Federal Reserve Bank of New York
Staff Reports

Liquidity Policies and Systemic Risk

Tobias Adrian
Nina Boyarchenko

Staff Report No. 661
December 2013

Liquidity Policies and Systemic Risk
Tobias Adrian and Nina Boyarchenko
Federal Reserve Bank of New York Staff Reports, no. 661
December 2013
JEL classification: E02, E32, G00, G28

Abstract

The growth of wholesale-funded credit intermediation has motivated liquidity regulations. We analyze a dynamic stochastic general equilibrium model in which liquidity and capital regulations interact with the supply of risk-free assets. In the model, the endogenously time-varying tightness of liquidity and capital constraints generates intermediaries' leverage cycle, influencing the pricing of risk and the level of risk in the economy. Our analysis focuses on liquidity policies' implications for household welfare. Within the context of our model, liquidity requirements are preferable to capital requirements, as tightening liquidity requirements lowers the likelihood of systemic distress without impairing consumption growth. In addition, we find that intermediate ranges of risk-free asset supply achieve higher welfare.

Key words: liquidity regulation, systemic risk, DSGE, financial intermediation

---

Adrian, Boyarchenko: Federal Reserve Bank of New York (e-mail: tobias.adrian@ny.frb.org, nina.boyarchenko@ny.frb.org). The views expressed in this paper are those of the authors and do not necessarily reflect the position of the Federal Reserve Bank of New York or the Federal Reserve System.

1 Introduction

While the role of liquidity transformation in the financial sector is well understood in partial equilibrium settings (e.g. Diamond and Dybvig [1983] and Kahn and Santos [2005]), there are only few, recent examples of dynamic general equilibrium models that incorporate systemic liquidity crisis (e.g. Angeloni and Faia [2013], Gertler and Kiyotaki [2012], and Martin, Skeie, and Von Thadden [2013]). These models don't typically study the role of liquidity requirements such as the Liquidity Coverage Ratio (LCR). The LCR has been developed in reaction to the growth of wholesale-funded credit intermediation outside of commercial banks which limits central banks' ability to act as lender of last resort (see Adrian and Shin [2010] and Adrian and Ashcraft [2012]).

The LCR was proposed by the Basel Committee on Banking Supervision (BCBS) in 2010 (see BCBS [2010] and BCBS [2013]) as a minimum requirement for the liquidity insurance of bank holding companies (BHC). Because the LCR is imposed at the holding company level, it covers entities such as broker-dealer and derivatives subsidiaries which do not have access to the discount window, in addition to the commercial bank subsidiary. The LCR, which must exceed unity, is the ratio of haircutted liquid assets to liabilities that are expected to evaporate in liquidity stress. Haircuts are higher for less liquid assets, while runoff rates for liabilities are higher for items that are less stable. Haircuts and runoff rates are calibrated to liquidity at a 30 day time horizon.

In this paper, we incorporate liquidity policies in the theory of Adrian and Boyarchenko [2012], which explicitly models endogenous and systemic risk in an equilibrium setting with liquidity risk. We study the equilibrium impact of liquidity requirements, capital requirements, and the risk-free asset supply. In our framework, the LCR is a requirement for banks to hold a certain amount of liquid assets in proportion to the short term riskiness of liabilities. Both the liquidity requirement and the capital requirement impact the risk taking of intermediaries. In equilibrium, these constraints interact with the total supply of the risk-free asset in determining the pricing of risk, and the equilibrium amount of risk. Prudential capital and liquidity regulation thus affect the systemic risk return tradeoff between the pricing of risk and the level of systemic risk.

Within the context of our model, liquidity requirements are a preferable prudential policy tool relative to capital requirements, as tightening liquidity requirements lowers the likelihood of systemic distress, without impairing consumption growth. In contrast, capital requirements trade off consumption growth and distress probabilities. In addition, we find that intermediate ranges of risk-free asset supply achieve higher welfare. This is because very low levels of the risk-free asset make liquidity requirements costly, while a very high supply of risk-free assets countermand the effects of prudential liquidity regulation.

1.1 Related literature

While the literature on macroprudential policies in dynamic general equilibrium models is recent, it is growing rapidly (see Goodhart, Kashyap, Tsomocos, and Vardoulakis [2012], Angelini, Neri, and Panetta [2011], Angeloni and Faia [2013], Korinek [2011], Bianchi and Mendoza [2011], Nuño and Thomas [2012], and Farhi and Werning [2013]). The distinguishing feature of the current paper is to focus on the LCR, while other papers primarily focus on capital requirements and monetary policy. An exception to that is presented by Goodhart et al. [2012], which studies liquidity requirements in general equilibrium. While our analysis focuses on the LCR, Goodhart et al. [2012] analyze the welfare implications of the Net Stable Funding Ratio (NSFR). The LCR is calibrated to potential liquidity shortages at the 30 day horizon, while the NSFR is calibrated to illiquidity at longer run horizons. The NSFR was proposed by BCBS [2010] in 2010, but has not been implemented, while the LCR is in the process of implementation (see BCBS [2013]). Goodhart et al. [2012] find the NSFR to be a good pre-emptive macroprudential tool, in comparison to cyclical variation in capital requirements or underwriting standards. The focus of our analysis is on the unconditional calibration of the LCR, which is more in line with the current approach to liquidity regulation by the BCBS.

There is some literature on liquidity regulation in partial equilibrium settings. Farhi, Golosov, and Tsyvinski [2009] provide a justification for liquidity requirements within a Diamond and Dybvig [1983] banking system. Cao and Illing [2010] show that liquidity requirements are advantageous in dealing with systemic liquidity risk relative to capital requirements. Perottia and Suarez [2011] argue that Pigouvian taxation is preferable to liquidity requirements due to lower distortions. Ratnovski [2009] points out that liquidity requirements can mitigate moral hazard due to public liquidity provision via lender of last resort facilities. Rochet [2008] provides an overview of liquidity regulation within the context of the banking literature. The main difference between our approach and this work is that we consider the impact of liquidity regulations within a dynamic macroeconomy, while the banking literature considers only particle equilibrium effects within static settings. Our main findings rely crucially on the endogeneity of asset risk and return which is totally missed in partial equilibrium settings. Our approach is thus macroprudential, while the banking literature typically employs a microprudential perspective.

Liquidity requirements have long played a central role in monetary economics. However, the focus of liquidity regulation has traditionally been on reserve requirements. For example, the credit channel of monetary transmission by Bernanke and Blinder [1988] relies on the scarcity of bank reserves. Indeed, in the U.S., the Federal Reserve used changes in reserve requirements as a policy instrument until the early 1990s, and emerging market economies tend to use variation in central bank liquidity requirements as a policy tool to this day. The type of liquidity requirement that we are studying here is conceptually distinct from reserve requirements, as it does not necessarily involve money or deposits. Indeed, the LCR can be satisfied by only holding liquid securities such as Treasury bills, and would apply to institutions such as broker-dealers that do not issue any deposits.

Our approach is closely related to the intermediary asset pricing theories of He and Krishnamurthy [2013] and Brunnermeier and Sannikov [2012], who explicitly introduce a financial sector into dynamic models of the macroeconomy. Our approach differs in important aspects from that work. Most importantly, we assume that the capital constraint on financial intermediaries is risk based. In contrast, He and Krishnamurthy [2013] have a constraint on outside equity financing without any constraint on leverage. Brunnermeier and Sannikov [2012] require intermediaries to manage leverage in a way to make their liabilities instantaneously riskless. Our setting gives rise to pro-cyclical intermediary leverage, a fact that is strongly supported by the data, as Adrian and Boyarchenko [2012, 2013] show. In contrast, He and Krishnamurthy [2013] and Brunnermeier and Sannikov [2012] exhibit countercyclical leverage.

2 The Model

We consider a continuous time, infinite horizon economy. Uncertainty is described by a two-dimensional, standard Brownian motion $Z_t = [Z_{at},\ Z_{\xi t}]'$ for $t \geq 0$, defined on a completed probability space $(\Omega, \mathcal{F}, \mathbb{P})$, where $\mathcal{F}$ is the augmented filtration generated by Z_t. There are three types of agents in the economy: (passive) producers, financially sophisticated intermediaries and unsophisticated households.

2.1 Production

There is a single consumption good in the economy, produced continuously. We assume that physical capital is an input into the production of the consumption good, so that the total output in the economy at date $t \geq 0$ is

$$Y_t = A_t K_t,$$

where K_t is the aggregate amount of capital in the economy at time t, and the stochastic productivity of capital $\{A_t = e^{a_t}\}_{t \geq 0}$ follows a geometric diffusion process of the form

$$da_t = \bar{a}dt + \sigma_a dZ_{at}.$$

The stock of physical capital in the economy depreciates at a constant rate λ_k, so that the total physical capital in the economy evolves as

$$dK_t = (I_t - \lambda_k)\, K_t dt,$$

where I_t is the reinvestment rate per unit of capital in place. There is a fully liquid market for physical capital in the economy, in which both the financial intermediaries and the households are allowed to participate. We denote by $p_{kt} A_t$ the price of one unit of capital at time $t \geq 0$ in terms of the consumption good.

2.2 Financial Intermediaries

There is a unit mass of identical, infinitely lived financial intermediaries in the economy. The financial intermediaries serve two functions in the economy. First, they generate new capital through investment in the productive sector. As in Brunnermeier and Sannikov [2012], we assume that the intermediaries have access to a superior investment technology relative to households. Thus, the intermediaries serve an important role in propagating growth in the economy. Second, since intermediaries accumulate wealth through retained earnings, they provide risk-bearing capacity to the households. By issuing risky debt to the households, the financial intermediaries increase market completeness and improve risk-sharing within the economy.

As in our previous work, we assume that the intermediaries are debt-financed, which allows us to abstract from modeling the dividend payment decision ("consumption") of the intermediary sector and to assume that an intermediary invests maximally if the opportunity arises. In particular, financial intermediaries create new capital through capital investment. Denote by k_t the physical capital held by the representative intermediary at time t and by $i_t A_t$ the investment rate per unit of capital. Then, without trade between households and intermediaries, the stock of capital held by the representative intermediary would evolve according to

$$dk_t = (\Phi(i_t) - \lambda_k)\, k_t dt.$$

Here, $\Phi(\cdot)$ reflects the costs of (dis)investment. We assume that $\Phi(0) = 0$, so in the absence of new investment, capital depreciates at the economy-wide rate λ_k. Notice that the above formulation implies that costs of adjusting capital are higher in economies with a higher level of capital productivity, corresponding to the intuition that more developed economies are more specialized. We follow Brunnermeier and Sannikov [2012] in assuming that investment carries quadratic adjustment costs, so that Φ has the form

$$\Phi(i_t) = \phi_0 \left(\sqrt{1 + \phi_1 i_t} - 1 \right),$$

for positive constants ϕ_0 and ϕ_1.

Each unit of capital owned by the intermediary produces $A_t(1-i_t)$ units of output net of investment. As a result, the total return from one unit of intermediary capital invested in physical capital is given by

$$dr_{kt} = \underbrace{\frac{(1-i_t)\,A_t k_t}{k_t p_{kt} A_t} dt}_{\text{dividend-price ratio}} + \underbrace{\frac{d\,(k_t p_{kt} A_t)}{k_t p_{kt} A_t}}_{\text{capital gains}} = dR_{kt} + \left(\Phi(i_t) - \frac{i_t}{p_{kt}}\right) dt.$$

Here, dR_{kt} is the return on holding capital earned by the representative household in the economy. Compared to the households, the financial intermediaries earn an extra return to holding firm capital to compensate them for the cost of investment. This extra return is partially passed on to the households as coupon payments on the intermediaries' debt.

The intermediaries finance their investment in new capital projects by issuing risky floating coupon bonds to the households. To keep the balance sheet structure of the financial institutions time-invariant, we assume that the bonds mature at a constant rate λ_b, so that the time t probability of a bond maturing before time $t+dt$ is $\lambda_b dt$. Notice that this corresponds to an infinite-horizon version of the "stationary balance sheet" assumption of Leland and Toft [1996]. Denoting by б_t the issuance rate of bonds at time t, the stock of bonds b_t on a representative intermediary's balance sheet evolves as

$$db_t = (\text{б}_t - \lambda_b)\, b_t dt.$$

Each unit of debt issued by the intermediary pays $C_d A_t$ units of output until maturity and A_t units of output at maturity. Similarly to the capital stock in the economy, the risky bonds are liquidly traded, with the price of one unit of intermediary debt at time t in terms of the consumption good given by $p_{bt} A_t$. The total net cost of one unit of intermediary debt is therefore given by

$$dr_{bt} = \underbrace{\frac{(C_d + \lambda_b - \text{б}_t p_{bt})\,A_t b_t}{b_t p_{bt} A_t} dt}_{\text{dividend-price ratio}} + \underbrace{\frac{d\,(b_t p_{bt} A_t)}{b_t p_{bt} A_t}}_{\text{capital gains}} = dR_{bt}.$$

Thus, the cost of debt to the intermediary equals the return on holding bank debt for the households.

