Online Appendix to "Time-Varying Inflation Risk and Stock Returns"

In this Online Appendix, we present more detail for the block-bootstrap procedure as well as the derivation of our model. We finally report results from a variety of robustness checks.

1 Bootstrap Algorithm

The block-bootstrap algorithm associated to the regressions of Tables 2, 4 and 5 in the paper consists of the following steps.

1. In each replication $m = 1, \ldots, 1000$, we construct pseudo-samples for both consumption growth and inflation by drawing with replacement T_m overlapping two-year blocks from:

$$\{\Delta C^m_{t+1:t+24}, \Pi^m_{t+1:t+24}\},\ t = s^m_1, s^m_2, \ldots, s^m_{T_m} \tag{OA.1}$$

   where the time indices, $s^m_1, s^m_2, \ldots, s^m_{T_m}$, are drawn randomly from the original time sequence $1, \ldots, T$. The two-year block size is chosen to preserve the (auto-) correlation between consumption growth and inflation in the data and to respect the estimation setup in Equations (2) and (3) of the paper. Additionally, it is a way to conserve the size of the cross-section in the resampled CRSP file (see Step 3 below). We join these blocks to construct a monthly time series matching the length of the sample from July 1967 to December 2014.

2. For $m = 1, \ldots, 1000$, we run the two-stage tests described in Section 2 for the artificial data:

$$\Delta C^m_{t+1:t+K} = d^{c,K}_{m,0} + d^{c,K}_{m,1}(\widehat{a^K_{m,t}} + \widehat{b^K_{m,t}}\Pi^m_t) + e^m_{t+1:t+K}, \text{ where} \tag{OA.2}$$

$$\Delta C^m_{s+1:s+K} = a^K_{m,t} + b^K_{m,t}\Pi^m_s + e^m_{s+1:s+K},\ s = 1, ..., t-K, \tag{OA.3}$$

and save the estimates $d^{c,K}_{m,0}$, $d^{c,K}_{m,1}$, and $b^K_{m,t}$, for $K = 1, 3, 6, 12$. The bootstrapped standard errors reported in Table 2 are calculated as the standard deviation of $d^{c,K}_{m,0}$ and $d^{c,K}_{m,1}$ over the 1000 bootstrap replications. The bootstrap estimates, $b^{12}_{m,t}$, represents the artificial nominal-real covariance that is going to be used to get the bootstrapped standard errors for the remaining tables of the paper.

3. Using the same time indexes $s^m_1, s^m_2, \ldots, s^m_{T_m}$, we re-sample all firms $i = 1, \ldots, I$ in the CRSP file. To be consistent with the data, we bootstrap both returns, $R_{t+1} = \{R_{1,t+1}, R_{2,t+1}, \ldots, R_{I,t+1}\}'$, and firm characteristics, $Z_t = \{MV_t, BM_t, MOM_t\}$, with, e.g., $MV_t = \{MV_{1,t}, MV_{2,t}, \ldots, MV_{I,t}\}'$, such that:

$$\{R^m_{t+1:t+24}, Z^m_{t:t+24-1}\},\ t = s^m_1, s^m_2, \ldots, s^m_{T_m}. \tag{OA.4}$$

   Notice that the characteristics are lagged by one month just like in the data. We join these blocks to construct 1000 artificial CRSP files matching the length of our sample.

4. In each replication, we estimate at the end of month t and for each artificial stock i its exposure to ARMA(1,1)-innovations in inflation, denoted $u^m_{\Pi,t+1}$. The ARMA model is estimated for the inflation series described in Step 1. The inflation betas are estimated using the WLS-Vasicek procedure described in Section 3.1.1 of the paper. We require that an artificial stock return series has at least 24 out of the last 60 months of returns available to estimate inflation beta, $\beta^m_{\Pi,i,t}$. Since many stocks have some missing returns in the CRSP file, due to late introduction or early exit, the overlapping block-bootstrap reduces the number of firms that satisfy this requirement relative to the data. However, we end up with about two-thirds of the number of firms that we use in the data in each bootstrapped cross-section.

5. For $m = 1, \ldots, 1000$ and at the end of each month t, we then sort the artificial stocks on these inflation betas and their market values to construct the ten value-weighted size-controlled inflation beta-sorted portfolios that feature prominently in the paper, $R^m_{p,t+1} = \{R^m_{High,t+1}, R^m_{2,t+1}, \ldots, R^m_{Low,t+1}\}$. The three bootstrap estimates of the inflation

risk premium are constructed as follows. First, we take the high-minus-low spreading portfolio from this sort: $R^m_{IP^{HL}_t,t+1} = R^m_{High,t+1} - R^m_{Low,t+1}$. Second, we regress the artificial ARMA(1,1)-innovations in inflation, $u^m_{\Pi,t+1}$, on the inflation sorted portfolios to construct the maximum correlation inflation-mimicking portfolio:

$$u^m_{\Pi,t+1} = intercept_m + weights'_m \times R^m_{p,t+1} + e^m_{t+1}, \tag{OA.5}$$

such that $R^m_{IP^{MC}_t,t+1}$ is the portfolio return $weights'_m \times R^m_{p,t+1}$. Finally, we run a cross-sectional regression of returns on lagged inflation betas, where we control for the firm characteristics:

$$R^m_{i,t+1} = l_{m,0,t} + l_{m,\Pi,t}\beta^m_{\Pi,i,t} + l_{Z,t}Z^m_{n,t} + u^m_{t+1}. \tag{OA.6}$$

The time series of coefficient estimates, $l_{m,\Pi,t}$, represents our third estimate of the inflation risk premium $R^m_{IP^{CS}_t,t+1}$.

6. For each replication, we then run the predictive regression described in Section 3.2 of the paper. That is, we regress returns on the artificial inflation portfolios and risk premiums (compounded over horizons $K = 1, 3, 12$ months) on the lagged nominal-real covariance (i.e., the bootstrap coefficient estimate $b^{12}_{m,t}$ from Step 2 above) using:

$$R^m_{p,t+1:t+K} = L_{m,0} + L_{m,NRC}b^{12}_{m,t} + \varepsilon^m_{t+1:t+K}. \tag{OA.7}$$

Note that the timing in the different steps of the bootstrap is consistent with the data, so that the left-hand side returns are observed strictly after the consumption and inflation numbers used to estimate the right-hand side nominal-real covariance. We use the standard deviation of the estimates $L_{m,0}$ and $L_{m,NRC}$ over the 1000 bootstrap replications as the standard error for the predictive regressions of Tables 4.

7. Finally, we run the pooled predictive regressions described in Section 4.1 of the paper. That is, we regress returns on the artificial inflation beta-sorted decile portfolios, $p = High, 2, 3, ..., Low$ on their time-varying inflation betas ($\beta^m_{\Pi,p,t}$), the nominal-real

covariance, and an interaction:

$$R^m_{p,t+1:t+K} = L_{m,0} + L_{m,\beta_\Pi}\beta^m_{\Pi,p,t} + L_{m,NRC}b^{12}_{m,t} + L_{m,\beta_\Pi \times NRC}(\beta^m_{\Pi,p,t} \times b^{12}_{m,t}) + \varepsilon^m_{t+1:t+K}. \tag{OA.8}$$

Because this regression includes an interaction term, the bootstrap may contain samples m that suffer from multicollinearity. To address this problem, we evaluate the joint significance of the variables by performing inference on the predicted risk premia when $\beta^m_{\Pi,p,t}$ and $b^{12}_{m,t}$ are at plus or minus one standard deviation from their respective means in the block-bootstrapped sample. We use the standard deviation of these predicted risk premia to calculate the t-statistics for the estimates report in Panel B of Table 5.

2 Model Derivations

This section present detailed derivations of our model results, including expressions for the moments that we target in our calibration.

2.1 Setup

The representative agent has preferences given by the recursive utility function of Epstein and Zin (1989) and Kreps and Porteus (1978),

$$U_t(W_t) = \left((1-\delta) C_t^{1-1/\psi} + \delta E_t \left[U_{t+1}(W_{t+1})^{1-\gamma} \right]^{\frac{1-1/\psi}{1-\gamma}} \right)^{\frac{1}{1-1/\psi}},$$

where W_t is real aggregate wealth and C_t is real aggregate consumption. The constant $\delta \in (0,1)$ is the discount rate, $\gamma > 0$ is the coefficient of relative risk aversion and $\psi > 0$ is the elasticity of intertemporal substitution (EIS). The first order condition for the representative agent's problem implies that the gross return $R_{i,t+1}$ on any tradable asset i satisfies the Euler equation

$$1 = E_t \left[M_{t+1} R_{i,t+1} \right],$$

with a stochastic discount factor M_{t+1} given by

$$m_{t+1} \equiv \log M_{t+1} = \theta \log \delta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{c,t+1}, \tag{OA.1}$$

where

$$\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}, \tag{OA.2}$$

and lowercase letters denote logarithms, so that $\Delta c_t = \ln C_t - \ln C_{t-1}$ and $r_{c,t} = \log(R_{c,t})$.

The processes for real consumption growth, Δc_t, inflation, π_t, the nominal-real covariance, φ_t, and real dividend growth for asset i, $\Delta d_{i,t}$, are exogenous and given by

$$\begin{aligned}
\pi_{t+1} &= \mu_\pi + \rho_\pi (\pi_t - \mu_\pi) + \phi_\pi u_{t+1} + \xi_\pi u_t, && \text{(OA.3)} \\
\Delta c_{t+1} &= \mu_c + \rho_c (\pi_t - \mu_\pi) + \sigma_c \eta_{t+1} + \xi_c \varphi_{t-1} u_t, && \text{(OA.4)} \\
\varphi_{t+1} &= \varphi_0 + v (\varphi_t - \varphi_0) + \sigma_w w_{t+1}, && \text{(OA.5)} \\
\Delta d_{i,t+1} &= \mu_i + \rho_i (\pi_t - \mu_\pi) + \sigma_i \eta_{t+1} + \xi_i \varphi_{t-1} u_t, && \text{(OA.6)} \\
& u_t, \eta_t, w_t \text{ iid standard normal.}
\end{aligned}$$

2.2 The Nominal-Real Covariance

The covariance between time-$(t+1)$ inflation and consumption conditional on times t and $t-1$ are

$$\begin{aligned}
Cov_t(\Delta c_{t+1}, \pi_{t+1}) &= 0, \\
Cov_{t-1}(\Delta c_{t+1}, \pi_{t+1}) &= \phi_\pi \rho_c (\xi_\pi + \phi_\pi \rho_\pi) + \xi_c (\rho_\pi \phi_\pi + \xi_\pi) \varphi_{t-1}.
\end{aligned}$$

Conditional predictability of consumption growth with inflation is given by

$$Cov_t(\Delta c_{t+1+j}, \pi_{t+1}) = \begin{cases} \phi_\pi^2 \rho_c + \phi_\pi \xi_c \varphi_t & , \quad \text{if } j = 1 \\ \phi_\pi \rho_c \rho_\pi^{j-2} (\xi_\pi + \phi_\pi \rho_\pi) & , \quad \text{if } j > 1 \end{cases},$$

and, using that

$$Var_t\left(\pi_{t+1}\right) = \phi_\pi^2,$$

we get

$$\begin{aligned} b_t^K &= \frac{Cov_t\left(\sum_{j=1}^K \Delta c_{t+1+j}, \pi_{t+1}\right)}{Var_t\left(\pi_{t+1}\right)} \\ &= \left(1 + \left(\frac{\xi_\pi}{\phi_\pi} + \rho_\pi\right)\frac{\rho_\pi^{K-1} - 1}{\rho_\pi - 1}\right)\rho_c + \frac{\xi_c}{\phi_\pi}\varphi_t \\ &= h_K + \frac{\xi_c}{\phi_\pi}\varphi_t, \end{aligned} \tag{OA.7}$$

where we have defined

$$h_K \equiv \left(1 + \left(\frac{\xi_\pi}{\phi_\pi} + \rho_\pi\right)\frac{\rho_\pi^{K-1} - 1}{\rho_\pi - 1}\right)\rho_c. \tag{OA.8}$$

Our main measure of the nominal-real covariance is

$$NRC_t^C \equiv b_t^{12}. \tag{OA.9}$$

2.3 Coefficients of the Wealth-Consumption Ratio

To price assets, we conjecture (and later verify) that the log wealth-consumption ratio is linear quadratic in the state variables π_t, φ_t, and u_t and has the following form

$$wc_t = A_0 + A_1\left(\pi_t - \mu_\pi\right) + A_2\varphi_{t-1}u_t + A_3u_t + A_4\left(\varphi_t - \varphi_0\right) + A_5\left(\varphi_t^2 - E\left[\varphi_t^2\right]\right), \tag{OA.10}$$

and that the price-dividend ratio for asset i is

$$pd_{i,t} = D_{i,0} + D_{i,1}\left(\pi_t - \mu_\pi\right) + D_{i,2}\varphi_{t-1}u_t + D_{i,3}u_t + D_{i,4}\left(\varphi_t - \varphi_0\right) + D_{i,5}\left(\varphi_t^2 - E\left[\varphi_t^2\right]\right), \tag{OA.11}$$

where

$$E\left[\varphi_t^2\right] = \varphi_0^2 + \frac{\sigma_w^2}{1 - v^2},$$

is the unconditional mean of φ_t^2.

A Campbell-Shiller approximation gives returns on the aggregate consumption claim (the wealth portfolio), $r_{c,t+1}$, and returns on any asset i, $r_{i,t+1}$,

$$r_{c,t+1} = \kappa_0 + \kappa_1 wc_{t+1} - wc_t + \Delta c_{t+1}, \tag{OA.12}$$

$$r_{i,t+1} = \kappa_{i,0} + \kappa_{i,1} pd_{i,t+1} - pd_{i,t} + \Delta d_{i,t+1}, \tag{OA.13}$$

with approximations constants

$$\kappa_1 = \frac{e^{E[wc_t]}}{e^{E[wc_t]}+1}, \tag{OA.14}$$

$$\kappa_0 = \log\left(e^{E[wc_t]}+1\right) - \frac{e^{E[wc_t]}}{e^{E[wc_t]}+1} E\left[wc_t\right], \tag{OA.15}$$

$$\kappa_{i,1} = \frac{e^{E[pd_{i,t}]}}{e^{E[pd_{i,t}]}+1}, \tag{OA.16}$$

$$\kappa_{i,0} = \log\left(e^{E[pd_{i,t}]}+1\right) - \frac{e^{E[pd_{i,t}]}}{e^{E[pd_{i,t}]}+1} E\left[pd_{i,t}\right]. \tag{OA.17}$$

Using equations (OA.12), and (OA.1), we get

$$m_{t+1} + r_{c,t+1} = \theta\left(\log\delta + \kappa_0\right) + \theta\left(1 - \frac{1}{\psi}\right)\Delta c_{t+1} + \theta\left(\kappa_1 wc_{t+1} - wc_t\right). \tag{OA.18}$$

The Euler equation for any asset i (including $i = c$) with lognormal returns is

$$0 = E_t\left[m_{t+1} + r_{i,t+1}\right] + \frac{1}{2} Var_t\left[m_{t+1} + r_{i,t+1}\right]. \tag{OA.19}$$

Plugging equation (OA.18) into (OA.19) with $i = c$, and evaluating conditional means and variances, the Euler equation for the consumption claim ($i = c$) can be expressed as

$$0 = C_0 + C_1\left(\pi_t - \mu_\pi\right) + C_2\varphi_{t-1}u_t + C_3 u_t + C_4\left(\varphi_t - \varphi_0\right) + C_5\left(\varphi_t^2 - E\left[\varphi_t^2\right]\right),$$

where C_i are constants that depend on A_0, A_1, A_2, A_3, A_4, A_5. In order for the Euler equation to be satisfied at all times, the coefficients C_i must be identically zero, which yields the following system of equations in A_0, A_1, A_2, A_3, A_4, A_5:

$$
\begin{aligned}
(\pi_t - \mu_\pi) \quad &: \quad 0 = \left(1 - \frac{1}{\psi}\right) \rho_c - A_1 \left(1 - \kappa_1 \rho_\pi\right), && \text{(OA.20)} \\
(\varphi_{t-1} u_t) \quad &: \quad 0 = \left(1 - \frac{1}{\psi}\right) \xi_c - A_2, && \text{(OA.21)} \\
(u_t) \quad &: \quad 0 = \kappa_1 \xi_\pi A_1 - A_3, && \text{(OA.22)} \\
(\varphi_t - \varphi_0) \quad &: \quad 0 = \theta \kappa_1^2 A_2 \left(A_3 + \phi_\pi A_1\right) + \left(v \kappa_1 - 1\right) A_4, && \text{(OA.23)} \\
&\quad\quad + \left(2 v \theta \kappa_1^2 \sigma_w^2\right) A_4 A_5 - 2 \kappa_1 \varphi_0 v \left(v - 1\right) \left(2 \theta \kappa_1 \sigma_w^2 A_5 + 1\right) A_5 \\
\left(\varphi_t^2 - E\left[\varphi_t^2\right]\right) \quad &: \quad 0 = \frac{\theta \kappa_1^2}{2} \left(A_2^2 + 4 v^2 \sigma_w^2 A_5^2\right) - A_5 \left(1 - v^2 \kappa_1\right), && \text{(OA.24)} \\
(const) \quad &: \quad 0 = \frac{\theta^2}{2} \left(\frac{1}{\psi} - 1\right)^2 \sigma_c^2 - \frac{\theta}{\psi} \mu_c + \theta \left(\kappa_1 - 1\right) A_0 + \theta \left(\kappa_0 + \mu_c + \ln \delta\right) \\
&\quad\quad + \frac{1}{2} \theta^2 \kappa_1^2 \left(\left(A_3 + \varphi_0 A_2 + \phi_\pi A_1\right)^2 + \sigma_w^2 \left(\left(A_4 + 2 \varphi_0 A_5\right)^2 + 2 \sigma_w^2 A_5^2\right)\right) \\
&\quad\quad + \frac{\theta^2 \kappa_1^2 \sigma_w^2}{2 \left(1 - v^2\right)} \left(A_2^2 + 4 v^2 \sigma_w^2 A_5^2\right). && \text{(OA.25)}
\end{aligned}
$$

We solve for A_1, A_2, A_3, A_4, A_5 in terms of A_0

$$
\begin{aligned}
A_1 &= \left(1 - \frac{1}{\psi}\right) \frac{\rho_c}{1 - \kappa_1 \rho_\pi}, && \text{(OA.26)} \\
A_2 &= \left(1 - \frac{1}{\psi}\right) \xi_c, && \text{(OA.27)} \\
A_3 &= \left(1 - \frac{1}{\psi}\right) \frac{\kappa_1 \xi_\pi \rho_c}{1 - \kappa_1 \rho_\pi}, && \text{(OA.28)} \\
A_4 &= \frac{A_2 \theta \kappa_1^2 \left(A_1 \phi_\pi + A_3\right) + 2 \kappa_1 \varphi_0 v \left(1 - v\right) \left(1 + 2 \theta \kappa_1 \sigma_w^2 A_5\right) A_5}{1 - \kappa_1 v \left(1 + 2 \theta \kappa_1 \sigma_w^2 A_5\right)}, && \text{(OA.29)} \\
A_5 &= \frac{1 - v^2 \kappa_1}{\theta \left(2 v \kappa_1 \sigma_w\right)^2} \left(1 - \sqrt{1 - \left(\frac{2 \left(1 - \gamma\right) v \kappa_1^2 \sigma_w \xi_c}{1 - v^2 \kappa_1}\right)^2}\right), && \text{(OA.30)}
\end{aligned}
$$

where we pick the negative root in the expression for A_5 (which comes from solving a quadratic equation) so that $\xi_c = 0$ implies the intuitive economic restriction that the nominal-real covariance φ_t has no effect on the wealth-consumption ratio when it does not affect consumption. Then we solve for A_0 numerically using equations (OA.15), (OA.14) and (OA.25).

