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Abstract

We show that Treasury bill auction procedures create classes of price-equivalent discount rates for bills with fewer than seventy-two days to maturity. We argue that it is inefficient for market participants to bid at a discount rate that is not the minimum rate in its class. The inefficiency of bidding at a rate other than the minimum is related to a quantity shortfall rather than an unexploited profit opportunity. Auction results for weekly offerings of four-week bills and occasional offerings of cash management bills show that market participants frequently bid at inefficient rates. However, they are more likely to bid at efficient rates than chance would suggest.

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I. Introduction

In this paper we assess bidding behavior in short-term U.S. Treasury bill auctions. Treasury Department rules for bill auctions provide that (1) competitive bids be specified in terms of a discount rate that is an integer multiple of one-half basis point, and that (2) the invoice price paid by a successful bidder be computed from the highest accepted bid rate and rounded to three digits of precision to the right of the decimal point. We show that these two provisions create classes of price-equivalent discount rates for bills with less than 72 days to maturity. Moreover, we argue that it is inefficient for market participants to bid at a discount rate that is not the minimum rate in its class.

The data show that market participants frequently bid at inefficient rates in weekly offerings of four-week bills and occasional offerings of cash management bills. Market participants thereby forgo opportunities to buy securities at prices at which they have indicated a willingness to buy. However, market participants are more likely to bid at efficient rates than chance would suggest, implying that at least some participants recognize the advantage of bidding efficiently. There is only weak evidence that bidding efficiency has increased over time, and bidding efficiency is not related to auction bid-to-cover ratios.

Other studies empirically examine Treasury security auctions in different contexts. Cammack (1991), Spindt and Stolz (1992), and Simon (1994a) find that securities tend to be underpriced at auction compared with prices in the when-issued market. Simon (1994b), Nyborg and Sundaresan (1996), and Goldreich (2003) relate underpricing to auction technique, whether single price or multiple price. No prior study examines price-equivalent discount rate classes and the implications of such classes for bidding behavior, presumably because the issue only became particularly relevant in 2001 with the Treasury's introduction of four-week bills.

II. Treasury Bill Auctions

Treasury bills are single-payment securities that pay a specified face amount on a stated maturity date and that trade at a discount to face amount until maturity. The difference between the face amount of a bill and its discounted market price is interest earned by an investor who holds the bill to maturity.

Bills are quoted in secondary market trading in terms of an annualized discount rate. The invoice price of a bill (expressed as a percentage of face amount) is computed by scaling the bill's discount rate by the fraction of a year remaining in the life of the bill (calculated with an actual-over-360 day count) and subtracting the result from 100. Market participants conventionally calculate an invoice price to six digits of precision to the right of the decimal point – or to 1 cent per $1 million face amount. A bill quoted at a discount rate of 2.11% with 87 days remaining to maturity thus has an invoice price of 99.490083% of face amount ($99.490083 = 100 - \frac{87}{360} \cdot 2.11$).

Treasury bills are currently auctioned weekly in three series. Thirteen- and 26-week bills are auctioned on Mondays and four-week bills are auctioned on Tuesdays. All three series are issued and paid for the following Thursday and mature 91, 182, and 28 days later, respectively.[1] The Treasury also issues cash management (CM) bills with varying maturities on an irregular schedule.

Market participants can submit either a single noncompetitive bid or one or more competitive bids in an auction. A noncompetitive bidder specifies a quantity of bills (up to $1

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[1] If the usual issue date of the bills is a holiday (e.g., Thanksgiving), the bills are issued the next day as 90-, 181-, and 27-day bills, respectively. If the usual maturity date of a bill is a holiday, the bill matures 1 day later.

million face amount) and agrees to purchase that quantity at the price paid by successful competitive bidders. A competitive bidder specifies a discount rate as well as a quantity and agrees to purchase up to the indicated quantity at the price derived from its bid rate or at any lower price. The discount rate on a competitive bid must be an integer multiple of one-half basis point.[2]