The key assumptions in this paper concern the regulatory constraints faced by the intermediary. First, as in Adrian and Boyarchenko [2012, 2013], we assume that intermediary borrowing is constrained by a risk-based capital constraint and, in particular, that the inside capital of the intermediary, w_t, is sufficient to absorb a shock to the asset-side of their balance sheet of α standard deviations

$$\alpha\sqrt{\frac{1}{dt}\left\langle k_t d\left(p_{kt}A_t\right)\right\rangle^2} \leq w_t, \tag{1}$$

where $\langle\cdot\rangle^2$ is the quadratic variation operator. The risk-based capital constraint implies a time-varying constraint on the intermediary's capital allocation choice, given by

$$\theta_{kt} \equiv \frac{p_{kt}A_t k_t}{w_t} \leq \frac{1}{\alpha\sqrt{\frac{1}{dt}\left\langle\frac{d(p_{kt}A_t)}{p_{kt}A_t}\right\rangle^2}}.$$

In our previous work, we make the case for using a value-at-risk (VaR) constraint to model the capital requirements faced by banking institutions, and show that it generates many empirical regularities, such as procyclicality of intermediary leverage and intermediated credit, and the positive price of risk associated with shock to intermediary leverage. Further, in Adrian and Boyarchenko [2013] we show that, even in an economy with two types of intermediaries which face different types of funding constraints, the VaR constraint generates procyclical leverage for the banking sector, as well as the empirical regularity that bank leverage leads asset growth for the whole financial system.

The novel assumption in this paper consists of the liquidity regulation faced by financial institutions. Similar to the new liquidity requirements proposed by the Basel Committee on Banking Supervision, we assume that the financial intermediaries are required to hold instantaneous risk-free debt ("cash") in proportion to their risky debt liabilities

$$A_t\mathcal{T}_t \geq \tilde{\Lambda} p_{bt} A_t b_t, \tag{2}$$

where $A_t\mathcal{T}_t$ is the value of cash held by the intermediaries and $\tilde{\Lambda}$ is the tightness of the liquidity constraint. As $\tilde{\Lambda}$ becomes smaller, the liquidity constraint becomes more relaxed, with the limiting case of $\tilde{\Lambda} = 0$ corresponding to an economy with no liquidity constraints. At the other extreme, when $\tilde{\Lambda} = 1$, intermediaries have to fully back their liabilities with liquid securities, corresponding to the traditional narrow model of banking.

Consider now the budget constraint of an intermediary in this economy, which holds the balance sheet in Figure 1. An intermediary in this economy holds capital investment projects (k_t) and cash ($\mathcal{T}_t$) on the asset side of its balance sheet and has bonds (b_t) on the liability side. In mathematical terms, we can express the corresponding budget constraint as

$$p_{kt}A_tk_t + A_t\mathcal{T}_t = p_{bt}A_tb_t + w_t, \tag{3}$$

where w_t is the implicit value of equity in the intermediary. Thus, in terms of flows, the intermediary's equity value evolves according to

$$\frac{dw_t}{w_t} = \theta_{kt}\left(dr_{kt} - r_{ft}dt\right) - \theta_{bt}\left(dr_{bt} - r_{ft}dt\right) + r_{ft}dt, \tag{4}$$

where r_{ft} is the economy-wide risk-free rate and θ_{bt} is the fraction of intermediary wealth allocated to issuing debt. Notice that, with this notation, we can represent the liquidity constraint as

$$1 + \theta_{bt} - \theta_t \geq \tilde{\Lambda}\theta_{bt},$$

or, equivalently,

$$\left(1 - \tilde{\Lambda}\right)\theta_{bt} \equiv \Lambda\theta_{bt} \geq \left(\theta_{kt} - 1\right).$$

As in He and Krishnamurthy [2012], we assume that the intermediary is myopic and maximizes a mean-variance objective of instantaneous wealth

$$\max_{\theta_{kt},\theta_{bt},i_t} \quad \mathbb{E}_t\left[\frac{dw_t}{w_t}\right] - \frac{\gamma}{2}\mathbb{V}_t\left[\frac{dw_t}{w_t}\right], \tag{5}$$

Figure 1: Intermediaries' Balance Sheet

Assets	**Liabilities**
Productive capital ($A_t p_{kt} k_t$)	Risky debt ($A_t p_{bt} b_t$)
Risk-free debt ($A_t \mathcal{T}_t$)	Inside equity (w_t)

subject to the dynamic intermediary budget constraint 4, the risk-based capital constraint constraint 1 and the liquidity constraint 2. Here, γ measures the degree of risk-aversion of the representative intermediary; when γ is close to zero, the intermediary is almost risk-neutral and chooses its portfolio each period to maximize the expected instantaneous growth rate. While in the Appendix we derive the optimal portfolio and investment choice for the financial intermediary for the general case, we focus on the case when γ is close to zero, so that the intermediary is always at either the risk-based capital constraint or the liquidity constraint. In particular, we have the following result.

Lemma 1. *The representative financial intermediary optimally invests in new projects at rate*

$$i_t = \frac{1}{\phi_1}\left(\frac{\phi_0^2\phi_1^2}{4}p_{kt}^2 - 1\right).$$

For nearly risk-neutral intermediaries (γ close to 0), the optimal allocation to firm capital is given by

$$\theta_{kt} = \min\left\{\frac{1}{\alpha\sqrt{\sigma_{ka,t}^2 + \sigma_{k\xi,t}^2}}, 1 + \Lambda\theta_{bt}\right\}.$$

While the intermediaries are not liquidity-constrained, the optimal debt-to-equity ratio is

$$\theta_{bt} = \gamma^{-1}\left(\sigma_{ba,t}^2 + \sigma_{b\xi,t}^2\right)^{-1}\left[-\left(\mu_{Rb,t} - r_{ft}\right) + \frac{\gamma\left(\sigma_{ka,t}\sigma_{ba,t} + \sigma_{k\xi,t}\sigma_{b\xi,t}\right)}{\alpha\sqrt{\sigma_{ka,t}^2 + \sigma_{k\xi,t}^2}}\right]$$

and the shadow cost of capital faced by the intermediary is

$$\zeta_{ct} = \mu_{Rk,t} + \Phi\left(i_t\right) + \frac{i_t}{p_{kt}} - r_{ft} - \frac{\gamma\sqrt{\sigma_{ka,t}^2 + \sigma_{k\xi,t}^2}}{\alpha} + \gamma\left(\sigma_{ka,t}\sigma_{ba,t} + \sigma_{k\xi,t}\sigma_{b\xi,t}\right)\theta_{bt}.$$

When the intermediary becomes liquidity-constrained, so that $\theta_{bt} = \Lambda^{-1}\left(\theta_{kt} - 1\right)$, *the fraction of intermediary equity allocated to capital is*

$$\theta_{kt} = det_t^{-1}\left[-\Lambda\left(\tilde{\mu}_{Rk,t} - r_{ft}\right) + \gamma\left(\sigma_{ka,t}\sigma_{ba,t} + \sigma_{k\xi,t}\sigma_{b\xi,t}\right) + \mu_{Rb,t} - r_{ft} - \gamma\Lambda^{-1}\left(\sigma_{ba,t}^2 + \sigma_{b\xi,t}^2\right)\right],$$

and the shadow cost of liquidity faced by the intermediary is

$$\begin{aligned}\zeta_{lt} &= det_t^{-1}\gamma\left(\tilde{\mu}_{Rk,t} - r_{ft}\right)\left[\left(\sigma_{ka,t}\sigma_{ba,t} + \sigma_{k\xi,t}\sigma_{b\xi,t}\right) - \Lambda^{-1}\left(\sigma_{ba,t}^2 + \sigma_{b\xi,t}^2\right)\right] \\ &\quad - det_t^{-1}\gamma\left(\mu_{Rb,t} - r_{ft}\right)\left[\left(\sigma_{ka,t}^2 + \sigma_{k\xi,t}^2\right) - \Lambda^{-1}\left(\sigma_{ka,t}\sigma_{ba,t} + \sigma_{k\xi,t}\sigma_{b\xi,t}\right)\right] \\ &\quad + det_t^{-1}\gamma^2\Lambda^{-1}\left(\sigma_{ka,t}\sigma_{b\xi,t} - \sigma_{k\xi,t}\sigma_{ba,t}\right)^2,\end{aligned}$$

where $det_t = -\gamma\Lambda^{-2}\left[\left(\Lambda\sigma_{ka,t} - \sigma_{ba,t}\right)^2 + \left(\Lambda\sigma_{k\xi,t} - \sigma_{b\xi,t}\right)^2\right]$. *In the case when both the capital and the liquidity constraints bind, the shadow cost of liquidity faced by the intermediary is*

$$\zeta_{lt} = \Lambda^{-1}\left[\left(\mu_{Rb,t} - r_{ft}\right) - \frac{\gamma\left(\sigma_{ka,t}\sigma_{ba,t} + \sigma_{k\xi,t}\sigma_{b\xi,t}\right)}{\alpha\sqrt{\sigma_{ka,t}^2 + \sigma_{k\xi,t}^2}} + \gamma\Lambda^{-1}\left(\sigma_{ba,t}^2 + \sigma_{b\xi,t}^2\right)\left(\frac{1}{\alpha\sqrt{\sigma_{ka,t}^2 + \sigma_{k\xi,t}^2}} - 1\right)\right],$$

and the shadow cost of capital faced by the intermediary is

$$\zeta_{ct} = \left(\tilde{\mu}_{Rk,t} - r_{ft}\right) - \frac{\gamma\sqrt{\sigma_{ka,t}^2 + \sigma_{k\xi,t}^2}}{\alpha} + \gamma\Lambda^{-1}\left(\sigma_{ka,t}\sigma_{ba,t} + \sigma_{k\xi,t}\sigma_{b\xi,t}\right)\left(\frac{1}{\alpha\sqrt{\sigma_{ka,t}^2 + \sigma_{k\xi,t}^2}} - 1\right) - \zeta_{lt}.$$

Proof. See Appendix A. □

Finally, notice that, since the debt issued by intermediaries is long-term and since the risk-based capital constraint does not keep the volatility of intermediary equity constant, the intermediary can become distressed and default on its debt to the households. We assume that distress occurs when the intermediary equity falls below an exogenously specified threshold, so that the (random) distress date of the intermediary is the first crossing time of the threshold

$$\tau_D = \inf_{t \geq 0} \left\{ w_t \leq \bar{\omega} p_{kt} A_t K_t \right\}.$$

Notice that, since the distress boundary grows with the scale of the economy, the intermediary can never outgrow the possibility of distress. When the intermediary is restructured, the management of the intermediary changes. The new management defaults of the debt of the previous intermediary, reducing leverage to $\underline{\theta}$, but maintains the same level of capital as before. The inside equity of the new intermediary is thus

$$w_{\tau_D^+} = \bar{\omega} \frac{\theta_{\tau_D}}{\underline{\theta}} p_{k\tau_D} A_{\tau_D} K_{\tau_D}.$$

We define the term structure of distress risk to be

$$\delta_t (T) = \mathbb{P} \left(\tau_D \leq T | \left(w_t, \theta_{kt} \right) \right).$$

Here, $\delta_t (T)$ is the time t probability of default occurring before time T. Notice that, since the fundamental shocks in the economy are Brownian, and all the agents in the economy have perfect information, the local distress risk is zero. We refer to the default of the intermediary as systemic risk, as there is a single representative intermediary in the economy, so its distress is systemic. In our simulations, we use parameter values for $\bar{\omega}$ that are positive (not zero), thus viewing intermediaries default state as a restructuring event.

2.3 Households

There is a unit mass of risk-averse, infinitely lived households in the economy. We assume that the households in the economy are identical, such that the equilibrium outcomes are determined by the decisions of the representative household. In addition to the productivity shock, Z_{at}, the representative household is also subject to a transitory discount rate shock, so that the representative household evaluates different consumption paths $\{c_t\}_{t\geq 0}$ according to

$$\mathbb{E}\left[\int_0^{+\infty} e^{-(\xi_t+\rho_h t)} \log c_t dt\right],$$

where ρ_h is the subjective time discount of the representative household, and c_t is the consumption at time t. Here, $\exp(-\xi_t)$ is the Radon-Nikodym derivative of the measure induced by households' time-varying preferences or beliefs with respect to the physical measure. For simplicity, we assume that $\{\xi_t\}_{t\geq 0}$ evolves as a Brownian motion, uncorrelated with the productivity shock, Z_{at}:

$$d\xi_t = \sigma_\xi dZ_{\xi t},$$

where $\{Z_{\xi t}\}$ is a standard Brownian motion of $(\Omega, \mathcal{F}_t, \mathbb{P})$, independent of Z_{at}. In the current setting, with households constrained in their portfolio allocation, $\exp(-\xi_t)$ can be interpreted as a time-varying liquidity preference shock, as in Allen and Gale [1994], Diamond and Dybvig [1983], and Holmström and Tirole [1998] or as a time-varying shock to the preference for early resolution of uncertainty, as in Bhamra, Kuehn, and Strebulaev [2010a,b]. In particular, when the households receive a positive $d\xi_t$ shock, their effective discount rate increases, leading to a higher demand for liquidity.

