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Money Market Funds Intermediation and Bank Instability

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Abstract

In recent years, U.S. banks have increasingly relied on deposits from financial intermediaries, especially money market funds (MMFs), which collect funds from large institutional investors and lend them to banks. Intermediation through MMFs allows investors to limit their exposure to a given bank. However, since MMFs are themselves subject to runs from their own investors, a banking system intermediated through MMFs is more unstable than one in which investors interact directly with banks. The mechanism through which instability arises in an MMF-intermediated financial system is the release of private information on bank assets, which is aggregated by MMFs and can lead them to withdraw en masse from a bank.

Key words: Money market funds, bank runs

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1 Introduction

In recent years, large global banks have increasingly relied on deposits from financial intermediaries, especially money market funds (MMFs). MMFs mainly collect funds from institutional and wholesale investors and lend them to banks.

Bank deposits of institutional and wholesale investors are not fully covered by deposit insurance. As a result, they need to limit their exposure to a single banking institution and diversify their portfolio of deposits. Intermediation by institutions such as MMFs allows large investors to reap gains from diversification, while saving on bank-monitoring costs.

In the U.S., MMFs have become a very popular financial instrument, comprising roughly 20 percent of all mutual fund assets at the end of 2012. Their assets under management grew from approximately $2 trillion in 2005 to $3 trillion at the end of 2008, although it contracted during the financial crisis (to $2.6 trillion at the end of 2012).[1] MMFs are key providers of short-term funding, especially to the financial sector. As Table 1 shows, in 2012 they were among the largest investors in some asset classes, financing 43 percent of financial commercial paper and 29 percent of certificates of deposit.

Table 1: MMF Investments by Asset Classes

Nonfinancial CP	Financial CP	Asset-backed commercial paper (ABCP)	Certificates of Deposit	Repurchase Agreements
43%	43%	38%	29%	33%
75bn	207bn	117bn	434bn	591bn

As a percentage of outstanding assets. June 2012. Source: McCabe et al. (2012).

In the U.S., MMFs offer demandable deposits (shares) redeemable at par, that is, with fixed net asset value (NAV). When the NAV (i.e., the value of the asset per share) falls below $0.995 ("breaks the buck"), the MMF is forced by SEC regulation to re-price all its shares. Hence, even small losses can start a run since investors have an incentive to redeem their shares before the MMF breaks the buck. In September 2008, the Reserve Primary Fund broke the buck due to its exposure to Lehman paper, causing a stampede of withdrawals across the sector. To stem the panic, the Federal Reserve provided a large amount of liquidity through emergency facilities and the Treasury Department guaranteed MMF liabilities. Figure 1 shows the sharp decrease of asset under management by MMFs in the U.S. after September 2008; importantly the decrease is almost entirely due to the behavior of institutional investors. A similar phenomenon was observed in August 2012 when institutional investors withdrew from U.S. MMFs due to their concerns about potential losses from exposure to European banks.

[1] For a description of the MMF industry, see McCabe et al. (2012).

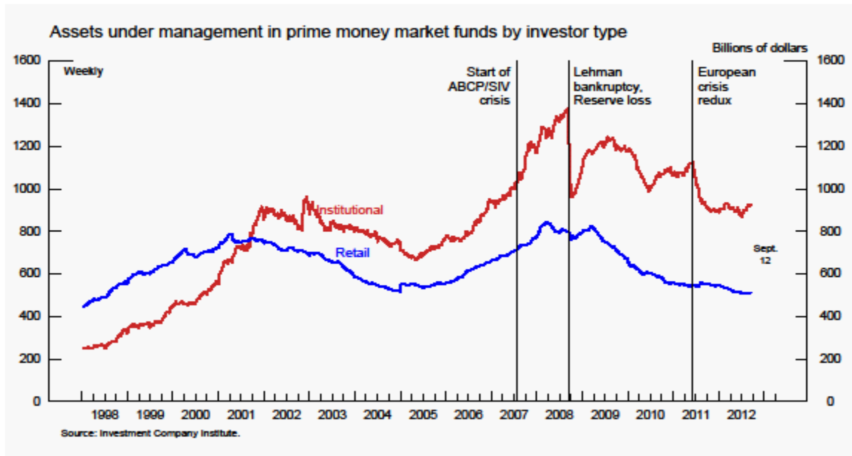


Figure 1: Assets under management in prime U.S. MMFs.

A banking system intermediated through MMFs can be more unstable than one in which large investors interact directly with banks. Since MMFs are themselves subject to run-like redemptions from their own investors, they may react to them by running the banks in which they have deposited, hence amplifying the impact of the initial redemptions. The instability of a financial system in which banks finance themselves through intermediaries such as MMFs is one of the driving forces behind the recent reform effort of the MMF industry by the SEC and the Financial Stability Oversight Council (FSOC).[2]

In this paper, we study an economy à la Diamond and Dybvig (1983) (DD hereafter) with two banks, whose long-term investments have stochastic and (perfectly) negatively correlated returns. Depositing in the two banks allows agents to reduce their risk through diversification. We consider two market structures: direct finance, where investors deposit directly into the banks, and MMF intermediation, where the relationship between investors and banks is intermediated through MMFs.[3]

In the model, bank bankruptcy may arise when a fraction of investors unexpectedly withdraw their funds. The withdrawal occurs either because investors have received a liquidity shock, or because they have received a perfectly informative (negative) signal on the return of the investment of one of the two banks. Under direct finance, unexpected withdrawals cause bank bankruptcy only if the amount withdrawn is large enough to force the bank into liquidation. In contrast, with MMF intermediation, when a fraction of investors unexpectedly redeem from the MMF, their actions

[2]See, for instance, Dudley (2012) and Geithner (2012).

[3]MMFs lend to banks mostly through unsecured commercial paper and other short-term investments (see Table 1). Nevertheless, our model captures the essential economic feature of short-term debt rollover through MMFs' decision to either keep or withdraw the money from the banks.

represent a (noisy) signal on the state of the world for the MMF. If this signal is strong enough, the MMF will run the bank, withdrawing all its funds and causing bankruptcy even if the fraction of the unexpected redemptions was small enough that bankruptcy would not have occurred under direct finance.

The instability of MMF intermediation stems from the fact that the negative information content of an unexpected redemption from an intermediary such as an MMF amplifies the effect of the redemption itself. Because of this, an economy intermediated by MMFs is more unstable than a direct-finance structure.

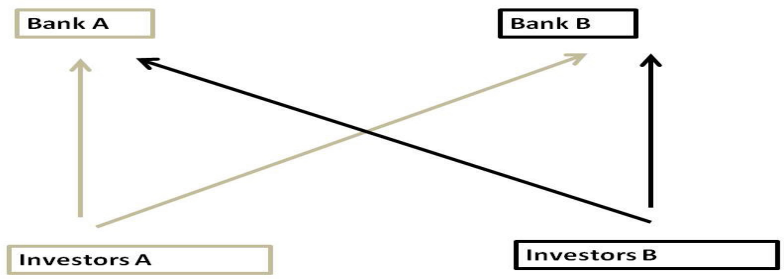
The amplification mechanism stems from the fact that MMFs themselves are subject to run-like redemptions because they offer investors demandable liabilities in order to satisfy their liquidity needs. When an MMF experiences large unexpected redemptions, it runs the bank to protect all its investors, and not just those initiating the redemptions. Because of the bank's fixed promise, the MMF, receiving negative information on the bank's assets, obtains a higher payoff for its investors if it runs than if it does not. Since the withdrawals of funds from the investors may be due to liquidity as opposed to informative reasons, bank bankruptcy may cause inefficient liquidation and a reduction in welfare.

The amplification result mentioned above may be more general than the case of MMF to which we apply it in this paper: a delegated monitor learns from the behavior of the agents it represents and by using this information may amplify the effect of their actions. This amplification may be inefficient if the delegated monitor takes value-destructing actions to increase the welfare of the agents it represents (for instance, in our model, the MMF forces the liquidation of banks' assets to increase the consumption of its shareholders, although this destroys value in the banks).

Note that in our economy, the run on the banking system does not occur because of agents' failure to coordinate as in DD. Here, instead, banks' fragility stems from stochastic fluctuations of the economy, which can be due to a liquidity shock as in Allen and Gale (2000); to the release of information on banks' asset returns as in Jacklin and Bhattacharya (1988); or to a combination of both as in Chari and Jagannathan (1988) and in our model. This is consistent with the features of the MMF industry. Indeed, the MMF run that started when the Primary Reserve Fund, which had invested in Lehman paper, broke the buck in September 2008, fits more the definition of a run caused by the sudden release of information in the market than an instance of self-fulfilling panic unrelated to economic fundamentals. Similarly, the "quiet run" by U.S. MMFs on European banks in the summer of 2012 was due to concern by U.S. investors on the assets held by European institutions (see Chernenko and Sunderam, 2013).