Suppose the Treasury offers to sell $15 billion (face amount) of 28-day bills and receives (1) competitive tenders at a variety of discount rates between 1.100% and 1.155% for a total of $33 billion of the bills, and (2) noncompetitive tenders for $2 billion of the bills. The noncompetitive tenders are filled in full; therefore, the Treasury has to sell $13 billion of the bills to competitive bidders. Beginning at the lowest bid discount rate of 1.100%, the Treasury accepts competitive tenders at progressively higher rates until it has accepted bids for at least $13 billion. The highest accepted discount rate is called the "stop." All competitive tenders at a rate lower than the stop are filled in full,[3] tenders at the stop are prorated, and tenders above the stop are rejected. The invoice price for all accepted competitive bids, as well as all noncompetitive bids, is computed from the stop and rounded to three digits of precision to the right of the decimal point.[4] For example, if the 28-day bill auction described earlier stops at a

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2 Code of Federal Regulations, title 31, part 356, section 12(c)(1)(i), July 31, 2003. Until April 18, 1983, auction bids had to be specified directly in terms of price (as a percentage of face amount) to three digits of precision to the right of the decimal point. Beginning with the auctions of 13- and 26-week bills on April 18, 1983, bids had to be specified in terms of a discount rate that was an integer multiple of one basis point. Bidding to one-half of a basis point was introduced for all bills other than CM bills in the November 10, 1997 auctions of 13- and 26-week bills, and was extended to CM bills beginning with the auction of 19-day bills on April 2, 2002.

3 Subject to a maximum of 35% of the amount offered, less the bidder's reportable net long position in the bill. Code of Federal Regulations, title 31, part 356, section 22(b), July 31, 2003.

4 Code of Federal Regulations, title 31, part 356, section 20 (c) and Appendix B (V)(A), July 31, 2003.

discount rate of 1.120%, the invoice price will be 99.913% of face amount, because $100 - \frac{28}{360} \cdot 1.120 = 99.912888\ldots$

Classes of Price-Equivalent Discount Rates

The convention of computing auction invoice prices to three digits of precision creates classes of price-equivalent discount rates for bills with less than 72 days to maturity. Consider, for example, the invoice price for a 28-day bill computed from discount rates ranging from 1.100% to 1.185%. As shown in Table 1, discount rates of 1.115%, 1.120%, and 1.125% lead to a common invoice price of 99.913% of face amount. Rates of 1.130% and 1.135% produce a common invoice price of 99.912%.

[Table 1]

Figure I shows invoice prices for a 28-day bill at discount rates ranging from 1.000% to 1.500%. Every equivalence class has either two or three members. Additionally, the classes follow a recurring pattern over a span of seven classes, with 3 rates in the first class, 3 rates in the second class, and then 2, 3, 2, 3, and finally 2 rates, respectively, for a total of 18 rates. (A complete sequence in this pattern is marked with solid balls in Figure I.) This occurs because, for a 28-day bill, 18 discount rate ticks of a half basis point equal exactly 7 invoice price ticks of 0.001% of face amount: $18 \cdot (28/360) \cdot (0.005) = 7 \cdot (0.001)$.

[Figure I]

Short-term bills with maturities other than 28 days exhibit similar equivalence classes and recurrence patterns. Figure II shows invoice prices for a 15-day bill. In this case the recurrence pattern has five classes, with 5 rates in each of the first four classes and 4 rates in the fifth class, for a total of 24 rates. The recurrence pattern spans 24 rates because, for a 15-day bill, 24 discount rate ticks of a half basis point equal 5 invoice price ticks of 0.001% of face amount: $24 \cdot (15/360) \cdot (0.005) = 5 \cdot (0.001)$. In general, if there are n discount rate ticks and m price ticks in a recurrence pattern for a bill maturing in k days, then $n \cdot (k/360) \cdot (0.005) = m \cdot (0.001)$. This implies that the number of discount rates and the number of price-equivalent classes in a recurrence pattern for a bill maturing in k days are the relatively prime values of n and m, respectively, that satisfy the equation $n \cdot (k/360) \cdot (0.005) = m \cdot (0.001)$, or $n/m = 72/k$.