The households finance their consumption through holdings of physical capital, holdings of risky intermediary debt, and short-term risk-free borrowing and lending. Unlike the intermediary sector, the households do not have access to the investment technology. Thus, without trade between the intermediaries and the households, the physical capital k_{ht} held by households would evolve according to

$$dk_{ht} = -\lambda_k k_{ht} dt.$$

When a household buys k_{ht} units of capital at price $p_{kt}A_t$, by Itô's lemma, the value of capital evolves according to

$$\frac{d(k_{ht}p_{kt}A_t)}{k_{ht}p_{kt}A_t} = \frac{dA_t}{A_t} + \frac{dp_{kt}}{p_{kt}} + \frac{dk_{ht}}{k_{ht}} + \left\langle \frac{dp_{kt}}{p_{kt}}, \frac{dA_t}{A_t} \right\rangle.$$

Each unit of capital owned by the household also produces A_t units of output, so the total return to one unit of household wealth invested in capital is

$$dR_{kt} = \underbrace{\frac{A_t k_{ht}}{k_{ht} p_{kt} A_t} dt}_{\text{dividend−price ratio}} + \underbrace{\frac{d\left(k_{ht} p_{kt} A_t\right)}{k_{ht} p_{kt} A_t}}_{\text{capital gains}} \equiv \mu_{Rk,t} dt + \sigma_{ka,t} dZ_{at} + \sigma_{k\xi,t} dZ_{\xi t}.$$

In addition to direct capital investment, the households can invest in risky intermediary debt. Similarly to the stock of debt issued by intermediaries, the risky debt holdings b_{ht} of households follow

$$db_{ht} = \left(б_t - \lambda_b\right) b_{ht} dt,$$

where, as before, $б_t$ is the issuance rate of new debt. Hence, the total return from one unit of household wealth invested in risky debt is

$$dR_{bt} = \underbrace{\frac{\left(C_d + \lambda_b - б_t p_{bt}\right) A_t b_{ht}}{b_{ht} p_{bt} A_t} dt}_{\text{dividend−price ratio}} + \underbrace{\frac{d\left(b_{ht} p_{bt} A_t\right)}{b_{ht} p_{bt} A_t}}_{\text{capital gains}} \equiv \mu_{Rb,t} dt + \sigma_{ba,t} dZ_{at} + \sigma_{b\xi,t} dZ_{\xi t}.$$

When a household with total wealth w_{ht} buys k_{ht} units of capital and b_{ht} units of risky intermediary debt, it invests the remaining $w_{ht} - p_{kt} k_{ht} - p_{bt} b_{ht}$ at the risk-free rate r_{ft}, so that household wealth evolves as

$$dw_{ht} = r_{ft} w_{ht} + p_{kt} A_t k_{ht} \left(dR_{kt} - r_{ft} dt\right) + p_{bt} A_t b_{ht} \left(dR_{bt} - r_{ft} dt\right) - c_t dt. \tag{6}$$

We assume that the households face no-shorting constraints, such that

$$k_{ht} \geq 0$$
$$b_{ht} \geq 0.$$

Thus, the households solve

$$\max_{\{c_t, k_{ht}, b_{ht}\}} \mathbb{E}\left[\int_0^{+\infty} e^{-(\xi_t + \rho_h t)} \log c_t dt\right], \tag{7}$$

subject to the household wealth evolution 6 and the no-shorting constraints. We have the following result.

Lemma 2. *The household's optimal consumption choice satisfies*

$$c_t = \left(\rho_h - \frac{\sigma_\xi^2}{2}\right) w_{ht}.$$

In the unconstrained region, the household's optimal portfolio choice is given by

$$\begin{bmatrix} \pi_{kt} \\ \pi_{bt} \end{bmatrix} = \left(\begin{bmatrix} \sigma_{ka,t} & \sigma_{k\xi,t} \\ \sigma_{ba,t} & \sigma_{b\xi,t} \end{bmatrix} \begin{bmatrix} \sigma_{ka,t} & \sigma_{ba,t} \\ \sigma_{k\xi,t} & \sigma_{b\xi,t} \end{bmatrix} \right)^{-1} \begin{bmatrix} \mu_{Rk,t} - r_{ft} \\ \mu_{Rb,t} - r_{ft} \end{bmatrix} - \sigma_\xi \begin{bmatrix} \sigma_{ka,t} & \sigma_{ba,t} \\ \sigma_{k\xi,t} & \sigma_{b\xi,t} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Proof. See Appendix A. □

Thus, the household with the time-varying beliefs chooses consumption as a myopic investor but with a lower rate of discount. The optimal portfolio choice of the household, on the other hand, also includes an intratemporal hedging component for variations in the Radon-Nikodym derivative, $\exp(-\xi_t)$. Since intermediary debt is locally risk-less, however, households do not self-insure against intermediary default. Appendix A provides also the optimal portfolio choice in the case when the household is constrained. In our simulations, the household never becomes constrained as the intermediary wealth never reaches zero.

2.4 Equilibrium

To define the equilibrium in this economy, we assume that monetary policy is implemented via risk-free government debt, with the supply of debt a constant fraction of aggregate wealth in the economy. While the households can short the risk-free debt in the economy, the intermediaries are constrained to maintain a positive position in the risk-free debt through the liquidity constraint. The risk-free rate in the economy adjusts to clear the risk-free debt market. Comparative statics with respect to the exogenous supply of risk-free debt allows us to evaluate the trade-off between static monetary, capital and liquidity policies.

Definition 1. *An equilibrium in this economy is a set of price processes* $\{p_{kt}, p_{bt}, r_{ft}\}_{t\geq 0}$*, a set of household decisions* $\{k_{ht}, b_{ht}, c_t\}_{t\geq 0}$*, and a set of intermediary decisions* $\{k_t, b_t, i_t, \theta_{kt}, \theta_{bt}\}_{t\geq 0}$ *such that the following apply:*

1. *Taking the price processes and the intermediary decisions as given, the household's choices solve the household's optimization problem 7, subject to the household budget constraint 6.*

2. *Taking the price processes and the household decisions as given, the intermediary's choices solve the intermediary optimization problem 5, subject to the intermediary wealth evolution 3, the risk-based capital constraint 1 and the liquidity constraint 2.*

3. *The capital market clears:*

$$K_t = k_t + k_{ht}.$$

4. *The risky bond market clears:*

$$b_t = b_{ht}.$$

5. *The risk-free debt market clears:*

$$\mathcal{B} p_{kt} A_t K_t = (1 - \pi_{kt} - \pi_{bt})\, w_{ht} + (1 - \theta_{kt} + \theta_{bt})\, w_t.$$

6. *The goods market clears:*

$$c_t = A_t \left(K_t - i_t k_t\right).$$

Notice that the bond markets' clearing conditions imply

$$(1 + \mathcal{B})\, p_{kt} A_t K_t = w_{ht} + w_t.$$

Figure 2: Equilibrium Market Clearing Conditions

Market	Intermediaries	Households	Total
Capital	k_t	k_{ht}	K_t
Consumption	$i_t k_t A_t$	c_t	$A_t K_t$
Risky Debt	$-b_t$	b_{ht}	0
Risk-Free Debt	$\mathcal{T}_t A_t$	$\mathcal{T}_{ht} A_t$	$\mathcal{B} A_t$

Notice also that the aggregate capital in the economy evolves as

$$dK_t = -\lambda_k K_t dt + \Phi\left(i_t\right) k_t dt.$$

We characterize the equilibrium in terms of the evolutions of three state variables: the leverage of intermediaries, θ_{kt}, and the relative wealth of intermediaries in the economy, ω_t. This representation allows us to characterize the equilibrium outcomes as a solution to a system of algebraic equations, which can easily be solved numerically. In particular, we represent the evolution of the state variables as

$$\begin{aligned}\frac{d\theta_{kt}}{\theta_{kt}} &= \mu_{\theta t} dt + \sigma_{\theta\xi,t} dZ_{\xi t} + \sigma_{\theta a,t} dZ_{at} \\ \frac{d\omega_t}{\omega_t} &= \mu_{\omega t} dt + \sigma_{\omega\xi,t} dZ_{\xi t} + \sigma_{\omega a,t} dZ_{at}.\end{aligned}$$

We can then express all the other equilibrium quantities in terms of the state variables, the debt-to-equity ratio of the intermediaries θ_{bt} and the sensitivities of the return to holding capital to output and liquidity shocks, $\sigma_{ka,t}$ and $\sigma_{k\xi,t}$. We solve for these last two equilibrium quantities numerically as solutions to the system of equations that

1. Equates θ_{kt} and θ_{bt} to the solution to the optimal portfolio allocation choices of the representative intermediary;

2. Equates the expected return to holding one unit of capital from the equilibrium returns process to the expected return to holding one unit of capital from the equilibrium price of capital.

The other equilibrium quantities can be expressed as follows.

1. Equilibrium price of capital, p_{kt}, (from goods market clearing) and optimal capital investment policy, i_t, (from intermediaries' optimization) as a function of the state variables only;

2. From capital market clearing, household allocation to capital π_{kt} as a function of state variables only;

3. From debt market clearing, household allocation to debt, π_{bt}, as a function of state variables and θ_{bt};

4. From the equilibrium evolution of the price of capital, the sensitivities of the intermediaries' leverage to output and liquidity shocks, $\sigma_{\theta a,t}$ and $\sigma_{\theta \xi,t}$, as a function of the state variables and $\sigma_{ka,t}$ and $\sigma_{k\xi,t}$;

5. From the equilibrium evolution of intermediaries' wealth, the sensitivities of the return to holding risky debt to output and liquidity shocks, $\sigma_{ba,t}$ and $\sigma_{b\xi,t}$, as a function of the state variables and θ_{bt}, $\sigma_{ka,t}$ and $\sigma_{k\xi,t}$;

6. From the households' optimal portfolio choice, expected excess return to holding capital, $\mu_{Rk,t} - r_{ft}$, and to holding debt, $\mu_{Rb,t} - r_{ft}$, as a function of the state variables and θ_{bt}, $\sigma_{ka,t}$ and $\sigma_{k\xi,t}$;

7. From the equilibrium evolution of intermediaries' wealth, the expected growth rate of intermediaries' wealth share, $\mu_{\omega t}$ as a function of the state variables and θ_{bt}, $\sigma_{ka,t}$ and $\sigma_{k\xi,t}$;

8. From the equilibrium evolution of capital, the expected growth rate of banks' leverage, $\mu_{\theta t}$ as a function of the state variables and $\sigma_{ka,t}$ and $\sigma_{k\xi,t}$;

9. From the households' Euler equation, the risk-free rate r_{ft} as a function of the state variables and θ_{bt}, $\sigma_{ka,t}$ and $\sigma_{k\xi,t}$.

The details of the solution are relegated to Appendix B.

3 Welfare

We illustrate the outcomes of the model by focusing on the welfare implications of different levels of the policy parameters α, Λ, and $\mathcal{B}$. In Figures 3–7, we present contour plots of endogenous variables as a function of the policy parameters. In those plots, darker shading corresponds to lower levels of the respective endogenous variable. The equilibrium outcomes are computed using the parameters summarized in Table 1.

Table 1: Parameters

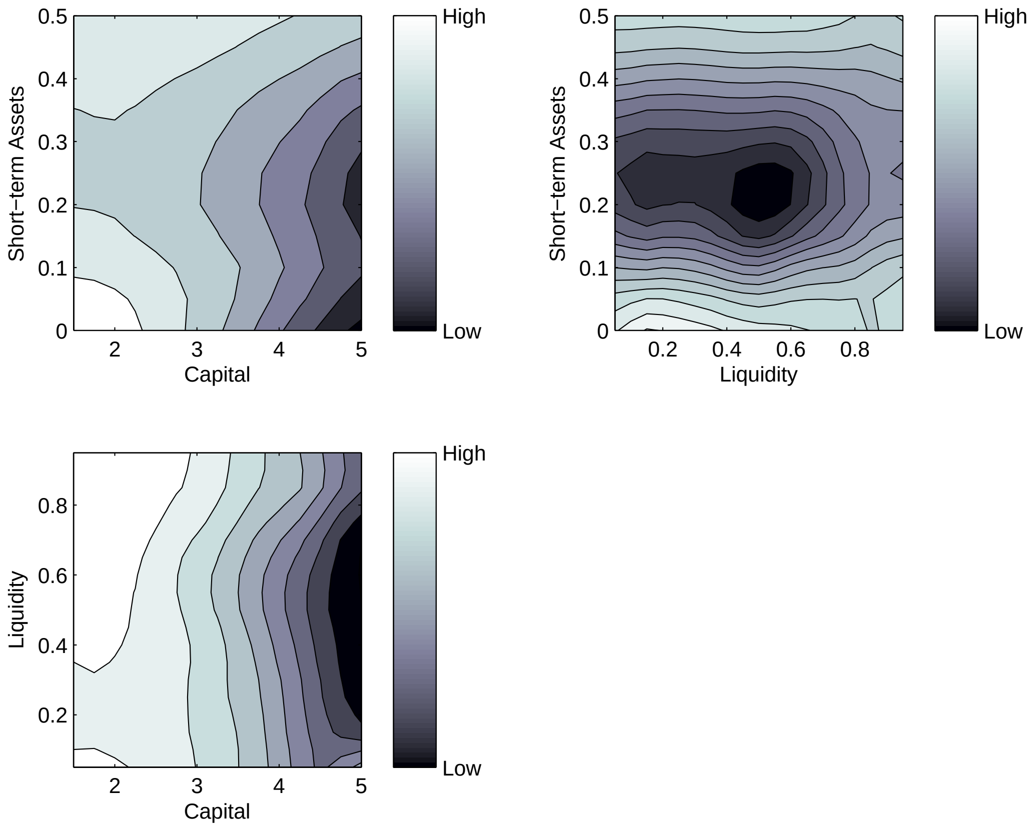
Parameter	Notation	Value
Expected growth rate of productivity	$\bar{a}$	0.0651
Volatility of growth rate of productivity	σ_a	0.388
Volatility of liquidity shocks	σ_ξ	0.0388
Discount rate of intermediaries	ρ	0.06
Effective discount rate of households	$\rho_h - \sigma_\xi^2/2$	0.05
Fixed cost of capital adjustment	ϕ_0	0.1
	ϕ_1	20
Depreciation rate of capital	λ_k	0.03

NOTES: Parameters used in simulations. The parameters of the productivity growth process ($\bar{a}$, σ_a), the parameters of the investment technology (ϕ_0, ϕ_1), subjective discount rates (ρ_h, ρ), and depreciation (λ_k) are taken from Brunnermeier and Sannikov [2012].