Section 2 describes our model and characterizes the economic function of MMF intermediation. Section 3 studies the effect of an unexpected withdrawal of funds from the financial system. Section 4 shows that an MMF-intermediated financial system is more fragile than one with direct finance. Section 5 concludes. The proofs are in the appendix.

Figure 2: The economy with direct finance.



2 The Model

2.1 Technology and Preferences

We describe our economy first with direct finance and then with MMF intermediation. There are two regions, A and B. In each region, there is a continuum of (wholesale or institutional)[4] investors of mass M, which can be interpreted as uninsured wholesale investors,[5] for a total population $2M$. Each investor is endowed with one unit of unique good. In each of the two regions there is one bank, Bank A and Bank B. The structure of the economy is depicted in Figure 2.

There are three dates, 0, 1, and 2, and a unique good that can be consumed, stored, or invested. Everyone in the economy can use storage, which returns one unit of the good at date $t+1$, for each unit invested at date t, $t = 0, 1$. In contrast, the investment technology is available only to banks. We consider an economy where the returns of the investments of two banks can be either high or low and are perfectly negatively correlated. We assume this in order to maximize the gains from diversification and simplify the model. The returns of the investment technology of the two banks per unit invested are as follows:

	Return Bank A	Return Bank B
Probability 1/2	R^H	R^L
Probability 1/2	R^L	R^H

with $R^H > 1 > R^L$. Since for each bank the probability of the investment technology yielding a

[4]We focus on the behavior of institutional investors (who lack insurance both when they invest in MMF and, largely, when they deposit in banks) since, as shown in Figure 1, they were the main drivers of MMF fragility during the recent crisis.

[5]Agents supplying funds to banks are normally referred to as depositors, who deposit or withdraw their funds. In contrast, agents supplying funds to MMFs are normally referred to as investors, who purchase or redeem shares of the MMF. In order not to saddle the reader, from now on we will use the term "investors," who may "supply" or "withdraw" their funds from the MMF or from the bank in reference to both direct finance and the MMF-intermediated economy.

high or a low return is $\frac{1}{2}$, the net present value of a unit of investment is the same for the two banks. As a result, it is optimal to supply an equal amount of funds to both banks at date 0. Investment can also be liquidated at date 1, in which case it returns $0 \leq r \leq R^L$ per unit invested.[6]

In each of the two regions, investors are subject to preference shocks: with probability π investors must consume at date 1 ("impatient" investors), and with probability $1-\pi$ they must consume at date 2 ("patient" investors). The realization of the shock to their preferences at date 1 is private information. For simplicity's sake, we assume that investors have logarithmic utility function, so that their expected utility is

$$\pi \log(c_1) + (1-\pi)\log(c_2),$$

where c_1 and c_2 denote date-1 and date-2 consumption, respectively. From the law of large numbers, a fraction π of agents consume at date 1 and a fraction $(1-\pi)$ at date 2.

2.2 The Optimal Contract with Direct Finance

As is standard in this literature, banks are subject to a zero-profit condition and, under direct finance, choose the contract to maximize the expected utility of their investors.

To simplify notation, we express all quantity variables per unit supplied to the banking system. In particular, we denote by i the total investment per unit by the two banks. Moreover, we assume that

$$\frac{R^H + R^L}{2} > 1, \tag{1}$$

that is, the expected net present value of each bank's investment is positive. This condition, as we show in the appendix, guarantees that the optimal level of investment i is positive, since the risk of banks' long-term technologies can be completely diversified away.

The optimal contract and the optimal investment level are: $c_1 = 1$, $c_2^H = R^H$, $c_2^L = R^L$, and $i = 1-\pi$, where c_2^H and c_2^L represent date-2 consumption if the bank has a high and a low return, respectively.[7] The optimal contract implies that the banks store enough funds to satisfy withdrawals from impatient investors only and invest all the remaining funds in the long-term technology. Note that since banks have perfect negatively correlated returns, under the optimal contract, investors deposit an equal amount in each bank; the banks, in turn, invest a fraction $1-\pi$ in its long-run technology, thus allowing patient investors a deterministic return $\frac{c_2^H + c_2^L}{2} = \frac{R^H + R^L}{2}$.

The diversification opportunities that arise from investing in both banks may turn into a source of fragility if a bank is not viable on its own given the contracts that it offers. Formally, this occurs if patient investors do not want to withdraw funds from one bank only and wait in the other versus

[6] The assumption is natural if we interpret the banks as investing in financial assets at different maturities—such as a loan—that it tries to sell in an unmodeled market; because of asymmetric information, market participants may not want to purchase the loans at a price $r > R^L$.

[7] See the appendix for the derivation.

withdrawing from both banks. That is, if:

$$\log(0.5c_1 + 0.5c_1) > 0.5 \log(0.5c_1 + 0.5c_2^H) + 0.5 \log(0.5c_1 + 0.5c_2^L). \tag{2}$$

Given the optimal contract, condition (2) becomes:

$$\log(1) = 0 > 0.5 \log(0.5 + 0.5R^H) + 0.5 \log(0.5 + 0.5R^L),$$

which is satisfied as long as

$$R^H < \frac{3 - R^L}{1 + R^L}. \tag{3}$$

Intuitively, condition (3) requires that R^H cannot be greater than (or equal to) $\frac{3 - R^L}{1 + R^L}$ because, otherwise, each bank would be so profitable that the contract it offers can stand on its own. Note that condition (1) for an interior solution for i and the condition (3) that contracts are not viable separately establish a range for R^H :

$$2 - R^L < R^H < \frac{3 - R^L}{1 + R^L},$$

which has a solution for any R^L.[8]

2.3 MMF Intermediation

The structure of the economy with MMF intermediation is similar to the one under direct finance except that, in each region A and B, there is one MMF—MMF A and MMF B—which channels the funds of its region to the two banks. Each MMF maximizes the expected utility of its investors by investing in banks' deposits (recall that only banks can invest in the long-term technology) and/or into the storage technology. The structure of the economy with MMF intermediation is depicted in Figure 3.

The risk-diversification problem does not change when we introduce MMFs in the economy. As a result, under the optimal contract, the final consumption for impatient and patient investors is the same as with direct finance. It is easy to show that this can be accomplished as long as the contracts that the two banks offer to the MMFs are the same as those offered to the wholesale investors with direct finance. Analogously, the contracts that the MMFs offer to their investors simply aggregate the payouts from the two banks: the contract per unit invested that each MMF offers is $c_1^{MMF} = 1$ and $c_2^{MMF} = \frac{R^H + R^L}{2}$. That is, the MMFs offer their investors claims redeemable at par at date 1. Finally, MMFs must share all the funds they collect from their investors equally between the

[8] The equality:

$$2 - R^L = \frac{3 - R^L}{1 + R^L},$$

has two equal roots $R^L = 1$, and it is always satisfied for $R^L > 0$.

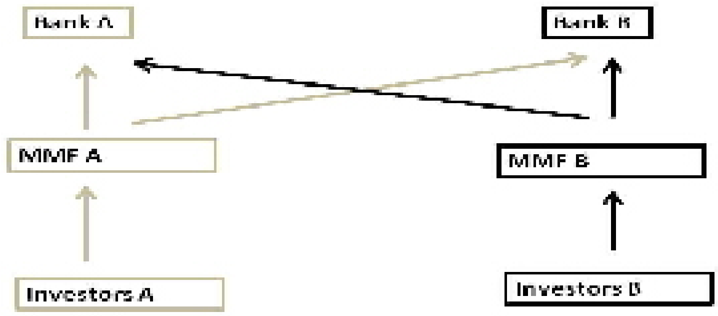


Figure 3: The economy with MMF intermediation.

two banks.

In order to understand the role of MMF intermediation in the economy, let us consider the case in which banks must be monitored or screened otherwise, their return is zero at date 2. The need to monitor the banks could arise from the fact that the opacity of bank loans allows them to underreport the return to the long-term technology or offers scope for moral hazard to bank managers. It is well known that since monitoring has a fixed-cost dimension, the duplication of monitoring costs that direct finance entails may be reduced when funds are intermediated through a delegated monitor (Diamond 1984).

For instance, let us assume that the cost to monitor each bank by a depositor or by an MMF is ε. Unlike banks, MMFs do not need to be monitored because they invest in deposit contracts offered by banks, which are less opaque than loans. Of course this is an extreme assumption: it can be thought of as the result of the normalization of the higher cost to monitoring banks versus MMFs, which is generally the case since MMFs invest in fixed-income securities (see Table 1).