[Figure II]

If the term of a bill exceeds 71 days, every equivalence class has only a single element, i.e., the invoice price associated with a given discount rate is not associated with any other discount rate. This follows because half a basis point on a bill maturing in 72 days or more is worth at least 0.001% of the face amount of the bill ($(k/360) \cdot (0.005) \geq 0.001$ when $k \geq 72$), so the invoice prices associated with two consecutive discount rates will differ by at least the minimum invoice price unit.

If the term of a bill is less than 37 days, every equivalence class has at least two elements. This follows because two discount rate ticks of a half basis point each are equivalent to no more than a single price tick of 0.001% of face amount for a bill maturing in less than 37 days ($2 \cdot (k/360) \cdot (0.005) \leq 0.001$ when $k \leq 36$), so every discount rate must be in the equivalence class of the rate half a basis point higher and/or the equivalence class of the rate half a basis point lower. The largest equivalence class occurs for a one-day bill, where there are 72 discount rates in each class (when k =1, the relatively prime values of n and m that satisfy the equation n/m = 72/k are n = 72 and m = 1).[5]

III. An Efficient Bidding Strategy

We argue that it is inefficient for a market participant to bid competitively at a discount rate that is not the minimum rate in its equivalence class. For example, suppose an investor is planning to bid for $10 million face amount of 28-day bills. The investor may be better off, and will never be worse off, bidding at 1.115% in lieu of 1.120% or 1.125%. (Table 1 shows that these three rates lie in a common equivalence class, with 1.115% the minimum rate in the class.)

To see this result, observe that all three bids are tantamount to a statement by the investor that at an invoice price of 99.913% of face amount (or at any lower invoice price) it will be better off buying bills than not buying bills and that – up to its quantity limit of $10 million – it will be better off buying more bills than fewer bills.

We consider first the case where the investor's decision to bid at 1.115%, 1.120%, or 1.125% does not affect the discount rate at which the auction stops.

- If the auction stops at a rate below 1.115%, its tender will be rejected regardless of which of the three price-equivalent bids it submits.
- If the auction stops at 1.115%, successful bidders will pay an invoice price of 99.913% of face amount. If the investor bid 1.115% it will receive a prorated award at that price, but if it bid 1.120% or 1.125% its tender will be rejected even

5 The Treasury auctioned a one-day CM bill on April 22, 1999.

though it was willing to pay the auction invoice price for up to $10 million of the bills. Thus, if the auction stops at 1.115% the investor will be better off if it bid 1.115% than if it bid either 1.120% or 1.125%.

- If the auction stops at 1.120%, successful bidders will again pay 99.913% of face amount. If the investor bid 1.115% it will receive its full bid amount at that price, if it bid 1.120% it will receive a prorated award, and if it bid 1.125% its tender will be rejected even though it was willing to pay the auction invoice price for up to $10 million of the bills. Thus, if the auction stops at 1.120% the investor will be better off if it bid 1.115% than if it bid either 1.120% or 1.125%.[6]
- If the auction stops at 1.125%, successful bidders will yet again pay 99.913% of face amount. If the investor bid 1.115% or 1.120% it will receive its full bid amount at that price. If it bid 1.125% it will receive a prorated award. Thus, if the auction stops at 1.125% the investor will be equally well off if it bid either 1.115% or 1.120% and will be worse off if it bid 1.125%.
- Finally, if the auction stops at a discount rate in excess of 1.125% the investor will buy $10 million of the bills at a price lower than 99.913% of face amount regardless of which of the three price-equivalent bids it submits.

Thus, no matter where the auction stops, the investor will be no worse off, and may be better off, bidding 1.115% rather than 1.120% or 1.125%.

[6] It is possible, but highly improbable, that the bidding at the stop could be just enough to complete the sale of the bills and that bids at the stop are not prorated, in which case the investor will be equally well off bidding either 1.115% or 1.120%.