3.1 Capital Regulation and Supply of Risk-Free Debt

We begin by considering the tradeoff between the tightness of the capital constraint, α, and the amount of risk-free debt $\mathcal{B}$ supplied by the government, setting the tightness of the liquidity constraint $\tilde{\Lambda} = 0.25$, so that, for every dollar of defaultable liabilities, the financial intermediary has to hold at least 25 cents of risk-free assets. The top left panel of Figure

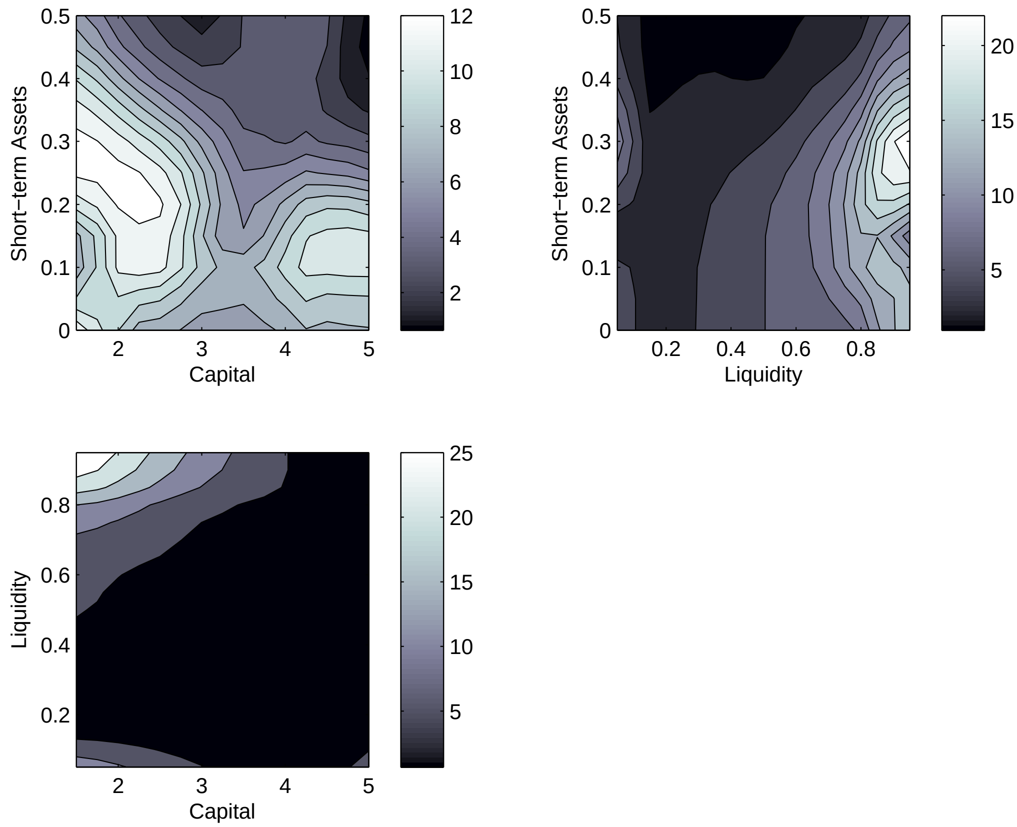
Figure 3: Household Welfare



Notes: Household welfare as a function of the tightness of the capital constraint, α, the tightness of the liquidity constraint, Λ, and the supply of risk-free debt in the economy, $\mathcal{B}$. Outcomes are computed using 1000 simulation paths, of 80 years each.

3 shows that household welfare is highest for loose capital constraints and a low supply of the risk-free debt. Intuitively, when the supply of risk-free debt is low, the equilibrium risk-free rate in the economy is high. Thus, if the intermediaries face a liquidity coverage ratio constraint in this environment, issuing debt is costly for the intermediaries. Indeed, the top left panel of Figure 4 shows that the equilibrium debt-to-equity ratio chosen by the intermediary is highest for a moderate supply of risk-free debt and moderate levels of tightness of the capital constraint. For the low levels of the supply of risk-free debt and the relatively loose capital constraint that maximize expected household welfare, the intermediaries choose lower leverage. Holding the tightness of the capital constraint fixed, as the government increases the supply of risk-free debt, the intermediaries initially increase the

Figure 4: Debt-to-equity Ratios



Notes: Debt-to-equity ratio, θ_{bt}, of the financial sector as a function of the tightness of the capital constraint, α, the tightness of the liquidity constraint, Λ, and the supply of risk-free debt in the economy, $\mathcal{B}$. Outcomes are computed using 1000 simulation paths, of 80 years each.

supply of risky debt to the households, but, for high enough levels of the supply of risk-free debt, intermediaries decrease their debt issuance. This is similar to the safety multiplier effect of government debt discussed by Weymuller [2013]: when capital constraints are relatively loose and supply of risk-free debt is moderately low, intermediaries improve the risk-sharing capabilities of households by issuing risky debt. As the supply of risk-free debt in the economy increases, the marginal cost of the liquidity constraint decreases, increasing the capability of intermediaries to increase leverage. This increases the vulnerability of intermediaries, making households less willing to lend to the intermediaries, decreasing the equilibrium level of intermediary leverage. Similarly, as the capital constraint becomes tighter, intermediaries are prevented from increasing leverage by regulatory constraints.

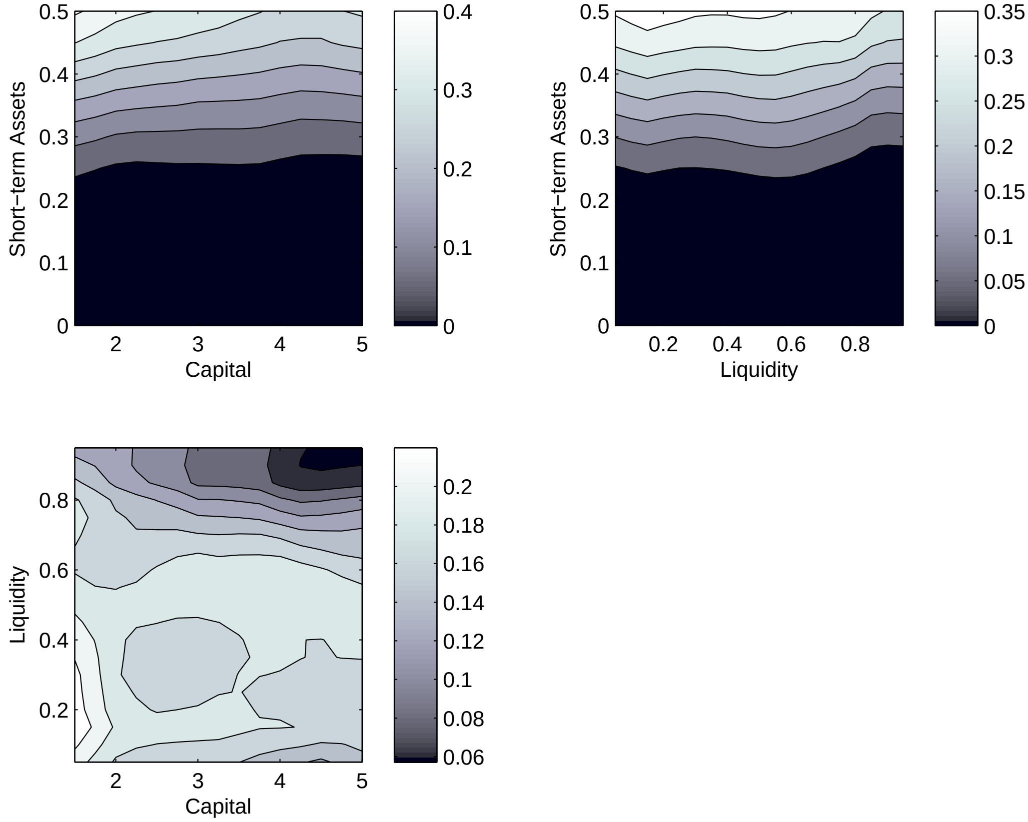
The top left panel of Figure 5 studies the relationship between the probability of the intermediary becoming distressed within six months and the supply of risk-free debt and the tightness of the capital constraint. As the supply of risk-free debt in the economy increases, the intermediaries become more vulnerable, and the distress probability increases. This is consistent with the intuition above: increases in the supply of risk-free debt make the liquidity coverage ratio less costly for the intermediaries, which allows for more risk taking opportunities. In particular, looser supply of risk-free credit to the economy increases the volatility of the return to capital, as shown in the top left panel of Figure 6. While the volatility paradox of Brunnermeier and Sannikov [2012] and Adrian and Boyarchenko [2012] is preserved for small supply of risk-free debt in the economy, with low levels of distress probability corresponding to high levels of local volatility, large supply of the risk-free debt breaks this negative relationship. The top row of Figure 7 shows that, when risk-free debt is in high supply, the transmission of the household liquidity shock is amplified, increasing the sensitivity $\sigma_{k\xi,t}$ of the return on capital to the liquidity shock. Since large supply of risk-free debt decreases the equilibrium risk-free rate, the households substitute away from risk-free debt toward holding the risky securities.

Finally, consider the expected average consumption growth rate as a function of supply of risk-free debt and the tightness of the capital constraint, plotted in the top left panel of Figure 8. Just like the expected household welfare, expected consumption growth is highest for small supplies of risk-free debt and loose capital constraints.

3.2 Liquidity Regulation and Supply of Risk-Free Debt

We turn now to the tradeoff between the tightness of the liquidity constraint, Λ, and the amount of risk-free debt supplied by the government, setting the tightness of the capital constraint $\alpha = 2.5$. The top right panel of Figure 3 shows that the expected household welfare is lowest for intermediate levels of both the tightness of the liquidity constraint and the supply of risk-free debt. Examining the corresponding trade-off in terms of consumption growth and the distress probability (top right panel of Figure 8 and 5, respectively), we see that low levels of expected welfare correspond to high probability of distress and low expected consumption growth. As the supply of risk-free debt decreases, expected consumption growth increases.

Figure 5: Distress probability

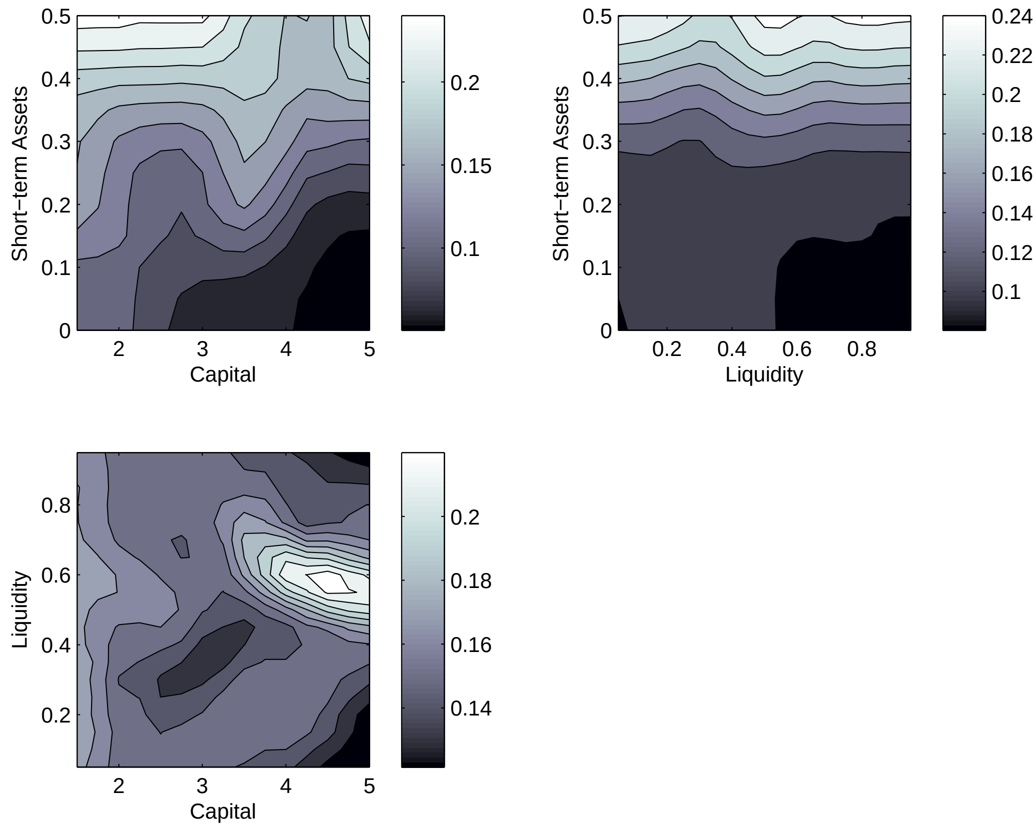


NOTES: Probability of the financial sector becoming distressed within six months as a function of the tightness of the capital constraint, α, the tightness of the liquidity constraint, Λ, and the supply of risk-free debt in the economy, $\mathcal{B}$. Outcomes are computed using 1000 simulation paths, of 80 years each.

