  $\alpha_F^*$  and  $\alpha_F^{\flat}$  are then immediate. The second implication of Lemma A.2 is that for any  $\epsilon > 0$ , there exists  $\bar{\rho} > 0$  such that for all  $\rho$  with  $0 < |\rho| < \bar{\rho}$ all of the other six roots of  $Q(\cdot;\rho)$  lie within  $\epsilon$  of  $-1$ ,  $-\alpha^{K}$ , or  $\pm \alpha^{K}i$ . Hence, for such  $\rho$ ,  $\alpha_{L}^{*}$ and  $\alpha_L^{\flat}$  are roots with multiplicity 1, and they are uniquely defined.

We can now check SOCs: for the leader in Lemma A.3 and the follower in Lemma A.4.

**Lemma A.3.** For  $|\rho| > 0$  sufficiently small, the candidate signaling pairs  $(\alpha_L^*, \alpha_F^*)$  and  $(\alpha_L^{\flat}, \alpha_F^{\flat})$  satisfy (A.18) and are the only candidate signaling pairs that do.

*Proof.* First, we show that  $(\alpha_L^*, \alpha_F^*)$  satisfy (A.18) for sufficiently small  $|\rho| > 0$ . As  $\rho \to 0$ , the left hand side of (A.18) tends to  $\sigma^2 - (\alpha^K)^2 \phi - 2\alpha^K \phi = -2\sigma \sqrt{ }$  $\overline{\phi}$  < 0, where we have used that  $\alpha_L^* \to \alpha^K$  by Corollary A.1. A nearly identical calculation shows  $(\alpha_L^{\flat}, \alpha_F^{\flat})$  also satisfy  $(A.18)$  for sufficiently small  $|\rho| > 0$ .

The remaining candidates for equilibria are associated with the real roots of (A.31) other than  $\alpha_L^*, \alpha_L^{\flat}$ . By Lemma A.2, as  $\rho \to 0$ , these roots must converge to the other roots of  $Q(\cdot; 0)$ , namely  $-1$ ,  $-\alpha^{K}$ , or  $\pm \alpha^{K}i$ . Any root of  $Q(\cdot; \rho)$  that is in a sufficiently small neighborhood of  $\pm \alpha^{K}i$  has a nonzero complex component, and is not an equilibrium candidate. Therefore, we need only consider candidates in neighborhoods of  $-1$  or  $-\alpha^{K}$ . In the first case, for any  $\alpha_F \in {\alpha_{F,1}, \alpha_{F,2}}$ , the left hand side of (A.18) converges to  $\sigma^2 - (-1)^2 \phi - 2(-1) \phi = \sigma^2 + \phi > 0$ . In the second case, for any  $\alpha_F \in {\alpha_{F,1}, \alpha_{F,2}}$ , the left hand side of (A.18) converges to √  $\sigma^2-\left(-\alpha^K\right)^2\phi-2\left(-\alpha^K\right)\phi=2\sigma^2$  $\overline{\phi} > 0$ . Thus, for  $|\rho| > 0$  sufficiently small, all roots of  $Q(\cdot; \rho)$  other than  $\alpha_L^*$  and  $\alpha_L^{\flat}$  violate the leader's SOC.  $\Box$ 

**Lemma A.4.** For  $|\rho| > 0$  sufficiently small, the candidate signaling pair  $(\alpha_L^*, \alpha_F^*)$  satisfies (A.19), while the pair  $(\alpha_L^{\flat}, \alpha_F^{\flat})$  does not.

*Proof.* For the pair  $(\alpha_L^*, \alpha_F^*)$ , the left hand side of  $(A.19)$  tends to  $-[(\alpha_K)^2 \phi^2 + \sigma^2 \phi] < 0$  as  $\rho \to 0$ . For the pair  $(\alpha_L^{\flat}, \alpha_F^{\flat})$ , however, it tends to  $(\alpha^K)^2 \phi^2 + \sigma^2 \phi > 0$ , violating (A.19).

From Lemmas A.3 and A.4, we conclude that for  $|\rho| > 0$  sufficiently small,  $(\alpha_L^*, \alpha_F^*)$  is the unique candidate signaling pair satisfying both (A.18) and (A.19). Hence, in any linear equilibrium,  $(\alpha_L, \alpha_F)$  must equal  $(\alpha_L^*, \alpha_F^*)$ .

To conclude, observe that as  $\rho \to 0$ ,  $\phi(1+\alpha_L^*) + \rho \to \phi(1+\alpha^K) > 0$ , allowing us to apply the "converse" part of Proposition A.1 when  $|\rho|$  is sufficiently small, giving us existence. Since we have already shown that  $0 < \alpha_L^* < \alpha^K$  if  $\rho > 0$ , (A.17) implies  $-\delta_L > \alpha^K$  in this case, and likewise when  $\rho < 0$ , we have  $\alpha_L^* > \alpha^K$  which implies  $0 < -\delta_L < \alpha^K$ .  $\Box$ 

By the results above, a unique PBS equilibrium exists if  $\rho$  is (i) positive or (ii) sufficiently close to zero. Thus,  $\rho := \inf \{ \rho' \in [-\phi, \phi] : a PBS$  equilibrium exists for all  $\rho \in [\rho', \phi] \} < 0$ and  $\rho_0 := \inf \{ \rho' \in [-\phi, \phi] : \text{a unique PBS equilibrium exists for all } \rho \in [\rho', \phi] \} < 0, \text{ where }$  $\rho_0 \geq \rho$  is obvious. To show that  $\rho > -\phi$ , we invoke the following result.

**Proposition A.5.** Fix  $\sigma, \phi > 0$ . There exists  $\hat{\rho} \in (-\phi, 0)$  such that if  $\rho < \hat{\rho}$ , there is no PBS equilibrium.

*Proof.* The proof is based on the following two lemmas.

**Lemma A.5.** There is no  $[-\phi, \phi]$ -valued sequence  $(\rho_n)_{n \in \mathbb{N}}$  that converges to  $-\phi$  and has the property that there is an associated sequence of PBS equilibria such that  $(\alpha_{F,n})_{n\in\mathbb{N}}$  is bounded.

Proof. Suppose by way of contradiction that there exists such a sequence with associated PBS equilibria indexed by n. We claim that  $(\alpha_{L,n})_{n\in\mathbb{N}}$  is bounded. To see this, take n sufficiently large that  $\rho_n \neq 0$ , and note that the right hand side of (A.20) must be bounded, since it equals  $\alpha_{F,n}$  which we have supposed is bounded. Since the numerator on the right hand side is cubic while the denominator is quadratic, it must be that  $(\alpha_{L,n})_{n\in\mathbb{N}}$  is bounded.

Given that  $(\alpha_{F,n})_{n\in\mathbb{N}}$  and  $(\alpha_{L,n})_{n\in\mathbb{N}}$  are both bounded, we can pass to a subsequence such  $\alpha_{F,n} \to \overline{\alpha}_F \geq 0$  and  $\alpha_{L,n} \to \overline{\alpha}_L \geq 0$ , where the inequalities follow from  $\alpha_{F,n}, \alpha_{L,n} \geq 0$  in PBS equilibria by definition. Then taking limits in  $(A.20)$ , we have

$$
\overline{\alpha}_F = \sqrt{\frac{\sigma^2}{\phi} + \overline{\alpha}_L^2} > \overline{\alpha}_L.
$$
\n(A.32)

The right hand side of (A.18) then has limit

$$
\sigma^2 + \overline{\alpha}_L^2 \phi - 2\overline{\alpha}_L[-\phi(1+\overline{\alpha}_F) + \phi(1+\overline{\alpha}_L)] = \sigma^2 + \overline{\alpha}_L^2 \phi + 2\overline{\alpha}_L \phi(\overline{\alpha}_F - \overline{\alpha}_L) > 0, \quad (A.33)
$$

where  $\overline{\alpha}_F - \overline{\alpha}_L > 0$  by (A.32). But since (A.18) is satisfied for all n, this limit must be  $\Box$ nonpositive, a contradiction.