With direct finance, in each region, depositors, who have mass M, have funds M to deposit, and therefore spend $(2\varepsilon)(M)$ in monitoring costs. As a result, the amount of deposits coming from the investors of each region is $M-(2\varepsilon)(M)=(1-2\varepsilon)(M)$. It is immediate to show that depositors will deposit $\frac{(1-2\varepsilon)(M)}{2}$ in each bank to maximize gain from diversification.

With MMF-intermediated finance, each MMF collects M from the depositors of its region only. Similarly to depositors, each MMF must monitor the banks to prevent them from under-reporting the return to the long-term technology and or to prevent moral hazard by banks managers. Hence each MMF spends 2ε to monitor the banks. This cost is passed onto the depositors. Therefore, with MMF intermediation, the funds that depositors of each region deposit in their MMF and that the MMF invests in the banks are $M-2\varepsilon$, which is greater than $(1-2\varepsilon)M$ as long as $M>1$. Hence, MMF intermediation is potentially valuable in saving monitoring costs to depositors, and as a result, in increasing their level of consumption, while at the same time allowing them to enjoy the gains from diversification.[9]

Since monitoring costs are higher under direct finance than under MMF intermediation, the aggregate amount of funds invested in the banking system will be lower. In the rest of the paper, however, we disregard this and carry out our analysis per unit deposited in each bank. None of our results is affected by this, since as we will show later, the occurrence of a run depends on the proportion of funds (and not by the amount of funds) unexpectedly withdrawn early by investors.

3 An Unexpected Withdrawal of Funds

3.1 Information Arrival

The fragility of an MMF-intermediated system can be captured by considering the effect of an unexpected withdrawal of funds in the economy with direct finance and in that with MMF intermediation.

Let us assume that at date 1 some patient investors unexpectedly withdraw their funds. Similarly to Chari and Jagannathan (1988), the unexpected withdrawal can happen either because of information reasons or because of a shock to agents' preferences. That is, investors unexpectedly withdraw early either because they have received a liquidity shock, i.e., some previously patient investors become impatient and must consume at date 1, or because they have received a perfectly informative signal that the return of the investment of the bank in their region is R_L. In either case the withdrawal stems from a shock to the fundamentals of the economy. Remember that wholesale investors' deposits and MMF deposits into banks are (largely) not covered by deposit insurance in the U.S.. Because of this, both investors and MMFs have an incentive to react to negative news about a bank's investment.

This withdrawal, which is unexpected at date 0 and hence is in excess of the liquidity available at date 1, has a different impact on the stability of the system under direct finance and under MMF-intermediated finance.

Under direct finance, the fraction of funds withdrawn in excess of π may be sufficiently low so as not to push the bank into insolvency and therefore not to alter the equilibrium described above. However, when investors unexpectedly redeem from the MMF, their actions represent a noisy signal on the state of the world for the MMF to interpret. From the size of the unexpected redemptions in excess of π, the MMF will update its prior belief on the return of the long-term investment of the bank in its own region, and it may run that bank by pulling all its funds away, thereby pushing it into bankruptcy.

More formally, we assume that a positive measure of patient investors q from region A unexpectedly withdraw their funds at date 1 from Bank A or, under MMF intermediation, from the MMF in region A.[10] This assumption is in the spirit of Allen and Gale (2000), who consider the

[9] Note also that diversification can be achieved through the interbank market; it is easy to show, however, that MMF intermediation saves on monitoring costs also with respect to interbank finance.

[10] Of course, since everything is symmetrical, nothing would change if the unexpected withdrawal of funds occurred

realization of an additional state of nature that was assigned a probability zero at date 0.[11]

As a result of the shock q, the total amount of withdrawals at date 1 from region A's investors is $\pi + (1-\pi)q$. Note that, since we assumed that condition (3) holds, that is, that Bank B's contract is not viable on its own, agents receiving negative information about Bank A will also withdraw from Bank B in the direct-finance case.[12]

In a DD economy, a run occurs because of agents' failure to coordinate. Here, instead, similarly to Jacklin and Bhattacharya (1988), Chari and Jagannathan (1988), and Allen and Gale (2000), the fragility of the financial system stems from stochastic fluctuations of the economy. In other words, we focus on "essential" bank runs, which cannot be ruled out by assuming that agents are able to coordinate. In the remainder of the paper, we follow the approach used by Allen and Gale (2000): we assume that agents are able to coordinate on the non-run superior equilibrium when it exists; and we study whether this equilibrium is more resilient to shocks under direct finance or under MMF-intermediation.

3.2 Information Aggregation and Updating

As we mention before, the unexpected withdrawal of funds can stem from a liquidity shock or from the release of negative information on Bank A's assets. Similarly to Chari and Jagannathan (1988), we assume that the probability that the unexpected withdrawal $(1-\pi)q$ is informative is increasing in q:

$$Pr(\{\text{Shock due to informational reason}\}) = f(q), \qquad f'(q) > 0$$

that is, the higher the fraction of withdrawals in excess of π, the more likely it is that it happens for information reasons. This is consistent with the idea that when there is a release of negative information to the market, we observe large withdrawal of funds.

Note that in order to keep the algebra simple, from now on we will consider the case

$$f(q) = q$$

that is, the probabilities that the unexpected withdrawal is informative or that it is due to a preference shock are q and $(1-q)$, respectively.[13] All the results we present, however, hold for any increasing function $f(q)$.

Under MMF intermediation, MMF A sees the unexpected withdrawal of funds by its investors

in region B.

[11] More recently, Gennaioli et. al (2012) argued that investors may not take into consideration certain highly improbable risks, such as the probability that the share price of a money market mutual fund may fall below 1.

[12] Note that if patient investors are negatively informed they will always find it convenient to withdraw because $c_1 > c_2^L$ (as we explain below, even if Bank A goes into bankruptcy, informed patient investors will always be able to recoup c_1).

[13] In an addendum available on request from the authors, we show that $f(q) = q$ can be derived from a simple informational structure.

and interprets this as (imperfect) bad news on the return of the assets of Bank A. In particular, after observing the unexpected withdrawal $(1-\pi)q$, the MMF A updates the joint probabilities that Bank A and Bank B have a high or a low return in the following manner:[14]

$$\begin{aligned} Pr(R_A^H, R_B^H|q) &= 0(q) + 0(1-q) = 0, \\ Pr(R_A^H, R_B^L|q) &= 0(q) + 0.5(1-q) = 0.5(1-q), \\ Pr(R_A^L, R_B^L|q) &= 0.5(q) + 0(1-q) = 0.5q, \\ Pr(R_A^L, R_B^H|q) &= 0.5(q) + 0.5(1-q) = 0.5. \end{aligned}$$

Note that when the shock is informative, the release of information is not about the state of nature; rather, it is about the return of one of the two banks (Bank A).[15] In other words, the probability of the return of Bank B being high or low is not affected by the arrival of negative information on Bank A. In fact, after observing $(1-\pi)q$, the conditional probability of Bank B being good or bad remains

$$Pr(R_B^L|q) = Pr(R_B^H|q) = 0.5.$$

Nevertheless, since the contract that Bank B offers is not viable on its own, because of condition (3), the destruction of diversification opportunity stemming from the release of information on Bank A may also send Bank B into bankruptcy, an issue that we will analyze below.

The fact that unexpected withdrawal is an imperfect signal on the bank long-term investment generates "confounding" as in Chari and Jagannathan (1988). If withdrawals were always due to the release of information, any realization of q, however small, would generate the collapse of Bank A and also the collapse of Bank B because of assumption (3).

Finally, we assume that the MMF does not learn the realization of the banks' asset return during its regular activity of monitoring. As will be apparent from the rest of the paper, should we remove this assumption, the fragility of an MMF-intermediated system would be even greater.

3.3 Bankruptcy

The excess withdrawal of funds matters because it may cause bank bankruptcy. To study its impact on the banking system, we need to make some assumptions on how the banks' assets are split in case of bankruptcy. In particular we assume that:

- Banks abide by the sequential service constraint when facing withdrawals at date 1, both under direct finance and in the MMF-intermediated economy.

- Patient investors withdrawing their funds early do so at the beginning of the queue. This captures the notion that since they are potentially informed about bank asset returns, they are

[14] In order to properly define the probabilities conditional to the unexpected excess withdrawal q, one can think of the unexpected excess withdrawal as being an event with a low enough probability that the optimal contracts (under both structures) are arbitrarily close to those described in the previous section.

[15] In other words, the zero-probability event can be thought of as a change in the returns to the long-run technologies in the two states of nature, which become (R_A^L, R_B^L) and (R_A^L, R_B^H).

able to jump ahead of the line.[16] The assumption reflects the fact that institutional investors are prone to run in a crisis (See Figure 1).