2.4 Coefficients of the Price-Dividend Ratio

The calculation is analogous to that for the A_i but instead of using the Euler equation for $r_{c,t}$, we use the one for $r_{i,t}$. The Euler equation for asset i can be expressed as

$$0 = F_{i,0} + F_{i,1}(\pi_t - \mu_\pi) + F_{i,2}\varphi_{t-1}u_t + F_{i,3}u_t + F_{i,4}(\varphi_t - \varphi_0) + F_{i,5}\left(\varphi_t^2 - E\left[\varphi_t^2\right]\right), \quad \text{(OA.31)}$$

where $F_{i,j}$ are constants that depend on $D_{i,0}, D_{i,1}, D_{i,2}, D_{i,3}, D_{i,4}, D_{i,5}$. In order for the Euler equation to be satisfied, the coefficients $F_{i,j}$ must be identically zero, which yields the following system of five equations in the five unknowns $D_{i,1}, D_{i,2}, D_{i,3}, D_{i,4}, D_{i,5}$

$$\begin{aligned}
(\pi_t - \mu_\pi) \quad &: \quad 0 = \rho_i - D_{i,1}(1 - \kappa_{i,1}\rho_\pi) \\
&\qquad + (\theta - 1)(\rho_c + A_1(\kappa_1\rho_\pi - 1)) - \frac{\theta}{\psi}\rho_c, && \text{(OA.32)} \\
(\varphi_{t-1}u_t) \quad &: \quad 0 = \xi_i - D_{i,2} + (\theta - 1)(\xi_c - A_2) - \frac{\theta}{\psi}\xi_c, && \text{(OA.33)} \\
(u_t) \quad &: \quad 0 = (\theta - 1)(\kappa_1\xi_\pi A_1 - A_3) + \xi_\pi\kappa_{i,1}D_{i,1} - D_{i,3}, && \text{(OA.34)} \\
(\varphi_t - \varphi_0) \quad &: \quad 0 = 2D_{i,5}v\kappa_{i,1}\varphi_0(1 - v) - D_{i,4}(1 - v\kappa_{i,1}) \\
&\qquad - (A_4 - \kappa_1 v(A_4 + 2A_5\varphi_0(1 - v)))(\theta - 1) \\
&\qquad + (\kappa_{i,1}D_{i,2} + \kappa_1 A_2(\theta - 1))(\kappa_{i,1}D_{i,3} + \kappa_1 A_3(\theta - 1)) \\
&\qquad + \phi_\pi(\kappa_{i,1}D_{i,2} + \kappa_1 A_2(\theta - 1))(\kappa_{i,1}D_{i,1} + \kappa_1 A_1(\theta - 1)) \\
&\qquad + 2v\sigma_w^2(\kappa_{i,1}D_{i,4} + \kappa_1 A_4(\theta - 1))(\kappa_{i,1}D_{i,5} + \kappa_1 A_5(\theta - 1)) \\
&\qquad - 4v\varphi_0\sigma_w^2(\kappa_{i,1}D_{i,5} + \kappa_1 A_5(\theta - 1))^2(v - 1), && \text{(OA.35)} \\
\left(\varphi_t^2 - E\left[\varphi_t^2\right]\right) \quad &: \quad 0 = \left(2v^2\sigma_w^2\kappa_{i,1}^2\right)D_{i,5}^2 \\
&\qquad + \left(v^2\kappa_{i,1}\left(4\kappa_1\sigma_w^2 A_5(\theta - 1) + 1\right) - 1\right)D_{i,5} \\
&\qquad + \frac{1}{2}(\kappa_{i,1}D_{i,2} - \kappa_1 A_2 + \theta\kappa_1 A_2)^2 \\
&\qquad + A_5(\theta - 1)\left(v^2\kappa_1\left(2\kappa_1\sigma_w^2 A_5(\theta - 1) + 1\right) - 1\right), && \text{(OA.36)}
\end{aligned}$$

and one equation for $D_{i,0}$

$$
\begin{aligned}
\left(1-\kappa_{i,1}\right) D_{i,0} \quad = \quad & \kappa_{i,0}+\mu_i+\theta \ln \delta-\frac{\theta}{\psi} \mu_c+(\theta-1)\left(\kappa_0+\mu_c-A_0\left(1-\kappa_1\right)\right) \\
& +\frac{1}{2}\left(\sigma_i+\sigma_c\left(\theta-\frac{\theta}{\psi}-1\right)\right)^2 \\
& +\frac{1}{2} \phi_\pi^2\left(\kappa_{i,1} D_{i,1}+(\theta-1) \kappa_1 A_1\right)^2 \\
& +\frac{1}{2} \kappa_1 \sigma_w^2 A_4(\theta-1)\left(2 \kappa_{i,1} D_{i,4}+\kappa_1(\theta-1)\left(A_4+4(1-v) \varphi_0 A_5\right)\right) \\
& +\sigma_w^2\left(2 \varphi_0^2\left(1-v^2\right)+\sigma_w^2\right)\left(\kappa_{i,1} D_{i,5}+\kappa_1 A_5(\theta-1)\right)^2 \\
& +\kappa_{i,1} \varphi_0\left(\kappa_{i,1} D_{i,2}+(\theta-1) \kappa_1 A_2\right)\left(\phi_\pi D_{i,1}+D_{i,3}\right) \\
& +\kappa_1(\theta-1) \varphi_0\left(\kappa_{i,1} D_{i,2}+(\theta-1) \kappa_1 A_2\right)\left(A_3+\phi_\pi A_1\right) \\
& +\frac{1}{2} \kappa_{i,1}\left(\kappa_{i,1} D_{i,3}+(\theta-1) \kappa_1 A_3\right)\left(2 \phi_\pi D_{i,1}+D_{i,3}\right) \\
& +\frac{1}{2} \kappa_1(\theta-1)\left(\kappa_{i,1} D_{i,3}+(\theta-1) \kappa_1 A_3\right)\left(A_3+2 \phi_\pi A_1\right) \\
& +\frac{1}{2}\left(\varphi_0^2+\frac{\sigma_w^2}{1-v^2}\right) \kappa_1^2(\theta-1)^2\left(A_2^2+4 v^2 \sigma_w^2 A_5^2\right) \\
& +\left(\varphi_0^2+\frac{\sigma_w^2}{1-v^2}\right) \kappa_{i,1} \kappa_1(\theta-1)\left(A_2 D_{i,2}+4 v^2 \sigma_w^2 A_5 D_{i,5}\right) \\
& +\kappa_{i,1}^2\left(D_{i,2}^2+4 v^2 \sigma_w^2 D_{i,5}^2\right) \\
& +\frac{1}{2} \sigma_w^2 \kappa_{i,1} D_{i,4} \kappa_{i,1}\left(4 \varphi_0 D_{i,5}+D_{i,4}\right) \\
& +\sigma_w^2 \kappa_{i,1} \sigma_w^2 \kappa_1 \varphi_0(\theta-1)\left(A_4 D_{i,5}+A_5 D_{i,4}\right) \\
& +2 \sigma_w^2 v \kappa_1^2 \varphi_0 A_4 A_5(\theta-1)^2 . \qquad \text{(OA.37)}
\end{aligned}
$$

We solve for $D_{i,j}$ for $j=1,2,3,4,5$ in terms of $D_{i,0}$ in closed form, picking the negative root in the solution of the quadratic equation for $D_{i,5}$, as we did for the solution of the system of equations for A_i. We then solve for $D_{i,0}$ numerically using equations (OA.17), (OA.16) and (OA.37).

2.5 Conditional Pricing

We first write the SDF in terms of state variables $(\pi_t - \mu_\pi, \varphi_{t-1}u_t, u_t, \varphi_t - \varphi_0, \varphi_t^2 - E[\varphi_t^2])$ and innovations $\left(u_{t+1}, w_{t+1}, \frac{w_{t+1}^2-1}{\sqrt{2}}, \eta_{t+1}\right)$. Note that the innovations are mean zero, variance one and orthogonal to each other (even though w_{t+1} and w_{t+1}^2 are not independent) since

$$E_t\left[w_{t+1}\left(w_{t+1}^2-1\right)\right] = E_t\left[w_{t+1}^3\right] - E_t\left[w_{t+1}\right] = 0.$$

The innovation to the SDF is

$$\begin{aligned} m_{t+1} - E_t\left[m_{t+1}\right] \;=\;& \kappa_1(\theta-1)\left(\phi_\pi A_1 + \varphi_t A_2 + A_3\right)u_{t+1} \\ &+\kappa_1\sigma_w(\theta-1)\left(A_4 + 2\varphi_0 A_5(1-v) + 2vA_5\varphi_t\right)w_{t+1} \\ &+\sqrt{2}\kappa_1\sigma_w^2(\theta-1)A_5\left(\frac{w_{t+1}^2-1}{\sqrt{2}}\right) \\ &+\left(\theta\left(1-\frac{1}{\psi}\right)-1\right)\sigma_c\eta_{t+1}. \end{aligned} \tag{OA.38}$$

We can then write

$$m_{t+1} - E_t\left[m_{t+1}\right] = -\lambda_{u,t}u_{t+1} - \lambda_{w,t}w_{t+1} - \lambda_{2w}\left(\frac{w_{t+1}^2-1}{\sqrt{2}}\right) - \lambda_\eta\eta_{t+1},$$

where the prices of risk for $\left(u_{t+1}, w_{t+1}, \frac{w_{t+1}^2-1}{\sqrt{2}}, \eta_{t+1}\right)$ are

$$\begin{aligned}
\lambda_{u,t} &\equiv -\frac{Cov_t\left(m_{t+1}, u_{t+1}\right)}{Var_t\left(u_{t+1}\right)} \\
&= -\kappa_1\left(\theta-1\right)\left(\phi_\pi A_1+\varphi_t A_2+A_3\right) \\
&= \kappa_1\left(\gamma-\frac{1}{\psi}\right)\left(\frac{\rho_c\left(\phi_\pi+\kappa_1\xi_\pi\right)}{1-\rho_\pi\kappa_1}+\xi_c\varphi_t\right), && \text{(OA.39)} \\
\lambda_{w,t} &\equiv -\frac{Cov_t\left(m_{t+1}, w_{t+1}\right)}{Var_t\left(w_{t+1}\right)} \\
&= -\kappa_1\sigma_w\left(\theta-1\right)\left(A_4+2\varphi_0 A_5\left(1-v\right)+2vA_5\varphi_t\right), && \text{(OA.40)} \\
\lambda_{2w} &\equiv -\frac{Cov_t\left(m_{t+1}, \left(w_{t+1}^2-1\right)/\sqrt{2}\right)}{Var_t\left(\left(w_{t+1}^2-1\right)/\sqrt{2}\right)} \\
&= -\sqrt{2}\kappa_1\sigma_w^2\left(\theta-1\right)A_5, && \text{(OA.41)} \\
\lambda_{\eta} &\equiv -\frac{Cov_t\left(m_{t+1}, \eta_{t+1}\right)}{Var_t\left(\eta_{t+1}\right)} \\
&= \gamma\sigma_c. && \text{(OA.42)}
\end{aligned}$$

For returns of asset i, we have

$$r_{i,t+1} = E_t\left[r_{i,t+1}\right]+\beta_{u,it}u_{t+1}+\beta_{w,i}w_{t+1}+\beta_{2w,i}\left(\frac{w_{t+1}^2-1}{\sqrt{2}}\right)+\beta_{\eta,i}\eta_{t+1}, \tag{OA.43}$$

where the conditional return is

$$\begin{aligned}
E_t\left[r_{i,t+1}\right] &= \kappa_{i,0}+\mu_i-D_{i,0}\left(1-\kappa_{i,1}\right) \\
&\quad +\left(\rho_i-D_{i,1}\left(1-\kappa_{i,1}\rho_\pi\right)\right)\left(\pi_t-\mu_\pi\right) \\
&\quad -u_t\left(D_{i,3}-\xi_\pi\kappa_{i,1}D_{i,1}\right)+\left(\xi_i-D_{i,2}\right)\varphi_{t-1}u_t \\
&\quad +\left(2D_{i,5}v\kappa_{i,1}\varphi_0\left(1-v\right)-D_{i,4}\left(1-v\kappa_{i,1}\right)\right)\left(\varphi_t-\varphi_0\right) \\
&\quad -D_{i,5}\left(1-v^2\kappa_{i,1}\right)\left(\varphi_t^2-E\left[\varphi_t^2\right]\right), && \text{(OA.44)}
\end{aligned}$$

and the quantities of risk are given by the betas

$$
\begin{aligned}
\beta_{u,it} &\equiv \frac{Cov_t\left(u_{t+1}, r_{i,t+1}\right)}{Var_t\left(u_{t+1}\right)} \\
&= \kappa_{i,1}\left(\phi_\pi D_{i,1} + D_{i,2}\varphi_t + D_{i,3}\right) \\
&= \kappa_{i,1}\left(\left(\frac{\rho_i}{\rho_c} - \frac{1}{\psi}\right)\frac{\left(\phi_\pi + \xi_\pi\kappa_{i,1}\right)\rho_c}{1-\rho_\pi\kappa_{i,1}} + \left(\frac{\xi_i}{\xi_c} - \frac{1}{\psi}\right)\xi_c\varphi_t\right), \qquad \text{(OA.45)} \\
\beta_{w,it} &\equiv \frac{Cov_t\left(w_{t+1}, r_{i,t+1}\right)}{Var_t\left(w_{t+1}\right)} \\
&= \kappa_{i,1}\sigma_w\left(D_{i,4} + 2\varphi_0 D_{i,5}\left(1-v\right) + 2vD_{i,5}\varphi_t\right), \qquad \text{(OA.46)} \\
\beta_{2w,i} &\equiv \frac{Cov_t\left(\left(w_{t+1}^2 - 1\right)/\sqrt{2}, r_{i,t+1}\right)}{Var_t\left(\left(w_{t+1}^2 - 1\right)/\sqrt{2}\right)} \\
&= \sqrt{2}\kappa_{i,1}\sigma_w^2 D_{i,5}, \qquad \text{(OA.47)} \\
\beta_{\eta,i} &\equiv \frac{Cov_t\left(\eta_{t+1}, r_{i,t+1}\right)}{Var_t\left(\eta_{t+1}\right)} \\
&= \sigma_i. \qquad \text{(OA.48)}
\end{aligned}
$$

Expected excess returns for asset i can then be written

$$
\begin{aligned}
-Cov_t\left(m_{t+1}, r_{i,t+1}\right) &= -E_t\left[\left(m_{t+1} - E_t\left[m_{t+1}\right]\right)\left(r_{i,t+1} - E_t\left[r_{i,t+1}\right]\right)\right] \\
&= \lambda_{u,t}\beta_{u,it} + \lambda_{w,t}\beta_{w,it} + \lambda_{2w}\beta_{2w,i} + \lambda_\eta\beta_{\eta,i}.
\end{aligned}
$$

Of course, we can always set $i = c$ and study the consumption portfolio:

$$
r_{c,t+1} = E_t\left[r_{c,t+1}\right] + \beta_{u,ct}u_{t+1} + \beta_{w,ct}w_{t+1} + \beta_{2w,c}\left(\frac{w_{t+1}^2 - 1}{\sqrt{2}}\right) + \beta_{\eta,c}\eta_{t+1},
$$

where

$$
\begin{aligned}
E_t\left[r_{c,t+1}\right] &= \kappa_0 + \mu_c - A_0\left(1-\kappa_1\right) \\
&+ \left(\rho_c - A_1\left(1-\kappa_1\rho_\pi\right)\right)\left(\pi_t - \mu_\pi\right) \\
&+ \left(\xi_\pi\kappa_1 A_1 - A_3\right)u_t \\
&+ \left(\xi_c - A_2\right)\varphi_{t-1}u_t \\
&+ \left(2v\kappa_1\varphi_0\left(1-v\right)A_5 - \left(1-v\kappa_1\right)A_4\right)\left(\varphi_t - \varphi_0\right) \\
&- A_5\left(1-v^2\kappa_1\right)\left(\varphi_t^2 - E\left[\varphi_t^2\right]\right),
\end{aligned}
$$

and where the quantities of risk are given by the betas

$$
\begin{aligned}
\beta_{u,ct} &= \frac{Cov_t\left(u_{t+1}, r_{c,t+1}\right)}{Var_t\left(u_{t+1}\right)} \\
&= \kappa_1\left(\phi_\pi A_1 + A_2\varphi_t + A_3\right), \\
\beta_{w,ct} &= \frac{Cov_t\left(w_{t+1}, r_{c,t+1}\right)}{Var_t\left(w_{t+1}\right)} \\
&= \kappa_1\sigma_w\left(\left(A_4 + 2\varphi_0 A_5\left(1-v\right)\right) + 2vA_5\varphi_t\right), \\
\beta_{2w,c} &= \frac{Cov_t\left(\left(w_{t+1}^2 - 1\right)/\sqrt{2}, r_{c,t+1}\right)}{Var_t\left(\left(w_{t+1}^2 - 1\right)/\sqrt{2}\right)} \\
&= \sqrt{2}\kappa_1\sigma_w^2 A_5, \\
\beta_{\eta,c} &= \frac{Cov_t\left(\eta_{t+1}, r_{c,t+1}\right)}{Var_t\left(\eta_{t+1}\right)} \\
&= \sigma_c.
\end{aligned}
$$

Expected excess returns for asset c can then be written

$$
\begin{aligned}
-Cov_t\left(m_{t+1}, r_{c,t+1}\right) &= -E_t\left[\left(m_{t+1} - E_t\left[m_{t+1}\right]\right)\left(r_{c,t+1} - E_t\left[r_{c,t+1}\right]\right)\right] \\
&= \lambda_{u,t}\beta_{u,ct} + \lambda_{w,t}\beta_{w,ct} + \lambda_{2w}\beta_{2w,c} + \lambda_\eta\beta_{\eta,c}.
\end{aligned}
$$

Inflation betas and the price of risk with respect to the shock $u_{\Pi,t+1} \equiv \phi_\pi u_{t+1}$ are

$$\begin{aligned} \beta_{\Pi,i,t} &= \phi_\pi \beta_{u,it}, && \text{(OA.49)} \\ \lambda_{\Pi,t} &= \frac{\lambda_{u,t}}{\phi_\pi}. && \text{(OA.50)} \end{aligned}$$

2.6 Derivation of equations (17) and (18)

First, we compute innovations for π_{t+2+j}, φ_{t+2+j} and φ^2_{t+2+j}:

$$\begin{aligned} \pi_{t+2+j} &= \mu_\pi + \rho_\pi \left(\pi_{t+1+j} - \mu_\pi\right) + \phi_\pi u_{t+2+j} + \xi_\pi u_{t+1+j}, \\ E_{t+1+j}\left[\pi_{t+2+j}\right] &= \mu_\pi + \rho_\pi \left(\pi_{t+1+j} - \mu_\pi\right) + \xi_\pi u_{t+1+j}, \\ \pi_{t+2+j} - E_{t+1+j}\left[\pi_{t+2+j}\right] &= \phi_\pi u_{t+2+j}, \end{aligned}$$

$$\begin{aligned} \varphi_{t+2+j} &= \varphi_0 + v\left(\varphi_{t+1+j} - \varphi_0\right) + \sigma_w w_{t+2+j}, \\ E_{t+1+j}\left[\varphi_{t+2+j}\right] &= \varphi_0 + v\left(\varphi_{t+1+j} - \varphi_0\right), \\ \varphi_{t+2+j} - E_{t+1+j}\left[\varphi_{t+2+j}\right] &= \sigma_w w_{t+2+j}, \end{aligned}$$

$$\begin{aligned} \varphi^2_{t+2+j} &= \left(\varphi_0 + v\left(\varphi_{t+1+j} - \varphi_0\right)\right)^2 + \sigma_w^2 w^2_{t+2+j} \\ &\quad +2\left(\varphi_0 + v\left(\varphi_{t+1+j} - \varphi_0\right)\right)\sigma_w w_{t+2+j}, \\ E_{t+1+j}\left[\varphi^2_{t+2+j}\right] &= \left(\varphi_0 + v\left(\varphi_{t+1+j} - \varphi_0\right)\right)^2 + \sigma_w^2, \\ \varphi^2_{t+2+j} - E_{t+1+j}\left[\varphi^2_{t+2+j}\right] &= \sqrt{2}\sigma_w^2 \left(\frac{w^2_{t+2+j} - 1}{\sqrt{2}}\right) \\ &\quad +2\left(\varphi_0 + v\left(\varphi_{t+1+j} - \varphi_0\right)\right)\sigma_w w_{t+2+j}. && \text{(OA.51)} \end{aligned}$$