**Lemma A.6.** There is no  $[-\phi, \phi]$ -valued sequence  $(\rho_n)_{n \in \mathbb{N}}$  that converges to  $-\phi$  and has the property that there is an associated sequence of PBS equilibria such that  $(\alpha_{F,n}) \to +\infty$ .

Proof. Suppose by way of contradiction that there were such a sequence. From the expression for  $\alpha_{F,n}$  in (A.20), it must be that  $\alpha_{L,n} \to +\infty$ . We claim that  $\frac{\alpha_{F,n}}{\alpha_{L,n}} \to 1$ . To obtain this, divide through (A.20) by  $\alpha_{L,n}$  to get

$$
\frac{\alpha_{F_n}}{\alpha_{L,n}} = \frac{(\rho_n + \phi + \phi \alpha_{L,n})(\alpha_L^2 \phi - \sigma^2)}{\rho_n \alpha_{L,n} [\sigma^2 - \alpha_{L,n}(1 + \alpha_{L,n})\phi]} \to 1.
$$

We now show that  $(A.18)$  eventually fails. The right hand side of  $(A.18)$  is

$$
\sigma^2 + \alpha_{L,n}^2 \phi - 2\alpha_{L,n} [\phi + \rho_n + \alpha_{L,n} (\rho_n \alpha_{F,n} / \alpha_{L,n} + \phi)]. \tag{A.34}
$$

Since  $\phi + \rho_n \to 0$  and  $\frac{\alpha_{F,n}}{\alpha_{L,n}} \to 1$ , for any  $\epsilon > 0$ , the expression in square brackets in (A.34) is less than  $\epsilon \alpha_{L,n}$  for sufficiently large n. Hence, (A.34) is eventually greater than  $\sigma^2 + \alpha_{L,n}^2 \phi$  $2\epsilon \alpha_{L,n}^2$ , which is positive for  $\epsilon < \phi/2$ , violating (A.18), contradicting equilibrium.  $\Box$ 

The existence of  $\hat{\rho} > -\phi$  then follows immediately from Lemmas A.5 and A.6, since if there is no such  $\hat{\rho}$  there would exist a sequence  $(\rho_n)_{n\in\mathbb{N}}$  with  $\rho_n \to -\phi$  and an associated sequence of PBS equilibria such that either (i)  $\alpha_{F,n} \to +\infty$  along some subsequence (which is ruled out by Lemma A.6) or (ii)  $(\alpha_{F,n})_{n\in\mathbb{N}}$  is bounded (ruled out by Lemma A.5). Since Proposition A.4 shows that a PBS equilibrium exists for some  $\rho < 0$ , we have  $\rho < 0$ .  $\Box$ 

For any  $\hat{\rho}$  as in Proposition A.5,  $\rho \geq \hat{\rho} > -\phi$ . This concludes the proof of Theorem 1.

### A.4 Monotonicity of leader's strategy coefficients

The following result establishes the decreasing patterns of  $\alpha_L$  and  $\delta_L$  with respect to  $\rho$  shown in Figure 1. Note that Proposition A.2 established that when  $\rho = 0$ ,  $\alpha_L = \alpha^K = -\delta_L$ .

**Proposition A.6.** Suppose  $\rho > \rho_0$ , where  $\rho_0 < 0$  was defined in the proof of Theorem 1. Then in the unique PBS equilibrium,  $\alpha_L$  and  $\delta_L$  are decreasing in  $\rho$ .

*Proof.* Due to the identity (A.17), it is sufficient to prove the claim for  $\alpha_L$ . First suppose  $\rho > 0$ . The right hand side of (A.20) crosses the left hand side from above at  $\alpha_L$ . Moreover, when  $\rho > 0$ , the right hand side is (positive and) decreasing in  $\rho$  at  $\alpha_L$  while the left hand side is increasing in  $\rho$ . Hence,  $\alpha_L$  is decreasing in  $\rho$ . In turn, when  $\rho < 0$ , the right hand side of  $(A.20)$  crosses the left hand side from below; the left hand side is decreasing in  $\rho$ ; and the right hand side is increasing in  $\rho$  at  $\alpha_L$ . Hence, again,  $\alpha_L$  is unambiguously decreasing in  $\rho$ . The result then follows since  $\alpha_L$  is continuous in  $\rho$  at  $\rho = 0$  by Corollary A.1.  $\Box$ 

### A.5 Proof of Proposition 1

By Proposition A.1,  $\alpha_F$  must satisfy (A.13), so either  $\alpha_F = \alpha_{F,1} := \sqrt{\frac{\sigma^2}{\gamma_1^F}}$  or  $\alpha_F = \alpha_{F,2} :=$  $-\sqrt{\frac{\sigma^2}{\gamma_1^F}}$ . Since  $\alpha_F > 0$  in any PBS equilibrium (by definition),  $\alpha_F = \alpha_{F,1}$ , and then  $(\beta_F, \delta_F)$ are characterized by  $(A.14)-(A.15)$ .

For the rest of the proof, consider  $\rho \neq 0$ . To sign  $\beta_F$ , recall that  $\alpha_F, \alpha_L > 0$  and  $|\rho| \leq \phi$ , so  $sign(\beta_F) = -sign(\rho)$  via (A.14). Similarly, from (A.15),  $sign(\delta_F) = sign((\alpha_L + \delta_L)\rho \alpha_L \phi - (\phi - \rho)$ ). This is unambiguously negative, since  $(\alpha_L + \delta_L) \rho \leq 0$  by Theorem 1, and since  $\alpha_L \phi > 0$  and  $\phi - \rho \geq 0$  by assumption.

We now establish that  $\beta_F < 1$ . For  $\rho > 0$ , this is immediate since  $\beta_F < 0$ . For  $\rho < 0$ , note that by using (A.14), (A.16) can be written as  $\alpha_L = \frac{\sigma^2}{\phi \alpha}$  $\frac{\sigma^2}{\phi \alpha_L} + \frac{\beta_F}{1-\beta}$  $\frac{\beta_F}{1-\beta_F}$ . Now recall from the proof of Theorem 1 that in a PBS equilibrium,  $\alpha_L > \alpha^K$ , and thus  $\alpha_L > \frac{(\alpha^K)^2}{\alpha^L}$  $\frac{(\alpha K)^2}{\alpha_L} = \frac{\sigma^2}{\phi \alpha_L}$  $\frac{\sigma^2}{\phi \alpha_L}$ . It follows that  $\frac{\beta_F}{1-\beta_F} > 0$ , and thus  $\beta_F \in (0,1)$ . For the case  $\rho = 0$ , we already showed above that  $\beta_F = 0$  in the unique linear equilibrium, also satisfying the inequality  $\beta_F < 1$ .