Intuitively, for a given level of the tightness of the liquidity constraint, as the supply of risk-free debt decreases, debt issuance becomes more costly for intermediaries, reducing the probability of distress. Costly debt issuance, however, decreases the profitability of the intermediaries. This reduces their availability to invest in new capital projects, decreasing expected consumption.

We can see the effect of relaxing the supply of risk-fee debt more clearly in the top right panel of Figure 4. As the supply of risk-free debt increases, the debt-to-equity ratio of intermediaries increases. Similarly, as the liquidity constraint is relaxed, intermediaries can issue more debt. Notice, however, that the debt-to-equity ratio is maximized for a moderate supply of the risk-free debt. The top right panel of Figure 6 shows that, as the supply of

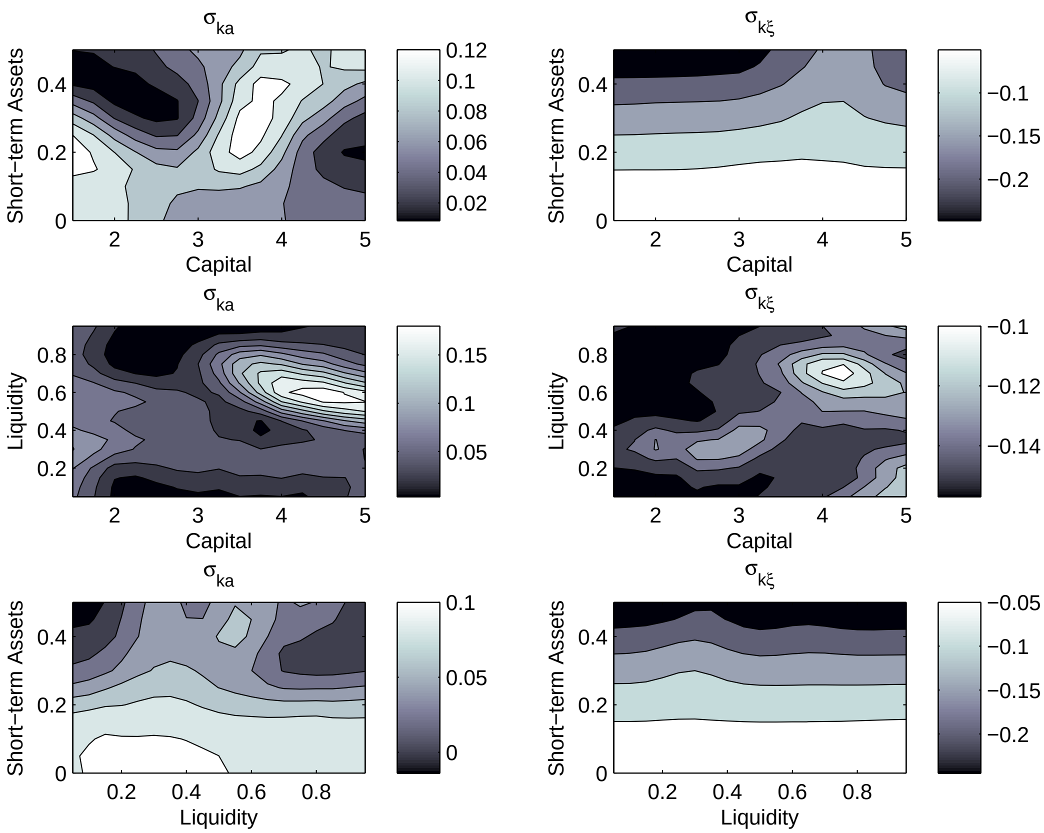
Figure 6: Local Volatility



NOTES: Instantaneous volatility of the return to capital, $\sqrt{\sigma_{ka,t}^2 + \sigma_{k\xi,t}^2}$, as a function of the tightness of the capital constraint, α, the tightness of the liquidity constraint, Λ, and the supply of risk-free debt in the economy, $\mathcal{B}$. Outcomes are computed using 1000 simulation paths, of 80 years each.

risk-free debt increases, the volatility of the return to holding capital increases. Intuitively, higher supply of risk-free debt increases the overall wealth in the economy. Since more wealth can be allocated to the risky capital, local volatility increases. The bottom row of Figure 7 shows that the overall increases in return volatility is due to increased sensitivity to the liquidity shocks, $\sigma_{k\xi,t}$. Thus, as the intermediary issues more debt, the transmission of liquidity shocks through the intermediaries to the risky capital return increases.

Figure 7: Exposures of Return to Capital to Fundamental Shocks

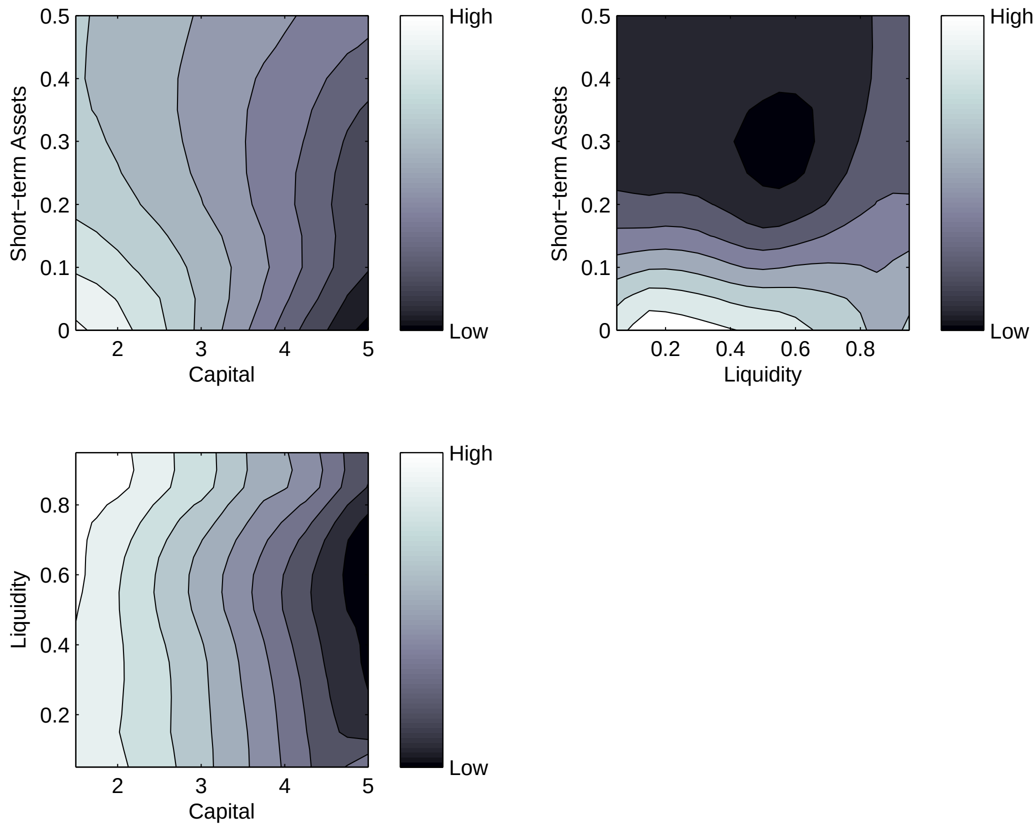


NOTES: Exposures $\sigma_{ka,t}$ and $\sigma_{k\xi,t}$ of return to capital to fundamental shocks Z_{at} and $Z_{\xi t}$ as a function of the tightness of the capital constraint, α, the tightness of the liquidity constraint, Λ, and the supply of risk-free debt in the economy, $\mathcal{B}$. Outcomes are computed using 1000 simulation paths, of 80 years each.

3.3 Capital and Liquidity Regulation

Finally, consider the tradeoff between liquidity and capital regulation. The bottom left panel of Figure 3 shows that there is a tradeoff between the tightness of the capital constraint, α, and the tightness of the liquidity constraint, Λ: high levels of household welfare are achieved for at loose capital requirements and tight liquidity requirements. The bottom left panel of Figure 8 reveals that the high levels of expected welfare correspond to high levels of expected consumption growth. The bottom left panel of Figure 5 shows that the probability of distress is lowest when both liquidity and capital constraints are tight, indicating the systemic risk return tradeoff of Adrian and Boyarchenko [2012]: while consumption growth is highest for

Figure 8: Consumption Growth



NOTES: Average annual consumption growth rate as a function of the tightness of the capital constraint, α, the tightness of the liquidity constraint, Λ, and the supply of risk-free debt in the economy, $\mathcal{B}$. Outcomes are computed using 1000 simulation paths, of 80 years each.

loose capital constraints, looser capital constraints increase the probability of systemic risk. Tighter liquidity requirements further reduce the likelihood of distress. Comparison of the lower panels of Figures 3, 8, and 5 shows that the impact of this tradeoff is maximizing welfare when capital and liquidity requirements are set such that distress probability is in an intermediate, and consumption growth is highest, which is achieved with relatively loose capital but strict liquidity requirements. Figure 4 shows that the welfare maximizing combination of capital and liquidity requirements corresponds to a high degree of leverage, indicating that the danger of high leverage is compensated by strong liquidity requirements. Local volatility is lowest for relatively high capital requirements, and intermediate levels of liquidity, which corresponds to a high distress probability, as shown in Figures 5 and 6. In choosing between capital and liquidity requirements, the volatility paradox of Adrian and Boyarchenko [2012] and Brunnermeier and Sannikov [2012] is thus present. Lower distress probability can be achieved by tightening capital and liquidity requirements, but that increases local volatility. In fact, Figure 7 shows that the (absolute value of) dependence of local volatility on both the liquidity and the productivity shocks increases when capital constraints are loosened.

4 Conclusion

Since the financial crisis, bank regulators have been developing liquidity regulations such as the liquidity coverage ratio. Little is known about the welfare implications of such regulations. The interaction of liquidity regulations with capital requirements and the supply of risk-free assets within the macroeconomy is even less researched. In conducting such analysis, we uncover notable interactions between capital and liquidity regulations and the supply of risk-free assets. General equilibrium considerations are paramount in determining household welfare, debt-to-equity ratios, and return volatilities, demonstrating the desirability of a macroprudential approach to regulation.

Within the context of our model, liquidity requirements are a preferable prudential policy tool relative to capital requirements, as tightening liquidity requirements lowers the likelihood of systemic distress, without impairing consumption growth. In contrast, capital requirements trade off consumption growth and distress probabilities. In addition, we find that intermediate ranges of risk-free asset supply achieve higher welfare. This is because very low levels of the risk-free asset make liquidity requirements costly, while a very high supply of risk-free assets countermand the effects of prudential liquidity regulation.

Our key findings can be summarized as follows:

- The probability of systemic distress is lowered by tighter capital or liquidity requirements, which are substitutes in that respect.
- Consumption growth declines in the tightness of capital requirements, but the link between consumption growth and liquidity requirements is ambiguous. There is thus a

systemic risk-return tradeoff with respect to capital requirements, but not necessarily with respect to liquidity requirements. Welfare tends to be highest with relatively loose capital requirements, and strict liquidity requirements.

- A larger supply of risk-free assets increases the probability of systemic distress, as it increases intermediaries' ability to take on risk. An increase of risk-free assets also tends to lower consumption growth via its impact on the risk-free rate. However, the impact of the amount of short term debt on welfare importantly depends on interactions with the level of liquidity and capital requirements. Intermediate levels of short term asset supply tends to be welfare maximizing.

- Higher leverage tends to be associated with looser capital requirements, tighter liquidity requirements, and intermediate levels of short term asset supply.

The impact of liquidity regulation on growth, systemic risk, and local volatility thus has to be analyzed in conjunction with the tightness of capital regulation and the supply of the risk-free asset.

References

Tobias Adrian and Adam Ashcraft. Shadow banking: a review of the literature. *New Palgrave Dictionary in Economics*, 2012.

Tobias Adrian and Nina Boyarchenko. Intermediary Leverage Cycles and Financial Stability. Federal Reserve Bank of New York Staff Report No. 567, 2012.

Tobias Adrian and Nina Boyarchenko. Intermediary Balance Sheets. Federal Reserve Bank of New York Staff Report No. 651, 2013.

Tobias Adrian and Hyun Shin. The changing nature of financial intermediation and the financial crisis of 2007–2009. *Annual Review of Economics*, 2:603–618, 2010.

Franklin Allen and Douglas Gale. Limited market participation and volatility of asset prices. *American Economic Review*, 84:933–955, 1994.

Paolo Angelini, Stefano Neri, and Fabio Panetta. Monetary and Macroprudential Policies. Bank of Italy Staff Report Number 801, 2011.

Ignazio Angeloni and Ester Faia. Capital regulation and monetary policy with fragile banks. *Journal of Monetary Economics*, 60(3):311–324, 2013.

BCBS. *Basel III: International framework for liquidity risk measurement, standards and monitoring*. Bank for International Settlements, 2010.

BCBS. *Basel III: The Liquidity Coverage Ratio and liquidity risk monitoring tools*. Bank for International Settlements, 2013.

Ben S. Bernanke and Alan S. Blinder. Credit, money, and aggregate demand. *American Economic Review*, 78(2):435–39, 1988.

Harjoat S. Bhamra, Lars-Alexander Kuehn, and Ilya A. Strebulaev. The Levered Equity Risk Premium and Credit Spreads: A Unified Approach. *Review of Financial Studies*, 23 (2):645–703, 2010a.

Harjoat S. Bhamra, Lars-Alexander Kuehn, and Ilya A. Strebulaev. The Aggregate Dynamics of Capital Structure and Macroeconomic Risk. *Review of Financial Studies*, 23(12):4175–4241, 2010b.