We use these innovations to compute innovations in wc_{t+2+j}

$$
\begin{aligned}
wc_{t+2+j} &= A_0 + A_1\left(\pi_{t+2+j} - \mu_\pi\right) + A_2\varphi_{t+1+j}u_{t+2+j} + A_3 u_{t+2+j} \\
&\quad + A_4\left(\varphi_{t+2+j} - \varphi_0\right) + A_5\left(\varphi^2_{t+2+j} - E\left[\varphi^2_t\right]\right), \\
E_{t+1+j}\left[wc_{t+2+j}\right] &= A_0 + A_1\left(E_{t+1+j}\left[\pi_{t+2+j}\right] - \mu_\pi\right) + A_2 E_{t+1+j}\left[\varphi_{t+1+j}u_{t+2+j}\right] \\
&\quad + A_3 E_{t+1+j}\left[u_{t+2+j}\right] + A_4\left(E_{t+1+j}\left[\varphi_{t+2+j}\right] - \varphi_0\right) \\
&\quad + A_5\left(E_{t+1+j}\left[\varphi^2_{t+2+j}\right] - E\left[\varphi^2_t\right]\right), \\
wc_{t+2+j} - E_{t+1+j}\left[wc_{t+2+j}\right] &= A_1\left(\pi_{t+2+j} - E_{t+1+j}\left[\pi_{t+2+j}\right]\right) + A_2\varphi_{t+1+j}u_{t+2+j} \\
&\quad + A_3 u_{t+2+j} + A_4\left(\varphi_{t+2+j} - E_{t+1+j}\left[\varphi_{t+2+j}\right]\right) \\
&\quad + A_5\left(\varphi^2_{t+2+j} - E_{t+1+j}\left[\varphi^2_{t+2+j}\right]\right) \\
&= A_1\left(\pi_{t+2+j} - E_{t+1+j}\left[\pi_{t+2+j}\right]\right) + A_2\varphi_{t+1+j}u_{t+2+j} \\
&\quad + A_3 u_{t+2+j} + A_4\left(\varphi_{t+2+j} - E_{t+1+j}\left[\varphi_{t+2+j}\right]\right) \\
&\quad + A_5\left(\varphi^2_{t+2+j} - E_{t+1+j}\left[\varphi^2_{t+2+j}\right]\right) \\
&= \left(A_1\phi_\pi + A_2\varphi_{t+1+j} + A_3\right)u_{t+2+j} \\
&\quad + \left(A_4 + 2A_5\left(\varphi_0 + v\left(\varphi_{t+1+j} - \varphi_0\right)\right)\right)\sigma_w w_{t+2+j} \\
&\quad + A_5\sqrt{2}\sigma^2_w\left(\frac{w^2_{t+2+j} - 1}{\sqrt{2}}\right). \qquad \text{(OA.52)}
\end{aligned}
$$

Innovations in consumption Δc_{t+2+j} are

$$
\begin{aligned}
\Delta c_{t+2+j} &= \mu_c + \rho_c\left(\pi_{t+1+j} - \mu_\pi\right) + \sigma_c\eta_{t+2+j} + \xi_c\varphi_{t+j}u_{t+1+j}, \\
E_{t+1+j}\left[\Delta c_{t+2+j}\right] &= \mu_c + \rho_c\left(\pi_{t+1+j} - \mu_\pi\right) + \xi_c\varphi_{t+j}u_{t+1+j}, \\
\Delta c_{t+2+j} - E_{t+1+j}\left[\Delta c_{t+2+j}\right] &= \sigma_c\eta_{t+2+j}. \qquad \text{(OA.53)}
\end{aligned}
$$

Since the shocks u_{t+2+j}, w_{t+2+j}, $(w^2_{t+2+j} - 1)/\sqrt{2}$ are independent of η_{t+2+j}, equations (OA.52) and (OA.53) imply

$$
Cov_{t+1+j}\left(\Delta c_{t+2+j}, wc_{t+2+j}\right) = 0. \qquad \text{(OA.54)}
$$

Equation (OA.18) gives

$$
\begin{aligned}
m_{t+2+j} + r_{c,t+2+j} &= \theta\left(\log\delta + \kappa_0\right) + \theta\left(1 - \frac{1}{\psi}\right)\Delta c_{t+2+j} + \theta\left(\kappa_1 wc_{t+2+j} - wc_{t+1+j}\right) \\
&= \theta\left(\log\delta + \kappa_0\right) - \theta wc_{t+1+j} + \theta\left(1 - \frac{1}{\psi}\right)\Delta c_{t+2+j} + \theta\kappa_1 wc_{t+2+j}
\end{aligned} \tag{OA.55}
$$

Equations (OA.53) and (OA.54) then imply

$$
\begin{aligned}
Var_{t+1+j}\left(m_{t+2+j} + r_{c,t+2+j}\right) &= Var_{t+1+j}\left(\theta\left(1 - \frac{1}{\psi}\right)\Delta c_{t+2+j} + \theta\kappa_1 wc_{t+2+j}\right) \\
&= \theta^2\left(1 - \frac{1}{\psi}\right)^2 Var_{t+1+j}\left(\Delta c_{t+2+j}\right) \\
&\quad + \left(\theta\kappa_1\right)^2 Var_{t+1+j}\left(wc_{t+2+j}\right) \\
&\quad + 2\theta^2\kappa_1\left(1 - \frac{1}{\psi}\right) Cov_{t+1+j}\left(\Delta c_{t+2+j}, wc_{t+2+j}\right) \\
&= \theta^2\left(1 - \frac{1}{\psi}\right)^2 \sigma_c^2 + \left(\theta\kappa_1\right)^2 Var_{t+1+j}\left(wc_{t+2+j}\right).
\end{aligned}
$$

The innovations in $Var_{t+1+j}\left(m_{t+2+j} + r_{c,t+2+j}\right)$ are

$$
\left(E_{t+1} - E_t\right) Var_{t+1+j}\left(m_{t+2+j} + r_{c,t+2+j}\right) = \left(\theta\kappa_1\right)^2 \left(E_{t+1} - E_t\right) Var_{t+1+j}\left(wc_{t+2+j}\right). \tag{OA.56}
$$

Plug (OA.55) evaluated at $j = 1$ into the first term of the Euler equation (OA.19) evaluated at $t + 2$ to get

$$
\begin{aligned}
0 &= E_{t+1}\left[m_{t+2} + r_{c,t+2}\right] + \frac{1}{2} Var_{t+1}\left[m_{t+2} + r_{c,t+2}\right], \\
0 &= E_{t+1}\left[\theta\left(\log\delta + \kappa_0\right) + \theta\left(1 - \frac{1}{\psi}\right)\Delta c_{t+2} + \theta\left(\kappa_1 wc_{t+2} - wc_{t+1}\right)\right] \\
&\quad + \frac{1}{2} Var_{t+1}\left[m_{t+2} + r_{c,t+2}\right].
\end{aligned}
$$

Solving for wc_{t+1} and iterating forward

$$\begin{aligned}
wc_{t+1} &= E_{t+1}\left[\log\delta + \kappa_0 + \left(1-\frac{1}{\psi}\right)\Delta c_{t+2} + \kappa_1 wc_{t+2}\right] + \frac{1}{2\theta}Var_{t+1}\left[m_{t+2}+r_{c,t+2}\right] \\
&= \log\delta + \kappa_0 + \left(1-\frac{1}{\psi}\right)E_{t+1}\left[\Delta c_{t+2}\right] + \frac{1}{2\theta}Var_{t+1}\left[m_{t+2}+r_{c,t+2}\right] + \kappa_1 E_{t+1}\left[wc_{t+2}\right] \\
&= \log\delta + \kappa_0 + \left(1-\frac{1}{\psi}\right)E_{t+1}\left[\Delta c_{t+2}\right] + \frac{1}{2\theta}Var_{t+1}\left[m_{t+2}+r_{c,t+2}\right] \\
&\quad +\kappa_1 E_{t+1}\left[\log\delta + \kappa_0 + \left(1-\frac{1}{\psi}\right)E_{t+2}\left[\Delta c_{t+3}\right] + \frac{1}{2\theta}Var_{t+2}\left[m_{t+3}+r_{c,t+3}\right] + \kappa_1 E_{t+2}\left[wc_{t+3}\right]\right] \\
&= (1+\kappa_1)(\log\delta+\kappa_0) \\
&\quad + \left(1-\frac{1}{\psi}\right)\left[E_{t+1}\left[\Delta c_{t+2}\right] + \kappa_1 E_{t+2}\left[\Delta c_{t+3}\right]\right] \\
&\quad + \frac{1}{2\theta}E_{t+1}\left[Var_{t+1}\left[m_{t+2}+r_{c,t+2}\right] + \kappa_1 Var_{t+2}\left[m_{t+3}+r_{c,t+3}\right]\right] \\
&\quad + \kappa_1^2 E_{t+1}\left[wc_{t+3}\right] \\
&= ... \\
&= (\log\delta+\kappa_0)\sum_{j=0}^{\infty}\kappa_1^j + \left(1-\frac{1}{\psi}\right)E_{t+1}\sum_{j=0}^{\infty}\kappa_1^j E_{t+1+j}\left[\Delta c_{t+2+j}\right] \\
&\quad + \frac{1}{2\theta}E_{t+1}\sum_{j=0}^{\infty}\kappa_1^j Var_{t+1+j}\left[m_{t+2+j}+r_{c,t+2+j}\right] + \lim_{j\to\infty}\kappa_1^j E_{t+1+j} wc_{t+2+j}. \qquad \text{(OA.57)}
\end{aligned}$$

Assuming the that the bubble term vanishes and applying $(E_{t+1} - E_t)$ to both sides gives

$$\begin{aligned}
(E_{t+1}-E_t)\,wc_{t+1} &= \left(1-\frac{1}{\psi}\right)(E_{t+1}-E_t)\sum_{j=0}^{\infty}\kappa_1^j E_{t+1+j}\left[\Delta c_{t+2+j}\right] \\
&\quad + \frac{1}{2\theta}(E_{t+1}-E_t)\sum_{j=0}^{\infty}\kappa_1^j Var_{t+1+j}\left[m_{t+2+j}+r_{c,t+2+j}\right]. \qquad \text{(OA.58)}
\end{aligned}$$

Using (OA.56) in (OA.58) gives

$$\begin{aligned}
(E_{t+1} - E_t)\, wc_{t+1} &= \left(1 - \frac{1}{\psi}\right)(E_{t+1} - E_t) \sum_{j=0}^{\infty} \kappa_1^j E_{t+1+j}\left[\Delta c_{t+2+j}\right] \\
&\quad + \frac{1}{2\theta}(E_{t+1} - E_t) \sum_{j=0}^{\infty} \kappa_1^j Var_{t+1+j}\left[m_{t+2+j} + r_{c,t+2+j}\right] \\
&= \left(1 - \frac{1}{\psi}\right)(E_{t+1} - E_t) \sum_{j=0}^{\infty} \kappa_1^j E_{t+1+j}\left[\Delta c_{t+2+j}\right] \\
&\quad + \frac{\theta \kappa_1^2}{2}(E_{t+1} - E_t) \sum_{j=0}^{\infty} \kappa_1^j Var_{t+1+j}\left(wc_{t+2+j}\right). \qquad \text{(OA.59)}
\end{aligned}$$

Plugging (OA.12) into (OA.1) and using (OA.2) gives

$$\begin{aligned}
m_{t+1} &= \theta \log \delta - \frac{\theta}{\psi} \Delta c_{t+1} - (1-\theta)\, r_{c,t+1} \\
&= \theta \log \delta + (\theta - 1)\, \kappa_0 - (\theta - 1)\, wc_t - \gamma \Delta c_{t+1} - (1-\theta)\, \kappa_1 wc_{t+1}.
\end{aligned}$$

Applying $(E_{t+1} - E_t)$ to both sides and using (OA.59) gives equations (17)-(18)

$$\begin{aligned}
m_{t+1} - E_t m_{t+1} &= -\gamma\,(E_{t+1} - E_t)\,\Delta c_{t+1} - (1-\theta)\,\kappa_1\,(E_{t+1} - E_t)\, wc_{t+1} \\
&= -\gamma(E_{t+1} - E_t)\Delta c_{t+1} \\
&\quad -(1-\theta)\kappa_1 \Bigg(\left(1 - \frac{1}{\psi}\right)(E_{t+1} - E_t) \sum_{j=0}^{\infty} \kappa_1^j E_{t+1+j}\left[\Delta c_{t+2+j}\right] \\
&\quad + \frac{\theta \kappa_1^2}{2}(E_{t+1} - E_t) \sum_{j=0}^{\infty} \kappa_1^j Var_{t+1+j}\left(wc_{t+2+j}\right) \Bigg) \\
&= -\gamma N_{C,t+1} - (1-\theta)\,\kappa_1 \left(\left(1 - \frac{1}{\psi}\right) N_{PATH,t+1} + \frac{\theta \kappa_1^2}{2} N_{RISK,t+1} \right),
\end{aligned}$$

where we define

$$
\begin{aligned}
N_{C,t+1} &\equiv (E_{t+1} - E_t)\,\Delta c_{t+1}, \\
N_{PATH,t+1} &\equiv (E_{t+1} - E_t) \sum_{j=0}^{\infty} \kappa_1^j E_{t+1+j}\left[\Delta c_{t+2+j}\right], \\
N_{RISK,t+1} &\equiv (E_{t+1} - E_t) \sum_{j=0}^{\infty} \kappa_1^j Var_{t+1+j}\left(wc_{t+2+j}\right).
\end{aligned}
$$

2.7 Shocks Driving $N_{C,t+1}$, $N_{PATH,t+1}$, and $N_{RISK,t+1}$

Using equation (11), news to $N_{C,t+1}$ depends only on the shock η_{t+1}, such that

$$
-\gamma N_{C,t+1} = -\gamma \sigma_c \eta_{t+1}. \tag{OA.60}
$$

$N_{PATH,t+1}$ in equation (19) can be obtained by direct computation using equation (11) and the law of iterated expectations:

$$
\begin{aligned}
N_{PATH,t+1} &= (E_{t+1} - E_t) \sum_{j=0}^{\infty} \kappa_1^j E_{t+1+j}\left[\Delta c_{t+2+j}\right] \\
&= (E_{t+1} - E_t) \sum_{j=0}^{\infty} \kappa_1^j \Delta c_{t+2+j} \\
&= (E_{t+1} - E_t)\,\Delta c_{t+2} + (E_{t+1} - E_t) \sum_{j=1}^{\infty} \kappa_1^j \Delta c_{t+2+j} \\
&= (\phi_\pi \rho_c + \xi_c \varphi_t)\, u_{t+1} + \sum_{j=1}^{\infty} \kappa_1^j \rho_\pi^{j-1} \rho_c \left(\xi_\pi + \rho_\pi \phi_\pi\right) u_{t+1} \\
&= \left(\frac{\phi_\pi \rho_c}{1 - \rho_\pi \kappa_1} + \xi_c \varphi_t + \frac{\rho_c \kappa_1 \xi_\pi}{1 - \rho_\pi \kappa_1} \right) u_{t+1}.
\end{aligned} \tag{OA.61}
$$

We now show that $N_{RISK,t+1}$ depends only on the nominal-real covariance shock w_{t+1}.

Equations (18) and (OA.38) give

$$\begin{aligned} m_{t+1} - E_t\left[m_{t+1}\right] &= -\gamma N_{C,t+1} - (1-\theta)\,\kappa_1 \left(\left(1-\frac{1}{\psi}\right) N_{PATH,t+1} + \frac{\theta\kappa_1^2}{2} N_{RISK,t+1}\right) \\ &= \kappa_1\left(\theta-1\right)\left(\phi_\pi A_1 + \varphi_t A_2 + A_3\right) u_{t+1} \\ &\quad + \left(\theta\left(1-\frac{1}{\psi}\right) - 1\right)\sigma_c \eta_{t+1} \\ &\quad + \kappa_1 \sigma_w \left(\theta-1\right)\left(A_4 + 2\varphi_0 A_5\left(1-v\right) + 2vA_5\varphi_t\right) w_{t+1} \\ &\quad + \sqrt{2}\kappa_1\sigma_w^2\left(\theta-1\right) A_5 \left(\frac{w_{t+1}^2-1}{\sqrt{2}}\right). \end{aligned}$$

Solving for $\frac{\kappa_1^3}{2}\theta\left(\theta-1\right) N_{RISK,t+1}$ gives

$$\begin{aligned} +\frac{\kappa_1^3}{2}\theta\left(\theta-1\right) N_{RISK,t+1} &= \kappa_1\left(\theta-1\right)\left(\phi_\pi A_1 + \varphi_t A_2 + A_3\right) u_{t+1} \\ &\quad - \left(\theta-1\right)\kappa_1\left(1-\frac{1}{\psi}\right) N_{PATH,t+1} \\ &\quad + \left(\theta\left(1-\frac{1}{\psi}\right) - 1\right)\sigma_c \eta_{t+1} + \gamma N_{C,t+1} \\ &\quad + \kappa_1 \sigma_w \left(\theta-1\right)\left(A_4 + 2\varphi_0 A_5\left(1-v\right) + 2vA_5\varphi_t\right) w_{t+1} \\ &\quad + \sqrt{2}\kappa_1\sigma_w^2\left(\theta-1\right) A_5 \left(\frac{w_{t+1}^2-1}{\sqrt{2}}\right). \end{aligned}$$

Using (OA.26)-(OA.28), (OA.60) and (OA.61), the last equation becomes

$$\begin{aligned} \frac{\kappa_1^3}{2}\theta(\theta-1)N_{RISK,t+1} &= \kappa_1 \sigma_w \left(\theta-1\right)\left(A_4 + 2\varphi_0 A_5\left(1-v\right) + 2vA_5\varphi_t\right) w_{t+1} \\ &\quad + \sqrt{2}\kappa_1\sigma_w^2\left(\theta-1\right) A_5 \left(\frac{w_{t+1}^2-1}{\sqrt{2}}\right), \end{aligned} \tag{OA.62}$$

which shows $N_{RISK,t+1}$ depends on the shock w_{t+1} but neither on η_{t+1} nor u_{t+1}.