Next, we verify that in any linear equilibrium (PBS or otherwise), the follower's strategy has the form  $\theta^F = \alpha_F (X_0^F - M_1^F)$  for  $\alpha_F = \alpha_{F,1}$  or  $\alpha_F = \alpha_{F,2}$ , and as argued above, in a PBS

equilibrium,  $\alpha_F = \alpha_{F,1}$ . First, express  $M_1^F$  in terms of  $P_1$  and  $\mu$  by using (A.5) to replace the surprise term  $\Psi_1 - \mu(\alpha_L + \delta_L)$  in (A.3):

$$
M_1^F = \mu + \frac{\alpha_L \rho}{\alpha_L^2 \phi + \sigma^2} \frac{P_1 - P_0}{\Lambda_1},\tag{A.35}
$$

where  $P_0$  is linear in  $\mu$  (see (A.1)). Substituting (A.35) into  $\theta^F = \alpha_{F,i}(X_0^F - M_1^F), i \in \{1, 2\},\$ then yields an expression for the follower's strategy in which the coefficient on  $X_0^F$  is  $\alpha_{F,i}$ , and the coefficients on  $(P_1, \mu)$  equal  $(\beta_{F,i}, \delta_{F,i})$  when  $(A.14)-(A.15)$  hold. This confirms that the follower's strategy has the stated form.

### A.6 Proof of Proposition 2

For both parts (a) and (b), we focus on PBS equilibria, i.e. linear equilibria in which

$$
\theta^L := \alpha_L \xi^L + \delta_L \mu + \eta_L
$$
  

$$
\theta^F := \alpha_F \xi^F + \beta_F P_1 + \delta_F \mu + \eta_F = \alpha_F (\xi^F - M_1^F),
$$

for  $\xi \in \{V, \zeta\}$ , where  $M_1^F := \mathbb{E}[\xi^F | \mathcal{F}_1]$  (see below), and where  $\alpha_L, \alpha_F > 0$ . A straightforward adaptation of the steps from the baseline analysis can be used to show that the follower's strategy must be a "gap strategy," i.e. one of the form  $\theta^F = \alpha_F(\xi^F - \mathbb{E}[\xi^F|\mathcal{F}_1])$ , in any linear equilibrium; here we restrict attention to equilibria with this property to simplify the exposition.

We show specifically that there exists a unique PBS equilibrium whenever  $\rho$  is not too negative. The leader trades according to

$$
\theta^L = \alpha^K (\xi^L - \mu) + \eta_L,\tag{A.36}
$$

where, in closed form:  $\alpha^{K} = \sigma/\sqrt{\phi}$ ;  $\eta_{L} = X_{0}^{L} \frac{\beta_{F}}{1-\beta_{F}}$  $\frac{\beta_F}{1-\beta_F};\ \beta_F=-\frac{\rho\alpha_F}{\phi(1+\alpha_L)}$  $\frac{\rho \alpha_F}{\phi(1+\alpha_L)+\rho};$  and  $\alpha_F = \sqrt{\frac{\sigma^2}{\gamma_1^F}}$ . In particular,  $\mathbb{E}[\theta^L | \mathcal{F}_0] = \eta_L \leq 0$  if and only if  $\rho \leq 0$ , with strict inequality if  $\rho > 0$ .

#### A.6.1 Part (a)

Since the effort technology is unchanged, it continues to be optimal to choose effort equal to the number of shares held; thus the firm's final value will be  $V^L + V^F + X_T^L + X_T^F$ , where  $X_T^i = X_0^i + \theta^i$  as before. Hence, the objective of activist i reduces to

$$
\sup_{\theta^i} \mathbb{E}[(V^L + V^F + X^i_T + X^{\text{-}i}_T)X^i_T - P_{t(i)}\theta^i - \frac{1}{2}(X^i_T)^2|V^i, \mathcal{F}_{t(i)-1}, \theta^i].
$$

Learning and pricing Conjecturing linear strategies (with a gap strategy for the follower), the ex ante expectation of firm value is

$$
P_0 = X_0^L + X_0^F + \eta_L + (2 + \alpha_L + \delta_L)\mu,
$$

where we have used that the follower's expected trade is 0 from an ex ante perspective. Since the type distribution is unchanged, the players' private prior beliefs about each other's initial positions have the same form as in the baseline model, with  $V^L$  and  $V^F$  playing the role of  $X_0^L$  and  $X_0^F$ , respectively.

Given  $\Psi_1$ , the MM's updated belief about  $V^L$  is

$$
M_1^L := \mathbb{E}[V^L|\mathcal{F}_1] = \mu + \frac{\alpha_L \phi}{\alpha_L^2 \phi + \sigma^2} \left\{ \Psi_1 - \mu(\alpha_L + \delta_L) - \eta_L \right\}.
$$

And the MM's updated belief about  $V^F$  is

$$
M_1^F := \mathbb{E}[V^F|\mathcal{F}_1] = \mu + \frac{\alpha_L \rho}{\alpha_L^2 \phi + \sigma^2} \left\{ \Psi_1 - \mu(\alpha_L + \delta_L) - \eta_L \right\}.
$$

Since the MM expects the follower to trade 0 conditional on first period order flow,

$$
P_1 = X_0^L + X_0^F + \eta_L + \mathbb{E}[V^L + V^F + \theta^L | \Psi_1]
$$
  
=  $X_0^L + X_0^F + \eta_L + M_1^L (1 + \alpha_L) + \delta_L \mu + M_1^F$   
=  $P_0 + \Lambda_1 \{ \Psi_1 - \mu(\alpha_L + \delta_L) - \eta_L \},$ 

where  $\Lambda_1 := \frac{\alpha_L[\rho + (1 + \alpha_L)\phi]}{\alpha^2 \phi + \sigma^2}$  $\frac{\rho + (1 + \alpha_L)\rho}{\alpha_L^2 \phi + \sigma^2}$ . This is equivalent to  $\Lambda_1$  in the baseline model, using the identity that  $\beta_F$  satisfies in a gap strategy.

The MM's posterior belief about  $(V^L, V^F)$  has covariance matrix  $\begin{pmatrix} \gamma_1^L & \rho_1 \\ \vdots & \vdots \\ \gamma_k^L & \eta_k^L \end{pmatrix}$  $\rho_1 \quad \gamma_1^F$  $\setminus$ , where

$$
\gamma_1^L = \frac{\phi \sigma^2}{\alpha_L^2 \phi + \sigma^2}, \qquad \gamma_1^F = \frac{\alpha_L^2 [\phi^2 - \rho^2] + \phi \sigma^2}{\alpha_L^2 \phi + \sigma^2}, \qquad \rho_1 = \frac{\rho \sigma^2}{\alpha_L^2 \phi + \sigma^2}.
$$

The follower's mean posterior belief about the leader's component  $V^L$  is

$$
Y_1^F := Y_0^F + \frac{\alpha_L \nu_0^F}{\alpha_L^2 \nu_0^F + \sigma^2} \underbrace{\left\{ \frac{P_1 - P_0}{\Lambda_1} + \alpha_L (\mu - Y_0^F) \right\}}_{=\Psi_1 - (\alpha_L Y_0^F + \delta_L \mu) - \eta_L}.
$$

Note that unlike in the baseline model analysis, here the updating is about the leader's type

rather than the leader's terminal position.