Javier Bianchi and Enrique Mendoza. Overborrowing, Financial Crises and Macro-prudential Policy. *IMF Working Paper*, 11/24, 2011.

Markus K. Brunnermeier and Yuliy Sannikov. A Macroeconomic Model with a Financial Sector. *American Economic Review*, forthcoming, 2012.

Jin Cao and Gerhard Illing. Regulation of systemic liquidity risk. *Financial Markets and Portfolio Management*, 24(1):31–48, 2010.

John Cox and Chi–fu Huang. Optimal Consumption and Portfolio Policies when Asset Prices follow a Diffusion Process. *Journal of Economic Theory*, 49:33–83, 1989.

Jakša Cvitanić and Ioannis Karatzas. Convex Duality in Constrained Portfolio Optimization. *The Annals of Applied Probability*, 2(4):767–818, 1992.

Douglas W. Diamond and Philip H. Dybvig. Bank runs, deposit insurance and liquidity. *Journal of Political Economy*, 93(1):401–419, 1983.

Emmanuel Farhi and Iván Werning. A theory of macroprudential policies in the presence of nominal rigidities. Technical report, National Bureau of Economic Research Working Paper 19313, 2013.

Emmanuel Farhi, Mikhail Golosov, and Aleh Tsyvinski. A theory of liquidity and regulation of financial intermediation. *The Review of Economic Studies*, 76(3):973–992, 2009.

Mark Gertler and Nobuhiro Kiyotaki. Banking, Liquidity, and Bank Runs in an Infinite Horizon Economy. Unpublished working papers, Princeton University, 2012.

Charles A. E. Goodhart, Anil K. Kashyap, Dimitrios P. Tsomocos, and Alexandros P. Vardoulakis. Financial Regulation in General Equilibrium. NBER Working Paper No. 17909, 2012.

Zhiguo He and Arvind Krishnamurthy. A Macroeconomic Framework for Quantifying Systemic Risk. Unpublished working paper, 2012.

Zhiguo He and Arvind Krishnamurthy. Intermediary Asset Pricing. *American Economic Review*, 103(2):732–770, 2013.

Bengt Holmström and Jean Tirole. Private and public supply of liquidity. *Journal of Political Economy*, 106(1):1–40, 1998.

Charles Kahn and Joao Santos. Allocating bank regulatory powers: Lender of last resort, deposit insurance and supervision. *European Economic Review*, 49(8):2107–2136, 2005.

Anton Korinek. Systemic risk-taking: amplification effects, externalities, and regulatory responses. ECB Working Paper, 2011.

Hayne E. Leland and Klaus Bjerre Toft. Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads. *Journal of Finance*, 51(3):987–1019, 1996.

Antoine Martin, David Skeie, and Elu Von Thadden. The fragility of short-term secured funding markets. *Journal of Economic Theory*, forthcoming, 2013.

Galo Nuño and Carlos Thomas. Bank Leverage Cycles. Unpublished ECB Working Paper, 2012.

Enrico Perottia and Javier Suarez. A pigovian approach to liquidity regulation. *International Journal of Central Banking*, forthcoming, 2011.

Lev Ratnovski. Bank liquidity regulation and the lender of last resort. *Journal of Financial Intermediation*, 18(4):541–558, 2009.

Jean-Charles Rochet. Liquidity regulation and the lender of last resort. *Financial Stability Review*, 11:45–52, 2008.

Charles-Henri Weymuller. Banks as Safety Multipliers: A Theory of Safe Assets Creation. Working Paper, Harvard University, 2013.

A Proofs

A.1 Intermediaries' Optimization

Recall that the representative intermediary maximizes a mean-variance objective of the instantaneous growth rate of wealth

$$\max_{\theta_{kt},\theta_{bt},i_t} \quad \frac{1}{dt}\mathbb{E}_t\left[\frac{dw_t}{w_t}\right] - \frac{1}{dt}\frac{\gamma}{2}\mathbb{V}_t\left[\frac{dw_t}{w_t}\right],$$

subject to the risk-based leverage constraint

$$\theta_{kt} \leq \alpha^{-1}\left(\sigma_{ka,t}^2 + \sigma_{k\xi,t}^2\right)^{-\frac{1}{2}},$$

the liquidity constraint

$$\Lambda\theta_{bt} \geq \left(\theta_{kt} - 1\right),$$

and the dynamic budget constraint

$$\frac{dw_t}{w_t} = r_{ft}dt + \theta_{kt}\left(dr_{kt} - r_{ft}dt\right) - \theta_{bt}\left(dr_{bt} - r_{ft}dt\right).$$

Notice first that, since the investment choice only enters the expected growth rate of wealth, we can directly take the first order condition with respect to investment to obtain

$$\Phi\left(i_t\right)' = p_{kt}^{-1},$$

or

$$i_t = \frac{1}{\phi_1}\left(\frac{\phi_0^2\phi_1^2}{4}p_{kt}^2 - 1\right).$$

Consider now the intermediaries' optimal portfolio choice. We have

$$\mathbb{E}_t\left[\frac{dw_t}{w_t}\right] = r_{ft}dt + \theta_{kt}\left(\mu_{Rk,t} + \Phi\left(i_t\right) - \frac{i_t}{p_{kt}} - r_{ft}\right)dt - \theta_{bt}\left(\mu_{Rb,t} - r_{ft}\right)dt.$$

Similarly,

$$\mathbb{V}_t\left[\frac{dw_t}{w_t}\right] = \left(\theta_{kt}\sigma_{ka,t} - \theta_{bt}\sigma_{ba,t}\right)^2 dt + \left(\theta_{kt}\sigma_{k\xi,t} - \theta_{bt}\sigma_{b\xi,t}\right)^2 dt.$$

Denote by ζ_{ct} the time t Lagrange multiplier on the risk-based capital constraint and by ζ_{lt} the time t Lagrange multiplier on the liquidity constraint, so that the Lagrangian for the intermediaries' problem is given by

$$\mathcal{L}_t = r_{ft} + \theta_{kt}\left(\tilde{\mu}_{Rk,t} - r_{ft}\right) - \theta_{bt}\left(\mu_{Rb,t} - r_{ft}\right) - \frac{\gamma}{2}\left(\theta_{kt}\sigma_{ka,t} - \theta_{bt}\sigma_{ba,t}\right)^2$$
$$- \frac{\gamma}{2}\left(\theta_{kt}\sigma_{k\xi,t} - \theta_{bt}\sigma_{b\xi,t}\right)^2 + \zeta_{ct}\left(\frac{1}{\alpha\sqrt{\sigma_{ka,t}^2 + \sigma_{k\xi,t}^2}} - \theta_{kt}\right) + \zeta_{lt}\left(\Lambda\theta_{bt} - \left(\theta_{kt} - 1\right)\right),$$

where we have denoted $\tilde{\mu}_{Rk,t} = \mu_{Rk,t} + \Phi\left(i_t\right) - \frac{i_t}{p_{kt}}$. Taking the first order conditions, we obtain

$$0 = \left(\tilde{\mu}_{Rk,t} - r_{ft}\right) - \gamma\sigma_{ka,t}\left(\theta_{kt}\sigma_{ka,t} - \theta_{bt}\sigma_{ba,t}\right) - \gamma\sigma_{k\xi,t}\left(\theta_{kt}\sigma_{k\xi,t} - \theta_{bt}\sigma_{b\xi,t}\right) - \zeta_{ct} - \zeta_{lt}$$
$$0 = -\left(\mu_{Rb,t} - r_{ft}\right) + \gamma\sigma_{ba,t}\left(\theta_{kt}\sigma_{ka,t} - \theta_{bt}\sigma_{ba,t}\right) + \gamma\sigma_{b\xi,t}\left(\theta_{kt}\sigma_{k\xi,t} - \theta_{bt}\sigma_{b\xi,t}\right) + \Lambda\zeta_{lt},$$

or, in matrix form,

$$\begin{bmatrix} \tilde{\mu}_{Rk,t} - r_{ft} \\ -\left(\mu_{Rb,t} - r_{ft}\right) \end{bmatrix} = \gamma \begin{bmatrix} \left(\sigma_{ka,t}^2 + \sigma_{k\xi,t}^2\right) & -\left(\sigma_{ka,t}\sigma_{ba,t} + \sigma_{k\xi,t}\sigma_{b\xi,t}\right) \\ -\left(\sigma_{ka,t}\sigma_{ba,t} + \sigma_{k\xi,t}\sigma_{b\xi,t}\right) & \left(\sigma_{ba,t}^2 + \sigma_{b\xi,t}^2\right) \end{bmatrix} \begin{bmatrix} \theta_{kt} \\ \theta_{bt} \end{bmatrix}$$
$$+ \begin{bmatrix} 1 & 1 \\ 0 & -\Lambda \end{bmatrix} \begin{bmatrix} \zeta_{ct} \\ \zeta_{lt} \end{bmatrix}.$$

A.1.1 Case 1: Neither constraint binds

In this case, $\zeta_{lt} = \zeta_{ct} = 0$, and we can use the first order conditions to solve for the optimal portfolio choice of the representative intermediary

$$\begin{bmatrix} \theta_{kt} \\ \theta_{bt} \end{bmatrix} = \gamma^{-1} \begin{bmatrix} \left(\sigma_{ka,t}^2 + \sigma_{k\xi,t}^2\right) & -\left(\sigma_{ka,t}\sigma_{ba,t} + \sigma_{k\xi,t}\sigma_{b\xi,t}\right) \\ -\left(\sigma_{ka,t}\sigma_{ba,t} + \sigma_{k\xi,t}\sigma_{b\xi,t}\right) & \left(\sigma_{ba,t}^2 + \sigma_{b\xi,t}^2\right) \end{bmatrix}^{-1} \begin{bmatrix} \tilde{\mu}_{Rk,t} - r_{ft} \\ -\left(\mu_{Rb,t} - r_{ft}\right) \end{bmatrix},$$

or, equivalently,

$$\theta_{kt} = \gamma^{-1}\left(\sigma_{ka,t}\sigma_{b\xi,t} - \sigma_{k\xi,t}\sigma_{ba,t}\right)^{-2}\left[\left(\tilde{\mu}_{Rk,t} - r_{ft}\right)\left(\sigma_{ba,t}^2 + \sigma_{b\xi,t}^2\right) - \left(\mu_{Rb,t} - r_{ft}\right)\left(\sigma_{ka,t}\sigma_{ba,t} + \sigma_{k\xi,t}\sigma_{b\xi,t}\right)\right]$$
$$\theta_{bt} = \gamma^{-1}\left(\sigma_{ka,t}\sigma_{b\xi,t} - \sigma_{k\xi,t}\sigma_{ba,t}\right)^{-2}\left[\left(\tilde{\mu}_{Rk,t} - r_{ft}\right)\left(\sigma_{ka,t}\sigma_{ba,t} + \sigma_{k\xi,t}\sigma_{b\xi,t}\right) - \left(\mu_{Rb,t} - r_{ft}\right)\left(\sigma_{ka,t}^2 + \sigma_{k\xi,t}^2\right)\right].$$

Thus, when the intermediary is unconstrained, the optimal portfolio choice of the intermediary corresponds to the standard solution for the portfolio allocation of a mean-variance investor with CRRA coefficient γ.

A.1.2 Case 2: Liquidity constraint binds but not the capital constraint

When the liquidity constraint binds, $\Lambda\theta_{bt} = \theta_{kt} - 1$, or, equivalently, $\theta_{bt} = \Lambda^{-1}(\theta_{kt} - 1)$. Substituting into the first order conditions, we obtain

$$\begin{bmatrix} \tilde{\mu}_{Rk,t} - r_{ft} - \gamma\Lambda^{-1}(\sigma_{ka,t}\sigma_{ba,t} + \sigma_{k\xi,t}\sigma_{b\xi,t}) \\ -(\mu_{Rb,t} - r_{ft}) + \gamma\Lambda^{-1}\left(\sigma_{ba,t}^2 + \sigma_{b\xi,t}^2\right) \end{bmatrix} = \gamma\theta_{kt}\begin{bmatrix} \left(\sigma_{ka,t}^2 + \sigma_{k\xi,t}^2\right) - \Lambda^{-1}(\sigma_{ka,t}\sigma_{ba,t} + \sigma_{k\xi,t}\sigma_{b\xi,t}) \\ -(\sigma_{ka,t}\sigma_{ba,t} + \sigma_{k\xi,t}\sigma_{b\xi,t}) + \Lambda^{-1}\left(\sigma_{ba,t}^2 + \sigma_{b\xi,t}^2\right) \end{bmatrix} + \zeta_{lt}\begin{bmatrix} 1 \\ -\Lambda \end{bmatrix}.$$