Using equations (OA.26)-(OA.30) and (OA.39)-(OA.42), equations (OA.60), (OA.61) and

(OA.62) can be written as

$$\begin{aligned}\gamma N_{C,t+1} &= \lambda_\eta \eta_{t+1}, \\ \kappa_1 (1-\theta)\left(1-\frac{1}{\psi}\right) N_{PATH,t+1} &= \lambda_{u,t} u_{t+1}, \\ \frac{\kappa_1^3}{2}\theta(\theta-1) N_{RISK,t+1} &= -\lambda_{w,t} w_{t+1} - \lambda_{2w}\left(\frac{w_{t+1}^2-1}{\sqrt{2}}\right). \end{aligned} \tag{OA.63}$$

2.8 Derivation of equation (22)

Using equation (OA.13) in equation (OA.19) and solving for $pd_{i,t+1}$ gives

$$pd_{i,t+1} = E_{t+1}\left[m_{t+2} + \kappa_{i,0} + \Delta d_{i,t+2} + \kappa_{i,1} pd_{i,t+2}\right] + \frac{1}{2} Var_{t+1}\left[m_{t+2} + \kappa_{i,1} pd_{i,t+2} + \Delta d_{i,t+2}\right]. \tag{OA.64}$$

Iterating forward, we get

$$\begin{aligned} pd_{i,t+1} &= E_{t+1}\left[m_{t+2} + \kappa_{i,0} + \Delta d_{i,t+2}\right] + \frac{1}{2} Var_{t+1}\left[m_{t+2} + \kappa_{i,1} pd_{i,t+2} + \Delta d_{i,t+2}\right] \\ &\quad + \kappa_{i,1} E_{t+1}\left[pd_{i,t+2}\right] \\ &= E_{t+1}\left[m_{t+2} + \kappa_{i,0} + \Delta d_{i,t+2}\right] + \frac{1}{2} Var_{t+1}\left[m_{t+2} + \kappa_{i,1} pd_{i,t+2} + \Delta d_{i,t+2}\right] \\ &\quad + \kappa_{i,1} E_{t+1}\Big[E_{t+2}\left[m_{t+3} + \kappa_{i,0} + \Delta d_{i,t+3}\right] + \frac{1}{2} Var_{t+2}\left[m_{t+3} + \kappa_{i,1} pd_{i,t+3} + \Delta d_{i,t+3}\right] \\ &\quad + \kappa_{i,1} E_{t+2}\left[pd_{i,t+3}\right]\Big] \\ &= E_{t+1}\left[m_{t+2} + \kappa_{i,0} + \Delta d_{i,t+2}\right] + \kappa_{i,1} E_{t+1} E_{t+2}\left[m_{t+3} + \kappa_{i,0} + \Delta d_{i,t+3}\right] \\ &\quad + \frac{1}{2} Var_{t+1}\left[m_{t+2} + \kappa_{i,1} pd_{i,t+2} + \Delta d_{i,t+2}\right] + \frac{1}{2}\kappa_{i,1} E_{t+1} Var_{t+2}\left[m_{t+3} + \kappa_{i,1} pd_{i,t+3} + \Delta d_{i,t+3}\right] \\ &\quad + \kappa_{i,1} E_{t+1}\left[pd_{i,t+3}\right] \\ &= \ldots \\ &= E_{t+1} \sum_{q=0}^{\infty} \kappa_{i,1}^q E_{t+1+q}\left[m_{t+2+q} + \kappa_{i,0} + \Delta d_{i,t+2+q}\right] \\ &\quad + \frac{1}{2} E_{t+1} \sum_{q=0}^{\infty} \kappa_{i,1}^q Var_{t+1+q}\left[m_{t+2+q} + \kappa_{i,1} pd_{i,t+2+q} + \Delta d_{i,t+2+q}\right] \\ &\quad + \lim_{q\to\infty} \kappa_{i,1}^q E_{t+1}\left[pd_{i,t+2+q}\right]. \end{aligned} \tag{OA.65}$$

Applying $(E_{t+1} - E_t)$ to both sides and assuming that the bubble term is zero

$$\begin{aligned}
(E_{t+1} - E_t) pd_{i,t+1} &= (E_{t+1} - E_t) \sum_{q=0}^{\infty} \kappa_{i,1}^{q} E_{t+1+q} [m_{t+2+q}] \\
&+ \frac{1}{2} (E_{t+1} - E_t) \sum_{q=0}^{\infty} \kappa_{i,1}^{q} Var_{t+1+q} [m_{t+2+q}] \qquad \text{(OA.66)} \\
&+ (E_{t+1} - E_t) \sum_{q=0}^{\infty} \kappa_{i,1}^{q} E_{t+1+q} [\Delta d_{i,t+2+q}] \\
&+ \frac{\kappa_{i,1}^{2}}{2} (E_{t+1} - E_t) \sum_{q=0}^{\infty} \kappa_{i,1}^{q} Var_{t+1+q} [pd_{i,t+2+q}] \\
&+ \frac{1}{2} (E_{t+1} - E_t) \sum_{q=0}^{\infty} \kappa_{i,1}^{q} Var_{t+1+q} [\Delta d_{i,t+2+q}] \\
&+ \kappa_{i,1} (E_{t+1} - E_t) \sum_{q=0}^{\infty} \kappa_{i,1}^{q} Cov_{t+1+q} [m_{t+2+q}, pd_{i,t+2+q}] \\
&+ \kappa_{i,1} (E_{t+1} - E_t) \sum_{q=0}^{\infty} \kappa_{i,1}^{q} Cov_{t+1+q} [pd_{i,t+2+q}, \Delta d_{i,t+2+q}] \qquad \text{(OA.67)}
\end{aligned}$$

Applying $(E_{t+1} - E_t)$ to both sides of equation (OA.13) and using equation (OA.67) gives equation (22)

$$\begin{aligned}
(E_{t+1} - E_t) r_{i,t+1} &= \kappa_{i,1} (E_{t+1} - E_t) pd_{i,t+1} + (E_{t+1} - E_t) \Delta d_{i,t+1} \\
&= \kappa_{i,1} \left(N_{PATH,t+1}^{i} + N_{m,t+1}^{i} + \frac{1}{2} N_{RISK,t+1}^{i} \right) + N_{D,t+1}^{i},
\end{aligned}$$

where we define

$$\begin{aligned}
N^i_{PATH,t+1} &\equiv (E_{t+1}-E_t)\sum_{q=0}^{\infty}\kappa_{i,1}^q E_{t+1+q}\left[\Delta d_{i,t+2+q}\right], \\
N^i_{m,t+1} &\equiv (E_{t+1}-E_t)\sum_{q=0}^{\infty}\kappa_{i,1}^q E_{t+1+q}\left[m_{t+2+q}\right], \\
N^i_{RISK,t+1} &\equiv (E_{t+1}-E_t)\sum_{q=0}^{\infty}\kappa_{i,1}^q Var_{t+1+q}\left[m_{t+2+q}+\kappa_{i,1}pd_{i,t+1}+\Delta d_{i,t+1}\right] \\
&= \frac{1}{2}(E_{t+1}-E_t)\sum_{q=0}^{\infty}\kappa_{i,1}^q Var_{t+1+q}\left[m_{t+2+q}\right] \\
&\quad +\frac{\kappa_{i,1}^2}{2}(E_{t+1}-E_t)\sum_{q=0}^{\infty}\kappa_{i,1}^q Var_{t+1+q}\left[pd_{i,t+2+q}\right] \\
&\quad +\frac{1}{2}(E_{t+1}-E_t)\sum_{q=0}^{\infty}\kappa_{i,1}^q Var_{t+1+q}\left[\Delta d_{i,t+2+q}\right] \\
&\quad +\kappa_{i,1}(E_{t+1}-E_t)\sum_{q=0}^{\infty}\kappa_{i,1}^q Cov_{t+1+q}\left[m_{t+2+q},pd_{i,t+2+q}\right] \\
&\quad +\kappa_{i,1}(E_{t+1}-E_t)\sum_{q=0}^{\infty}\kappa_{i,1}^q Cov_{t+1+q}\left[pd_{i,t+2+q},\Delta d_{i,t+2+q}\right] \\
&\quad +\kappa_{i,1}(E_{t+1}-E_t)\sum_{q=0}^{\infty}\kappa_{i,1}^q Cov_{t+1+q}\left[m_{t+2+q},\Delta d_{i,t+2+q}\right], \\
N^i_{D,t+1} &\equiv (E_{t+1}-E_t)\Delta d_{i,t+1}.
\end{aligned}$$

2.9 Shocks Driving $N^i_{PATH,t+1}, N^i_{m,t+1}, N^i_{RISK,t+1}$ and $N^i_{D,t+1}$

$N^i_{PATH,t+1}$ can be computed directly from equations (OA.3)-(OA.6):

$$\begin{aligned}
N^i_{PATH,t+1} &\equiv (E_{t+1}-E_t)\sum_{q=0}^{\infty}\kappa_{i,1}^q E_{t+1+q}\left[\Delta d_{i,t+2+q}\right] \\
&= (E_{t+1}-E_t)\sum_{q=0}^{\infty}\kappa_{i,1}^q\left(\mu_i+\rho_i\left(\pi_{t+1+q}-\mu_\pi\right)+\xi_i\varphi_{t+q}u_{t+1+q}\right) \\
&= \left(\frac{\rho_i\left(\phi_\pi+\kappa_{i,1}\xi_\pi\right)}{1-\rho_\pi\kappa_{i,1}}+\xi_i\varphi_t\right)u_{t+1}.
\end{aligned}$$

and depends only on u_{t+1}.

Iterating equation (OA.3) forward

$$\begin{aligned}
\pi_{t+j} &= \mu_\pi + \rho_\pi \left(\pi_{t+j-1} - \mu_\pi\right) + \phi_\pi u_{t+j} + \xi_\pi u_{t+j-1} \\
&= \mu_\pi + \rho_\pi \left(\rho_\pi \left(\pi_{t+j-2} - \mu_\pi\right) + \phi_\pi u_{t+j-1} + \xi_\pi u_{t+j-2}\right) + \phi_\pi u_{t+j} + \xi_\pi u_{t+j-1} \\
&= ... \\
&= \mu_\pi + \rho_\pi^j \left(\pi_t - \mu_\pi\right) + \phi_\pi u_{t+j} - \rho_\pi^j \phi_\pi u_t + \sum_{q=0}^{j-1} \rho_\pi^q \left(\rho_\pi \phi_\pi + \xi_\pi\right) u_{t+j-1-q}, \quad \text{(OA.68)}
\end{aligned}$$

gives

$$\left(E_{t+1} - E_t\right) \pi_{t+1+j} = \begin{cases} \phi_\pi u_{t+1} & , \text{ if } j = 0 \\ \rho_\pi^{j-1} \left(\rho_\pi \phi_\pi + \xi_\pi\right) u_{t+1} & , \text{ if } j > 0 \end{cases}.$$

It follows that

$$\begin{aligned}
\left(E_{t+1} - E_t\right) \sum_{q=0}^{\infty} \kappa_{i,1}^q \pi_{t+1+j} &= \left(E_{t+1} - E_t\right) \pi_{t+1} + \left(E_{t+1} - E_t\right) \sum_{q=1}^{\infty} \kappa_{i,1}^q \pi_{t+1+q} \\
&= \left(\phi_\pi + \frac{\kappa_{i,1} \left(\rho_\pi \phi_\pi + \xi_\pi\right)}{1 - \kappa_{i,1} \rho_\pi}\right) u_{t+1}. \quad \text{(OA.69)}
\end{aligned}$$

Using equation (OA.12) in equation (OA.1) gives

$$\begin{aligned}
E_{t+1+j}\left[m_{t+2+j}\right] &= \theta \log \delta + (\theta - 1) \kappa_0 - \gamma E_{t+1+j} \Delta c_{t+2+j} - (\theta - 1) wc_{t+1+j} \\
&\quad + (\theta - 1) \kappa_1 E_{t+1+j} wc_{t+2+j}.
\end{aligned}$$

Plugging (OA.4) and (OA.10) into the last equation and computing expectations

$$\begin{aligned}
E_{t+1+j}\left[m_{t+2+j}\right] &= \theta\log\delta+(\theta-1)\,\kappa_0-\gamma\left(\mu_c+\rho_c\left(\pi_{t+1+j}-\mu_\pi\right)+\xi_c\varphi_{t+j}u_{t+1+j}\right) \\
&\quad -(\theta-1)\Big(A_0+A_1\left(\pi_{t+1+j}-\mu_\pi\right)+A_2\varphi_{t+j}u_{t+1+j}+A_3u_{t+1+j} \\
&\quad +A_4\left(\varphi_{t+1+j}-\varphi_0\right)+A_5\left(\varphi^2_{t+1+j}-E\left[\varphi^2_t\right]\right)\Big) \\
&\quad +(\theta-1)\,\kappa_1\Big(A_0+A_1\left(\rho_\pi\left(\pi_{t+1+j}-\mu_\pi\right)+\xi_\pi u_{t+1+j}\right) \\
&\quad +A_4\left(E_{t+1+j}\varphi_{t+2+j}-\varphi_0\right)+A_5\left(E_{t+1+j}\varphi^2_{t+2+j}-E\left[\varphi^2_t\right]\right)\Big) \\
&= \theta\log\delta+(\theta-1)\,\kappa_0-\gamma\mu_c-(\theta-1)\,A_0+(\theta-1)\,\kappa_1A_0 \\
&\quad -\frac{\rho_c}{\psi}\left(\pi_{t+1+j}-\mu_\pi\right) \\
&\quad +\left((\theta-1)\left(\kappa_1A_1\xi_\pi-A_3\right)-\frac{1}{\psi}\xi_c\varphi_{t+j}\right)u_{t+1+j} \\
&\quad -(\theta-1)\,A_4\left(\varphi_{t+1+j}-\varphi_0\right)-(\theta-1)\,A_5\left(\varphi^2_{t+1+j}-E\left[\varphi^2_t\right]\right) \\
&\quad +(\theta-1)\,\kappa_1A_4\left(E_{t+1+j}\varphi_{t+2+j}-\varphi_0\right) \\
&\quad +(\theta-1)\,\kappa_1A_5\left(E_{t+1+j}\varphi^2_{t+2+j}-E\left[\varphi^2_t\right]\right). \qquad \text{(OA.70)}
\end{aligned}$$

$N^i_{m,t+1}$ can now be computed using equations (OA.69) and (OA.70)

$$
\begin{aligned}
N^i_{m,t+1} &= (E_{t+1}-E_t)\sum_{q=0}^{\infty}\kappa_{i,1}^q E_{t+1+j}\left[m_{t+2+j}\right] \\
&= -\frac{\rho_c}{\psi}\left(E_{t+1}-E_t\right)\sum_{q=0}^{\infty}\kappa_{i,1}^q \pi_{t+1+j} \\
&\quad +\left(E_{t+1}-E_t\right)\sum_{q=0}^{\infty}\kappa_{i,1}^q\left((\theta-1)\left(\kappa_1 A_1\xi_\pi - A_3\right)-\frac{1}{\psi}\xi_c\varphi_{t+j}\right)u_{t+1+j} \\
&\quad +\left(E_{t+1}-E_t\right)\sum_{q=0}^{\infty}\kappa_{i,1}^q\left[-(\theta-1)A_4\left(\varphi_{t+1+j}-\varphi_0\right)-(\theta-1)A_5\left(\varphi^2_{t+1+j}-E\left[\varphi^2_t\right]\right)\right] \\
&\quad +\left(E_{t+1}-E_t\right)\sum_{q=0}^{\infty}\kappa_{i,1}^q\Big[(\theta-1)\kappa_1 A_4\left(E_{t+1+j}\varphi_{t+2+j}-\varphi_0\right) \\
&\quad +(\theta-1)\kappa_1 A_5\left(E_{t+1+j}\varphi^2_{t+2+j}-E\left[\varphi^2_t\right]\right)\Big] \\
&= -\frac{\rho_c}{\psi}\frac{\kappa_{i,1}\left(\rho_\pi\phi_\pi+\xi_\pi\right)}{1-\kappa_{i,1}\rho_\pi}u_{t+1} \\
&\quad +\left((\theta-1)\left(\kappa_1 A_1\xi_\pi - A_3\right)-\frac{\rho_c}{\psi}\phi_\pi-\frac{1}{\psi}\xi_c\varphi_t\right)u_{t+1} \\
&\quad +\left(E_{t+1}-E_t\right)\sum_{q=0}^{\infty}\kappa_{i,1}^q\left[-(\theta-1)A_4\left(\varphi_{t+1+j}-\varphi_0\right)-(\theta-1)A_5\left(\varphi^2_{t+1+j}-E\left[\varphi^2_t\right]\right)\right] \\
&\quad +\left(E_{t+1}-E_t\right)\sum_{q=0}^{\infty}\kappa_{i,1}^q\Big[(\theta-1)\kappa_1 A_4\left(E_{t+1+j}\varphi_{t+2+j}-\varphi_0\right) \\
&\quad +(\theta-1)\kappa_1 A_5\left(E_{t+1+j}\varphi^2_{t+2+j}-E\left[\varphi^2_t\right]\right)\Big].
\end{aligned}
\tag{OA.71}
$$

By equation (OA.5), the innovations $(E_{t+1}-E_t)\,\varphi_{t+j+2}$ and $(E_{t+1}-E_t)\,\varphi^2_{t+j+2}$ only depend on the shock w_{t+1}, so we can write equation (OA.71) as

$$
\begin{aligned}
N^i_{m,t+1} &= -\frac{\rho_c}{\psi}\frac{\kappa_{i,1}\left(\rho_\pi\phi_\pi+\xi_\pi\right)}{1-\kappa_{i,1}\rho_\pi}u_{t+1} \\
&\quad +\left((\theta-1)\left(\kappa_1 A_1\xi_\pi - A_3\right)-\frac{\rho_c}{\psi}\phi_\pi-\frac{1}{\psi}\xi_c\varphi_t\right)u_{t+1} \\
&\quad +R\left(w_{t+1}\right),
\end{aligned}
\tag{OA.72}
$$

where $R(\cdot)$ is a function of w_{t+1} but not of (η_{t+1}, u_{t+1}).

By direct computation, equation (OA.6) implies that

$$\begin{aligned} N_{D,t+1}^{i} &\equiv \left(E_{t+1}-E_{t}\right)\Delta d_{i,t+1} \\ &= \sigma_{i}\eta_{t+1}, \end{aligned} \tag{OA.73}$$

and thus $N_{D,t+1}^{i}$ depends only on the shock η_{t+1}.

Finally, by noting that $N_{RISK,t+1}^{i}$ depends only on variances and covariances and that φ_t is the only source of heteroskedasticity in the model, $N_{RISK,t+1}^{i}$ depends only on the shock w_{t+1}.

2.10 Regression from Table 6

The goal of this section is to compute the coefficient $L_{\beta_{\Pi}\times NRC,m}$ in regression (9) when the regression is run in the model (instead of in the data, as is done in Table 6).

2.10.1 Preliminary Calculations

Iterating equation (OA.5) forward, we have that for $j>1$

$$\begin{aligned} \varphi_{t+j}-\varphi_{0} &= v\left(\varphi_{t+j-1}-\varphi_{0}\right)+\sigma_{w}w_{t+j} \\ &= v^{2}\left(\varphi_{t+j-2}-\varphi_{0}\right)+v\sigma_{w}w_{t+j-1}+\sigma_{w}w_{t+j} \\ &= ... \\ &= v^{j}\left(\varphi_{t}-\varphi_{0}\right)+v^{j-1}\sigma_{w}w_{t+1}+...+v\sigma_{w}w_{t+j-1}+\sigma_{w}w_{t+j} \\ &= v^{j}\left(\varphi_{t}-\varphi_{0}\right)+\sum_{q=0}^{j-1}v^{q}\sigma_{w}w_{t+j-q}. \end{aligned} \tag{OA.74}$$

Squaring both sides gives

$$\varphi_{t+j}^2 = \left(\varphi_0 + v^j\left(\varphi_t - \varphi_0\right)\right)^2 + 2\left(\varphi_0 + v^j\left(\varphi_t - \varphi_0\right)\right)\sum_{q=0}^{j-1} v^q \sigma_w w_{t+j-q} + \left(\sum_{q=0}^{j-1} v^q \sigma_w w_{t+j-q}\right)^2 . \tag{OA.75}$$

Taking expectations conditional on time t in equations (OA.68), (OA.74) and (OA.75) gives

$$E_t\left[\pi_{t+s-1}\right] - \mu_\pi = \begin{cases} \pi_t - \mu_\pi & , \text{ if } s = 1 \\ \rho_\pi^{s-1}\left(\pi_t - \mu_\pi\right) + \rho_\pi^{s-2}\xi_\pi u_t & , \text{ if } s > 1 \end{cases} ,$$

$$E_t\left[\varphi_{t+s-1} - \varphi_0\right] = \begin{cases} \varphi_t - \varphi_0 & , \text{ if } s = 1 \\ v^{s-1}\left(\varphi_t - \varphi_0\right) & , \text{ if } s > 1 \end{cases} ,$$

$$E_t\left[\varphi_{t+s-1}^2 - E\left[\varphi_t^2\right]\right] = \begin{cases} \varphi_t^2 - E\left[\varphi_t^2\right] & , \text{ if } s = 1 \\ \left(\varphi_0 + v^{s-1}\left(\varphi_t - \varphi_0\right)\right)^2 + \sigma_w^2 \frac{v^{2(s-1)}-1}{\nu^2-1} - E\left[\varphi_t^2\right] & , \text{ if } s > 1 \end{cases} .$$

Because φ_τ and u_t are independent for all t, τ , we have

$$E_t\left[\varphi_{t+s-2} u_{t+s-1}\right] = \begin{cases} \varphi_{t-1} u_t & , \text{ if } s = 1 \\ 0 & , \text{ if } s > 1 \end{cases} .$$