- Analogously and for the same reasons, if one MMF makes unexpected withdrawals, it is first in the queue with respect to the other MMF.[17]

Bankruptcy in either market structures is result of an essential bank run, that is, a run due to either the release of private information or to a shock to investors' preferences. It is not the result of a sunspot unrelated to economic fundamentals (e.g., a wave of pessimism), but stems from the unexpected withdrawal of funds $(1-\pi)q$ by patient investors and, in the MMF-intermediated economy, from the information that such a withdrawal conveys to the MMF.

We now contrast the effect of the unexpected withdrawals of funds $(1-\pi)\,q$, under direct finance and in a MMF-intermediated economy, and we study how it may cause bank runs in the two economies.

3.3.1 Bankruptcy with Direct Finance

In the case of direct finance, the unexpected withdrawal of funds from Bank A will push it into bankruptcy if the proportion q of patient investors who withdraw their funds at date 1 is such that:

$$(\pi + \frac{(1-\pi)}{2}q)c_1 > 1 - i + ri. \tag{4}$$

That is, bankruptcy will occur when the bank's date-1 liabilities, per unit deposited in the bank, (LHS of 4) exceed its date-1 assets (RHS).[18] Given the optimal contract described above, condition (4) becomes:

$$\pi + \frac{(1-\pi)}{2}q > \pi + r(1-\pi).$$

Hence, the bank goes bankrupt if and only if $q > 2r$. Note that since $q \in (0,1)$, a necessary condition for bankruptcy to occur under direct finance is

$$r < \frac{1}{2}. \tag{5}$$

From now on, we concentrate on realizations of q such that the bank does not go bankrupt with direct finance, and we show that the same realizations of q may instead trigger bankruptcy under MMF intermediation. That is, we assume:

$$q \leq 2r. \tag{6}$$

[16] Note that liquidity withdrawers will also try to jump ahead of the queue as they are aware that the bank/MMF may not be able to serve latecomers if there are excess withdrawals at date 1.

[17] These assumptions allow us to characterize the equilibrium in the economy in the simplest possible way. As will be clear, however, the fragility of an MMF-intermediated economy does not stem from the particular bankruptcy assumption that we adopted, but from the ability of MMFs to aggregate information among their investors.

[18] Observe that only $\frac{(1-\pi)}{2}q$ patient investors withdraw from Bank A since the other $\frac{(1-\pi)}{2}$ patient investors have deposited in Bank B.

Let us make two observations. First, since the proportion $\frac{(1-\pi)q}{2}$ of patient investors who unexpectedly withdraw at date 1 are early in the queue, they are able to withdraw $c_1 = 1$ as long as

$$\frac{(1-\pi)q}{2} \leq \pi + r(1-\pi), \tag{7}$$

where the LHS of (7) is the amount of funds withdrawn at date 1 by the patient investors, and the RHS are the bank's assets at date 1.[19] Obviously, as long as the level of withdrawals is such that the bank is not pushed into bankruptcy, that is, as long as $q \leq 2r$, the bank is always able to pay $c_1 = 1$ to the patient investors withdrawing their funds early. As a result, as mentioned above, a patient investor knowing that the return of Bank A's assets is low (because he received the information shock) finds it optimal to withdraw since $c_1 > c_2^L$.

Second, if bankruptcy occurs, impatient investors (from both Banks A and B) do not necessarily get c_1 since there are not enough resources in the bank, even after liquidating all the long-term assets. Moreover, even if bankruptcy does not occur, patient investors do not receive the optimal contract at date 2 since some (or all) of the long term investment has been liquidated.

As a final remark, observe that since $r \leq R^L$, then Bank A will never liquidate all its long-term assets when it observes an excess withdrawal of funds; it only liquidates whatever is needed to repay the proportion of patient investors who withdraw their funds early.

3.3.2 Bankruptcy with MMF Intermediation

Even if both the MMFs and the banks issue the same claims demandable at par at date 1, upon observing unexpected redemptions MMF A behaves differently from the bank in the direct-finance case. In fact, the MMF can withdraw its funds from Bank A at the contract $c_1 = 1$, while in direct finance, when Bank A liquidates early to meet the unexpected withdrawal, it does so at $r < 1$. Therefore, if after observing the unexpected redemptions, MMF A believes that Bank A's return is low with high enough probability, it withdraws all its funds, and not only what is needed to meet the unexpected redemptions. This amplification mechanism makes the MMF-intermediated structure more unstable than direct finance.

Of course, the fact that MMF A withdraws all its funds from Bank A does not necessarily imply that Bank A is bankrupt, which only happens when:

$$(\pi + \frac{(1-\pi)}{2})c_1 > 1 - i + ri,$$

which, given the optimal contract, becomes:

[19] The condition (7), when computed for the highest possible level of withdrawal, i.e., for $q = 1$, becomes:

$$(1-\pi) < 2(\pi + r(1-\pi)),$$

which is always true for $\pi > \frac{1}{3}$.

$$\pi + \frac{(1-\pi)}{2} > \pi + r(1-\pi),$$
$$\text{or} \quad r < \frac{1}{2}.$$

Note that $r < \frac{1}{2}$ is the same as condition (5), which makes bankruptcy possible in direct finance for a high enough realization of q.

3.4 The MMF Reaction to an Unexpected Withdrawals of Funds

We now investigate the continuation equilibrium of MMF A after observing an unexpected withdrawal of funds. MMF A will be able to offer c_1 to all its investors withdrawing their funds at date 1 as long as:

$$(\pi + (1-\pi)q)\, c_1 \leq 2(\pi + r(1-\pi)), \tag{8}$$

where the LHS of (8) is the overall withdrawal of funds from MMF A, and the RHS is the value of the combined assets of both banks A and B at date 1. MMF A is able to offer c_1 because it has demandable claims on both banks and it free rides on the claims of the other MMF.

Since $c_1 = 1$ under the optimal contract, (8) becomes

$$(1-\pi)q \leq \pi + 2r(1-\pi). \tag{9}$$

Note that since we are only considering realizations of q such that the banking system does not go bankrupt under direct finance[20] (i.e., $q \leq 2r$), condition (9) becomes:

$$(1-\pi)2r \leq \pi + 2r(1-\pi),$$

which, as in the case of the analogous condition with direct finance, is always satisfied. As a result, in MMF A both impatient and patient investors redeeming early receive c_1. This allows us to study the reaction of the MMF upon observing the unexpected redemptions disregarding the welfare of the investors redeeming early.

In particular, MMF A must choose the proportion by which it meets the unexpected redemptions $(1-\pi)q$ by withdrawing funds from Bank A and from Bank B. These proportions, which we denote by λ and $(1-\lambda)$, are the results of the MMF re-optimization upon observing the unex-

[20] In contrast, for $q = 1$, the condition would be:

$$1 - \pi < \pi + 2r - 2r\pi$$
$$\pi > \left(\frac{1-2r}{2+2r}\right)$$

Note that, in this case, if the MMF withdraws all its funds early at the rate c_1 from Bank A and B, the banks will never go bankrupt. This is because all their combined assets $2(\pi + r(1-\pi))$ are equal to or greater than the MMF maximum withdrawal, which is equal to $\pi + (1-\pi)$.

pected withdrawal of funds. Since the MMF knows it is able to pay its investors redeeming early the amount $c_1 = 1$, the proportions λ and $(1-\lambda)$ are derived only by looking at the welfare of the remaining patient investors (i.e., $(1-\pi)(1-q)$). Note that although the excess withdrawal of funds occurs in region A (and, with probability q, it reflects bad information on Bank A's long-term investment), in general the MMF will decide to meet the unexpected redemptions by pulling funds from both banks. The reason is that although the expected return on Bank A assets has decreased (whereas that on Bank B assets has not), it may not be optimal to meet the unexpected redemptions exclusively from Bank A, as the two banks provide a hedge one against the other. The optimal λ is given by

$$\lambda = \max\left(\frac{(\frac{R^H}{R^L}-1)(1-q)+\frac{r(\frac{R^H}{R^L}+1)}{1-\pi}-q}{(2-q)(\frac{R^H}{R^L}-1)}, 1\right). \tag{10}$$

In the interest of space, we do not report the derivation of the optimal level of λ in the text of the paper, but describe it in the appendix. Note that, as shown in the appendix, if $q = 0$, then $\lambda = \frac{1}{2}$; that is, in the limit, when the unexpected withdrawal of funds does not contain any information on Bank A, it is met by withdrawing funds equally from both banks. Note also that if $q = 1$, then $\lambda = 1$; that is, if the unexpected withdrawal is so high that the MMF knows that the return on Bank A assets is low, it is met by withdrawing from Bank A only.