Thus, the intermediaries' optimal allocation to capital is given in this case by

$$\theta_{kt} = det_t^{-1}\left[-\Lambda(\tilde{\mu}_{Rk,t} - r_{ft}) + \gamma(\sigma_{ka,t}\sigma_{ba,t} + \sigma_{k\xi,t}\sigma_{b\xi,t}) + \mu_{Rb,t} - r_{ft} - \gamma\Lambda^{-1}\left(\sigma_{ba,t}^2 + \sigma_{b\xi,t}^2\right)\right],$$

and the Lagrange multiplier on the liquidity constraint by

$$\begin{aligned} \zeta_{lt} &= det_t^{-1}\gamma(\tilde{\mu}_{Rk,t} - r_{ft})\left[(\sigma_{ka,t}\sigma_{ba,t} + \sigma_{k\xi,t}\sigma_{b\xi,t}) - \Lambda^{-1}\left(\sigma_{ba,t}^2 + \sigma_{b\xi,t}^2\right)\right] \\ &\quad - det_t^{-1}\gamma(\mu_{Rb,t} - r_{ft})\left[\left(\sigma_{ka,t}^2 + \sigma_{k\xi,t}^2\right) - \Lambda^{-1}(\sigma_{ka,t}\sigma_{ba,t} + \sigma_{k\xi,t}\sigma_{b\xi,t})\right] \\ &\quad + det_t^{-1}\gamma^2\Lambda^{-1}(\sigma_{ka,t}\sigma_{b\xi,t} - \sigma_{k\xi,t}\sigma_{ba,t})^2, \end{aligned}$$

where

$$det_t = -\gamma\Lambda^{-2}\left[(\Lambda\sigma_{ka,t} - \sigma_{ba,t})^2 + (\Lambda\sigma_{k\xi,t} - \sigma_{b\xi,t})^2\right].$$

A.1.3 Case 3: Capital constraint binds but not the liquidity constraint

In this case, $\zeta_{lt} = 0$ and $\theta_{kt} = \bar{\theta}_{kt} \equiv \alpha^{-1}\left(\sigma_{ka,t}^2 + \sigma_{k\xi,t}^2\right)^{-\frac{1}{2}}$. Substituting into the first order conditions, we obtain

$$\begin{bmatrix} \zeta_{ct} \\ \theta_{bt} \end{bmatrix} = \begin{bmatrix} 1 & -\gamma(\sigma_{ka,t}\sigma_{ba,t} + \sigma_{k\xi,t}\sigma_{b\xi,t}) \\ 0 & \gamma\left(\sigma_{ba,t}^2 + \sigma_{b\xi,t}^2\right) \end{bmatrix}^{-1}\begin{bmatrix} \tilde{\mu}_{Rk,t} - r_{ft} - \gamma\left(\sigma_{ka,t}^2 + \sigma_{k\xi,t}^2\right)\bar{\theta}_{kt} \\ -(\mu_{Rb,t} - r_{ft}) + \gamma(\sigma_{ka,t}\sigma_{ba,t} + \sigma_{k\xi,t}\sigma_{b\xi,t})\bar{\theta}_{kt} \end{bmatrix},$$

so that intermediary leverage is given by

$$\theta_{bt} = \gamma^{-1}\left(\sigma_{ba,t}^2 + \sigma_{b\xi,t}^2\right)^{-1}\left[-(\mu_{Rb,t} - r_{ft}) + \gamma(\sigma_{ka,t}\sigma_{ba,t} + \sigma_{k\xi,t}\sigma_{b\xi,t})\bar{\theta}_{kt}\right]$$

and the Lagrange multiplier on the capital constraint by

$$\zeta_{ct} = \tilde{\mu}_{Rk,t} - r_{ft} - \gamma\left(\sigma_{ka,t}^2 + \sigma_{k\xi,t}^2\right)\bar{\theta}_{kt} + \gamma(\sigma_{ka,t}\sigma_{ba,t} + \sigma_{k\xi,t}\sigma_{b\xi,t})\theta_{bt}.$$

A.1.4 Case 4: Both constraints bind

When both constraints bind, $\theta_{kt} = \bar{\theta}_{kt}$ and $\theta_{bt} = \Lambda^{-1}\left(\bar{\theta}_{kt} - 1\right)$. Substituting into the first order conditions, we obtain

$$\begin{bmatrix} 1 & 1 \\ 0 & -\Lambda \end{bmatrix}\begin{bmatrix} \zeta_{ct} \\ \zeta_{lt} \end{bmatrix} = \begin{bmatrix} \tilde{\mu}_{Rk,t} - r_{ft} - \gamma\left(\sigma_{ka,t}^2 + \sigma_{k\xi,t}^2\right)\bar{\theta}_{kt} + \gamma\Lambda^{-1}(\sigma_{ka,t}\sigma_{ba,t} + \sigma_{k\xi,t}\sigma_{b\xi,t})\left(\bar{\theta}_{kt} - 1\right) \\ -(\mu_{Rb,t} - r_{ft}) + \gamma(\sigma_{ka,t}\sigma_{ba,t} + \sigma_{k\xi,t}\sigma_{b\xi,t})\bar{\theta}_{kt} - \gamma\Lambda^{-1}\left(\sigma_{ba,t}^2 + \sigma_{b\xi,t}^2\right)\left(\bar{\theta}_{kt} - 1\right) \end{bmatrix}.$$

Thus the Lagrange multiplier on the liquidity constraint is given by

$$\zeta_{lt} = \Lambda^{-1}\left[(\mu_{Rb,t} - r_{ft}) - \gamma\left(\sigma_{ka,t}\sigma_{ba,t} + \sigma_{k\xi,t}\sigma_{b\xi,t}\right)\bar{\theta}_{kt} + \gamma\Lambda^{-1}\left(\sigma_{ba,t}^2 + \sigma_{b\xi,t}^2\right)\left(\bar{\theta}_{kt} - 1\right)\right],$$

and the Lagrange multiplier on the capital constraint by

$$\zeta_{ct} = (\tilde{\mu}_{Rk,t} - r_{ft}) - \gamma\left(\sigma_{ka,t}^2 + \sigma_{k\xi,t}^2\right)\bar{\theta}_{kt} + \gamma\Lambda^{-1}\left(\sigma_{ka,t}\sigma_{ba,t} + \sigma_{k\xi,t}\sigma_{b\xi,t}\right)\left(\bar{\theta}_{kt} - 1\right) - \zeta_{lt}.$$

A.2 Households' Optimization

Recall that the representative household maximizes expected discounted utility of consumption

$$\max_{c_t, k_{ht}, b_{ht}} \mathbb{E}\left[\int_0^{+\infty} e^{-\xi_t - \rho_h t}\log c_t dt\right],$$

subject to the dynamic budget constraint

$$\frac{dw_{ht}}{w_{ht}} = \pi_{kt}\left(dR_{kt} - r_{ft}dt\right) + \pi_{bt}\left(dR_{bt} - r_{ft}dt\right) + r_{ft}dt - \frac{c_t}{w_{ht}}dt,$$

and no shorting constraints

$$\begin{aligned} \pi_{kt} &\geq 0 \\ \pi_{bt} &\geq 0. \end{aligned}$$

Instead of solving the dynamic optimization problem, we follow Cvitanić and Karatzas [1992] and rewrite the household problem in terms of a static optimization. Cvitanić and Karatzas [1992] extend the Cox and Huang [1989] martingale method approach to constrained optimization problems, such as the one that the households face in our economy.
Define $K = \mathbb{R}_+^2$ to be the convex set of admissible portfolio strategies and introduce the support function of the set $-K$ to be

$$\begin{aligned} \delta(x) = \delta(x|\, K) &\equiv \sup_{\vec{\pi}\in K}\left(-\vec{\pi}'x\right) \\ &= \begin{cases} 0, & x \in K \\ +\infty, & x \notin K \end{cases}. \end{aligned}$$

We can then define an auxiliary unconstrained optimization problem for the household, with the returns in the auxiliary asset market defined as

$$\begin{aligned} r_{ft}^v &= r_{ft} + \delta(\vec{v}_t) \\ dR_{kt}^v &= \left(\mu_{Rk,t} + v_{1t} + \delta(\vec{v}_t)\right)dt + \sigma_{ka,t}dZ_{at} + \sigma_{k\xi,t}dZ_{\xi,t} \\ dR_{bt}^v &= \left(\mu_{Rb,t} + v_{2t} + \delta(\vec{v}_t)\right)dt + \sigma_{ba,t}dZ_{at} + \sigma_{b\xi,t}dZ_{\xi,t}, \end{aligned}$$

for each $\vec{v}_t = [v_{1t}\ v_{2t}]'$ in the space $V(K)$ of square-integrable, progressively measurable processes taking values in K. Corresponding to the auxiliary returns processes is an auxiliary state-price density

$$\frac{d\eta_t^v}{\eta_{t-}^v} = -\left(r_{ft} + \delta\left(\vec{v}_t\right)\right)dt - \left(\vec{\mu}_{Rt} - r_{ft} + \vec{v}_t\right)'\left(\sigma'_{Rt}\right)^{-1}d\vec{Z}_t,$$

where

$$\vec{\mu}_{Rt} = \begin{bmatrix} \mu_{Rk,t} \\ \mu_{Rb,t} \end{bmatrix}; \quad \sigma_{Rt} = \begin{bmatrix} \sigma_{ka,t} & \sigma_{k\xi,t} \\ \sigma_{ba,t} & \sigma_{b\xi,t} \end{bmatrix}; \quad \vec{Z}_t = \begin{bmatrix} Z_{at} \\ Z_{\xi t} \end{bmatrix}.$$

The auxiliary unconstrained problem of the representative household then becomes

$$\max_{c_t} \mathbb{E}\left[\int_0^{+\infty} e^{-\xi_t - \rho_h t} \log c_t dt\right]$$

subject to the static budget constraint:

$$w_{h0} = \mathbb{E}\left[\int_0^{+\infty} \eta_t^v c_t dt\right].$$

The solution to the original constrained problem is then given by the solution to the unconstrained problem for the v that solves the dual problem

$$\min_{v \in V(K)} \mathbb{E}\left[\int_0^{+\infty} e^{-\xi_t - \rho_h t}\tilde{u}\left(\lambda \eta_t^v\right) dt\right],$$

where $\tilde{u}(x)$ is the convex conjugate of $-u(-x)$

$$\tilde{u}(x) \equiv \sup_{z>0}\left[\log(zx) - zx\right] = -\left(1 + \log x\right)$$

and λ is the Lagrange multiplier of the static budget constraint. Cvitanić and Karatzas [1992] show that, for the case of logarithmic utility, the optimal choice of v satisfies

$$\begin{aligned} v_t^* &= \arg\min_{x \in K}\left\{2\delta(x) + \left|\left|\left(\vec{\mu}_{Rt} - r_{ft} + x\right)'\sigma_{Rt}^{-1}\right|\right|^2\right\} \\ &= \arg\min_{x \in K}\left|\left|\left(\vec{\mu}_{Rt} - r_{ft} + x\right)'\sigma_{Rt}^{-1}\right|\right|^2. \end{aligned}$$

Thus,

$$\begin{aligned} v_{1t} &= \begin{cases} 0, & \mu_{Rk,t} - r_{ft} \geq 0 \\ r_{ft} - \mu_{Rk,t}, & \mu_{Rk,t} - r_{ft} < 0 \end{cases} \\ v_{2t} &= \begin{cases} 0, & \mu_{Rb,t} - r_{ft} \geq 0 \\ r_{ft} - \mu_{Rb,t}, & \mu_{Rb,t} - r_{ft} < 0 \end{cases}. \end{aligned}$$

Consider now solving the auxiliary unconstrained problem. Taking the first order condition, we obtain

$$[c_t]: \quad 0 = \frac{e^{-\xi_t - \rho_h t}}{c_t} - \lambda \eta_t^v,$$

or

$$c_t = \frac{e^{-\xi_t - \rho_h t}}{\lambda \eta_t^v}.$$

Substituting into the static budget constraint, we obtain

$$\eta_t^v w_{ht} = \mathbb{E}_t \left[\int_t^{+\infty} \eta_s^v c_s ds \right] = \mathbb{E}_t \left[\int_t^{+\infty} \frac{e^{-\xi_s - \rho_h s}}{\lambda} ds \right] = \frac{e^{-\xi_t - \rho_h t}}{\lambda \left(\rho_h - \sigma_\xi^2 / 2 \right)}.$$

Thus

$$c_t = \left(\rho_h - \frac{\sigma_\xi^2}{2} \right) w_{ht}.$$

To solve for the household's optimal portfolio allocation, notice that:

$$\frac{d \left(\eta_t^v w_{ht} \right)}{\eta_t^v w_{ht}} = -\rho_h dt - d\xi_t + \frac{1}{2} d\xi_t^2 = \left(-\rho_h + \frac{1}{2} \sigma_\xi^2 \right) dt - \sigma_\xi dZ_{\xi t}.$$

On the other hand, applying Itô's lemma, we obtain

$$\frac{d \left(\eta_t^v w_{ht} \right)}{\eta_t^v w_{ht}} = \frac{d\eta_t^v}{\eta_t^v} + \frac{dw_{ht}}{w_{ht}} + \frac{dw_{ht}}{w_{ht}} \frac{d\eta_t^v}{\eta_t^v}.$$

Equating the coefficients on the Brownian terms, we obtain

$$\vec{\pi}_t' = \left(\vec{\mu}_{Rt} - r_{ft} + \vec{v}_t \right)' \left(\sigma_{Rt}' \sigma_{Rt} \right)^{-1} - \sigma_\xi \begin{bmatrix} 0 & 1 \end{bmatrix} \sigma_{Rt}^{-1}.$$

B Equilibrium Derivation

In this Appendix, we provide the details of the derivation of the equilibrium outcomes.