After seeing  $\Psi_2$ , the market maker again updates beliefs about  $V^L$  and  $V^F$ :

$$
M_2^F := M_1^F + \frac{\alpha_F \gamma_1^F}{\alpha_F^2 \gamma_1^F + \sigma^2} \Psi_2 \quad \text{and} \quad M_2^L := M_1^L + \frac{\alpha_F \rho_1}{\alpha_F^2 \gamma_1^F + \sigma^2} \Psi_2.
$$

The price is then

$$
P_2 = P_1 + \Psi_2 \underbrace{\frac{\alpha_F [(1 + \alpha_L)\rho_1 + (1 + \alpha_F)\gamma_1^F]}{\alpha_F^2 \gamma_1^F + \sigma^2}}_{=: \Lambda_2}.
$$

This  $\Lambda_2$  is equivalent to the one in the baseline model, where the extra  $1 + \alpha_L$  now makes up for the one that was "missing" in the new  $\rho_1$ .

FOC's and PBS equilibrium The follower's first order condition is

$$
0 = \mathbb{E}[V^{L} + \theta^{L}|V^{F}, \mathcal{F}_{1}] + V^{F} + X_{0}^{L} + X_{0}^{F} + \theta^{F} - P_{1} - 2\Lambda_{2}\theta^{F}
$$
  
\n
$$
\implies \theta^{F} = \frac{Y_{1}^{F}(1 + \alpha_{L}) + \delta_{L}\mu + \eta_{L} + V^{F} + X_{0}^{L} + X_{0}^{F} - P_{1}}{2\Lambda_{2} - 1}.
$$

It is straightforward to check that the RHS is equivalent to  $\alpha_F(V^F - M_1^F)$  for  $\alpha_F =$  $\sqrt{\sigma^2/\gamma_1^F}$ . The remaining coefficients are  $\beta_F = -\frac{\rho \alpha_F}{\phi(1+\alpha_F)}$  $\frac{\rho\alpha_F}{\phi(1+\alpha_L)+\rho}$  and  $\delta_F = \frac{(\alpha_L+\delta_L)\rho-\alpha_L\phi-(\phi-\rho)}{\phi(1+\alpha_L)+\rho}$  $\frac{\phi_L(\rho - \alpha_L \phi - (\rho - \rho)}{\phi(1 + \alpha_L) + \rho} \alpha_F,$ and  $\eta_F = -\beta_F (X_0^L + X_0^F + \eta_L).$ 

The leader's FOC is

$$
0 = -\mathbb{E}_{L}[P_{0} + \Lambda_{1}\{\Psi_{1} - \mathbb{E}[\Psi_{1}]\}|\theta^{L}] - \theta\Lambda_{1}
$$
  
+ 
$$
\underbrace{(X_{0}^{L} + \theta^{L}) + V^{L} + Y_{0}^{L} + \mathbb{E}_{L}[X_{T}^{F}|\theta^{L}]}_{=\mathbb{E}[\text{firm value}|V^{L},\theta^{L}]}\n+ \underbrace{(X_{0}^{L} + \theta^{L}) \frac{\partial \mathbb{E}_{L}[X_{T}^{F}|\theta^{L}]}{\partial \theta^{L}}}_{\text{value of manipulation}}\n= -\mathbb{E}_{L}[P_{1}|\theta^{L}] - \theta^{L}\Lambda_{1} + V^{L} + Y_{0}^{L}(1 + \alpha_{F}) + \beta_{F}\mathbb{E}_{L}[P_{1}|\theta^{L}] + \delta_{F}\mu + \eta_{F} + (X_{0}^{L} + \theta)(1 + \beta_{F}\Lambda_{1}).
$$
\n(A.37)

Matching coefficients on  $V^L$  and  $\mu$  and the intercept yields three equations. After substituting in the follower's strategy derived above, it is easy to verify that the coefficients  $(\alpha_L, \delta_L) = (\alpha^K, -\alpha^K)$  solve the  $V^L$ - and  $\mu$ - components of the FOC, and these are the only solutions with  $\alpha_L > 0$  when  $\rho$  is positive or sufficiently close to 0. Note that in the baseline model, for positive correlation,  $\alpha_L < \sigma/\sqrt{\phi}$ . The greater sensitivity to type here comes from the fact that in the original model, higher types had a greater benefit of manipulation since they, by definition, had more initial shares.

The equation derived from the intercept yields  $\eta_L = -X_0^L \frac{\rho \alpha_F}{\rho(1+\alpha_F)+\phi(1+\alpha_L)} = X_0^L \frac{\beta_F}{1-\beta_F}$  $\frac{\beta_F}{1-\beta_F}.$ Since  $\beta_F < 1$  by the leader's SOC (see below) and  $\beta_F$  has the opposite sign of  $\rho$ , so does  $\eta_L$ : the leader sells (buys) on average when correlation is positive (negative).

The second order conditions are the same as before:

$$
1 - 2\Lambda_1(1 - \beta_F) < 0, \text{ for } i = L,
$$
\n
$$
1 - 2\Lambda_2 < 0, \text{ for } i = F.
$$

By direct substitution of our closed form solution, these can be rewritten in terms of  $(\phi, \rho, \sigma)$ , and it is easy to check that they are satisfied whenever  $\rho \geq \rho$ , for some  $\rho \in [-\phi, 0)$ . Also,  $\Lambda_1 > 0$  by inspection, so the leader's SOC implies  $\beta_F < 1$ .

#### A.6.2 Part (b)

Given the cost function, trader i's optimal effort is  $X_T^i + \zeta^i$ . Hence, trader i's objective is

$$
\sup_{\theta^i} \mathbb{E}[(X_T^i + X_T^{-i} + \zeta^i + \zeta^{-i})X_T^i - P_{t(i)}\theta^i - \frac{1}{2}(X_T^i + \zeta^i)^2 + \zeta^i(X_T^i + \zeta^i)|\zeta^i, \mathcal{F}_{t(i)-1}, \theta^i].
$$

For each trader, this objective is the same as in the variation from part (a) of the proposition, with  $\zeta^i$  in place of  $V^i$ , except for a  $\frac{(\zeta^i)^2}{2}$  $\frac{y^2}{2}$  term which is not strategically relevant. The information structure is also the same. Thus, the equilibria are the same as in part (a), and the leader trades according to (A.36).

### A.7 Proof of Proposition 3

Part (i) Ex ante expected firm value is  $\mathbb{E}[W^L + W^F] = \mathbb{E}[X_0^L + \theta^L + X_0^F + \theta^F] = 2\mu + \mathbb{E}[\theta^L],$ where we have used that terminal efforts coincide with terminal positions and that  $\mathbb{E}[\theta^F] = 0$ . The inequality  $\mathbb{E}[W^L + W^F] \leq 2\mu$  is therefore equivalent to  $\mathbb{E}[\theta^L] \leq 0$ , which holds iff  $\rho \leq 0$ (with strict inequality if  $\rho \neq 0$ ) by Theorem 1. Moreover, since  $\mathbb{E}[\theta^L] = (\alpha_L + \delta_L)\mu$ , we have  $\mathbb{E}[W^L + W^F] = (2 + \alpha_L + \delta_L)\mu$ , which is monotone decreasing in  $\rho$  by Proposition A.6.

Part (ii) We show that  $\alpha_L + \delta_L > -1$ . Using (A.17), we have  $\alpha_L + \delta_L = \alpha_L - \frac{\sigma^2}{\phi \alpha_L}$  $\frac{\sigma^2}{\phi \alpha_L} =: h(\alpha_L).$ Note that h is increasing in  $\alpha_L$  for  $\alpha_L > 0$ , and from the proof of Proposition A.3,  $\alpha_L > \hat{\alpha}$ . By direct calculation,  $h(\hat{\alpha}) = -1$ , so we are done.