Since u_t and w_t are independent, we have

$$\begin{aligned} Cov\left(\pi_t, \varphi_t\right) &= 0, \\ Cov\left(u_t, \varphi_t\right) &= 0, \\ Cov\left(\varphi_{t-1} u_t, \varphi_t\right) &= E\left[\varphi_{t-1} u_t\left(\varphi_t - \varphi_0\right)\right] \\ &= E\left[E_t\left[\varphi_{t-1} u_t\left(\varphi_t - \varphi_0\right)\right]\right] \\ &= E\left[\varphi_{t-1} E_t\left[u_t\left(\varphi_t - \varphi_0\right)\right]\right] \\ &= E\left[\varphi_{t-1} E_t\left[u_t\right] E_t\left[\varphi_t - \varphi_0\right]\right] \\ &= 0, \end{aligned} \tag{OA.76}$$

and

$$\begin{aligned}
Cov\left(\pi_t, \varphi_t^2\right) &= 0, \\
Cov\left(u_t, \varphi_t^2\right) &= 0, \\
Cov\left(\varphi_{t-1}u_t, (\varphi_t - \varphi_0)^2\right) &= Cov\left(\varphi_{t-1}u_t, \varphi_0^2 - 2\varphi_0\varphi_t + \varphi_t^2\right) \\
&= Cov\left(\varphi_{t-1}u_t, \varphi_0^2\right) \\
&\quad -2\varphi_0 Cov\left(\varphi_{t-1}u_t, \varphi_t\right) \\
&\quad +Cov\left(\varphi_{t-1}u_t, \varphi_t^2\right) \\
&= Cov\left(\varphi_{t-1}u_t, \varphi_t^2\right) \\
&= E\left[\varphi_{t-1}u_t\left(\varphi_t^2 - E\left[\varphi_0^2\right]\right)\right] \\
&= E\left[\varphi_{t-1}E_t\left[u_t\left(\varphi_t^2 - E\left[\varphi_0^2\right]\right)\right]\right] \\
&= E\left[\varphi_{t-1}E_t\left[u_t\right]E_t\left[\varphi_t^2 - E\left[\varphi_0^2\right]\right]\right] \\
&= 0. \qquad \text{(OA.77)}
\end{aligned}$$

Define $\tilde{\theta}_t^{(12)}$ to be the nominal-real covariance $\theta_t^{(12)}$ normalized to have mean equal to zero and standard deviation equal to one, and $\tilde{\beta}_{u,it}$ to be the inflation beta $\beta_{u,it}$ minus its unconditional mean:

$$\begin{aligned}
\tilde{\theta}_t^{(12)} &\equiv \frac{\theta_t^{(12)} - E\left[\theta_t^{(12)}\right]}{std\left(\theta_t^{(12)}\right)}, \\
\tilde{\beta}_{u,it} &\equiv \beta_{u,it} - E\left[\beta_{u,it}\right].
\end{aligned}$$

Using equations (OA.5), (16) and (OA.45), we find

$$\tilde{\theta}_t^{(12)} = \frac{sign\left(\xi_c\right)}{sign\left(\phi_\pi\right)} \frac{\sqrt{1-\nu^2}}{\left|\sigma_w\right|}\left(\varphi_t - \varphi_0\right), \qquad \text{(OA.78)}$$

$$\tilde{\beta}_{u,it} = \kappa_{i,1}\left(\xi_i - \frac{\xi_c}{\psi}\right)\left(\varphi_t - \varphi_0\right). \qquad \text{(OA.79)}$$

and therefore

$$\tilde{\beta}_{u,it} \times \tilde{\theta}_t^{(12)} = \frac{sign\left(\xi_c\right)}{sign\left(\phi_\pi\right)} \frac{\kappa_{i,1}\sqrt{1-\nu^2}}{\left|\sigma_w\right|} \left(\xi_i - \frac{\xi_c}{\psi}\right) \left(\varphi_t - \varphi_0\right)^2 . \qquad \text{(OA.80)}$$

2.10.2 Running the Regression

We let the asset $i = m$ be the asset that represents the aggregate stock market in the model. We can write (log) returns cumulated from $t+1$ to $t+12$ as the sum of expected and unexpected components as of time t

$$\begin{aligned} r_{m,t+1:t+12} &= \sum_{s=1}^{12} r_{m,t+s} \\ &= \sum_{s=1}^{12} E_t\left[r_{m,t+s}\right] + \sum_{s=1}^{12} \left(r_{m,t+s} - E_t\left[r_{m,t+s}\right]\right). \end{aligned} \qquad \text{(OA.81)}$$

Using equations (OA.43), (OA.44) and (OA.20)-(OA.23) gives

$$\begin{aligned} \sum_{s=1}^{12} E_t\left[r_{m,t+1:t+12}\right] &= R_0 + R_1\left(\pi_t - \mu_\pi\right) + R_2\varphi_{t-1}u_t + R_3 u_t \\ &\quad + R_4\left(\varphi_t - \varphi_0\right) + R_5\left(\varphi_t^2 - E\left[\varphi_t^2\right]\right), \qquad \text{(OA.82)} \\ \sum_{s=1}^{12}\left(r_{m,t+s} - E_t\left[r_{m,t+s}\right]\right) &= \sum_{s=1}^{12}\left(\beta_{u,m,t+s-1}u_{t+s} + \beta_{w,m}w_{t+s}\right) \\ &\quad + \sum_{s=1}^{12}\left(\beta_{2w,m}\left(\frac{w_{t+s}^2 - 1}{\sqrt{2}}\right) + \beta_{\eta,m}\eta_{t+s}\right) \qquad \text{(OA.83)} \end{aligned}$$

where the constants R_i are given by

$$
\begin{aligned}
R_0 &= 12\left(\kappa_{m,0}+\mu_m-D_{m,0}\left(1-\kappa_{m,1}\right)\right), \\
R_1 &= \left(\rho_m-D_{m,1}\left(1-\kappa_{m,1}\rho_\pi\right)\right)\left(\sum_{s=1}^{12}\rho_\pi^{s-1}\right), \\
R_2 &= \xi_m-D_{m,2}, \\
R_3 &= \xi_\pi\left(\kappa_{m,1}\sum_{s=1}^{12}\rho_\pi^{s-1}+\frac{1-\rho_m}{\rho_\pi}\left(1-\sum_{s=1}^{12}\rho_\pi^{s-1}\right)\right)-D_{m,3}, \\
R_4 &= -2\varphi_0 v D_{m,5} v^{s-2}\sum_{s=1}^{12}\left(1-v^{s-1}-\kappa_{m,1}v\left(1-v^s\right)\right)-D_{m,4}\left(1-v\kappa_{m,1}\right)\left(\sum_{s=1}^{12}v^{s-1}\right), \\
R_5 &= -D_{m,5}\left(1-v^2\kappa_{m,1}\right)\left(\sum_{s=1}^{12}v^{2(s-1)}\right).
\end{aligned} \tag{OA.84}
$$

Defining

$$
\begin{aligned}
\varepsilon_{t+1:t+12} &\equiv \sum_{s=1}^{12}\left(\beta_{u,m,t+s-1}u_{t+s}+\beta_{w,m}w_{t+s}+\beta_{2w,m}\left(\frac{w_{t+s}^2-1}{\sqrt{2}}\right)+\beta_{\eta,m}\eta_{t+s}\right), \\
\varkappa_t &\equiv R_1\left(\pi_t-\mu_\pi\right)+R_2\varphi_{t-1}u_t+R_3u_t, \\
L_2 &\equiv -\frac{1}{\kappa_{m,1}}\frac{sign\left(\phi_\pi\right)}{sign\left(\xi_c\right)}\frac{|\sigma_w|}{\sqrt{1-\nu^2}}\frac{R_5}{\xi_c/\psi-\xi_m}, \\
L_1 &\equiv R_4+2R_5\varphi_0, \\
L_0 &\equiv R_0-\frac{\sigma_w^2}{1-v^2}R_5,
\end{aligned} \tag{OA.85}
$$

and using equations (OA.80), (OA.82) and (OA.83), we can write equation (OA.81) as

$$
r_{m,t+1:t+12}=L_0+L_1\left(\varphi_t-\varphi_0\right)+L_2\left(\tilde{\beta}_{u,mt}\times\tilde{\theta}_t^{(12)}\right)+\varkappa_t+\varepsilon_{t+1:t+12}. \tag{OA.86}
$$

Under the null of the model, running regression (OA.86) with or without including the regressors $\varkappa_t$ gives the same estimate $\hat{L}_0, \hat{L}_1, \hat{L}_2$ for L_0, L_1, L_2 since $\varkappa_t$ is uncorrelated to the other regressors by equations (OA.76)-(OA.77), and uncorrelated to the error term by the law of iterated expectations and the fact that $E_t\left[\varepsilon_{t+1:t+12}\right]=0$:

$$\begin{aligned} E\left[\varkappa_t \varepsilon_{t+1:t+12}\right] &= E\left[E_t\left[\varkappa_t \varepsilon_{t+1:t+12}\right]\right] \\ &= E\left[\varkappa_t E_t\left[\varepsilon_{t+1:t+12}\right]\right] \\ &= 0. \end{aligned}$$

Since we are interested in $\hat{L}_2$, we therefore run regression (OA.86) in the model without including $\varkappa_t$

$$r_{m,t+1:t+12}=L_0+L_1\left(\varphi_t-\varphi_0\right)+L_2\left(\tilde{\beta}_{u,mt}\times\tilde{\theta}_t^{(12)}\right)+\varepsilon_{t+1:t+12}, \tag{OA.87}$$

and obtain estimates $\hat{L}_0,\hat{L}_1,\hat{L}_2$.

In the data, we run the regression in equation (9) reproduced next for convenience:

$$r_{m,t+1:t+12}=L_{0,m}+L_{\beta_\Pi,m}\tilde{\beta}_{u,mt}+L_{NRC,m}\tilde{\theta}_t^{(12)}+L_{\beta_\Pi\times NRC,m}\left(\tilde{\beta}_{u,mt}\times\tilde{\theta}_t^{(12)}\right)+\varepsilon_{t+1:t+12}. \tag{OA.88}$$

We now show that, under the null of the model, the estimate $\hat{L}_{\beta_\Pi\times NRC,m}$ from regression (OA.88) is the same as the estimate $\hat{L}_2$ obtained from regression (OA.87). By equations (OA.78)-(OA.79),

$$\begin{aligned} L_{0,m}+L_{\beta_\Pi,m}\tilde{\beta}_{u,mt}+L_{NRC,m}\tilde{\theta}_t^{(12)} &= L_{0,m}+L_{\beta_\Pi,m}\kappa_{i,1}\left(\xi_i-\frac{\xi_c}{\psi}\right)\left(\varphi_t-\varphi_0\right) \\ &\quad +L_{NRC,m}\frac{sign\left(\xi_c\right)}{sign\left(\phi_\pi\right)}\frac{\sqrt{1-\nu^2}}{\left|\sigma_w\right|}\left(\varphi_t-\varphi_0\right) \\ &= L_{0,m} \\ &\quad +\left[L_{\beta_\Pi,m}\kappa_{i,1}\left(\xi_i-\frac{\xi_c}{\psi}\right)+L_{NRC,m}\frac{sign\left(\xi_c\right)}{sign\left(\phi_\pi\right)}\frac{\sqrt{1-\nu^2}}{\left|\sigma_w\right|}\right]\left(\varphi_t-\varphi_0\right), \end{aligned}$$

so regression (OA.88) can be written as

$$\begin{aligned} r_{m,t+1:t+12} &= L_{0,m}+\left[L_{\beta_\Pi,m}\kappa_{i,1}\left(\xi_i-\frac{\xi_c}{\psi}\right)+L_{NRC,m}\frac{sign\left(\xi_c\right)}{sign\left(\phi_\pi\right)}\frac{\sqrt{1-\nu^2}}{\left|\sigma_w\right|}\right]\left(\varphi_t-\varphi_0\right) \\ &\quad +L_{\beta_\Pi\times NRC,m}\left(\tilde{\beta}_{u,mt}\times\tilde{\theta}_t^{(12)}\right)+\varepsilon_{t+1:t+12}, \end{aligned}$$

where we can then identify

$$\begin{aligned}
\hat{L}_0 &= \hat{L}_{0,m}, \\
\hat{L}_1 &= \hat{L}_{\beta_\Pi,m}\kappa_{i,1}\left(\xi_i - \frac{\xi_c}{\psi}\right) + \hat{L}_{NRC,m}\frac{sign\left(\xi_c\right)}{sign\left(\phi_\pi\right)}\frac{\sqrt{1-\nu^2}}{\left|\sigma_w\right|}, \\
\hat{L}_2 &= \hat{L}_{\beta_\Pi \times NRC,m}.
\end{aligned}$$

Note that although $\hat{L}_{\beta_\Pi,m}$ and $\hat{L}_{NRC,m}$ cannot be separately identified in the model by running regression (OA.87), the coefficient we seek, $\hat{L}_{\beta_\Pi \times NRC,m}$, is identified by $\hat{L}_2$.

Plugging (OA.85) into (OA.84), we find

$$\begin{aligned}
L_2 &= L_{\beta_\Pi \times NRC,m} \\
&= \frac{1}{\kappa_{m,1}}\frac{sign\left(\phi_\pi\right)}{sign\left(\xi_c\right)}\frac{\left|\sigma_w\right|}{\sqrt{1-\nu^2}}\frac{D_{m,5}\left(1-v^2\kappa_{m,1}\right)\left(\sum_{s=1}^{12} v^{2(s-1)}\right)}{\xi_c/\psi - \xi_m}.
\end{aligned} \tag{OA.89}$$

Note also that

$$\begin{aligned}
\frac{cov\left(r_{m,t+1:t+12}, \tilde{\beta}_{u,it}\times\tilde{\theta}_t^{(12)}\right)}{var\left(\tilde{\beta}_{u,it}\times\tilde{\theta}_t^{(12)}\right)} &= \frac{cov\left(L_1\left(\varphi_t-\varphi_0\right), \tilde{\beta}_{u,mt}\times\tilde{\theta}_t^{(12)}\right)}{var\left(\tilde{\beta}_{u,it}\times\tilde{\theta}_t^{(12)}\right)} \\
&\quad + \frac{cov\left(L_2, \tilde{\beta}_{u,mt}\times\tilde{\theta}_t^{(12)}\right)}{var\left(\tilde{\beta}_{u,it}\times\tilde{\theta}_t^{(12)}\right)} \\
&\quad + \frac{cov\left(L_0 + \varkappa_t + \varepsilon_{t+1:t+12}, \tilde{\beta}_{u,it}\times\tilde{\theta}_t^{(12)}\right)}{var\left(\tilde{\beta}_{u,it}\times\tilde{\theta}_t^{(12)}\right)} \\
&= L_1\frac{cov\left(\varphi_t-\varphi_0, \tilde{\beta}_{u,it}\times\tilde{\theta}_t^{(12)}\right)}{var\left(\tilde{\beta}_{u,it}\times\tilde{\theta}_t^{(12)}\right)} + L_2 + \frac{cov\left(\varepsilon_{t+1:t+12}, \tilde{\beta}_{u,it}\times\tilde{\theta}_t^{(12)}\right)}{var\left(\tilde{\beta}_{u,it}\times\tilde{\theta}_t^{(12)}\right)} \\
&= \frac{sign\left(\xi_c\right)}{sign\left(\phi_\pi\right)}\frac{\kappa_{i,1}\sqrt{1-\nu^2}}{\left|\sigma_w\right|}\left(\xi_i-\frac{\xi_c}{\psi}\right)L_1\frac{cov\left(\varphi_t-\varphi_0,\left(\varphi_t-\varphi_0\right)^2\right)}{var\left(\tilde{\beta}_{u,it}\times\tilde{\theta}_t^{(12)}\right)} \\
&\quad + L_2 + \frac{sign\left(\xi_c\right)}{sign\left(\phi_\pi\right)}\frac{\kappa_{i,1}\sqrt{1-\nu^2}}{\left|\sigma_w\right|}\left(\xi_i-\frac{\xi_c}{\psi}\right)\frac{cov\left(\varepsilon_{t+1:t+12},\left(\varphi_t-\varphi_0\right)^2\right)}{vvar\left(\tilde{\beta}_{u,it}\times\tilde{\theta}_t^{(12)}\right)} \\
&= L_2,
\end{aligned}$$

since

$$
\begin{aligned}
cov\left(\varphi_t - \varphi_0, (\varphi_t - \varphi_0)^2\right) &= 0, \\
\frac{cov\left(\varepsilon_{t+1:t+12}, (\varphi_t - \varphi_0)^2\right)}{var\left((\varphi_t - \varphi_0)^2\right)} &= 0.
\end{aligned}
$$

It then follows that $L_{\beta_\Pi \times NRC,m}$ can actually be computed from a univariate regression of $r_{m,t+1:t+12}$ on $\tilde{\beta}_{u,it} \times \tilde{\theta}_t^{(12)}$ since $\tilde{\beta}_{u,it} \times \tilde{\theta}_t^{(12)}$ is uncorrelated to the rest of the regressors.

2.11 Unconditional and Conditional Predictability of Consumption with Inflation

We show how to compute, in the model, the coefficients $d_0^{u,K}, d_1^{u,K}, d_0^{c,K}, d_1^{c,K}$ and the R^2 of the regressions in equations (1) and (3) and Table 2.

Using equation (OA.68), we have

$$
\begin{aligned}
\Delta c_{t+1+j} &= \mu_c + \rho_c\left(\pi_{t+j} - \mu_\pi\right) + \sigma_c \eta_{t+1+j} + \xi_c \varphi_{t-1+j} u_{t+j} \\
&= \mu_c + \rho_c\left(\pi_{t+j} - \mu_\pi\right) + \sigma_c \eta_{t+1+j} + \xi_c \varphi_{t-1+j} u_{t+j} \\
&= \mu_c + \rho_c\left(\rho_\pi^j\left(\pi_t - \mu_\pi\right) + \phi_\pi u_{t+j} - \rho_\pi^j \phi_\pi u_t + \sum_{q=0}^{j-1} \rho_\pi^q\left(\rho_\pi \phi_\pi + \xi_\pi\right) u_{t+j-1-q}\right) \\
&\quad + \sigma_c \eta_{t+1+j} + \xi_c\left(\varphi_0 + v^j\left(\varphi_{t-1} - \varphi_0\right) + \sum_{q=0}^{j-1} v^q \sigma_w w_{t-1+j-q}\right) u_{t+j}, \qquad \text{(OA.90)}
\end{aligned}
$$

and therefore

$$
\begin{aligned}
E_t\left[\Delta c_{t+1+j}\right] &= \mu_c+\rho_c\left(\rho_\pi^j\left(\pi_t-\mu_\pi\right)+\phi_\pi E_t\left[u_{t+j}\right]-\rho_\pi^j\phi_\pi E_t\left[u_t\right]+\sum_{q=0}^{j-1}\rho_\pi^q\left(\rho_\pi\phi_\pi+\xi_\pi\right)E_t\left[u_{t+j-1-q}\right]\right)\\
&\quad+\sigma_c E_t\left[\eta_{t+1+j}\right]+\xi_c\left(\varphi_0+v^j\left(\varphi_{t-1}-\varphi_0\right)+\sum_{q=0}^{j-1}v^q\sigma_w E_t\left[w_{t-1+j-q}\right]\right)E_t\left[u_{t+j}\right]\\
&= \mu_c+\rho_c\left(\rho_\pi^j\left(\pi_t-\mu_\pi\right)-\rho_\pi^j\phi_\pi u_t+\left(\rho_\pi\phi_\pi+\xi_\pi\right)E_t\left[u_{t+j-1}\right]\right)\\
&\quad+\rho_c\sum_{q=1}^{j-1}\rho_\pi^q\left(\rho_\pi\phi_\pi+\xi_\pi\right)E_t\left[u_{t+j-1-q}\right] \qquad \text{(OA.91)}\\
&= \mu_c+\rho_c\left(\rho_\pi^j\left(\pi_t-\mu_\pi\right)-\rho_\pi^j\phi_\pi u_t+\rho_\pi^{j-1}\left(\rho_\pi\phi_\pi+\xi_\pi\right)u_t\right). \qquad \text{(OA.92)}
\end{aligned}
$$