Finally, recall that the total amount withdrawn at date 1 by MMF A is $\pi+(1-\pi)q$. Since both banks have invested in the long-term asset a fraction $i = 1-\pi$ of each unit deposited, the overall liquidation of the long-term assets per unit deposited in the banking system is $\frac{(1-\pi)q}{r}$. Given the optimal fraction λ withdrawn by MMF A from Bank A, the withdrawal per unit of deposit from Bank A is $\lambda(\pi+(1-\pi)q)$ and that from Bank B is $(1-\lambda)(\pi+(1-\pi)q)$. Moreover, the amount of Bank A's assets liquidated at date 1 is $\lambda\frac{(1-\pi)q}{r}$; and the amount of Bank B's assets liquidated at date 1 is $(1-\lambda)\,\frac{(1-\pi)q}{r}$. Because of the liquidation of both banks' assets to meet the unexpected withdrawal of funds, the payoffs that the banks can afford to offer at date 2 will change; the payoffs offered by Bank A are:

$$\widehat{c}^{H}_{2,A} = \max(R^H(1-\lambda\frac{(1-\pi)q}{r}), 0), \tag{11}$$

$$\widehat{c}^{L}_{2,A} = \max(R^L(1-\lambda\frac{(1-\pi)q}{r}), 0), \tag{12}$$

and those offered by Bank B are:

$$\widehat{c}^{H}_{2,B} = \max(R^H(1-(1-\lambda)\frac{(1-\pi)q}{r}), 0), \tag{13}$$

$$\widehat{c}^{L}_{2,B} = \max(R^L(1-(1-\lambda)\frac{(1-\pi)q}{r}), 0). \tag{14}$$

4 The Fragility of MMF Intermediation

Having characterized the continuation equilibrium conditional on the unexpected withdrawals of funds at date 1, we now show that there are levels of withdrawals and redemptions such that there is no bank bankruptcy with direct finance, but bankruptcy occurs with MMF intermediation.

We are looking for a condition on q such that MMF A, after having received the unexpected redemptions, prefers to withdraw all its holdings from Bank A and trigger its liquidation,[21] as opposed to liquidating only the minimum from both banks to satisfy the unexpected redemptions and keep the rest in the banks.

Recall that MMF A maximizes the expected utility of the investors from region A only. The expected utility of the MMF A investors if the MMF decides to withdraw only $(1-\pi)q$, and not to force Bank A's liquidation, is:

$$EU^{\text{Non-Liquidation}} = \underbrace{(\pi + (1-\pi)q)\, u\,(c_1)}_{0} + (1 - (\pi + (1-\pi)q)) \cdot \tag{15}$$
$$\left[0.5(1-q)u(\frac{\widehat{c}^{H}_{2,A} + \widehat{c}^{L}_{2,B}}{2}) + 0.5u(\frac{\widehat{c}^{H}_{2,B} + \widehat{c}^{L}_{2,A}}{2}) + 0.5qu(\frac{\widehat{c}^{L}_{2,B} + \widehat{c}^{L}_{2,A}}{2}) \right],$$

where $\widehat{c}^{H}_{2,i}, \widehat{c}^{L}_{2,i}$ $i = A, B$ are the payouts of Banks A and B at date 2 after MMF A withdraws its funds, from equations (11), (12), (13), and (14). Note that since the MMF knows it can pay its investors redeeming at date 1 the amount $c_1 = 1$, the first term drops out from (15).

In contrast, the expected utility of MMF A investors' if the MMF decides to force Bank A into liquidation is:

$$EU^{\text{Liquidation}} = (\pi + (1-\pi)q)\, u\,(\widehat{c}_1) + \tag{16}$$
$$(1 - (\pi + (1-\pi)q)) \left[0.5u(\frac{c^{H}_{2} + \widehat{c}_1}{2}) + 0.5u(\frac{c^{L}_{2} + \widehat{c}_1}{2}) \right],$$

where

$$\widehat{c}_1 \equiv \min(\frac{\pi + r(1-\pi)}{\frac{\pi}{2} + \frac{(1-\pi)}{2}}, 1) = \min(2\left[\pi + r(1-\pi)\right], 1), \tag{17}$$

is how much the MMF obtains if it forces Bank A to liquidate all its assets at date 1. Note that (17) is the level of consumption that MMF A is able to pay to those withdrawing at date 1: $\pi + r(1-\pi)$ are Bank A's assets at date 1 if it liquidates all its long-term investment; in the denominator $\pi/2$ is the measure of impatient investors that MMF A satisfies with its deposits in Bank A, and $(1-\pi)/2$ is the measure of patient investors that MMF A satisfies with its deposits in Bank A. Note that if the MMF A forces Bank A into bankruptcy, it will not necessarily be able to pay all its investors $c_1 = 1$ and, therefore, generally, $\widehat{c}_1 < c_1$.[22]

[21] Since we assumed that $r < \frac{1}{2}$, Bank A goes bankrupt if MMF A withdraws all its assets (see Section 3.2.2).

[22] MMF B never withdraws in excess of π since it has no information on the return of the long-term asset.

At date 1, MMF A withdraws from Bank A all its holdings (as opposed to only the unexpected withdrawal $(1-\pi)q$) if the expected utility of its investors upon total withdrawal from Bank A ($EU^{\text{Liquidation}}$) is greater than the expected utility upon keeping funds in Bank A ($EU^{\text{Non-Liquidation}}$).[23] Thus, we can establish a level of q such that MMF A, after having observed the unexpected withdrawal $(1-\pi)q$, prefers to withdraw all its holdings from Bank A and trigger its liquidation (if $r < \frac{1}{2}$), as opposed to liquidating only the minimum from both banks to satisfy the unexpected withdrawal of funds.

The following proposition compares the stability of MMF intermediation and direct finance.

Proposition 1: For any values of R^H and R^L satisfying condition (1) and for $\pi \geq \frac{0.5-r}{1-r}$, there is an interval of realizations of q for which bankruptcy occurs with MMF intermediation, but not with direct finance.

The proof is in the appendix.

Proposition 1 establishes that a MMF-intermediated system is more fragile than direct finance. That is, there is a set of realizations of q such that a MMF-intermediated system collapses whereas an economy under direct finance would not.

This happens because MMFs have demandable claims on the banks and give their investors demandable liabilities in order to satisfy their liquidity needs, which, in turn, makes MMFs liabilities subject to run-like redemptions. The unexpected early redemptions may contain negative information on Bank A's assets, which may make it optimal for the MMF to run the bank. Note that the MMF decides to run the bank in order to protect all its investors, and not just those unexpectedly withdrawing early. Indeed, given the bank's fixed promise at date 1, the MMF obtains a higher payoff for its investors if it runs than if it does not. The reason is that MMF A's demandable claims on both banks allow it to free ride on the claims of the other MMF in both banks to satisfy the excess redemptions from depositors in region A. Furthermore since the unexpected early redemptions may be due to liquidity as opposed to informative reasons, bank bankruptcy under MMF intermediation may cause inefficient liquidation of the long-term investment.

Observe that the higher instability of a MMF-intermediated economy is not due to the lack of insurance for MMF investments. Indeed, we are comparing MMF intermediation with a direct finance structure in which all depositors are wholesale investors and, therefore, uninsured. It is immediate to show that the higher fragility of MMF-intermediation would survive in an economy in which banks received some of their deposits from retail insured investors. Note also that, in this economy, MMF intermediation generates financial fragility even though MMFs maximize the welfare of their investors. The instability does not arise from any friction (such as agency problems), but simply from the ability of MMFs to aggregate private information and use it to the benefit of its investors.

[23] For simplicity's sake, we assume in the proof that MMF A prefers to liquidate when the inequality holds weakly.

As mentioned above, bankruptcy in an MMF-intermediated economy occurs because there is a threshold of q, such that any realization of q greater than that leads MMF A to withdraw its funds from Bank A and, as a result, Bank A collapses. In the following proposition we provide an upper bound for such a threshold.

Proposition 2: For any values of R^H and R^L satisfying condition (1) and for $\pi \geq \frac{0.5-r}{1-r}$, let us define by $\hat{q}$ the threshold such that any realization of q greater than $\hat{q}$ leads to bankruptcy under MMF intermediation. We can show that

$$\hat{q} \leq \tilde{q} \equiv \frac{\log \frac{(R^H+1)(R^L+1)}{(R^H+R^L)^2}}{\log \frac{2}{\frac{R^H}{R^L}+1}}.$$

The proof is in the appendix, where we also show that:

$$\frac{\partial \tilde{q}}{\partial R^L} > 0, \; \frac{\partial \tilde{q}}{\partial R^H} > 0,$$

that is, $\tilde{q}$ increases with both R^L and R^H. When R^L is higher, the negative information conveyed by the excess withdrawal is less important; therefore, a higher level of withdrawal is needed for the MMF to cause the bank's bankruptcy. Similarly, when R^H increases, the higher return in the high state of the world increases the expected utility from not withdrawing from the bank; as a result, a higher excess withdrawal is needed for the MMF to cause bankruptcy.