Suppose instead that the stop is affected by where the investor bids. Since the investor is choosing between bidding at 1.115%, 1.120%, or 1.125%, the stop cannot be below 1.115% nor above 1.125%. (If the auction stops at a rate below 1.115%, the decision of the investor to bid 1.115%, 1.120%, or 1.125% cannot affect the stop. Similarly, if the auction stops at a rate above 1.125%, the investor's decision to bid 1.115%, 1.120%, or 1.125% cannot affect the stop.) Since the auction stops at either 1.115%, 1.120%, or 1.125%, the invoice price of the bills is 99.913% of face amount regardless of where the investor bids. But then the investor's bid affects only how many bills it will get, and not the price it will pay. Since the investor is better off with more bills – up to its quantity limit of $10 million – it will be better off bidding 1.115%.

In most studies of economic efficiency an inefficiency is identified by the existence of an unexploited risk-adjusted profit opportunity. In contrast, the inefficiency of bidding 1.120% or 1.125% rather than 1.115% in the foregoing example arises because the higher discount rates may leave the investor buying fewer than the maximum possible amount of bills, up to its specified quantity limit, at an invoice price of 99.913% of face amount. That is, the inefficiency is related to a quantity shortfall rather than an unexploited profit opportunity.

IV. Data

The Treasury does not make public the complete record of bids that it receives in an auction, but it does announce the high (or stop) bid rate, the median bid rate (identified from the definition that 50% of the amount of accepted competitive bids are tendered at or below the median bid rate), and the "low" bid rate (identified from the definition that 5% of the amount of accepted competitive bids are tendered at or below the low bid rate). We collect the three rates for all auctions of four-week bills from the auction of the first four-week bill on July 31, 2001 until and including the auction on December 30, 2003. We also collect the rates for all auctions of CM bills between April 2, 2002 (when bidding for CM bills changed from integer multiples of a basis point to integer multiples of a half basis point) and the end of 2003.

Our data set consists of 148 bill auctions, including 126 auctions of four-week bills and 22 auctions of CM bills.[7] The four-week bill auctions include 116 auctions of 28-day bills, five auctions of 27-day bills (four-week bills issued on a Friday because the preceding day was a holiday), and five auctions of 29-day bills (four-week bills that matured on a Friday because the preceding day was a holiday). The CM bills had original maturities ranging from 2 to 19 days. Since every bill in the data set matured in less than 37 days, every equivalence class for every bill has at least two discount rates and therefore at least one inefficient discount rate. Auction descriptive statistics are presented in Table 2.

[Table 2]

V. Empirical Results

The results show that market participants frequently bid at inefficient discount rates in short-term Treasury bill auctions. As shown in Panel C of Table 3, 84 low bids (56.8% of the total sample of 148 low bids), 82 median bids (55.4%), and 67 high bids (45.3%) were tendered at inefficient discount rates. Panels A and B of the table show that the results are similar in the sub-samples of 28-day bills and bills with other original maturities.

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7 The Treasury auctioned four-week bills in every week in our sample period with the exception of September 11, 2001. The cancellation notice for the September 11, 2001 four-week bill auction is posted at ftp://ftp.publicdebt.treas.gov/of/ofconfirmcancel912.pdf. Fleming and Garbade (2002) describe the disruptions in the Treasury securities market following September 11.

[Table 3]

Finding that a large fraction of low bids are inefficient is not compelling evidence of sub-optimal bidding behavior. Market participants who specify low discount rates may be bidding aggressively in order to win a desired quantity of bills at whatever the auction stop turns out to be. That is, they may be submitting bids that are effectively noncompetitive for quantities in excess of $1 million face amount.[8] If the bids are effectively noncompetitive, then it does not matter whether they are efficient or not. The more interesting evidence is that almost half of the bids at what turns out to be the stop are inefficient. Contrary to our hypothesis, market participants do not abstain from tendering inefficient bids even when their bids affect the quantity of bills they are awarded.