Using equations (OA.3)-(OA.5), (OA.90), (OA.92), and the law of total covariance

$$
\begin{aligned}
Cov\left(\Delta c_{t+1+j},\pi_{t+1}\right) &= E\left[Cov_t\left(\Delta c_{t+1+j},\pi_{t+1}\right)\right]+cov\left(E_t\left[\Delta c_{t+1+j}\right],E_t\left[\pi_{t+1}\right]\right)\\
&= \begin{cases}\rho_\pi\rho_c Var\left(\pi_t\right)+\varphi_0\xi_c\left(\xi_\pi+\phi_\pi\rho_\pi\right)+\rho_c\xi_\pi\phi_\pi & \text{, if } j=0\\ \phi_\pi^2\rho_c+\phi_\pi\xi_c\varphi_0 & \text{, if } j=1\\ \phi_\pi\rho_c\rho_\pi^{j-2}\left(\xi_\pi+\phi_\pi\rho_\pi\right) & \text{, if } j>1\end{cases}\\
&\quad+\begin{cases}\rho_\pi\rho_c Var\left(\pi_t\right)+\rho_c\xi_\pi\phi_\pi+\varphi_0\xi_c\left(\xi_\pi+\phi_\pi\rho_\pi\right) & \text{, if } j=0\\ \rho_\pi^{j+1}\rho_c Var\left(\pi_t\right)+\rho_c\xi_\pi\rho_\pi^{j-1}\left(\xi_\pi+2\phi_\pi\rho_\pi\right) & \text{, if } j>0\end{cases}\\
&= \begin{cases}2\left(\rho_\pi\rho_c Var\left(\pi_t\right)+\varphi_0\xi_c\left(\xi_\pi+\phi_\pi\rho_\pi\right)+\rho_c\phi_\pi\xi_\pi\right) & \text{, if } j=0\\ \rho_c Var\left(\pi_t\right)+\xi_c\varphi_0\phi_\pi & \text{, if } j=1\\ \rho_c\rho_\pi^{j-1}Var\left(\pi_t\right)+\rho_\pi^{j-2}\rho_c\phi_\pi\xi_\pi & \text{, if } j>1\end{cases}.
\end{aligned}
$$

It follows that

$$\begin{aligned}
d_1^{u,K} &= \frac{Cov\left(\sum_{j=0}^{K-1}\Delta c_{t+1+j},\pi_t\right)}{Var(\pi_t)} \\
&= \frac{\sum_{j=0}^{K-1} Cov\left(\Delta c_{t+1+j},\pi_t\right)}{Var(\pi_t)} \\
&= \frac{Cov\left(\Delta c_{t+1},\pi_t\right)}{Var(\pi_t)} + \frac{Cov\left(\Delta c_{t+2},\pi_t\right)}{Var(\pi_t)} + \frac{\sum_{j=2}^{K-1} Cov\left(\Delta c_{t+1+j},\pi_t\right)}{Var(\pi_t)} \\
&= \frac{2\left(\rho_\pi\rho_c Var(\pi_t) + \varphi_0\xi_c\left(\xi_\pi + \phi_\pi\rho_\pi\right) + \rho_c\phi_\pi\xi_\pi\right)}{Var(\pi_t)} + \frac{\rho_c Var(\pi_t) + \xi_c\varphi_0\phi_\pi}{Var(\pi_t)} \\
&\quad + \frac{\sum_{j=2}^{K-1}\left(\rho_c\rho_\pi^{j-1}Var(\pi_t) + \rho_\pi^{j-2}\rho_c\phi_\pi\xi_\pi\right)}{Var(\pi_t)} \\
&= 2\rho_\pi\rho_c + \rho_c + \frac{\varphi_0\xi_c\left(\phi_\pi + 2\phi_\pi\rho_\pi + 2\xi_\pi\right) + \rho_c\phi_\pi\xi_\pi}{Var(\pi_t)} \\
&\quad + \rho_c\sum_{j=2}^{K-1}\rho_\pi^{j-1} + \frac{\rho_c\phi_\pi\xi_\pi}{Var(\pi_t)}\sum_{j=2}^{K-1}\rho_\pi^{j-2} \\
&= \rho_c\left(\frac{1-\rho_\pi^K}{1-\rho_\pi}\right)\left(1 + \frac{\xi_\pi\phi_\pi}{Var(\pi_t)}\left(\frac{1-\rho_\pi^{K-1}}{1-\rho_\pi^K}\right)\right) + \frac{\xi_c\phi_\pi}{Var(\pi_t)}\varphi_0, \qquad \text{(OA.93)}
\end{aligned}$$

and

$$\begin{aligned}
d_0^{u,K} &= E\left[\sum_{j=0}^{K-1}\Delta c_{t+1+j} - d_1^{u,K}\pi_t\right] \\
&= \sum_{j=0}^{K-1} E\left[\Delta c_{t+1+j}\right] - d_1^{u,K}E\left[\pi_t\right] \\
&= K\mu_c - d_1^{u,K}\mu_\pi. \qquad \text{(OA.94)}
\end{aligned}$$

Now we turn to the conditional regression of Table 2. Because of the way we constructed b_t^K, we can write

$$\sum_{j=0}^{K-1}\Delta c_{t+1+j} = a_t^K + b_t^K\pi_t + \varepsilon_{t+K},$$

where, using equations (OA.3) and (OA.92), we have

$$\begin{aligned}
a_t^K &= E_t\left[\sum_{j=1}^{K}\Delta c_{t+1+j} - b_t^K\pi_{t+1}\right] \\
&= \sum_{j=1}^{K} E_t\left[\Delta c_{t+1+j}\right] - b_t^K E_t\left[\pi_{t+1}\right] \\
&= \sum_{j=1}^{K}\left(\mu_c + \rho_c\left(\rho_\pi^j\left(\pi_t - \mu_\pi\right) - \rho_\pi^j\phi_\pi u_t + \rho_\pi^{j-1}\left(\rho_\pi\phi_\pi + \xi_\pi\right)u_t\right)\right) \\
&\quad - b_t^K\left(\mu_\pi + \rho_\pi\left(\pi_t - \mu_\pi\right) + \xi_\pi u_t\right) \\
&= K\mu_c + \rho_\pi\rho_c\left(\pi_t - \mu_\pi + \frac{\xi_\pi}{\rho_\pi}u_t\right)\frac{1-\rho_\pi^K}{1-\rho_\pi} \\
&\quad - b_t^K\left(\mu_\pi + \rho_\pi\left(\pi_t - \mu_\pi\right) + \xi_\pi u_t\right).
\end{aligned} \tag{OA.95}$$

The residual ε_{t+K} is known at time $t+K$ and, because it is a regression residual of the conditional regression,

$$E_{t-1}\left[\varepsilon_{t+K}\right] = 0, \tag{OA.96}$$

$$Cov_{t-1}\left(a_t^K + b_t^K\pi_t, \varepsilon_{t+K}\right) = 0. \tag{OA.97}$$

By the law of total covariance and equations (OA.96)-(OA.97), we have

$$\begin{aligned}
Cov\left(a_t^K + b_t^K\pi_t, \varepsilon_{t+K}\right) &= Cov\left(E_{t-1}\left[a_t^K + b_t^K\pi_t\right], E_{t-1}\left[\varepsilon_{t+K}\right]\right) \\
&\quad + E\left[Cov_{t-1}\left(a_t^K + b_t^K\pi_t, \varepsilon_{t+K}\right)\right] \\
&= 0.
\end{aligned} \tag{OA.98}$$

Then, using (OA.98), we compute

$$\begin{aligned} d_1^{c,K} &= \frac{Cov\left(\sum_{j=0}^{K-1}\Delta c_{t+1+j}, a_t^K + b_t^K\pi_t\right)}{Var\left(a_t^K + b_t^K\pi_t\right)} \\ &= \frac{Cov\left(a_t^K + b_t^K\pi_t + \varepsilon_{t+K}, a_t^K + b_t^K\pi_t\right)}{Var\left(a_t^K + b_t^K\pi_t\right)} \\ &= \frac{Cov\left(a_t^K + b_t^K\pi_t, a_t^K + b_t^K\pi_t\right)}{Var\left(a_t^K + b_t^K\pi_t\right)} + \frac{Cov\left(\varepsilon_{t+K}, a_t^K + b_t^K\pi_t\right)}{Var\left(a_t^K + b_t^K\pi_t\right)} \\ &= \frac{Var\left(a_t^K + b_t^K\pi_t\right)}{Var\left(a_t^K + b_t^K\pi_t\right)} + \frac{Cov\left(\varepsilon_{t+K}, a_t^K + b_t^K\pi_t\right)}{Var\left(a_t^K + b_t^K\pi_t\right)} \\ &= 1. \end{aligned} \tag{OA.99}$$

By (OA.95), (OA.99), and the law of iterated expectations,

$$\begin{aligned} d_0^{c,K} &= E\left[\sum_{j=0}^{K-1}\Delta c_{t+1+j} - d_1^{c,K}\left(a_t^K + b_t^K\pi_t\right)\right] \\ &= E\left[\sum_{j=0}^{K-1}\Delta c_{t+1+j}\right] - E\left[a_t^K\right] - E\left[b_t^K\pi_t\right] \\ &= E\left[\sum_{j=0}^{K-1}\Delta c_{t+1+j}\right] - E\left[E_t\left[\sum_{j=1}^{K}\Delta c_{t+1+j} - b_t^K\pi_{t+1}\right]\right] - E\left[b_t^K\pi_t\right] \\ &= E\left[\sum_{j=0}^{K-1}\Delta c_{t+1+j}\right] - E\left[\sum_{j=1}^{K}\Delta c_{t+1+j} - b_t^K\pi_{t+1}\right] - E\left[b_t^K\pi_t\right] \\ &= \sum_{j=0}^{K-1}E\left[\Delta c_{t+1+j}\right] - \sum_{j=1}^{K}E\left[\Delta c_{t+1+j}\right] + E\left[b_t^K\pi_{t+1}\right] - E\left[b_t^K\pi_t\right]. \end{aligned} \tag{OA.100}$$

Using equations (OA.3)-(OA.5) and (OA.7), equation (OA.100) gives

$$\begin{aligned} d_0^{c,K} &= K\mu_c - K\mu_c + E\left[b_t^K\left(\pi_{t+1} - \pi_t\right)\right] \\ &= K\mu_c - K\mu_c + E\left[E_t\left[b_t^K\left(\pi_{t+1} - \pi_t\right)\right]\right] \\ &= E\left[b_t^K\left(E_t\left[\pi_{t+1}\right] - \pi_t\right)\right] \\ &= E\left[b_t^K\left(\mu_\pi + \rho_\pi\left(\pi_t - \mu_\pi\right) + \xi_\pi u_t - \pi_t\right)\right] \\ &= E\left[\left(h_K + \frac{\xi_c}{\phi_\pi}\varphi_t\right)\left(\left(\rho_\pi - 1\right)\left(\pi_t - \mu_\pi\right) + \xi_\pi u_t\right)\right] \\ &= E\left[h_K + \frac{\xi_c}{\phi_\pi}\varphi_t\right] E\left[\left(\rho_\pi - 1\right)\left(\pi_t - \mu_\pi\right) + \xi_\pi u_t\right] \\ &= E\left[h_K + \frac{\xi_c}{\phi_\pi}\varphi_t\right] \times 0 \\ &= 0. \end{aligned} \tag{OA.101}$$

2.12 Calibrated Moments in Table 10

In this section, we compute the target moments for calibration in Table 10.

Using equations (OA.3)-(OA.5), the moments for consumption and inflation in the "Descriptive Statistics" part of Table 10 are

$$
\begin{aligned}
E\left[\pi_t\right] &= \mu_\pi, \\
\sigma\left(\pi_t\right) &= \frac{\sqrt{\left|\xi_\pi^2+2\xi_\pi\rho_\pi\phi_\pi+\phi_\pi^2\right|}}{\sqrt{1-\rho_\pi^2}}, \\
AR(1) \text{ of } \pi_t &= \rho_\pi+\frac{\left(1-\rho_\pi^2\right)\xi_\pi\phi_\pi}{2\xi_\pi\rho_\pi\phi_\pi+\xi_\pi^2+\phi_\pi^2}, \\
E\left[\Delta c_t\right] &= \mu_c, \\
\sigma\left(\Delta c_t\right) &= \sqrt{\left|\sigma_c^2+\left(2\varphi_0\xi_c+\frac{\phi_\pi\rho_c}{1-\rho_\pi^2}\right)\phi_\pi\rho_c+\left(\frac{\sigma_w^2}{1-v^2}+\varphi_0^2\right)\xi_c^2+\frac{\xi_\pi\rho_c^2\left(\xi_\pi+2\phi_\pi\rho_\pi\right)}{1-\rho_\pi^2}\right|}, \\
AR(1) \text{ of } \Delta c_t &= \frac{\rho_c\left(1-\nu^2\right)\left(\phi_\pi\rho_\pi+\xi_\pi\right)\left(\rho_c\left(\phi_\pi+\xi_\pi\rho_\pi\right)+\xi_c\varphi_0\left(1-\rho_\pi^2\right)\right)}{\left(1-\rho_\pi^2\right)\left[\left(1-\nu^2\right)\left(\rho_c^2\sigma^2\left(\pi_t\right)+\left(\varphi_0\xi_c\left(\varphi_0\xi_c+2\phi_\pi\rho_c\right)+\sigma_c^2\right)\right)+\xi_c^2\sigma_w^2\right]}, \\
E\left[u_{\Pi,t}\right] &= 0, \\
\sigma\left(u_{\Pi,t}\right) &= \left|\phi_\pi\right|, \\
AR(1) \text{ of } u_{\Pi,t} &= 0, \\
corr\left(\pi_t,u_{\Pi,t}\right) &= \frac{\left|\phi_\pi\right|}{\sigma\left(\pi_t\right)}, \\
corr\left(\pi_t,\Delta c_t\right) &= \frac{\xi_\pi+\rho_\pi\phi_\pi}{\sigma\left(\pi_t\right)\sigma\left(\Delta c_t\right)}\left(\varphi_0\xi_c+\frac{\left(\phi_\pi+\xi_\pi\rho_\pi\right)\rho_c}{1-\rho_\pi^2}\right), \\
corr\left(\Delta c_t,u_{\Pi,t}\right) &= 0.
\end{aligned}
$$

The moments for the "Predictve Regressions of Consumption Growth on Inflation" part of Table 10 are given by equations (OA.94), (OA.93), (OA.101) , (OA.99) and by

$$
\begin{aligned}
AR(1) \text{ of } NRC_t^C &= v, \\
\sigma\left(NRC_t^C\right) &= \frac{1}{\sqrt{12}}\frac{1}{\sqrt{1-v^2}}\left|\frac{\xi_c\sigma_w}{\phi_\pi}\right|,
\end{aligned}
$$

which follow from using equations (OA.5), (OA.7)-(OA.9).

We now compute moments for assets $i=H,L,MKT$, where H and L represent in the model the highest and lowest inflation beta portfolios that we constructed in the data, and MKT is the market portfolio. Taking unconditional expectations in equation (OA.44) gives mean returns for asset i

$$E\left[r_{i,t}\right]=\kappa_{i,0}+\mu_i-D_{i,0}\left(1-\kappa_{i,1}\right).$$

Plugging equation (OA.44) into (OA.43) and computing the unconditional variance gives the variance of returns for asset i, denoted $\sigma^2\left(r_{i,t}\right)$

$$\begin{aligned}
\sigma^2\left(r_{i,t}\right) \;=\;& 2\varphi_0\phi_\pi D_{i,1}D_{i,2}\left(1-\rho_\pi\kappa_{1,1}+\kappa_{1,1}^2\right)+\varphi_0\left(\xi_1\varphi_0+\rho_1\phi_\pi\right)\left(\xi_1-2D_{i,2}\right)\\
&+\varphi_0^2\left(\kappa_{1,1}^2+1\right)D_{i,2}^2+\xi_1\varphi_0\rho_1\phi_\pi+\sigma_1^2\\
&-\frac{\phi_\pi}{1-\rho_\pi^2}\left(2\left(1-\rho_\pi\kappa_{1,1}\right)\left(\rho_1\phi_\pi+\left(1-\rho_\pi^2\right)\xi_1\varphi_0\right)D\right)\\
&+\frac{\phi_\pi}{1-\rho_\pi^2}\left(\phi_\pi D_{i,1}^2\left(\kappa_{1,1}^2-2\rho_\pi\kappa_{1,1}+1\right)-\rho_1^2\phi_\pi\right)\\
&+\frac{\sigma_w^2}{1-v^2}\left(\left(2\varphi_0D_{i,5}+D_{1,4}\right)^2\left(\kappa_{1,1}^2-2v\kappa_{1,1}+1\right)+\left(\xi_1-D_{i,2}\right)^2+D_{i,2}^2\kappa_{1,1}^2\right)\\
&+\frac{2\sigma_w^4}{\left(v^2-1\right)^2}\left(\kappa_{1,1}^2-2v^2\kappa_{1,1}+1\right)D_{i,5}^2\\
&+\frac{1}{1-\rho_\pi^2}\left(\left(\rho_1-D_{i,1}\left(1-\rho_\pi\kappa_{i,1}\right)\right)^2\right)\xi_\pi^2+\xi_\pi^2\kappa_{i,1}^2D_{i,1}^2\\
&+\frac{2\phi_\pi}{1-\rho_\pi^2}\left(\left(\rho_\pi-\kappa_{1,1}\right)\left(1-\rho_\pi\kappa_{1,1}\right)D_{i,1}^2+\left(\left(\kappa_{1,1}\rho_\pi^2-2\rho_\pi+\kappa_{1,1}\right)D_{i,1}+\rho_1\rho_\pi\right)\rho_1\right)\xi_\pi\\
&-2\kappa_{1,1}D_{1,1}\left(-\varphi_0\left(\xi_1-D_{i,2}\right)+D_{1,3}\right)\xi_\pi+\left(\kappa_{1,1}^2+1\right)D_{i,3}^2\\
&+2\left(\left(\left(\kappa_{1,1}^2+1\right)D_{i,2}-\xi_1\right)\varphi_0+\phi_\pi\left(\left(\kappa_{1,1}^2-\rho_\pi\kappa_{1,1}+1\right)D_{i,1}-\rho_1\right)\right)D_{i,3}.
\end{aligned}$$

The standard deviation of returns is simply $\sigma\left(r_{i,t}\right)$.