Note that the contract offered by Bank B may not be viable on its own, given the conditional information that the MMF has on Bank B returns (see Section 3.1). In particular, it is easy to show that if assumption (3) holds, bankruptcy of Bank A triggers bankruptcy of Bank B. The bankruptcy of Bank B stems from the loss of diversification opportunity once Bank A goes bankrupt. In other words, the presence of two banks offers wholesale investors hedging opportunities; however, when a bank is liquidated, this hedging opportunity vanishes, which may cause the other bank to be liquidated too. This channel of banking contagion due to loss of diversification has been studied by Cipriani, Martin and Parigi (2013).

4.1 Fragility: an Example

In this Section, we provide a numerical example of an economy in which an unexpected redemption $(1-\pi)q$ causes bankruptcy under MMF intermediation, but not under direct finance. The bankruptcy of one Bank under MMF intermediation causes the other bank to go bankrupt too (contagion).

Consider an economy where $R^L = 0.25$ and $R^H = 2$. Assume that the liquidation value $r = 0.249 < R^L$. Therefore, since $r < 1/2$, by condition (5) bankruptcy is possible under direct finance. Also assume that the fraction of impatient π equals 0.8. With a logarithmic utility function, the optimal contract offered by the banks is

$$c_1 = 1 \quad c_2^H = 2 \quad c_2^L = 0.25.$$

Moreover, since $R^H < \frac{3-R^L}{(1+R^L)} = 2.2$, by condition (3) the optimal contracts offered by Banks A and B are not viable separately.

Consider a level of excess withdrawal $q = 0.35$, which implies that the unexpected withdrawal is $(1-\pi)q = 0.2(0.35) = 0.07$. Since, $q < 2r = 0.498$, by condition (5) such level of unexpected withdrawal does not cause bankruptcy in an economy with direct finance.

What happens instead with MMF intermediation? Upon observing the unexpected redemption, the MMF A will update upwards the probability that return of Bank A long-term investment yields R^L.

Given this information, if MMF A decides to withdraw only what is needed to meet the unexpected withdrawal of funds from its investors $(1-\pi)q$, it would withdraw only from Bank A and nothing from Bank B (i.e., the optimal λ equals 1).

Because of this, upon observing the withdrawal, Bank A would only be able to offer

$$\widehat{c}_2^H = R^H(1 - \frac{(1-\pi)q}{r}) = 1.44 \quad \text{and} \quad \widehat{c}_2^L = R^L(1 - \frac{(1-\pi)q}{r}) = 0.18,$$

whereas Bank B would not have to modify its payouts. As a result, from (15) the expected utility of MMF A investors would be -0.10.

What would happen if MMF A decides to pull all its funds from Bank A? Since π is relatively high (0.8), MMF A would be able to pay $\widehat{c}_1 = c_1 = 1$ to all investors withdrawing early. As a result, from (16) the expected utility of MMF A investors would be -0.004, higher than if MMF A decides not to pull out all its funds from Bank A. Finally it is easy to verify that given the decision to pull out from Bank A, MMF A would also find it convenient to withdraw its funds from Bank B. That is, we found a level of unexpected withdrawals such that with direct finance there is no bankruptcy, whereas with MMF intermediation both Bank A and Bank B go bankrupt.

5 Conclusion

In this paper we show that MMF intermediation allows uninsured investors to limit their exposure to a single banking institution and reap the gains from diversification. However, a banking system intermediated through MMFs is more unstable than one in which investors interact directly with banks because MMFs have demandable claims on the banks and are themselves subject to runs from their own investors. The mechanism through which instability arises is the release of private information on bank assets, which is aggregated by MMFs and lead them to withdraw en masse from a bank.

Our results provide a theoretical underpinning for the idea that an MMF-intermediated financial system can be particularly fragile. This fragility has been the main driver of the recent regulatory efforts of the industry by the SEC and FSOC. Over the recent decades, banks have relied more and more on financial intermediaries, such as money markets funds, to finance their investment. Our results, suggest that this trend, while providing investors with valuable diversification opportunities, may increase the instability of the banking system.

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7 Appendix

7.1 The Optimal Contract with Direct Finance

We derive optimal contract as the solution to the planner problem in an economy with direct finance. Note that although each bank A and B offers potentially different contracts

$$c_1^A, c_2^{A,H}, c_2^{A,L}, c_1^B, c_2^{B,H}, c_2^{B,L}$$

it is trivial to show that, under the optimal contract, bank contracts would be identical and investors would invest an equal amount in each bank. Therefore, to simplify notation, we denote the optimal contract by c_1, c_2^H, c_2^L.

Denote with s storage to date 2 per unit of deposit. The optimal contract is the solution to the following optimization problem:

$$\begin{aligned}
& Max\ \pi u(c_1) + (1-\pi)[u(\frac{c_2^H + c_2^L}{2})], \\
& \text{w.r.t. } c_1, c_2^H, c_2^L, i, s \\
& \text{s.t.} \\
\text{date 1} \quad : \quad & \pi c_1 = 1 - i - s, \\
\text{date 2: } (1-\pi)c_2^H = & iR^H + s, \\
\text{date 2: } (1-\pi)c_2^L = & iR^L + s, \\
i + s \leq 1, \quad -i \leq 0, \quad & -s \leq 0,
\end{aligned}$$

where, recall, the second utility term comes from the fact that by investing $\frac{1}{2}$ in each bank, and since banks have perfect negative correlation, patient investors obtain a deterministic return at date 2.

Substituting the equality constraint

$$\begin{aligned}
& Max\ \pi u(\frac{1-i-s}{\pi}) + (1-\pi)[u(\frac{i(R^H + R^L) + 2s}{2(1-\pi)})], \\
& \text{w.r.t. } i, s \\
& s.t \\
i + s \leq 1, -i \leq 0, \quad & -s \leq 0.
\end{aligned}$$

The FONCs are:

$$\begin{aligned}
-u\prime(c_1) + u'(\frac{c_2^H + c_2^L}{2})\frac{(R^H + R^L)}{2} - \lambda = 0 \\
-u\prime(c_1) + u'(\frac{c_2^H + c_2^L}{2})2 - \mu = 0. \\
\lambda i = 0, \mu s = 0, \\
\text{where } \lambda, \mu \geqslant 0
\end{aligned}$$

which, with the natural log utility function, becomes

$$\begin{aligned}
-\frac{1}{c_1} + \frac{1}{c_2^H + c_2^L}\left(R^H + R^L\right) + \lambda = 0 \\
-\frac{1}{c_1} + \frac{1}{c_2^H + c_2^L}2 + \mu = 0. \\
\lambda i = 0, \mu s = 0, \\
\text{where } \lambda, \mu \geqslant 0.
\end{aligned}$$

There are three cases:

Case 1), with $s = 0, i > 0$.

Then the multiplier $\lambda = 0$ and the first constraint,

$$-\frac{1}{c_1} + \frac{1}{c_2^H + c_2^L}\left(R^H + R^L\right) = 0.$$

The solution to the optimization problem is interior and $i = 1 - \pi$.

The second constraint,

$$-\frac{1}{c_1} + \frac{1}{c_2^H + c_2^L} 2 + \mu = 0,$$

where $\mu > 0$. For the constraint to be satisfied, it must be the case that

$$-\frac{1}{c_1} + \frac{2}{c_2^H + c_2^L} < 0,$$

that is,

$$-\frac{1}{1} + \frac{2}{(R^H + R^L)} < 0,$$

or

$$R^H + R^L > 2,$$

which is the condition (1) for an interior solution.

Case 2), with $s > 0$, $i = 0$.

Then, the multiplier $\mu = 0$, and the second constraint,

$$-\frac{1}{c_1} + \frac{2}{c_2^H + c_2^L} = 0.$$

For the constraint to be satisfied, it must be the case that

$$-\frac{\pi}{1-s} + \frac{(1-\pi)}{s} = 0, \text{ that is,}$$

$$s = (1 - \pi).$$

The first constraint,

$$-\frac{1}{c_1} + \frac{1}{c_2^H + c_2^L}(R^H + R^L) + \lambda = 0,$$

which since $\lambda \geq 0$ implies

$$-\frac{\pi}{1-s} + \frac{(1-\pi)}{s}\frac{(R^H + R^L)}{2} \leqslant 0,$$

that is,

$$R^H + R^L \leqslant 2,$$

in which case the banks' net present value is smaller than zero, and the optimal contract implies zero investment in the long technology.