The cost of tendering an inefficient bid at what turns out to be the auction stop is that the bidder is awarded only a prorated quantity of bills even though it could have won its full bid amount at the same invoice price if it had bid a half basis point lower. This cost might be negligible if the average allocation for auctions that stop at inefficient discount rates is near 100%. However, the average allocation in the 67 auctions that stopped at inefficient rates was 47.5%. This implies that a market participant who bid at what turned out to be the stop in those auctions could have, on average, doubled its allotment if it had bid efficiently.

Since it is clear that market participants do not abstain from tendering inefficient bids, we examine the alternative proposition that participants make no distinction at all between efficient and inefficient bids. We know from the earlier discussion that 7 out of every 18

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8 For example, in the auction of $9 billion of 28-day bills on September 16, 2003, at least $447 million of competitive bids were tendered at or below the low bid rate of nine basis points. The median and high bids in the auction were 0.865% and 0.870%, respectively.

consecutive discount rates for a 28-day bill are efficient. If market participants make no distinction between efficient and inefficient discount rates, we would then expect that 38.9% of tenders (7 out of every 18) would be efficient over a random sample of auctions. However, the top panel of Table 3 shows that 66 out of 116 high bids in our sample (56.9%) are efficient. The probability of observing so many efficient bids by chance alone is less than 0.01%.[9]

Explaining the Variation in Bidding Efficiency

There is only weak evidence that the incidence of bidding efficiency in 28-day bill auctions has increased over time. Suppose the probability P that the high bid for a 28-day bill is efficient can be written as:

$$P = \int_{-\infty}^{\alpha+\beta\cdot I} n(z)\cdot dz, \tag{1}$$

where n(•) is the standard normal probability density function, α and β are scalar coefficients, and I indexes 28-day bill auctions such that I equals zero for the first auction and increases by one each week thereafter. The β coefficient in this probit model should be positive if the incidence of bidding efficiency has increased over time.[10] The coefficient estimate from our sample of 116 28-day bill auctions is 0.00589. The estimate has a standard error of 0.00332 and a p-value (on the null hypothesis that the coefficient is zero) of 0.0762.

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9 From the binomial distribution, if the probability that a single bid is efficient is 38.9%, the probability that more than 65 out of 116 statistically independent bids are efficient is approximately 0.0065%.

10 Probit models are discussed in Greene (1997, chap. 19).

In contrast, there is no evidence that the incidence of bidding efficiency depends on auction competitiveness. One proxy for auction competitiveness, suggested by Jegadeesh (1993) and also employed by Sundaresan (1994) and Jordan and Jordan (1997), is the ratio, R, of total bills bid for to total bills sold in an auction, i.e., the bid-to-cover ratio. Consider the probit model:

$$P = \int_{-\infty}^{\gamma+\delta\cdot R} n(z)\cdot dz, \tag{2}$$

where P is again the probability that the high bid for a 28-day bill is efficient. The estimate of δ is 0.0172 and has a standard error of 0.2173 and a p-value (on the null hypothesis that the coefficient is zero) of 0.9368.

The composition of market participants might also be related to bidding efficiency, with more sophisticated bidders presumably more likely to bid efficiently. The Treasury has historically released statistics on the allotment of coupon securities across various investor classes, but it does not release such statistics for bills. Since May 2003, however, the Treasury has released some statistics on bidder characteristics after all auctions, including those for bills. While there is insufficient data for statistical analysis, the data is nonetheless revealing. Primary dealers, who can be characterized as relatively sophisticated, were awarded an average of 91.2% of the amounts offered of four-week bills for their own accounts over the 35 auctions between May 6 and December 30, 2003.

VI. Conclusions

The evidence shows that market participants frequently bid at inefficient discount rates in short-term Treasury bill auctions. Prior to the introduction of four-week bills, the issue of bidding efficiency arose only in the context of cash management bills. CM bills have varying maturities and therefore varying patterns of efficient and inefficient discount rates. As a result, it may not be too surprising to find that market participants fail to pay close attention to the efficiency of their bids for CM bills.

The issue of bidding efficiency became more important following the introduction of weekly auctions of four-week bills in July 2001. Our results show that market participants bid inefficient discount rates in these auctions as well. However, our results also suggest that some market participants consider the matter of bidding efficiency important enough that, overall, there is a statistically significant tendency for efficient bid rates in 28-day bill auctions.