Part (iii) Fix  $\rho > 0$ . For part (iii.1), we begin with some useful preliminary observations. Recall from the proof of Proposition A.3 that  $\hat{\alpha} < \alpha_L < \alpha^K$ . But  $\lim_{\sigma \to +\infty} \frac{\hat{\alpha}}{\sigma} = 1/$ √  $\overline{\phi} =$ 

 $\lim_{\sigma \to +\infty} \frac{\alpha^K}{\sigma}$  $\frac{\alpha}{\sigma}$ , so  $\lim_{\sigma \to +\infty} \frac{\alpha_L}{\sigma} = 1/2$ √  $\overline{\phi}$ . Then by (A.17),  $\lim_{\sigma \to +\infty} \frac{\delta_L}{\sigma} = -1/$ √  $\overline{\phi}$ . These limits imply  $\lim_{\sigma \to +\infty} \alpha_L = +\infty$  and  $\lim_{\sigma \to +\infty} \delta_L = -\infty$ . Let  $x_L := \alpha_L/\sigma$  and  $x_F := \alpha_F/\sigma$ .

For the first limit in part (iii.1), recall from above that  $x_L$  converges to a positive constant as  $\sigma \to +\infty$ . Using the expression for  $\alpha_F$  in (A.20), it is easy to see that  $x_F$  also converges to a positive constant as  $\sigma \to +\infty$ . Now  $\mathbb{E}[\theta^L] = \mu(\alpha_L + \delta_L)$ , and from (A.16) and (A.17),  $\alpha_L + \delta_L = -\frac{\rho \alpha_F}{\phi(1+\alpha_L) + \rho(1+\alpha_F)} = -\frac{\rho x_F}{(\rho+\phi)/\sigma+\phi}$  $\frac{\rho x_F}{(\rho + \phi)/\sigma + \phi x_L + \rho x_F}$ , which converges to a negative constant as  $\sigma \rightarrow +\infty$  since both  $x_L$  and  $x_F$  converge to positive constants.

For the second limit in part (iii.1), note that  $\alpha_L - \alpha^K = \frac{\alpha_L}{\alpha_L + \alpha^K}$  $\alpha_L+\alpha^K$  $\left(\alpha_L - \frac{(\alpha^K)^2}{\alpha_I}\right)$  $\alpha_L$  . The first factor is  $\frac{x_L}{x_L+1/\sqrt{\phi}}$ , which has a finite positive limit as  $\sigma \to +\infty$ , and the second equals  $\alpha_L + \delta_L$ which, as just argued, converges to a finite negative limit. Hence  $\lim_{\sigma \to +\infty} {\{\alpha_L - \alpha^K\}} \in (-\infty, 0)$ .

For part (iii.2), from the proof of Proposition 6, in the PBS equilibrium,  $\alpha_L/\sigma$  converges to a positive constant as  $\sigma \to 0$ , so it follows that  $\lim_{\sigma \to 0} \alpha_L = 0$ . By (A.17),  $\delta_L/\sigma = -1/(\phi \alpha_L/\sigma)$  converges to a negative constant, and thus  $\lim_{\sigma \to 0} \delta_L = 0$ . Therefore,  $\lim_{\sigma\to 0} \mathbb{E}[\theta^L] = \lim_{\sigma\to 0} \{(\alpha_L + \delta_L)\mu\} = 0$ , and  $\lim_{\sigma\to 0} \{\alpha_L - \sqrt{\sigma^2/\phi}\} = 0 - 0 = 0$ .

### A.8 Proof of Proposition 4

We consider symmetric linear strategies of the form

$$
\theta^i = \alpha X_0^i + \beta \mu. \tag{A.38}
$$

We begin by characterizing belief updating and pricing, and then we use these to set up the best-response problem of either trader. We show that in any symmetric PBS equilibrium,  $\alpha = \frac{\sigma}{\sqrt{2\phi}}$ , and then we show that there exists  $\rho_0^{\rm sim} \in (-\phi, 0)$  such that for all  $\rho \in [\rho_0^{\rm sim}, \phi],$ there exists a unique symmetric PBS equilibrium.

After observing the total order flow, the market maker updates her beliefs about the activists' positions. Given the form of strategies and symmetry, it is sufficient for the market maker to only estimate the sum of initial positions. By the projection theorem,

$$
\mathbb{E}[X_0^i + X_0^j | \mathcal{F}_1] = 2\mu + \frac{\text{Cov}(X_0^i + X_0^j, \Psi_1)}{\text{Var}(\Psi_1)} \left\{ \Psi_1 - \left[ \underbrace{2\alpha\mu + 2\beta\mu}_{=\mathbb{E}[\theta^i + \theta^j]} \right] \right\}
$$
  
= 
$$
2\mu + \frac{2\alpha(\phi + \rho)}{2\alpha^2(\phi + \rho) + \sigma^2} \left\{ \Psi_1 - 2\mu(\alpha + \beta) \right\}.
$$

Hence,  $P_1$  is equal to

$$
P_1 = \mathbb{E}[W|\mathcal{F}_1] = \mathbb{E}[X_T^i + X_T^j|\mathcal{F}_1] = (1+\alpha)\mathbb{E}[X_0^i + X_0^j|\mathcal{F}_1] + 2\mu\beta
$$

$$
= P_0^S + \Lambda_1^S \left\{ \Psi_1 - 2\mu(\alpha + \beta) \right\},\,
$$

where  $P_0^S := 2\mu(1 + \alpha + \beta)$  is the ex ante expected firm value and  $\Lambda_1^S := (1 + \alpha) \frac{2\alpha(\phi + \rho)}{2\alpha^2(\phi + \rho)+1}$  $\sqrt{2\alpha^2(\phi+\rho)+\sigma^2}$ is Kyle's lambda.

Each activist then maximizes

$$
\sup_{\theta^i} \mathbb{E}\left[\frac{(X_0^i + \theta^i)^2 + 2X_T^{-i}(X_0^i + \theta^i)}{2} - P_1\theta^i | X_0^i, \theta^i\right].
$$
 (A.39)

The FOC is  $\frac{2(X_0^i+\theta^i)+2\mathbb{E}[X_T^{-i}|X_0^i]}{2}-\theta^i\frac{\partial P_1}{\partial \theta^i}-P_1=0$ . Plugging in the expression for  $\Lambda_1^S$ , evaluating at the conjectured strategy (A.38), and setting the coefficient on  $X_0^i$  to 0 yields an equation for  $\alpha$  with the following three roots:

$$
\alpha = \frac{\sigma}{\sqrt{2\phi}}, \quad -\frac{\sigma}{\sqrt{2\phi}}, \quad -1. \tag{A.40}
$$

Similarly, setting the coefficient on  $\mu$  to 0, we can pin down  $\beta$  from  $\alpha$  via the following equation

$$
\beta = \frac{\sigma^2}{2\sigma^2 - 4\alpha(1+\alpha)\phi}.\tag{A.41}
$$

Since the second and third roots are negative, we have a unique candidate for a symmetric PBS equilibrium.

Existence and uniqueness: For existence, we must check the SOC:  $1 - 2\Lambda_1^S \leq 0$ . Plugging in  $\alpha = \frac{\sigma}{\sqrt{2\phi}}$ , this condition is equivalent to the inequality

$$
\sigma^2 - 2\alpha(2+\alpha)(\rho+\phi) = \sigma^2 - 2\frac{\sigma}{\sqrt{2\phi}}\left(2+\frac{\sigma}{\sqrt{2\phi}}\right)(\phi+\rho) \le 0.
$$

The left hand side is decreasing and continuous in  $\rho$ , and it is strictly negative when  $\rho = 0$ , so there exists  $\rho_0^{\text{sim}} \in (-\phi, 0)$  such that the inequality is satisfied, and in turn a unique PBS equilibrium exists, whenever  $\rho \in [\rho_0^{\text{sim}}, \phi]$ .