We find the mean inflation beta for asset i, $E\left[\beta_{\Pi,i,t}\right]$, and its standard deviation, $\sigma\left(\beta_{\Pi,i,t}\right)$, by taking the mean and standard deviation in equation (OA.45) and using equation (OA.49)

$$\begin{aligned}
E\left[\beta_{\Pi,i,t}\right] &= \frac{\kappa_{1,i}}{\left|\phi_\pi\right|}\left(\varphi_0D_{i,2}+\phi_\pi D_{i,1}+D_{i,3}\right),\\
\sigma\left(\beta_{\Pi,i,t}\right) &= \frac{\kappa_{i,1}}{\sqrt{1-v^2}}\left|\frac{\sigma_wD_{i,2}}{\phi_\pi}\right|.
\end{aligned}$$

Also using equation (OA.45), the correlation in inflation betas for two assets i,j is

$$corr\left(\beta_{\Pi,i,t},\beta_{\Pi,j,t}\right)=sign\left(D_{i,2}D_{j,2}\right).$$

Denote the high-minus-low inflation portfolio by $i = IP^{HL}$, with returns defined by

$$r_{IP^{HL},t} \equiv r_{H,t} - r_{L,t}. \tag{OA.102}$$

Equations (OA.44) and (OA.43) give

$$\begin{aligned} r_{IP^{HL},t} &= r_{H,t+1} - r_{L,t+1} \\ &= E_t\left[r_{IP^{HL},t+1}\right] + \left(\beta_{u,Ht} - \beta_{u,Lt}\right) u_{t+1} \\ &\quad + \left(\beta_{w,H} - \beta_{w,L}\right) w_{t+1} + \left(\beta_{2w,H} - \beta_{2w,L}\right)\left(\frac{w_{t+1}^2 - 1}{\sqrt{2}}\right) \\ &\quad + \left(\beta_{\eta,H} - \beta_{\eta,L}\right) \eta_{t+1}, \end{aligned}$$

where

$$\begin{aligned} E_t\left[r_{IP^{HL},t+1}\right] &= E_t\left[r_{H,t+1}\right] - E_t\left[r_{L,t+1}\right] \\ &= \kappa_{H,0} + \mu_H - D_{H,0}\left(1 - \kappa_{H,1}\right) \\ &\quad - \left(\kappa_{L,0} + \mu_L - D_{L,0}\left(1 - \kappa_{L,1}\right)\right) \\ &\quad + \left(\rho_H - D_{H,1}\left(1 - \kappa_{H,1}\rho_\pi\right) - \left(\rho_L - D_{L,1}\left(1 - \kappa_{L,1}\rho_\pi\right)\right)\right)\left(\pi_t - \mu_\pi\right) \\ &\quad - \left(D_{H,3} - \xi_\pi \kappa_{H,1} D_{H,1} - \left(D_{L,3} - \xi_\pi \kappa_{L,1} D_{L,1}\right)\right) u_t \\ &\quad + \left(\xi_H - D_{H,2} - \left(\xi_L - D_{L,2}\right)\right) \varphi_{t-1} u_t \\ &\quad + \left(2D_{H,5} \upsilon \kappa_{H,1} \varphi_0 \left(1 - \upsilon\right) - D_{H,4}\left(1 - \upsilon \kappa_{H,1}\right)\right)\left(\varphi_t - \varphi_0\right) \\ &\quad - \left(2D_{L,5} \upsilon \kappa_{L,1} \varphi_0 \left(1 - \upsilon\right) - D_{L,4}\left(1 - \upsilon \kappa_{L,1}\right)\right)\left(\varphi_t - \varphi_0\right) \\ &\quad - \left(D_{H,5}\left(1 - \upsilon^2 \kappa_{H,1}\right) - \left(D_{L,5}\left(1 - \upsilon^2 \kappa_{L,1}\right)\right)\right)\left(\varphi_t^2 - E\left[\varphi_t^2\right]\right). \end{aligned} \tag{OA.103}$$

Equation (OA.45) can be used together with equations (OA.49) and (OA.102) to get

$$\begin{aligned}
\beta_{\Pi,IP^{HL},t} &= \frac{Cov\left(r_{IP^{HL},t}, u_{\Pi,t}\right)}{Var\left(u_{\Pi,t}\right)} \\
&= \frac{1}{|\phi_\pi|}\frac{Cov\left(r_{IP^{HL},t}, u_t\right)}{Var\left(u_t\right)} \qquad \text{(OA.104)} \\
&= \frac{1}{|\phi_\pi|}Cov\left(r_{H,t} - r_{L,t}, u_{\Pi,t}\right) \\
&= \frac{1}{|\phi_\pi|}Cov\left(r_{H,t}, u_{u,t}\right) - \frac{1}{|\phi_\pi|}Cov\left(r_{L,t}, u_{u,t}\right) \\
&= \frac{1}{|\phi_\pi|}\beta_{u,H,t} - \frac{1}{|\phi_\pi|}\beta_{u,L,t} \\
&= \beta_{\Pi,H,t} - \beta_{\Pi,L,t} \qquad \text{(OA.105)} \\
&= \frac{\kappa_{H,1}}{|\phi_\pi|}\left(\phi_\pi D_{H,1} + D_{H,2}\varphi_t + D_{H,3}\right) \\
&\quad - \frac{\kappa_{L,1}}{|\phi_\pi|}\left(\phi_\pi D_{L,1} + D_{L,2}\varphi_t + D_{L,3}\right). \qquad \text{(OA.106)}
\end{aligned}$$

The standard deviation of the inflation beta of IP^{HL} can be found using equation (OA.106)

$$\begin{aligned}
\sigma\left(\beta_{u,IP^{HL},t}\right) &= \sigma\left(\beta_{u,1t} - \beta_{u,2t}\right) \\
&= \frac{1}{\sqrt{1-v^2}}\left|\frac{\sigma_w\left(D_{1,2}\kappa_{1,1} - D_{2,2}\kappa_{2,1}\right)}{\phi_\pi}\right|.
\end{aligned}$$

The coefficient L^{NRC} of a regression of the returns of IP_t^{HL} on the standardized nominal-real covariance is

$$\begin{aligned}
L_{NRC,IP_t^{HL}} &= \frac{cov\left(r_{IP^{HL},t}, \tilde{\theta}_t^{(12)}\right)}{var\left(\tilde{\theta}_t^{(12)}\right)} \\
&= cov\left(r_{IP^{HL},t}, \tilde{\theta}_t^{(12)}\right),
\end{aligned}$$

where $\tilde{\theta}_t^{(12)}$ is defined in equation (OA.78). Using equations (OA.43), (OA.44), (OA.78), (OA.102) we get

$$\begin{aligned} L_{NRC,IP_t^{HL}} &= cov\left(r_{IP^{HL},t}, \tilde{\theta}_t^{(12)}\right) \\ &= -\left(1 - v\kappa_{1,1}\right) \frac{sign\left(\phi_\pi\right) sign\left(\xi_c\right) \left|\sigma_w\right|}{\sqrt{1-v^2}} \left(2\varphi_0 D_{1,5} + D_{1,4}\right) \\ &\quad + \left(1 - v\kappa_{2,1}\right) \frac{sign\left(\phi_\pi\right) sign\left(\xi_c\right) \left|\sigma_w\right|}{\sqrt{1-v^2}} \left(2\varphi_0 D_{2,5} + D_{2,4}\right). \end{aligned}$$

Last, the regression coefficient $L_{\beta_\Pi \times NRC,m}$ is given in equation (OA.89).

The numbers reported in Table 10 are annualized and in percentage points. More precisely, the numbers reported in Table 10 are obtained by using the formulas above and then by multiplying $E\left[\pi_t\right]$, $E\left[u_{\Pi,t}\right]$, $E\left[\Delta c_t\right]$, $E\left[r_{i,t}\right]$, $L_{NRC,IP_t^{HL}}$ by 100×12; $\sigma\left(\pi_t\right)$, $\sigma\left(u_{\Pi,t}\right)$, $\sigma\left(\Delta c_t\right)$, $\sigma\left(r_{i,t}\right)$ by $100 \times \sqrt{12}$; all correlations, $AR(1)$ coefficients, and R^2 by 100; the coefficient $L_{\beta_\Pi \times NRC,m}$ by 100 (no need to multiply by 12 because it was obtained using cumulative returns over 12 months).

3 Robustness Checks

Table OA.1: The Maximum Correlation Inflation Portfolio (IP_t^{MC})

This table presents the portfolio weights used to construct IP_t^{MC}. These weights are found by regressing the ARMA(1,1)-innovations in inflation on the ten inflation beta-sorted portfolios: $u_{\Pi,t+1} = intercept + weights' \times R_{t+1} + e_{t+1}$. For this regression we also present the adjusted R^2 and the p-value of a joint F-test: $h_0 : weights = 0_{10}$. Standard errors are Newey-West. The sample period is July 1967 to December 2014. We see that the Wald-test for the null hypothesis that the weights are jointly equal to zero rejects at the 0.1%-level. Furthermore, the R^2 of 5.35% is quite large relative to other mimicking portfolios for non-traded factors in the literature (see, e.g., Vassalou (2003) and Petkova (2006)).

	H	2	3	4	5	6	7	8	9	L		
weights×100	1.82	-0.29	-0.32	-1.52	0.89	-1.84	3.58	-2.20	-0.91	0.17	$R^2 \times 100$	5.34
t	(2.11)	(-0.31)	(-0.32)	(-1.29)	(0.70)	(-1.51)	(2.42)	(-1.68)	(-0.70)	(0.22)	p-val	0.0003

Table OA.2: The Inflation Risk Premium Split in Above and Below Average NRC_t^C Months
This table is similar to Table 3 of the paper, but presents annualized performance statistics for the inflation beta-sorted portfolios in months when NRC_t^C is below and above average as well as the difference between the two.

	H	2	3	4	5	6	7	8	9	L	IP_t^{HL}	IP_t^{MC}	IP_t^{CS}	p-val Mono.
							Below Average NRC_t^C							
Avg. Ret.	0.53	3.70	5.00	5.27	6.48	6.61	6.76	7.87	8.22	9.44	-8.92	-6.43	-7.90	(0.000)
t	(0.13)	(0.97)	(1.43)	(1.56)	(1.91)	(1.89)	(1.89)	(2.12)	(2.08)	(2.21)	(-3.20)	(-3.35)	(-3.05)	
Sharpe	0.02	0.18	0.27	0.29	0.36	0.35	0.35	0.39	0.39	0.41	-0.60	-0.62	-0.57	(0.006)
α_{CAPM}	-4.37	-0.89	0.57	0.97	2.12	2.14	2.23	3.19	3.27	4.22	-8.59	-5.43	-7.18	(0.002)
t	(-1.92)	(-0.46)	(0.38)	(0.69)	(1.60)	(1.50)	(1.46)	(1.94)	(1.82)	(2.01)	(-2.99)	(-2.95)	(-2.75)	
							Above Average NRC_t^C							
Avg. Ret.	12.61	12.87	12.29	11.20	11.70	10.88	10.19	9.49	9.62	9.55	3.07	-1.20	1.54	(0.052)
t	(2.35)	(2.76)	(2.83)	(2.59)	(2.72)	(2.56)	(2.37)	(2.22)	(2.23)	(2.08)	(1.08)	(-0.57)	(0.58)	
Sharpe	0.54	0.64	0.66	0.60	0.63	0.59	0.55	0.51	0.52	0.48	0.25	-0.13	0.14	(0.760)
α_{CAPM}	0.98	2.24	2.30	1.11	1.75	0.95	0.18	-0.41	-0.29	-1.03	2.00	0.07	0.64	(0.227)
t	(0.37)	(1.21)	(1.42)	(0.76)	(1.13)	(0.65)	(0.12)	(-0.26)	(-0.18)	(-0.57)	(0.68)	(0.03)	(0.23)	
							Above-Below Difference							
Avg. Ret.	12.09	9.16	7.30	5.94	5.22	4.28	3.43	1.62	1.40	0.11	11.98	5.23	9.44	(0.001)
t	(1.78)	(1.52)	(1.31)	(1.08)	(0.95)	(0.78)	(0.61)	(0.29)	(0.24)	(0.02)	(3.02)	(1.83)	(2.55)	
Sharpe	0.52	0.46	0.39	0.31	0.28	0.24	0.20	0.12	0.13	0.07	0.85	0.49	0.70	(0.006)
α_{CAPM}	5.35	3.13	1.74	0.14	-0.37	-1.19	-2.04	-3.61	-3.57	-5.24	10.59	5.50	7.82	(0.007)
t	(1.52)	(1.17)	(0.79)	(0.07)	(-0.18)	(-0.58)	(-0.94)	(-1.57)	(-1.45)	(-1.90)	(2.57)	(1.88)	(2.07)	

Table OA.3: Predicting the Inflation Risk Premium with the Nominal-Real Covariance: Robustness

This table presents two robustness checks for the results in Table 4 of the paper. We regress the returns of ten inflation beta-sorted portfolios as well as our three estimates of the inflation risk premium (IP_t^{HL}, IP_t^{MC}, and IP_t^{CS}) on the nominal-real covariance. Panel A presents results for two alternative measures of the nominal-real covariance: (i) the time-varying relation between inflation and industrial production growth, NRC_t^{IP}, and (ii) the negative of the stock market beta of the long-term bond, NRC_t^{-BB}. In Panel B, we present results for three subsamples: the first and second half of the sample (split around February 1991) as well as a sample from July 1967 to December 2007 (to exclude the period with interest rates at the zero lower bound). We report for each regression the estimated coefficients, with corresponding t-statistics in parentheses based on Newey-West standard errors with $K = 12$ lags, and the adjusted R^2 (in percentage points).

Panel A: Alternative Measures of the Nominal-Real Covariance

		H	2	3	4	5	6	7	8	9	L	IP_t^{HL}	IP_t^{MC}	IP_t^{CS}	p-val Mono.
							Inflation and Industrial Production (NRC_t^{IP})								
$K = 12$	L_0	5.88	7.66	8.09	7.99	9.08	8.82	8.68	9.03	9.35	10.24	-4.37	-4.25	-3.94	(0.044)
	t	(1.91)	(2.89)	(3.46)	(3.36)	(3.73)	(3.62)	(3.45)	(3.52)	(3.58)	(3.52)	(-1.67)	(-2.59)	(-2.32)	
	L_{NRC}	1.94	1.18	-0.05	-0.23	-0.48	-1.02	-1.24	-2.30	-2.80	-3.42	5.36	3.24	4.54	(0.001)
	t	(0.65)	(0.47)	(-0.02)	(-0.09)	(-0.19)	(-0.40)	(-0.47)	(-0.87)	(-1.04)	(-1.17)	(2.39)	(2.21)	(2.74)	
	R^2	0.36	0.09	-0.18	-0.17	-0.13	0.05	0.14	0.88	1.31	1.61	6.60	5.72	9.79	
							Stock Market Beta of Long-Term Bond (NRC_t^{-BB})								
$K = 12$	L_0	5.88	7.66	8.09	7.99	9.08	8.82	8.68	9.03	9.35	10.24	-4.37	-4.25	-3.94	(0.060)
	t	(1.92)	(2.92)	(3.46)	(3.36)	(3.74)	(3.61)	(3.44)	(3.49)	(3.54)	(3.47)	(-1.63)	(-2.55)	(-2.23)	
	L_{NRC}	3.22	2.71	1.07	0.87	1.05	0.38	0.25	-0.48	-0.45	-0.33	3.55	2.54	3.22	(0.032)
	t	(1.10)	(1.09)	(0.45)	(0.34)	(0.41)	(0.15)	(0.10)	(-0.18)	(-0.16)	(-0.11)	(1.79)	(1.84)	(2.12)	
	R^2	1.32	1.23	0.10	0.01	0.07	-0.15	-0.17	-0.13	-0.14	-0.16	2.80	3.45	4.85	

Panel B: Subsamples

		First Half			Second Half			Up to December 2007		
		IP_t^{HL}	IP_t^{MC}	IP_t^{CS}	IP_t^{HL}	IP_t^{MC}	IP_t^{CS}	IP_t^{HL}	IP_t^{MC}	IP_t^{CS}
$K = 12$	L_0	-4.09	-3.42	-5.49	-4.66	-5.11	-2.33	-6.67	-5.27	-5.38
	t	(-1.39)	(-1.64)	(-2.69)	(-1.19)	(-2.22)	(-0.87)	(-2.42)	(-3.03)	(-3.07)
	L_{NRC}	1.67	3.71	2.59	11.41	5.53	6.26	3.60	3.63	3.98
	t	(0.82)	(2.68)	(2.02)	(2.49)	(2.18)	(2.01)	(1.81)	(2.96)	(3.07)
	R^2	0.61	7.81	4.20	22.77	15.97	14.54	3.24	7.42	8.62

Table OA.4: Time-Varying Inflation Risk Premia in Pooled Regressions at Shorter Horizons

This table is similar to Panel A of Table 5 except that results are presented for the one- and three-month horizon. We present estimated coefficients (with asymptotic Driscoll and Kraay (1998) standard errors in parentheses) from various pooled predictive regressions of returns on the inflation beta-sorted decile portfolios on their inflation beta ($\beta_{\Pi,p,t}$), the nominal-real covariance (NRC_t^C), and an interaction. In Model [1], we set $L_{\beta_\Pi \times NRC} = 0$, whereas Model [2] estimates all three coefficients freely. For the latter model, Panel B presents the predicted risk premia (with asymptotic and bootstrapped standard errors in parentheses) in four distinct cases, i.e., when $\beta_{\Pi,p,t}$ and NRC_t^C equal plus or minus one standard deviation from their respective means in the pool. Models [3] to [5] in Panel A analyze which components of inflation betas interact with the nominal-real covariance. Model [3] replaces the interaction term, $\beta_{\Pi,p,t} \times NRC_t^C$, with the interaction between the nominal-real covariance and the portfolio-specific component of inflation betas, $\widehat{\beta_{\Pi,p}} = T^{-1} \sum_{t=1}^{T} \beta_{\Pi,p,t}$. Model [4] replaces the interaction term, $\beta_{\Pi,p,t} \times NRC_t^C$, with the interaction between the nominal-real covariance and the time-specific component of inflation betas, $\widehat{\beta_{\Pi,t}} = 10^{-1} \sum_{p=1}^{10} \beta_{\Pi,p,t}$. Model [5] includes both component-wise interaction terms. To accommodate interpretation, NRC_t^C is standardized in the time series to have mean zero and standard deviation equal to one, whereas $\beta_{\Pi,p,t}$ is demeaned in the pool. The sample period is July 1967 to December 2014.

	Horizon $K = 1$ Month					Horizon $K = 3$ Months				
Model	[1]	[2]	[3]	[4]	[5]	[1]	[2]	[3]	[4]	[5]
L_0	7.97	4.76	7.97	4.55	4.57	8.29	4.82	8.29	4.63	4.66
t	(2.69)	(1.36)	(2.69)	(1.24)	(1.25)	(3.09)	(1.66)	(3.10)	(1.54)	(1.55)
L_{β_Π}	-0.22	-1.12	-0.19	-1.24	-1.20	0.11	-0.88	0.15	-1.00	-0.95
t	(-0.19)	(-0.87)	(-0.15)	(-0.92)	(-0.89)	(0.11)	(-0.92)	(0.15)	(-1.01)	(-0.96)
L_{NRC}	-0.28	-0.96	-0.34	-0.91	-0.96	-1.12	-1.89	-1.19	-1.82	-1.88
t	(-0.07)	(-0.26)	(-0.09)	(-0.24)	(-0.26)	(-0.33)	(-0.57)	(-0.35)	(-0.56)	(-0.57)
$L_{\beta_\Pi \times NRC}$		2.07					2.27			
t		(1.88)					(2.53)			
$L_{\beta_{\Pi,p} \times NRC}$			1.87		1.64			2.21		1.95
t			(2.19)		(2.01)			(2.63)		(2.43)
$L_{\beta_{\Pi,t} \times NRC}$				2.21	2.19				2.39	2.37
t				(1.80)	(1.79)				(2.41)	(2.39)
R^2	0.02	0.69	0.06	0.68	0.71	0.05	2.08	0.21	1.99	2.11

Table OA.5: The Nominal-Real Covariance and the Average Risk Premium
This table is similar to Table 6 of the paper and presents coefficient estimates from time-series predictive regressions for the equal-weighted average return over the ten decile portfolios on the inflation beta of this average portfolio ($\beta_{\Pi,avg,t}$), the nominal-real covariance (NRC_t^C), and an interaction. We present this regression for the twelve-month horizon and standard errors are Newey-West(12). To accommodate interpretation, NRC_t^C is standardized and $\beta_{\Pi,avg,t}$ is demeaned. The sample period is July 1967 to December 2014.

	[1]	[2]
$L_{0,avg}$	8.48	5.06
t	(3.47)	(1.98)
$L_{\beta_\Pi,avg}$	0.81	-0.45
t	(0.75)	(-0.51)
$L_{NRC,avg}$	-2.53	-3.04
t	(-0.76)	(-1.12)
$L_{\beta_\Pi \times NRC,avg}$		2.38
t		(3.31)
R^2	0.90	8.44

Table OA.6: Time-Varying Inflation Risk Premia in Pooled Regressions Controlling for Market Beta

This table is similar to Panel A of Table 5 except that we now also control for each portfolio's market beta (estimated by regressing in month t each portfolio's returns on the CRSP value-weighted market portfolio over an expanding window of historical returns, similar to the way inflation betas are measured). We present estimated coefficients (with asymptotic Driscoll and Kraay (1998) standard errors in parentheses) from various pooled predictive regressions. In Model [1], we regress returns on the inflation beta-sorted decile portfolios on their market beta ($\beta_{MKT,p,t}$), the nominal-real covariance (NRC_t^C), and their interaction. In Model [2], we add inflation betas ($\beta_{\Pi,p,t}$) and their interaction with the nominal-real covariance. In Model [3], we decompose the inflation beta for the interaction with the nominal-real covariance in the portfolio- and time-specific components: $\widehat{\beta_{\Pi,p}} = T^{-1}\sum_{t=1}^{T}\beta_{\Pi,p,t}$ and $\widehat{\beta_{\Pi,t}} = 10^{-1}\sum_{p=1}^{10}\beta_{\Pi,p,t}$. To accommodate interpretation, NRC_t^C is standardized in the time series to have mean zero and standard deviation equal to one, whereas $\beta_{\Pi,p,t}$ is demeaned in the pool. The sample period is July 1967 to December 2014 and we focus on the twelve-month horizon.