It will be interesting to see whether bidding strategies change as market participants become more familiar with the peculiarities of bidding for short-term Treasury bills and as information on the economics of bidding efficiency becomes more widely disseminated. In early 2004, the Treasury announced that it would begin to compute invoice prices on auction awards to six digits of precision to the right of the decimal point starting in the second half of the year.[11] The change will result in a one-to-one mapping of bid rates to invoice prices for all securities and eliminate the anomalies identified in this paper. However, the announcement itself may lead some market participants to reassess their bidding strategies – and limit their bidding to efficient discount rates – before the change is adopted.

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[11] "February 2004 Quarterly Refunding Statement," Department of the Treasury, February 4, 2004, posted at www.treas.gov/press/releases/js1137.htm.

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Table 1. Invoice Price for a 28-Day Bill as a Function of the Discount Rate

Discount Rate	Invoice Price
1.100	99.914
1.105	99.914
1.110	99.914
1.115	99.913
1.120	99.913
1.125	99.913
1.130	99.912
1.135	99.912
1.140	99.911
1.145	99.911
1.150	99.911
1.155	99.910
1.160	99.910
1.165	99.909
1.170	99.909
1.175	99.909
1.180	99.908
1.185	99.908

Note: The table reports the U.S. Treasury Department's invoice price (as a percentage of face amount) for a 28-day bill as a function of the discount rate (in percent per annum). Note that the primary market invoice price is calculated to only three digits of precision to the right of the decimal point leading to classes of price-equivalent discount rates.

Table 2. Descriptive Statistics for Short-Term Bill Auctions

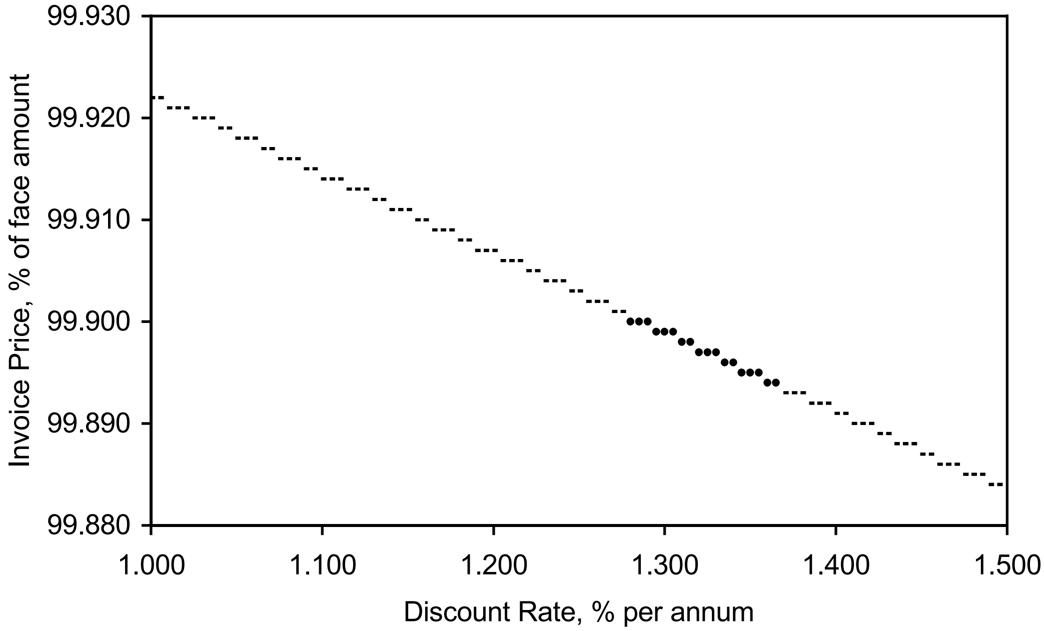
	Four-Week Bills	Cash Management Bills
Number of auctions	126	22
Offering amount	$16.33 billion	$16.64 billion
Bid-to-cover ratio	2.55	3.21
High bid – low bid	5.01 basis points	4.50 basis points
High bid – median bid	1.45 basis points	1.45 basis points
Allocation at high bid	45.72%	47.42%

Note: The table reports auction descriptive statistics for four-week and cash management bills. Averages are reported for all variables except the number of auctions. Offering amount and bid-to-cover ratio exclude amounts awarded to the Federal Reserve.