Payoff comparison: To compare payoffs to those in the sequential-move game, first consider  $\rho = 0$ . The equilibrium is characterized in Proposition A.2, and  $\alpha_L = \alpha_F = \sqrt{\frac{\sigma^2}{\phi}}$  $\frac{\sigma^2}{\phi}$ . The coefficient in the simultaneous-move game is  $\alpha_S := \sqrt{\frac{\sigma^2}{2\phi}}$  (see (A.40)), where  $\alpha_L = \alpha_F > \alpha_S$ .

To calculate the players' expected payoffs in the sequential case (which are the same

given  $\rho = 0$ , plug the equilibrium strategies into (4) to obtain

$$
\mathbb{E}\bigg[\frac{1}{2}\left(X_0^L\left(1+\sqrt{\frac{\sigma^2}{\phi}}\right)-\sqrt{\frac{\sigma^2}{\phi}}\mu\right)^2+\left(X_0^F+\sqrt{\frac{\sigma^2}{\phi}}(X_0^F-\mu)\right)\left(X_0^L+\sqrt{\frac{\sigma^2}{\phi}}(X_0^L-\mu)\right)\\-\left(P_0+\Lambda_1\left(\sqrt{\frac{\sigma^2}{\phi}}(X_0^L-\mu)+\sigma Z_1\right)\right)\sqrt{\frac{\sigma^2}{\phi}}(X_0^L-\mu)\bigg].
$$

Opening up the expectation and simplifying we can write the first line as  $\frac{1}{2}(\mu^2 + (\sigma + \sqrt{\frac{2\mu^2}{n}}))$ and simplifying we can write the first line as  $\frac{1}{2}(\mu^2 + (\sigma + \sqrt{\phi})^2) +$  $\mu^2$  and second line as  $-\frac{\sigma(\sigma+\sqrt{\phi})}{2}$  $\frac{f+\sqrt{\phi}}{2}$ . Hence, each trader's total expected payoff when  $\rho=0$  is

$$
\frac{1}{2} \left[ 3\mu^2 + \phi + \sigma \sqrt{\phi} \right]. \tag{A.42}
$$

Following similar steps for the simultaneous case, we can write the equilibrium payoff of player  $i$   $(i = 1, 2)$  as

$$
\mathbb{E}\bigg[\frac{1}{2}\left(X_0^i\left(1+\sqrt{\frac{\sigma^2}{2\phi}}\right)-\sqrt{\frac{\sigma^2}{2\phi}}\mu\right)^2+2\left(X_0^j+\sqrt{\frac{\sigma^2}{2\phi}}(X_0^j-\mu)\right)\left(X_0^i+\sqrt{\frac{\sigma^2}{2\phi}}(X_0^i-\mu)\right)\\-\left(P_0^S+\Lambda_1^S\left(\sqrt{\frac{\sigma^2}{2\phi}}(X_0^i-\mu)+\epsilon_i\right)\right)\sqrt{\frac{\sigma^2}{2\phi}}(X_0^i-\mu)\bigg].
$$

Opening up the expectation, the first line simplifies to  $\frac{1}{2} \left( \mu^2 + \frac{(\sigma + \sqrt{2\phi})^2}{2} \right)$  $\left(\frac{\sqrt{2\phi}}{2}\right)^2 + \mu^2$ , while the second line simplifies to  $-\frac{\sigma(\sigma+\sqrt{2\phi})}{4}$  $\frac{-\sqrt{2\varphi}}{4}$ , for a total expected payoff of

$$
\frac{1}{2}\left[3\mu^2 + \phi + \frac{\sigma\sqrt{2\phi}}{2}\right].
$$
\n(A.43)

Subtracting (A.43) from (A.42) yields  $\frac{1}{2}$   $\left(1 - \frac{1}{2}\right)$  $\sqrt{2}$  $\sqrt{2\over 2}\Big)$   $\sigma$ √  $\overline{\phi}$ , which is strictly positive. Therefore, the both players unambiguously prefers the sequential-move game when  $\rho = 0$ .

The same comparison extends to  $|\rho| >$  sufficiently small by continuity. Specifically, Proposition A.4 and the results above, establish existence and uniqueness for small  $|\rho|$ . For such  $|\rho|$ ,  $\alpha_L$  and  $\alpha_F$  in the sequential-move game are continuous in  $\rho$  at  $\rho = 0$  by Corollary A.1. After using (A.14), (A.15), and (A.17) to eliminate  $(\beta_F, \delta_F, \delta_L)$ , the players' payoffs can be written as continuous functions of  $(\rho, \alpha_L, \alpha_F)$  and are therefore continuous in  $\rho$  at  $\rho = 0.36$ For the simultaneous-move case, the equilibrium trading coefficients are independent of  $\rho$  as shown earlier, and payoffs are clearly continuous in  $\rho$ . Figure 4 illustrates.

 $36$ Full expressions for general  $\rho$  are available from the authors upon request.

### A.9 Proof of Proposition 5

Fix  $\mu, \sigma, \phi, \rho$ . Let  $\mu s_{\mu}$  denote the prior mean for each follower,  $\phi s_{\phi}$  the variance, and  $\rho s_{\rho}$ the covariance between the leader and each follower, where  $s_{\mu}, s_{\phi}, s_{\rho}$  will vary with N. The setup described in Section 5.2 is captured by  $s_{\mu} = 1/N$ ,  $s_{\phi} = 1/N^2$ , and  $s_{\rho} = 1/N$ .

Define  $\gamma_1^{\text{sum}} = N^2 \gamma_1^F$ , the market maker's posterior variance of the sum of all followers' positions. In any PBS equilibrium, the followers play gap strategies and their FOC yields  $\alpha_F = \sqrt{\frac{N\sigma^2}{N\gamma_1^{\text{sum}}}} = \sqrt{\frac{\sigma^2}{N\gamma_1^F}}$ . Incorporating this into the leader's FOC then yields the following equation generalizing (A.20):

$$
\frac{(N\rho s_{\rho} + \phi + \alpha_L \phi)(\sigma^2 - \alpha_L^2 \phi)}{N\rho s_{\rho}[\alpha_L(1 + \alpha_L)\phi - \sigma^2]} = \sqrt{\frac{\sigma^4 + \sigma^2 \alpha_L^2 \phi}{N[\phi s_{\phi} \sigma^2 + \alpha_L^2(\phi^2 s_{\phi} - (\rho s_{\rho})^2)]}}.
$$
(A.44)

Arguments similar to those earlier show that for  $\rho > 0$ , (A.44) has a solution  $\alpha_L$  in  $(\hat{\alpha}, \alpha^K)$ , there is no other solution for  $\alpha_L \geq 0$ , and SOCs are satisfied. The FOC also implies that the coefficient on  $\mu$  is  $\delta_L = -\frac{\sigma^2}{\phi \alpha}$  $\frac{\sigma^2}{\phi \alpha_L}$ . Hence, we have characterized the unique PBS equilibrium.