Model	[1]	[2]	[3]
L_0	6.31	18.22	18.46
	(0.60)	(1.80)	(1.52)
$L_{\beta_{MKT}}$	2.02	-12.00	-12.18
	(0.20)	(-1.19)	(-1.05)
$L_{\beta_{MKT}\times NRC}$	-6.46	3.71	3.49
	(-0.72)	(0.42)	(0.39)
$L_{\beta_{\Pi}}$		-0.82	-0.82
		(-0.97)	(-0.94)
L_{NRC}	6.10	-6.83	-6.56
	(0.60)	(-0.66)	(-0.63)
$L_{\beta_{\Pi}\times NRC}$		2.61	
		(3.69)	
$L_{\beta_{\Pi,p}\times NRC}$			3.28
			(2.34)
$L_{\beta_{\Pi,t}\times NRC}$			2.58
			(3.52)
R^2	0.53	9.34	9.27

Table OA.7: The timing premium in our model of the nominal-real covariance
This table shows the size of the *timing premium* for our model under three parameter configurations. The first column shows the timing premium under our baseline calibration found in Table 9. We display only the parameters that change across columns, the rest of the parameters are held fixed at the values shown in Table 9. The second column increases the persistence of inflation (ρ_π) and the degree of predictability that inflation has on future consumption (ρ_c) so that inflation mimics long-run risk as calibrated in Bansal, Kiku, and Yaron (2012) (in Table , the persistence of long-run risk is denoted by a and the degree of predictability that long-run risk has on future consumption is always -1). In the third column, we modify our baseline calibration by postulating a nominal-real covariance that is much more persistent (higher ν) and volatile (higher σ_w).

	Baseline	Inflation has long-run risk persistence	More volatile and persistent φ_t
ρ_π	0.799	0.979	0.799
ρ_c	-0.052	-1	-0.052
ν	0.9963	0.9963	0.999
σ_w	0.0029	0.0029	0.02
Timing premium	14.8%	59.0%	14.8%

Table OA.8: The timing premium in the long-run risk model
This table shows the size of the *timing premium* in the long-run risk model of Bansal, Kiku, and Yaron (2012) under three parameter configurations. The first two columns reproduce the results in Table 1 of Epstein, Farhi, and Strzalecki (2014) and show the timing premium in versions of the long-run risk model with and without stochastic volatility. In the third column, we compute the timing premium in the long-run risk model with stochastic volatility but replacing the coefficient of relative risk aversion (RRA) and the elasticity of intertemporal substitution (EIS) by the values of our main calibration in Table 9.

Parameter	LLR no stoch. vol.	LLR with stoch. vol.	Higher RRA and EIS
σ	0.0078	0.0078	0.0078
ϕ	0.044	0.044	0.044
a	0.9790	0.9790	0.9790
σ_w	0	0.23×10^{-5}	0.23×10^{-5}
ν	0	0.987	0.987
β	0.998	0.998	0.998
RRA	7.5 or 10	7.5 or 10	15
EIS	1.5	1.5	2
Timing premium	23% or 29%	24% or 31%	44.0%

Table OA.9: Controlling Ex Ante for Benchmark Factor Exposure when Estimating Inflation Beta

This table asks whether our results extend when we control ex ante (i.e., when estimating inflation beta) for stock's exposures to the benchmark asset-pricing factors of the CAPM (MKT), FF3M (MKT, SMB, HML), FFCM (MKT, SMB, HML, and MOM), and FF5M (MKT, SMB, HML, RMW, CMA). For each alternative sorting procedure, we calculate our three estimates of the inflation risk premium (IP_t^{HL}, IP_t^{MC}, and IP_t^{CS},) and regress overlapping twelve-month returns on the nominal-real covariance (NRC_t^C), as in Table 4 of the paper.

$R_{t+1:t+K}$		CAPM			FF3M			FFCM			FF5M		
		IP_t^{HL}	IP_t^{MC}	IP_t^{CS}	IP_t^{HL}	IP_t^{MC}	IP_t^{CS}P	IP_t^{HL}	IP_t^{MC}	IP_t^{CS}	IP_t^{HL}	IP_t^{MC}	IP_t^{CS}
$K = 12$	L_0	-4.47	-5.88	-4.17	-2.68	-3.94	-3.97	-2.50	-4.14	-3.71	-1.90	-2.97	-3.69
	t	(-1.89)	(-3.72)	(-2.59)	(-1.54)	(-3.89)	(-2.62)	(-1.50)	(-3.51)	(-2.47)	(-1.22)	(-3.44)	(-2.69)
	L_{NRC}	6.09	4.38	3.77	5.48	2.68	3.75	5.40	3.45	3.45	4.22	1.37	3.92
	t	(2.59)	(2.66)	(2.13)	(2.61)	(2.38)	(2.15)	(2.63)	(2.67)	(2.01)	(2.18)	(1.49)	(2.50)
	R^2	9.98	10.62	8.00	12.99	8.67	8.80	13.57	11.16	7.67	10.08	3.18	11.32

Table OA.10: Controlling Ex Post for Benchmark Factor Exposure when Predicting the Inflation Risk Premium

This table asks whether our conclusions on the time-varying inflation risk premium extend when we control ex post for exposure in our estimates of the inflation risk premium to the benchmark asset pricing factors. For this exercise, we regress the returns of IP_t^{HL}, IP_t^{MC}, and IP_t^{CS} on the nominal-real covariance (NRC_t^C) as well as on contemporaneous exposure to the CAPM (MKT), FF3M (MKT, SMB, HML), FFCM (MKT, SMB, HML, and MOM), and FF5M (MKT, SMB, HML, RMW, CMA). We focus on the annual horizon $K = 12$ and calculate overlapping twelve-month compounded returns on both the left- and right-hand side. We presents for each regression the estimated coefficients and the adjusted R^2 (in percentage points). To conserve space, we present t-statistics in parentheses (based on Newey-West standard errors with K lags) only for the intercept and the coefficient on NRC_t^C (L_0 and L_{NRC}).

		CAPM			FF3M			FFCM			FF5M		
$R_{t+1:t+K}$		IP_t^{HL}	IP_t^{MC}	IP_t^{CS}	IP_t^{HL}	IP_t^{MC}	IP_t^{CS}	IP_t^{HL}	IP_t^{MC}	IP_t^{CS}	IP_t^{HL}	IP_t^{MC}	IP_t^{CS}
$K = 12$	L_0	-4.98	-3.56	-4.30	-3.85	-2.31	-3.09	0.48	-0.98	-1.43	1.28	0.85	0.32
	t	(-1.62)	(-1.86)	(-2.16)	(-1.74)	(-1.83)	(-2.04)	(0.21)	(-0.68)	(-0.91)	(0.51)	(0.56)	(0.21)
	L_{NRC}	5.46	3.65	4.56	4.96	3.08	4.01	3.45	2.62	3.43	4.14	2.57	3.46
	t	(2.21)	(2.52)	(2.76)	(2.07)	(2.25)	(2.48)	(1.64)	(2.09)	(2.25)	(1.66)	(1.78)	(2.08)
	β_{MKT}	0.10	-0.11	0.06	0.16	-0.09	0.08	0.09	-0.11	0.05	-0.01	-0.20	-0.03
	β_{SMB}				-0.52	-0.31	-0.35	-0.54	-0.32	-0.36	-0.58	-0.35	-0.40
	β_{HML}				-0.07	-0.12	-0.11	-0.20	-0.16	-0.16	0.32	0.11	0.11
	β_{MOM}							-0.35	-0.11	-0.14			
	β_{RMW}										-0.87	-0.55	-0.68
	β_{CMA}										-0.52	-0.30	-0.23
	R^2	7.19	9.57	10.01	15.56	17.66	18.56	22.05	19.00	20.42	35.63	36.27	41.53

Table OA.11: Controlling for Benchmark Predictors when Predicting the Inflation Risk Premium

This table asks whether our conclusions on the time-varying inflation risk premium extend when we control for benchmark predictors. For this exercise, we regress the returns of IP_t^{HL}, IP_t^{MC}, and IP_t^{CS} on the nominal-real covariance (NRC_t^C) controlling for either the dividend yield (DY), default spread (DS), and term spread (TS) or the consumption-wealth ratio (CAY). All control variables are standardized. We present the coefficient estimates and, to conserve space, report t-statistics in parentheses (based on Newey-West standard errors with K lags) only for the intercept and the coefficient on NRC_t^C (L_0 and L_{NRC}).

$R_{t+1:t+K}$		IP_t^{HL}	IP_t^{MC}	IP_t^{CS}	IP_t^{HL}	IP_t^{MC}	IP_t^{CS}
$K = 12$	L_0	-4.37	-4.25	-3.94	-4.37	-4.25	-3.94
	t	(-1.76)	(-2.66)	(-2.35)	(-1.67)	(-2.63)	(-2.31)
	L_{NRC}	7.19	4.50	4.26	5.29	3.46	4.39
	t	(2.13)	(2.30)	(1.81)	(1.81)	(2.00)	(2.35)
	ζ_{DY}	2.34	1.36	-1.63			
	ζ_{DS}	2.53	0.42	2.05			
	ζ_{TS}	3.32	1.05	0.84			
	ζ_{CAY}				-0.24	-0.71	-0.30
	R^2	11.93	8.59	11.82	6.50	7.71	9.55

Table OA.12: Alternative Measures of Inflation Risk
This table asks whether our results extend for alternative measures of inflation risk. In column one, we repeat our benchmark specification using ARMA(1,1)-innovations in inflation. In column two, we use raw inflation. In column three, we use an AR(1)-model to proxy for inflation-innovations. In column four, we use the monthly change in annual inflation. In column five, we perform a truly out-of-sample exercise using real-time vintage CPI inflation. For this exercise, we skip a month after portfolio formation, thus taking into account the reporting delay in inflation data. In all cases, we calculate the returns of the High-minus-Low inflation beta portfolio (IP_t^{HL}). We present coefficient estimates from a regression of returns on the lagged nominal-real covariance (NRC_t^C) at the twelve-month horizon, with corresponding t-statistics in parentheses based on Newey-West standard errors with $K = 12$ lags.

		ARMA(1,1)	Inflation	AR(1)	Change in Annual Inflation	Real time
$K = 12$	L_0	-4.37	-3.89	-3.92	-4.88	-2.06
	t	(-1.68)	(-1.74)	(-1.73)	(-1.66)	(-0.96)
	L_{NRC}	5.38	7.35	5.08	6.87	7.84
	t	(2.22)	(3.25)	(2.33)	(2.56)	(3.35)
	R^2	6.65	14.30	7.26	7.84	16.12

Table OA.13: Alternative Sorting Procedures
Each column in this table presents results from an alternative sorting procedure. We construct a High-minus-Low inflation beta decile portfolio by sorting stocks on inflation betas estimated using (i) Weighted Least Squares plus Shrinkage (WLS) or Ordinary Least Squares (OLS), (ii) a sort where each portfolio comes from a double sort on beta and size (Size) or a single sort on inflation beta (No Size), and (iii) ARMA(1,1) innovations in inflation (ARMA) or inflation itself (Inflation). The table presents the predictive regression of the inflation risk premium on the nominal-real covariance, as in Table 4 of the paper. The first column reports the original results as a benchmark.

	WLS Size ARMA	WLS Size Inflation	WLS No Size ARMA	WLS No Size Inflation	OLS Size ARMA	OLS Size Inflation	OLS No Size ARMA	OLS No Size Inflation
L_0	-4.37	-3.89	-4.32	-3.07	-3.67	-2.71	-3.11	-1.60
t	(-1.68)	(-1.74)	(-1.50)	(-1.27)	(-1.46)	(-1.18)	(-1.10)	(-0.60)
L_{NRC}	5.38	7.35	4.80	6.08	5.71	8.52	5.38	8.96
t	(2.22)	(3.25)	(1.78)	(2.59)	(2.39)	(3.80)	(2.11)	(3.69)
R^2	6.65	14.30	4.21	8.61	7.78	17.33	5.29	14.60

Table OA.14: Predicting the Inflation Risk Premium with the Nominal-Real Covariance (Quarterly data)

This table is similar to Table 4 of the paper, except that we use quarterly instead of monthly data and, for brevity, we focus on the High-minus-Low inflation risk premium. We construct a High-minus-Low inflation beta portfolio by sorting stocks on the exposure in quarterly returns to quarterly ARMA(1,1)-innovations in inflation. Moreover, we use quarterly inflation and quarterly consumption growth to estimate the nominal-real covariance. We then run the regression of Equation (7). The nominal-real covariance is standardized and standard errors are Newey-West with K lags.

$R_{t+1:t+K}$	L_0	L_{NRC}	R^2
$K = 1Q$	-1.99	6.31	2.00
	(-0.66)	(2.07)	
$K = 2Q$	-1.92	6.25	3.90
	(-0.67)	(2.26)	
$K = 4Q$	-1.79	6.67	8.62
	(-0.69)	(2.80)	

Table OA.15: Pooled Regressions: Inflation Betas Linear in the Nominal-Real Covariance

Similar to Panel A of Table 5, this table presents pooled regressions of returns on the inflation beta-sorted decile portfolios on inflation beta, the nominal-real covariance, and their interaction. In this case, inflation betas are estimated as a linear function of the nominal-real covariance, i.e., $\beta_{\Pi,p,t} = \beta^0_{\Pi,p,t} + \beta^1_{\Pi,p,t} NRC^C_{t-1}$ (see Section 7 for more detail). We present estimated coefficients (with asymptotic Driscoll and Kraay (1998) standard errors in parentheses) from various pooled predictive regressions. To accommodate interpretation, NRC^C_t is standardized and $\beta_{\Pi,p,t}$ is demeaned in the pool. We focus on the twelve-month horizon.

Model	[1]	[2]	[3]	[4]	[5]
L_0	8.48	5.17	8.48	5.09	5.12
t	(3.47)	(2.24)	(3.47)	(2.18)	(2.19)
L_{β_Π}	0.45	-0.71	0.49	-0.75	-0.71
t	(0.65)	(-1.30)	(0.71)	(-1.34)	(-1.25)
L_{NRC}	-2.27	-1.69	-2.35	-1.66	-1.73
t	(-0.68)	(-0.71)	(-0.70)	(-0.70)	(-0.73)
$L_{\beta_\Pi \times NRC}$		1.64			
t		(4.25)			
$L_{\beta_{\Pi,p} \times NRC}$			2.72		2.28
t			(2.84)		(2.45)
$L_{\beta_{\Pi,t} \times NRC}$				1.68	1.67
t				(4.13)	(4.09)
R^2	0.89	11.36	1.60	10.95	11.44

Table OA.16: Pooled Regressions Controlling for Monetary Policy Regimes
This table is similar to Panel A of Table 5 except that we now include a dummy for each monetary policy regime identified in Campbell, Pflueger, and Viceira (2015): MPR_1 runs from the start of the sample to March 1977, MPR_2 runs from April 1977 to December 2000, and MPR_3 runs from January 2001 to December 2014. We present estimated coefficients (with asymptotic Driscoll and Kraay (1998) standard errors in parentheses) from various pooled predictive regressions. In Model [1], we include only the dummies for the monetary policy regimes. In Model [2], we add inflation betas ($\beta_{\Pi,p,t}$) and their interaction with the nominal-real covariance. In Model [3], we decompose the inflation beta for the interaction with the nominal-real covariance in a portfolio- and time-specific component: $\widehat{\beta_{\Pi,p}} = T^{-1}\sum_{t=1}^{T}\beta_{\Pi,p,t}$ and $\widehat{\beta_{\Pi,t}} = 10^{-1}\sum_{p=1}^{10}\beta_{\Pi,p,t}$. To accommodate interpretation, NRC_t^C is standardized in the time series to have mean zero and standard deviation equal to one, whereas $\beta_{\Pi,p,t}$ is demeaned in the pool. We focus on the twelve-month horizon.

Model	[1]	[2]	[3]
MPR_1	1.30	-1.17	-1.30
	(0.20)	(-0.18)	(-0.20)
MPR_2	10.03	5.54	5.43
	(3.89)	(1.86)	(1.79)
MPR_3	11.03	10.16	10.13
	(2.18)	(1.32)	(1.32)
L_{β_Π}		-1.42	-1.48
		(-1.41)	(-1.42)
L_{NRC}		-2.37	-2.37
		(-0.79)	(-0.79)
$L_{\beta_\Pi \times NRC}$		2.12	
		(3.12)	
$L_{\beta_{\Pi,p} \times NRC}$			1.95
			(2.13)
$L_{\beta_{\Pi,t} \times NRC}$			2.18
			(2.93)
R^2	2.72	10.19	10.21

Table OA.17: Exposure to Monetary Policy Shocks

To test whether our results are driven by exposure to monetary policy shocks, we regress returns of the ten inflation beta-sorted portfolios as well as the three inflation risk premiums (IP_t^{HL}, IP_t^{MC}, and IP_t^{CS}) on the monetary policy shocks of Romer and Romer (2004) in Panel A (sample period from March 1969 to December 2008, dictated by data availability) and changes in the federal funds rate in Panel B (from July 1967 to December 2014). For brevity, we report only the estimated coefficient on the monetary policy shocks (×100), which we denoted β_{MPS}, and we do so for the full sample as well as in the three monetary policy regimes of Campbell, Pflueger, and Viceira (2015), denoted MPR_s, $s = 1, 2, 3$.

	H	2	3	4	5	6	7	8	9	L	IP_t^{HL}	IP_t^{MC}	IP_t^{CS}
Panel A: Romer and Romer (2004) Monetary Policy Shocks													
Full Sample													
β_{MPS}	1.34	1.21	0.94	1.23	0.99	1.12	1.22	1.26	1.40	1.47	-0.13	-0.30	-0.49
t	(0.98)	(0.98)	(0.88)	(1.20)	(0.91)	(1.05)	(1.22)	(1.27)	(1.33)	(1.39)	(-0.19)	(-0.83)	(-0.99)
Monetary Policy Regimes of Campbell, Pflueger, and Viceira (2015)													
β_{MPS,MPR_1}	-0.57	-0.34	-0.30	-0.08	-0.74	-0.42	-0.54	0.08	-0.25	0.00	-0.57	-1.07	0.34
t	(-0.26)	(-0.15)	(-0.13)	(-0.03)	(-0.27)	(-0.15)	(-0.18)	(0.03)	(-0.08)	(0.00)	(-0.50)	(-1.02)	(0.26)
β_{MPS,MPR_2}	2.38	2.10	1.57	1.88	1.74	1.85	1.92	1.91	2.08	2.11	0.28	-0.14	-0.49
t	(1.77)	(1.75)	(1.52)	(1.91)	(1.70)	(1.81)	(2.17)	(2.10)	(2.11)	(2.14)	(0.42)	(-0.39)	(-1.25)
β_{MPS,MPR_3}	-4.59	-4.04	-2.54	-2.11	-2.45	-2.47	-1.57	-2.46	-1.46	-1.33	-3.26	0.03	-2.56
t	(-0.83)	(-0.84)	(-0.58)	(-0.59)	(-0.84)	(-0.90)	(-0.61)	(-1.07)	(-0.61)	(-0.47)	(-0.73)	(0.01)	(-0.61)
Panel B: Changes in Federal Funds Rate													
Full Sample													
β_{MPS}	-1.09	-1.12	-1.07	-1.26	-1.37	-1.32	-1.34	-1.43	-1.47	-1.30	0.21	0.54	0.56
t	(-1.93)	(-2.27)	(-2.34)	(-2.68)	(-2.89)	(-2.56)	(-2.68)	(-2.82)	(-2.78)	(-2.32)	(0.79)	(2.33)	(2.40)
Monetary Policy Regimes of Campbell, Pflueger, and Viceira (2015)													
β_{MPS,MPR_1}	-2.74	-2.63	-2.72	-3.05	-3.39	-3.25	-3.15	-3.36	-3.57	-3.11	0.37	1.30	1.09
t	(-1.94)	(-1.98)	(-1.90)	(-2.10)	(-2.13)	(-1.90)	(-1.73)	(-1.85)	(-1.97)	(-1.52)	(0.47)	(2.58)	(1.80)
β_{MPS,MPR_2}	-1.05	-0.95	-0.89	-1.06	-1.16	-1.12	-1.20	-1.24	-1.29	-1.18	0.13	0.29	0.50
t	(-1.79)	(-1.93)	(-2.04)	(-2.39)	(-2.65)	(-2.28)	(-2.63)	(-2.63)	(-2.58)	(-2.23)	(0.48)	(1.36)	(2.19)
β_{MPS,MPR_3}	6.27	2.48	2.92	3.09	3.87	3.74	4.53	3.60	4.77	4.83	1.45	2.73	-0.82
t	(1.07)	(0.50)	(0.66)	(0.71)	(1.01)	(1.03)	(1.21)	(1.03)	(1.34)	(1.16)	(0.46)	(1.49)	(-0.28)