Case 3), with $s > 0, i > 0$.

Then, both multipliers $\mu, \lambda = 0$, and the constraints become:

$$-\frac{1}{c_1} + \frac{1}{c_2^H + c_2^L} 2 = 0,$$

and

$$-\frac{1}{c_1} + \frac{1}{c_2^H + c_2^L}\left(R^H + R^L\right) = 0,$$

which can never be the case unless $R^H + R^L = 2$.

As mentioned in the text, we assumed that condition (1) holds, that is, $R^H + R^L > 2$, which implies $i > 0$, $s = 0$.

7.2 The Optimal Withdrawal by the MMF

Recall that MMF A chooses how much to withdraw from Banks A and B assuming that it can still obtain c_1 for all its investors redeeming early (that is, $\widehat{c}_1 = c_1$). This allows us to disregard the welfare of the investors redeeming early from the MMF A. As a result, the optimal withdrawal of MMF A from the two banks is the result of the following maximization problem:

$$
\begin{aligned}
&Max_{w.r.t.\ \widetilde{\lambda}}\ 0.5(1-q)u(\frac{c_2^{H,A}+c_2^{L,B}}{2}) + 0.5u(\frac{c_2^{H,B}+c_2^{L,A}}{2}) \qquad (18)\\
&+0.5qu(\frac{c_2^{L,A}+c_2^{L,B}}{2})\\
&\text{s.t.}\\
&c_2^{H,A} = \max(R^H(1-\widetilde{\lambda}\frac{(1-\pi)q}{r}),0),\\
&c_2^{L,A} = \max(R^L(1-\widetilde{\lambda}\frac{(1-\pi)q}{r}),0),\\
&c_2^{H,B} = \max(R^H(1-(1-\widetilde{\lambda})\frac{(1-\pi)q}{r}),0),\\
&c_2^{L,B} = \max(R^L(1-(1-\widetilde{\lambda})\frac{(1-\pi)q}{r}),0),
\end{aligned}
$$

where $\widetilde{\lambda}$, and $(1-\widetilde{\lambda})$ represent the fraction of withdrawal that the $MMF\ A$ will do in Bank A and B respectively, and $c_2^{i,j}$ represents date-2 consumption if the returns are low or high, $i = L, H$, by bank $j = A, B$. Let us analyze the three terms in (18) we need to maximize separately:

Term 1:

$$
\begin{aligned}
&\frac{c_2^{H,A}+c_2^{L,B}}{2} = \frac{(R^H(1-\widetilde{\lambda}\frac{(1-\pi)q}{r})) + (R^L(1-(1-\widetilde{\lambda})\frac{(1-\pi)q}{r}))}{2} =\\
&\frac{R^L}{2}(\frac{R^H}{R^L}(1-\widetilde{\lambda}\frac{(1-\pi)q}{r}) + (1-(1-\widetilde{\lambda})\frac{(1-\pi)q}{r})) =\\
&\frac{R^L}{2}(\frac{R^H}{R^L}+1) - \frac{R^L}{2}\frac{(1-\pi)q}{r}(\widetilde{\lambda}(\frac{R^H}{R^L}-1)+1),
\end{aligned}
$$

which is decreasing in $\widetilde{\lambda}$.

Term 2:

$$
\begin{aligned}
\frac{c_2^{H,B}+c_2^{L,A}}{2} &= \frac{(R^H(1-(1-\widetilde{\lambda})\frac{(1-\pi)q}{r})) + (R^L(1-\widetilde{\lambda}\frac{(1-\pi)q}{r}))}{2},\\
&= \frac{R^L}{2}(\frac{R^H}{R^L}(1-(1-\widetilde{\lambda})\frac{(1-\pi)q}{r}) + (1-\widetilde{\lambda}\frac{(1-\pi)q}{r})),\\
&= \frac{R^L}{2}(\frac{R^H}{R^L}+1) - \frac{R^L}{2}\frac{(1-\pi)q}{r}((1-\widetilde{\lambda})(\frac{R^H}{R^L}-1)+1),
\end{aligned}
$$

which is increasing in $\widetilde{\lambda}$.

Term 3:

$$\begin{aligned}\frac{c_2^{L,B} + c_2^{L,A}}{2} &= \frac{(R^L(1-(1-\widetilde{\lambda})\frac{(1-\pi)q}{r})) + (R^L(1-\widetilde{\lambda}\frac{(1-\pi)q}{r}))}{2} \\ &= R^L - \frac{R^L}{2}\frac{(1-\pi)q}{r},\end{aligned}$$

which is independent from $\widetilde{\lambda}$. Therefore, the solution to the maximization of (18) is the solution to the maximization of the first two terms. That is,

$$\begin{aligned}&0.5(1-q)u(\frac{R^L}{2}(\frac{R^H}{R^L}+1) - \frac{R^L}{2}\frac{(1-\pi)q}{r}(\widetilde{\lambda}(\frac{R^H}{R^L}-1)+1)) + \\ &0.5u(\frac{R^L}{2}(\frac{R^H}{R^L}+1) - \frac{R^L}{2}\frac{(1-\pi)q}{r}((1-\widetilde{\lambda})(\frac{R^H}{R^L}-1)+1)),\end{aligned}$$

or

$$\begin{aligned}&(1-q)u(\frac{R}{2}(\frac{R^H}{R^L}+1) - \frac{R}{2}\frac{(1-\pi)q}{r}(\widetilde{\lambda}(\frac{R^H}{R^L}-1)+1)) + \\ &u(\frac{R}{2}(\frac{R^H}{R^L}+1) - \frac{R}{2}\frac{(1-\pi)q}{r}((1-\widetilde{\lambda})(\frac{R^H}{R^L}-1)+1)),\end{aligned}$$

which, with a logarithmic utility function, becomes:

$$\begin{aligned}&(1-q)\log(\frac{R}{2}(\frac{R^H}{R^L}+1) - \frac{R}{2}\frac{(1-\pi)q}{r}(\widetilde{\lambda}(\frac{R^H}{R^L}-1)+1)) + \\ &\log(\frac{R^L}{2}(\frac{R^H}{R^L}+1) - \frac{R^L}{2}\frac{(1-\pi)q}{r}((1-\widetilde{\lambda})(\frac{R^H}{R^L}-1)+1)),\end{aligned}$$

which is equivalent to

$$\begin{aligned}&(1-q)\log(\frac{R^H}{R^L}+1 - \frac{(1-\pi)q}{r}(\widetilde{\lambda}(\frac{R^H}{R^L}-1)+1)) + \\ &\log(\frac{R^H}{R^L}+1 - \frac{(1-\pi)q}{r}((1-\widetilde{\lambda})(\frac{R^H}{R^L}-1)+1)).\end{aligned}$$

The FONC of the maximization problem is:

$$\begin{aligned}&(1-q)\frac{-\frac{(1-\pi)q}{r}(\frac{R^H}{R^L}-1)}{\frac{R^H}{R^L}+1-\frac{(1-\pi)q}{r}(\widetilde{\lambda}(\frac{R^H}{R^L}-1)+1)} + \\ &\frac{\frac{(1-\pi)q}{r}(\frac{R^H}{R^L}-1)}{(\frac{R^H}{R^L}+1)-\frac{(1-\pi)q}{r}((1-\widetilde{\lambda})(\frac{R^H}{R^L}-1)+1)} = 0,\end{aligned}$$

which is equivalent to:

$$(1-q)\left(\frac{R^H}{R^L}+1-\frac{(1-\pi)q}{r}((1-\widetilde{\lambda})(\frac{R^H}{R^L}-1)+1)\right)+$$
$$\left(\frac{R^H}{R^L}+1-\frac{(1-\pi)q}{r}(\widetilde{\lambda}(\frac{R^H}{R^L}-1)+1)\right)=0.$$

Then:

$$-(1-q)\left(\frac{R^H}{R^L}+1-\frac{(1-\pi)q}{r}((1-\widetilde{\lambda})(\frac{R^H}{R^L}-1)+1)\right)+ \tag{19}$$
$$\left(\frac{R^H}{R^L}+1-\frac{(1-\pi)q}{r}(\widetilde{\lambda}(\frac{R^H}{R^L}-1)+1)\right)=0.$$

Let us denote $W=\frac{(1-\pi)q}{r}$, and observe that $W\leq\frac{(1-\pi)2r}{r}<2$. To simplify notation, denote $A=(\frac{R^H}{R^L}+1)$ and $D=(\frac{R^H}{R^L}-1)=A-2$, so that equation (19) becomes:

$$-(1-q)(A-W((1-\widetilde{\lambda})D+1))+(A-W(\widetilde{\lambda}D+1))=0.$$

That is,

$$\widetilde{\lambda}=\frac{WD(1-q)+qA-qW}{(2-q)DW}, \tag{20}$$
$$=\frac{W(\frac{R^H}{R^L}-1)(1-q)+q(\frac{R^H}{R^L}+1)-qW}{(2-q)(\frac{R^H}{R^L}-1)W}.$$