We now turn to comparative statics wrt N. After plugging in our values for  $(s_{\mu}, s_{\phi}, s_{\rho})$ , (A.44) reduces to

$$
\frac{(\rho + \phi + \alpha_L \phi)(\sigma^2 - \alpha_L^2 \phi)}{\rho[\alpha_L(1 + \alpha_L)\phi - \sigma^2]} = \sqrt{\frac{N(\sigma^4 + \sigma^2 \alpha_L^2 \phi)}{\phi \sigma^2 + \alpha_L^2(\phi^2 - \rho^2)}}.
$$
\n(A.45)

When these intersect at  $\alpha_L \in (\hat{\alpha}, \alpha^K)$ , the left hand side crosses the right hand side from above. Then since the right hand side is increasing in N, the equilibrium value of  $\alpha_L$  is decreasing in N. It is also straightforward to show that the left side of  $(A.45)$  is decreasing in  $\alpha_L$  on  $(\hat{\alpha}, \infty)$ , so each side of  $(A.45)$  is increasing in N. Since the right hand side is precisely  $\alpha_F$ , this establishes that  $\alpha_F$  is increasing in N. Note that while the decay in  $\alpha_L$ raises  $\gamma_1^F$  in  $\alpha_F = \sqrt{\frac{\sigma^2}{N\gamma_1^F}}$  all else equal, this effect does not overturn the direct downward effect that larger N has on  $\gamma_1^F$ , as  $\gamma_1^F \leq \phi/N^2$  for any linear strategy of the leader.

Since the followers play gap strategies, ex ante firm value is still  $(2 + \alpha_L + \delta_L)\mu$  $(2+\alpha_L-\sigma^2/(\phi\alpha_L))\mu$  for all N. Since  $\alpha_L$  is decreasing in N, ex ante firm value is decreasing in N.

For later use, we show that  $\lim_{N\to\infty} \alpha_L = \hat{\alpha} > 0$ , where  $\hat{\alpha}$  was defined earlier as the positive root of  $\alpha_L(1+\alpha_L)\phi-\sigma^2$ . As  $N\to\infty$ , the right hand side of (A.45) explodes as the rest of the expression in the square root is bounded. Thus, the left hand side must also explode, which requires its denominator to vanish. Given that  $\alpha_L > 0$ , this implies that  $\alpha_L$ converges to  $\hat{\alpha}$ .

We now turn to the asymptotic result. The leader's expected payoff is

$$
\mathbb{E}\left[-P_1\theta_L + \frac{(X_0^L + \theta_L)^2}{2} + (X_0^L + \theta_L)N(X_0^F + \alpha_F(X_0^F - M_1^F))\right].
$$
 (A.46)

We simplify (A.46) one term at a time. The first term equals

$$
- \mathbb{E}[(P_0 + \Lambda_1[\Psi_1 - (\alpha_L + \delta_L)\mu])\theta_L]
$$
  
= 
$$
- \mathbb{E}[P_0(\alpha_L X_0^L + \delta_L \mu) + \Lambda_1 \alpha_L (X_0^L - \mu)(\alpha_L X_0^L + \delta_L \mu)]
$$
  
= 
$$
-[(2 + \alpha_L + \delta_L)(\alpha_L + \delta_L)\mu^2 + \Lambda_1 \alpha_L^2 \phi] =: S_1.
$$
 (A.47)

Since  $\alpha_L$  and  $\delta_L$  have finite limits as  $N \to \infty$ , and  $\Lambda_1 = \frac{\alpha_L(\rho + \phi(1 + \alpha_L))}{\sigma^2 + \alpha_L^2 \phi}$  $\frac{\rho+\phi(1+\alpha_L)}{\sigma^2+\alpha_L^2\phi}$  also has a finite limit, this term overall is therefore uniformly bounded in N.

The expectation of the second term in (A.46) equals

$$
S_2 := \frac{1}{2} \mathbb{E} \left[ (X_0^L (1 + \alpha_L) + \delta_L \mu)^2 \right] = \frac{1}{2} [(1 + \alpha_L + \delta_L)^2 \mu^2 + \phi (1 + \alpha_L)^2], \tag{A.48}
$$

which is also uniformly bounded in  $N$ .

Using that  $\mathbb{E}[X_0^F - M_1^F] = 0$  by the law of iterated expectations, the third term in (A.46) simplifies as:

$$
\mathbb{E}[(X_0^L(1+\alpha_L)+\delta_L\mu)N(X_0^F+\alpha_F(X_0^F-M_1^F))]
$$
\n=  $(1+\alpha_L)(1+\alpha_F)N\mathbb{E}[X_0^L X_0^F] + \delta_L N\mu^2 s_\mu - \mathbb{E}[X_0^L(1+\alpha_L)N\alpha_F M_1^F]$   
\n=  $(1+\alpha_L)(1+\alpha_F)N(\mu^2 s_\mu + \rho s_\rho) + \delta_L N\mu^2 s_\mu - \mathbb{E}[X_0^L(1+\alpha_L)N\alpha_F M_1^F]$   
\n=  $(1+\alpha_L)(1+\alpha_F)N(\mu^2 s_\mu + \rho s_\rho) + \delta_L N\mu^2 s_\mu$   
\n-  $\mathbb{E}[X_0^L(1+\alpha_L)N\alpha_F \left\{\mu s_\mu + \frac{\alpha_L \rho s_\rho}{\alpha_L^2 \phi + \sigma^2} [\alpha_L X_0^L + \delta_L \mu - (\alpha_L + \delta_L)\mu] \right\}.$  (A.49)

We now simplify the last term in (A.49):

$$
\mathbb{E}\left[X_0^L(1+\alpha_L)N\alpha_F\left\{\mu s_\mu + \frac{\alpha_L \rho s_\rho}{\alpha_L^2 \phi + \sigma^2}[\alpha_L X_0^L + \delta_L \mu - (\alpha_L + \delta_L)\mu]\right\}\right]
$$
\n
$$
= \mathbb{E}\left[X_0^L(1+\alpha_L)N\alpha_F\left\{\mu s_\mu + \frac{\alpha_L \rho s_\rho}{\alpha_L^2 \phi + \sigma^2}\alpha_L(X_0^L - \mu)\right\}\right]
$$
\n
$$
= (1+\alpha_L)N\alpha_F\mu^2 s_\mu + (1+\alpha_L)N\alpha_F\frac{\alpha_L \rho s_\rho}{\alpha_L^2 \phi + \sigma^2}\alpha_L \mathbb{E}[X_0^L(X_0^L - \mu)]
$$
\n
$$
= (1+\alpha_L)N\alpha_F\mu^2 s_\mu + (1+\alpha_L)N\alpha_F\frac{\alpha_L \rho s_\rho}{\alpha_L^2 \phi + \sigma^2}\alpha_L \text{Var}(X_0^L)
$$

$$
= (1 + \alpha_L)\alpha_F \mu^2 + (1 + \alpha_L)\alpha_F \frac{\alpha_L \rho}{\alpha_L^2 \phi + \sigma^2} \alpha_L \phi.
$$

Incorporating this in  $(A.49)$ , the third term of  $(A.46)$  equals

$$
S_3 := (1 + \alpha_L)(1 + \alpha_F)(\mu^2 + \rho) + \delta_L \mu^2 - \left[ (1 + \alpha_L)\alpha_F \mu^2 + (1 + \alpha_L)\alpha_F \frac{\alpha_L^2 \rho \phi}{\alpha_L^2 \phi + \sigma^2} \right]
$$
  
=  $(1 + \alpha_L)(\mu^2 + \rho) + \delta_L \mu^2 + \alpha_F \rho (1 + \alpha_L) \frac{\sigma^2}{\alpha_L^2 \phi + \sigma^2},$  (A.50)

where we have canceled N with  $1/N$  in  $s_{\mu}$  and  $s_{\rho}$ .