Table 3. Classification of Bids in Short-Term Bill Auctions

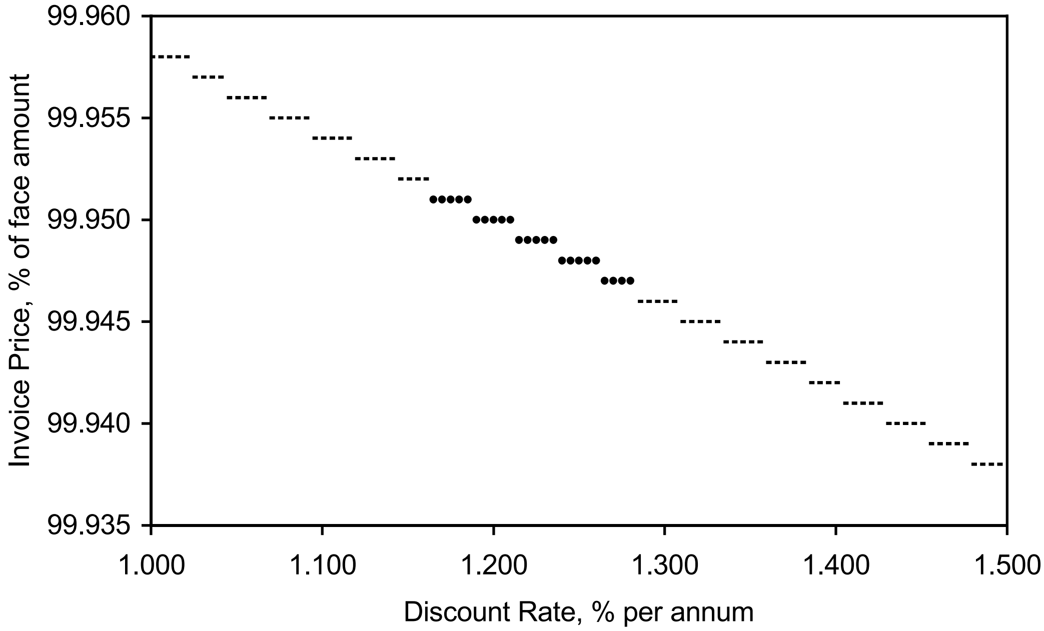
	Low Bid	Median Bid	High Bid
Panel A: 28-Day Bills (n=116)			
Efficient	53	51	66
Inefficient	63	65	50
Panel B: Other Bills (n=32)			
Efficient	11	15	15
Inefficient	21	17	17
Panel C: All Bills (n=148)			
Efficient	64	66	81
Inefficient	84	82	67

Note: The table classifies low, median, and high auction bids as efficient and inefficient for various groups of bills. An efficient bid is a bid at the minimum rate in a class of price-equivalent discount rates. Any bid that is not efficient is inefficient. Every equivalence class for every bill in the data set has at least two rates and therefore at least one inefficient rate.



Note: The figure shows the U.S. Treasury Department's invoice price for a 28-day bill as a function of the discount rate. The classes of price-equivalent discount rates follow a recurring pattern over a span of seven classes, with a complete sequence in this pattern marked by solid balls.

Figure I. Invoice Price for a 28-Day Bill as a Function of the Discount Rate



Note: The figure shows the U.S. Treasury Department's invoice price for a 15-day bill as a function of the discount rate. The classes of price-equivalent discount rates follow a recurring pattern over a span of five classes, with a complete sequence in this pattern marked by solid balls.

Figure II. Invoice Price for a 15-Day Bill as a Function of the Discount Rate