That is,

$$\widetilde{\lambda}=\frac{(\frac{R^H}{R^L}-1)(1-q)+\frac{r(\frac{R^H}{R^L}+1)}{(1-\pi)}-q}{(2-q)(\frac{R^H}{R^L}-1)}. \tag{21}$$

Note that, for simplicity's sake, we solved the maximization problem without imposing the condition that the proportion withdrawn from Bank A must be less than 1, and without explicitly considering the non-negativity of the payoff at date 2 that agents receive from both banks. Thus, the optimal level of withdrawal from Bank A is given by (10) that is

$$\lambda=\min\left(\frac{(\frac{R^H}{R^L}-1)(1-q)+\frac{r(\frac{R^H}{R^L}+1)}{1-\pi}-q}{(2-q)(\frac{R^H}{R^L}-1)},1\right).$$

Note that if $q=0$,

$$\lambda=\frac{W(\frac{R^H}{R^L}-1)(1-q)+q(\frac{R^H}{R^L}+1)-qW}{(2-q)(\frac{R^H}{R^L}-1)W}=\frac{W(\frac{R^H}{R^L}-1)}{2(\frac{R^H}{R^L}-1)W}=\frac{1}{2},$$

which means that if the unexpected redemption is low enough not to contain any information on Bank A, it will be met by withdrawing equally from both banks.

Note also that if $q = 1$, then

$$\lambda = \min\left(\frac{\frac{R^H}{R^L} + 1 - W}{(\frac{R^H}{R^L} - 1)W}, 1\right) = 1,$$

since $\frac{R^H}{R^L} + 1 < \frac{R^H}{R^L}W$ (recall that $W < 2$ and $\frac{R^H}{R^L} > 1$). This means that if the unexpected redemption is so high that the MMF knows that the return on Bank A assets is low, it will be met by withdrawing from Bank A only.

Finally, note that from (20),

$$\frac{d}{dq}\frac{WD(1-q) + qA - qW}{(2-q)DW} =$$
$$\frac{1}{DW(q-2)^2}(2A - 2W - DW) = \frac{1}{DW(q-2)^2}(2A - W(2+D)) =$$
$$\frac{1}{DW(q-2)^2}(2A - WA) = \frac{A}{DW(q-2)^2}(2 - W) > 0,$$

since $W < 2$. That is, the higher the level of unexpected redemptions q, the higher the proportion of funds withdrawn from Bank A as opposed to Bank B (since the probability that Bank A has a low return is higher).

7.3 Proof of Proposition 1

From the assumption $\pi > \frac{0.5-r}{1-r}$, we know from (17) that $\widehat{c}_1 = c_1 = 1$. For simplicity's sake, we assume in the proof that MMF prefers to liquidate when the inequality holds weakly.

The MMF run condition (15) $\leq$ (16) becomes:

$$\begin{aligned}
&(1 - (\pi + (1-\pi)q))\, 0.5 * \\
&\left[(1-q)\log(\frac{\widehat{c}^H_{2,A} + \widehat{c}^L_{2,B}}{2}) + \log(\frac{\widehat{c}^H_{2,B} + \widehat{c}^L_{2,A}}{2}) + q\log(\frac{\widehat{c}^L_{2,B} + \widehat{c}^L_{2,A}}{2})\right] \\
\leq\, &(1 - (\pi + (1-\pi)q))\, 0.5 \left[\log(\frac{c^H_2 + 1}{2}) + \log(\frac{c^L_2 + 1}{2})\right].
\end{aligned} \tag{22}$$

Since

$$\widehat{c}^H_{2,A}, \widehat{c}^H_{2,B} \leq c^H_2 = R^H$$
$$\widehat{c}^L_{2,A}, \widehat{c}^L_{2,B} \leq c^L_2 = R^L$$

and $(1-(\pi+(1-\pi)q)) \geq 0$ a fortiori condition (22) will be satisfied if:

$$(1-q)\log(\frac{R^H+R^L}{2})+\log(\frac{R^H+R^L}{2})+q\log(R^L) \leq \log(\frac{R^H+1}{2})+\log(\frac{R^L+1}{2}). \tag{23}$$

Hence the inequality (23) will hold iff:

$$\log(R^H+R^L)^{(2-q)}+\log(2R^L)^q \leq \log(R^H+1)+\log(R^L+1),$$

that is

$$(R^H+R^L)^{(2-q)}(2R^L)^q \leq (R^H+1)(R^L+1). \tag{24}$$

Assume that $r=\frac{1}{2}-\eta$ so that direct finance may lead to bankruptcy for q high enough. Consider $q=1-2\eta=2r$, so that for this realization of q there is no bankruptcy with direct finance (however small η is). Let us now show that bankruptcy will occur for MMF intermediation for η small enough.

Observe that for $q=1$ the inequality (24) becomes

$$(R^H+R^L)(2R^L) \leq (R^H+1)(R^L+1),$$

which is always satisfied for any values of R^H and R^L, since $R^L<1$. Thus, by continuity, there will be a value of η such that $q=1-2\eta$ has bankruptcy under MMF intermediation, but not under direct finance.

7.4 Proof of Proposition 2

From the proof of Proposition 1 we know that bankruptcy will occur if (24) is satisfied. By algebraic manipulation of (24) we obtain:

$$q \leq \tilde{q} \equiv \frac{\log\frac{(R^H+1)(R^L+1)}{(R^H+R^L)^2}}{\log\frac{2}{\frac{R^H}{R^L}+1}}. \tag{25}$$

Note that both the denominator and the numerator of the RHS of (25) are negative. The denominator is negative since $\frac{2}{\frac{R^H}{R^L}+1}<1$. It is easy to show that the numerator is also negative. To see that consider that

$$1<\frac{1}{2}\left(R^H+R^L\right),$$

by condition (1). Thus:

$$\frac{1}{2}\left(R^H+1\right)+\frac{1}{2}\left(R^L+1\right)<R^H+R^L,$$

which, because of the concavity of the log function, yields

$$\log \frac{(R^H+1)(R^L+1)}{(R^H+R^L)^2} < 0.$$

This shows that also the RHS of (25) > 0.

We also want to study the sign of the derivatives of $\tilde{q}$ with respect to R^L and R^H :

$$\frac{d\tilde{q}}{dR^L} = \frac{R^L+R^H+2}{R^L\left(\ln \frac{\frac{R^H}{R^L}+1}{2}\right)(R^H+1)(R^L+1)} > 0.$$

Furthermore, denote

$$N = \log \frac{(R^H+1)(R^L+1)}{(R^H+R^L)^2} \text{ and } D = \log\left(\frac{2R^L}{R^H+R^L}\right).$$

Then

$$\frac{\partial N}{\partial R^H} = \frac{(R^H+R^L)^2}{(R^H+1)(R^L+1)}\left[\frac{(R^L+1)(R^H+R^L)^2 - 2(R^H+R^L)(R^H+1)\left(R^L+1\right)}{(R^H+R^L)^4}\right] =$$
$$\frac{1}{(R^H+1)}\left[\frac{R^L-R^H-2}{R^H+R^L}\right] < 0,$$

and

$$\frac{\partial D}{\partial R^H} = -\frac{1}{R^H+R^L} < 0.$$

Hence

$$sign \frac{\partial \tilde{q}}{\partial R^H} = sign\left(\frac{\partial N}{\partial R^H}D - \frac{\partial D}{\partial R^H}N\right) =>$$
$$\frac{\partial N}{\partial R^H}D - \frac{\partial D}{\partial R^H}N = \left[\frac{\overbrace{R^L-R^H-2}^{<0}}{(R^H+1)(R^H+R^L)}\right] D + \frac{1}{R^H+R^L}N.$$

Recall that $D < 0$ and $N < 0$ and that $\tilde{q} \equiv \frac{N}{D} < 1$. Hence $|D| > |N|$. Thus sufficient condition for

$$sign\left(\frac{\partial N}{\partial R^H}D - \frac{\partial D}{\partial R^H}N\right) > 0$$

is that

$$\left|\frac{R^L - R^H - 2}{(R^H+1)(R^H+R^L)}\right| > \frac{1}{R^H+R^L} \Leftrightarrow$$
$$\left|\frac{R^L - R^H - 2}{R^H+1}\right| > 1$$

which is true since

$$\frac{R^L - R^H - 2}{R^H+1} = -1 - \frac{1-R^L}{R^H+1} < -1.$$