The leader's payoff is the sum of  $(A.47)$ ,  $(A.48)$ , and  $(A.50)$ :  $\Pi_L = S_1 + S_2 + S_3$ . To show that the rate of growth is  $\sqrt{N}$ , we calculate

$$
\lim_{N \to \infty} \frac{\Pi_L}{\sqrt{N}} = \lim_{N \to \infty} \frac{S_1}{\sqrt{N}} + \lim_{N \to \infty} \frac{S_2}{\sqrt{N}} + \lim_{N \to \infty} \frac{S_3}{\sqrt{N}}
$$

$$
= 0 + 0 + \lim_{N \to \infty} \frac{S_3}{\sqrt{N}}
$$

$$
= \left(\lim_{N \to \infty} \frac{\alpha_F}{\sqrt{N}}\right) \left(\lim_{N \to \infty} (1 + \alpha_L) \rho \frac{\sigma^2}{\alpha_L^2 \phi + \sigma^2}\right)
$$

,

where we have used that in  $S_3$ ,  $(1 + \alpha_L)(\mu^2 + \rho) + \delta_L \mu^2$  is uniformly bounded in N. To take limits in the last line, we use the fact that for  $\rho \in (0, \phi]$ ,  $\lim_{N \to \infty} \alpha_L = \hat{\alpha} > 0$ , as shown earlier in the proof. The two factors in the product then have limits

$$
\lim_{N \to \infty} \frac{\alpha_F}{\sqrt{N}} = \lim_{N \to \infty} \sqrt{\frac{(\sigma^4 + \sigma^2 \alpha_L^2 \phi)}{\phi \sigma^2 + \alpha_L^2 (\phi^2 - \rho^2)}} = \sqrt{\frac{(\sigma^4 + \sigma^2 \hat{\alpha}^2 \phi)}{\phi \sigma^2 + \hat{\alpha}^2 (\phi^2 - \rho^2)}}
$$

$$
\lim_{N \to \infty} (1 + \alpha_L) \rho \frac{\sigma^2}{\alpha_L^2 \phi + \sigma^2} = (1 + \hat{\alpha}) \rho \frac{\sigma^2}{\hat{\alpha}^2 \phi + \sigma^2}.
$$

Since these limits are positive and finite, so is their product, and we conclude that  $\Pi_L$ grows asymptotically at rate  $\sqrt{N}$ .

The following lemma formalizes the last statement of the proposition.

**Lemma A.7.** Assume  $\rho = \phi$ , and let  $\Pi_L^{seq}$  $L^{seq}_L$  and  $\Pi^{sim}_L$  denote the leader's payoff in the sequential- and simultaneous-move games, respectively. When N is sufficiently large, the leader's payoff advantage from going first is increasing in N. Specifically,  $\Pi_L^{seq}$ leader's payoff advantage from going first is increasing in N. Specifically,  $\Pi_L^{seq}$  and  $\Pi_L^{sim}$ <br>grow at rate  $\sqrt{N}$  asymptotically, and  $\lim_{N\to\infty} \frac{\Pi_L^{seq} - \Pi_L^{sim}}{\sqrt{N}} > 0$ .

*Proof.* Proposition 5 characterizes the asymptotics of  $\Pi_L^{\text{seq}}$ , so consider the simultaneousmove game. The FOCs lead to the following system of equations:  $\alpha_L = \frac{1-\frac{\rho}{\phi}\Lambda\alpha_F+\frac{\rho}{\phi}(1+\alpha_F)}{2\Lambda-1}$  $\frac{\alpha_F + \phi^{(1+\alpha_F)}}{2\Lambda - 1}, \alpha_F =$  $N(1-\frac{\rho}{\phi}\Lambda\alpha_L+\frac{\rho}{\phi}(1+\alpha_L))$  $\frac{\frac{L}{\phi}\Lambda\alpha_L+\frac{L}{\phi}(1+\alpha_L))}{(\frac{N+1}{\Lambda}-N)}$ , where  $\Lambda=\frac{(1+\alpha_L)(\phi\alpha_L+\rho\alpha_F)+(1+\alpha_F)(\phi\alpha_F+\rho\alpha_L)}{\phi(\alpha_L^2+\alpha_F^2)+2\alpha_L\alpha_F\rho+\sigma^2}$ .

For the case  $\rho = \phi$ , we obtain  $(\alpha_L, \alpha_F) = \left(\frac{\sigma}{\sqrt{(N+1)\phi}}, \frac{N\sigma}{\sqrt{(N+1)\phi}}\right)$  $(N+1)\phi$  $\setminus$ . The leader's payoff is again of the order  $\sqrt{N}$ , with coefficient  $\lim_{N\to\infty} \frac{\alpha_F}{\sqrt{N}}(1+\alpha_L) \text{Cov}(X_0^L, X_0^F) = \lim_{N\to\infty} \frac{\alpha_F}{\sqrt{N}}(1+\alpha_L)$  $\alpha_L$ ) $\phi = \sigma \sqrt{\phi}$ . To complete the proof, we show that this is strictly less than the correspond-√ ing coefficient in the sequential-move game, namely  $\sqrt{\frac{(\sigma^4 + \sigma^2 \hat{\alpha}^2 \phi)}{\phi \sigma^2}} (1 + \hat{\alpha}) \phi \frac{\sigma^2}{\hat{\alpha}^2 \phi + \hat{\alpha}^2 \phi}$  $\frac{\sigma^2}{\hat{\alpha}^2 \phi + \sigma^2}$ . By routine simplifications,

$$
\sigma\sqrt{\phi} \le \sqrt{\frac{(\sigma^4 + \sigma^2 \hat{\alpha}^2 \phi)}{\phi \sigma^2}} (1 + \hat{\alpha}) \phi \frac{\sigma^2}{\hat{\alpha}^2 \phi + \sigma^2}
$$
  
\n
$$
\iff 1 \le \sqrt{\sigma^2 + \hat{\alpha}^2 \phi} (1 + \hat{\alpha}) \frac{\sigma}{\hat{\alpha}^2 \phi + \sigma^2}
$$
  
\n
$$
\iff \sqrt{\sigma^2 + \hat{\alpha}^2 \phi} \le (1 + \hat{\alpha}) \sigma
$$
  
\n
$$
\iff \sigma^2 + \hat{\alpha}^2 \phi \le (1 + \hat{\alpha})^2 \sigma^2 \qquad \text{(since both sides are positive)}
$$
  
\n
$$
\iff 0 \le \hat{\alpha} [\hat{\alpha} (\sigma^2 - \phi) + 2\sigma^2].
$$

Since  $\hat{\alpha}$  solves  $\sigma^2 - \hat{\alpha}(1 + \hat{\alpha})\phi = 0$ , the right hand side is

$$
\hat{\alpha}[\hat{\alpha}(\sigma^2 - \phi) + 2\sigma^2] = \hat{\alpha}[\hat{\alpha}\sigma^2 + \hat{\alpha}^2\phi - \sigma^2 + 2\sigma^2] = \hat{\alpha}[\hat{\alpha}\sigma^2 + \hat{\alpha}^2\phi + \sigma^2] \ge 0,
$$

establishing the inequality.

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