B.1 Capital evolution

Recall that we defined the fraction of intermediary wealth allocated to capital as

$$\theta_{kt} = \frac{p_{kt} A_t k_t}{w_t}.$$

Applying Ito's lemma to the number of units of capital held by the intermediary

$$k_t = \theta_{kt}\omega_t K_t,$$

we obtain

$$\frac{dk_t}{k_t} = \frac{d\omega_t}{\omega_t} + \frac{d\theta_{kt}}{\theta_{kt}} + \frac{dK_t}{K_t} + \left\langle \frac{d\omega_t}{\omega_t}, \frac{d\theta_{kt}}{\theta_{kt}} \right\rangle.$$

Recall on the other hand that the capital held by the intermediary evolves as

$$\frac{dk_t}{k_t} = \left(\Phi\left(i_t\right) - \lambda_k\right) dt.$$

Equating coefficients, we obtain

$$\begin{aligned}
\sigma_{\theta a,t} &= -\sigma_{\omega a,t} \\
\sigma_{\theta \xi,t} &= -\sigma_{\omega \xi,t} \\
\mu_{\theta t} &= \Phi\left(i_t\right)\left(1 - \theta_{kt}\omega_t\right) - \mu_{\omega t} + \sigma_{\theta a,t}^2 + \sigma_{\theta \xi,t}^2.
\end{aligned}$$

B.2 Intermediary wealth evolution

Turn now to equilibrium evolution of intermediaries' wealth. Recall that we have defined the fraction of total wealth in the economy held by the intermediaries to be

$$\omega_t = \frac{w_t}{p_{kt}A_t K_t}.$$

Thus, applying Ito's lemma, we obtain

$$\frac{d\omega_t}{\omega_t} = \frac{dw_t}{w_t} - \frac{d\left(p_{kt}A_t\right)}{p_{kt}A_t} - \frac{dK_t}{K_t} + \left\langle \frac{d\left(p_{kt}A_t\right)}{p_{kt}A_t} \right\rangle^2 - \left\langle \frac{dw_t}{w_t}, \frac{d\left(p_{kt}A_t\right)}{p_{kt}A_t} \right\rangle.$$

Recall further

$$\begin{aligned}
\frac{dw_t}{w_t} &= \theta_{kt}\left(dr_{kt} - r_{ft}dt\right) - \theta_{bt}\left(dr_{bt} - r_{ft}dt\right) + r_{ft}dt \\
&= \theta_{kt}\left[\left(\mu_{Rk,t} + \Phi\left(i_t\right) - \frac{i_t}{p_{kt}} - r_{ft}\right)dt + \sigma_{ka,t}dZ_{a,t} + \sigma_{k\xi,t}dZ_{\xi,t}\right] \\
&- \theta_{bt}\left[\left(\mu_{Rb,t} - r_{ft}\right)dt + \sigma_{ba,t}dZ_{a,t} + \sigma_{b\xi,t}dZ_{\xi,t}\right] + r_{ft}dt,
\end{aligned}$$

and

$$\begin{aligned}
\frac{d\left(p_{kt}A_t\right)}{p_{kt}A_t} &= \left(\mu_{Rk,t} + \lambda_k - \frac{1}{p_{kt}}\right)dt + \sigma_{ka,t}dZ_{a,t} + \sigma_{k\xi,t}dZ_{\xi,t} \\
\frac{dK_t}{K_t} &= \left(\Phi\left(i_t\right)\theta_{kt}\omega_t - \lambda_k\right)dt.
\end{aligned}$$

Equating coefficients, we obtain

$$\begin{aligned}
\mu_{\omega t} &= \theta_{kt}\Phi\left(i_t\right)\left(1-\omega_t\right) - \theta_{bt}\left(\mu_{Rb,t} - r_{ft}\right) + \left(\theta_t - 1\right)\left(\mu_{Rk,t} - r_{ft}\right) + \frac{1}{p_{kt}}\left(1 - \theta_{kt} i_t\right) \\
&\quad + \left(1-\theta_{kt}\right)\left(\sigma_{ka,t}^2 + \sigma_{k\xi,t}^2\right) + \theta_{bt}\sigma_{ka,t}\sigma_{ba,t} + \theta_{bt}\sigma_{k\xi,t}\sigma_{b\xi,t} \\
\sigma_{\omega a,t} &= \left(\theta_{kt} - 1\right)\sigma_{ka,t} - \theta_{bt}\sigma_{ba,t} \\
\sigma_{\omega\xi,t} &= \left(\theta_{kt} - 1\right)\sigma_{k\xi,t} - \theta_{bt}\sigma_{b\xi,t}.
\end{aligned}$$

B.3 Equilibrium pricing kernel

Notice that we can express the household optimal portfolio allocation as

$$\begin{aligned}
\pi_{kt} &= \frac{p_{kt}A_t k_{ht}}{w_{ht}} = \frac{1-\theta_{kt}\omega_t}{1+\mathcal{B}-\omega_t} \\
\pi_{bt} &= \frac{p_{bt}A_t b_{ht}}{w_{ht}} = \frac{\theta_{bt}\omega_t}{1+\mathcal{B}-\omega_t}.
\end{aligned}$$

Substituting into the households' optimal portfolio allocation decision and solving for excess returns, we obtain

$$\begin{aligned}
\mu_{Rk,t} - r_{ft} &= \left(\sigma_{ka,t}^2 + \sigma_{k\xi,t}^2\right)\pi_{kt} + \left(\sigma_{ka,t}\sigma_{ba,t} + \sigma_{k\xi,t}\sigma_{b\xi,t}\right)\pi_{bt} + \sigma_{k\xi,t}\sigma_{\xi} \\
&= \left(\sigma_{ka,t}^2 + \sigma_{k\xi,t}^2\right)\frac{1-\theta_{kt}\omega_t}{1+\mathcal{B}-\omega_t} + \left(\sigma_{ka,t}\sigma_{ba,t} + \sigma_{k\xi,t}\sigma_{b\xi,t}\right)\frac{\theta_{bt}\omega_t}{1+\mathcal{B}-\omega_t} + \sigma_{k\xi,t}\sigma_{\xi} \\
\mu_{Rb,t} - r_{ft} &= \left(\sigma_{ba,t}^2 + \sigma_{b\xi,t}^2\right)\pi_{bt} + \left(\sigma_{ka,t}\sigma_{ba,t} + \sigma_{k\xi,t}\sigma_{b\xi,t}\right)\pi_{kt} + \sigma_{b\xi,t}\sigma_{\xi} \\
&= \left(\sigma_{ba,t}^2 + \sigma_{b\xi,t}^2\right)\frac{\theta_{bt}\omega_t}{1+\mathcal{B}-\omega_t} + \left(\sigma_{ka,t}\sigma_{ba,t} + \sigma_{k\xi,t}\sigma_{b\xi,t}\right)\frac{1-\theta_{kt}\omega_t}{1+\mathcal{B}-\omega_t} + \sigma_{b\xi,t}\sigma_{\xi}.
\end{aligned}$$

Thus, we can express the equilibrium pricing kernel in terms of the primitive shocks $(dZ_{at}, dZ_{\xi t})$ as

$$\frac{d\Lambda_t}{\Lambda_t} = -r_{ft}dt - \eta_{at}dZ_{at} - \eta_{\xi t}dZ_{\xi t},$$

where the price of risk of productivity shocks, η_{at}, is given by

$$\eta_{at} = \sigma_{ka,t}\frac{1-\theta_{kt}\omega_t}{1+\mathcal{B}-\omega_t} + \sigma_{ba,t}\frac{\theta_{bt}\omega_t}{1+\mathcal{B}-\omega_t},$$

and the price of risk of liquidity shocks, $\eta_{\xi t}$, by

$$\eta_{\xi t} = \sigma_{k\xi,t}\frac{1-\theta_{kt}\omega_t}{1+\mathcal{B}-\omega_t} + \sigma_{b\xi,t}\frac{\theta_{bt}\omega_t}{1+\mathcal{B}-\omega_t} + \sigma_{\xi}.$$

While it is natural to express the pricing kernel in terms of the primitive shocks in the economy, these shocks are not readily observable. Instead, we express the pricing kernel in terms of shocks to output and leverage. Define the standardized innovation to (log) output as

$$d\hat{y}_t \equiv \sigma_a^{-1} \left(d \log Y_t - \mathbb{E}_t \left[d \log Y_t \right] \right) = dZ_{at},$$

and the standardized innovation to the growth rate of leverage as

$$d\hat{\theta}_t \equiv \left(\sigma^2_{\theta a,t} + \sigma^2_{\theta \xi,t} \right)^{-\frac{1}{2}} \left(\frac{d\theta_{kt}}{\theta_{kt}} - \mathbb{E}_t \left[\frac{d\theta_{kt}}{\theta_{kt}} \right] \right) = \frac{\sigma_{\theta a,t}}{\sqrt{\sigma^2_{\theta a,t} + \sigma^2_{\theta \xi,t}}} dZ_{at} + \frac{\sigma_{\theta \xi,t}}{\sqrt{\sigma^2_{\theta a,t} + \sigma^2_{\theta \xi,t}}} dZ_{\xi t}.$$

Then

$$dZ_{\xi t} = \sqrt{1 + \frac{\sigma^2_{\theta a,t}}{\sigma^2_{\theta \xi,t}}} d\hat{\theta}_t - \frac{\sigma_{\theta a,t}}{\sigma_{\theta \xi,t}} d\hat{y}_t,$$

so that we can express the equilibrium pricing kernel as

$$\frac{d\Lambda_t}{\Lambda_t} = -r_{ft} dt - \eta_{yt} d\hat{y}_t - \eta_{\theta t} d\hat{\theta}_t,$$

where the price of risk of output, η_{yt}, is given by

$$\eta_{yt} = \eta_{at} - \frac{\sigma_{\theta a,t}}{\sigma_{\theta \xi,t}} \eta_{\xi t} = \left(\sigma_{ka,t} - \frac{\sigma_{\theta a,t}}{\sigma_{\theta \xi,t}} \sigma_{k\xi,t} \right) \frac{1 - \theta_{kt}\omega_t}{1 + \mathcal{B} - \omega_t} + \left(\sigma_{ba,t} - \frac{\sigma_{\theta a,t}}{\sigma_{\theta \xi,t}} \sigma_{b\xi,t} \right) \frac{\theta_{bt}\omega_t}{1 + \mathcal{B} - \omega_t} - \frac{\sigma_{\theta a,t}}{\sigma_{\theta \xi,t}} \sigma_\xi,$$

and the price of risk of leverage, $\eta_{\theta t}$, by

$$\eta_{\theta t} = \sqrt{1 + \frac{\sigma^2_{\theta a,t}}{\sigma^2_{\theta \xi,t}}} \eta_{\xi t} = \sqrt{1 + \frac{\sigma^2_{\theta a,t}}{\sigma^2_{\theta \xi,t}}} \left[\sigma_{k\xi,t} \frac{1 - \theta_{kt}\omega_t}{1 + \mathcal{B} - \omega_t} + \sigma_{b\xi,t} \frac{\theta_{bt}\omega_t}{1 + \mathcal{B} - \omega_t} + \sigma_\xi \right].$$

B.4 Equilibrium price of capital

Recall that the goods market clearing condition gives

$$c_t + i_t k_t A_t = A_t K_t,$$

and that the intermediaries' optimal investment policy is

$$i_t = \frac{1}{\phi_1} \left(\frac{\phi_0^2 \phi_1^2}{4} p_{kt}^2 - 1 \right).$$

Diving both sides by $A_t K_t$, we obtain

$$1 = \left(\rho_h - \frac{\sigma_\xi^2}{2} \right) p_{kt} \left(1 + \mathcal{B} - \omega_t \right) + \frac{1}{\phi_1} \left(\frac{\phi_0^2 \phi_1^2}{4} p_{kt}^2 - 1 \right) \theta_{kt} \omega_t.$$

Define

$$\beta = \frac{2}{\phi_0^2 \phi_1} \left(\rho_h - \frac{\sigma_\xi^2}{2} \right),$$

so that the price of capital solves

$$0 = p_{kt}^2 \theta_{kt} \omega_t + 2\beta \left(1 + \mathcal{B} - \omega_t\right) p_{kt} - \frac{4}{\phi_0^2 \phi_1} - \frac{4}{\phi_0^2 \phi_1^2} \theta_{kt} \omega_t,$$

or, equivalently,

$$p_{kt} = \frac{-\beta \left(1 + \mathcal{B} - \omega_t\right) + \sqrt{\beta^2 \left(1 + \mathcal{B} - \omega_t\right)^2 + \frac{4}{\phi_0^2 \phi_1^2} \theta_{kt} \omega_t \left(\phi_1 + \theta_{kt} \omega_t\right)}}{\theta_{kt} \omega_t}.$$

Applying Ito's lemma to the goods market clearing condition, we obtain

$$\begin{aligned} 0 &= 2 p_{kt}^2 \theta_{kt} \omega_t \frac{dp_{kt}}{p_{kt}} + p_{kt}^2 \theta_{kt} \omega_t \left\langle \frac{dp_{kt}}{p_{kt}} \right\rangle^2 + \left(p_{kt}^2 - \frac{4}{\phi_0^2 \phi_1^2} \right) \theta_{kt} \omega_t \frac{d\left(\theta_{kt} \omega_t\right)}{\theta_{kt} \omega_t} \\ &+ 2\beta \left(1 + \mathcal{B} - \omega_t\right) p_{kt} \frac{dp_{kt}}{p_{kt}} - 2\beta \omega_t p_{kt} \frac{d\omega_t}{\omega_t} - 2\beta \omega_t p_{kt} \left\langle \frac{d\omega_t}{\omega_t}, \frac{dp_{kt}}{p_{kt}} \right\rangle. \end{aligned}$$

Notice that we can express

$$\frac{dp_{kt}}{p_{kt}} = dR_{kt} - p_{kt}^{-1} dt - \left(\bar{a} + \frac{\sigma_a^2}{2} \right) dt - \sigma_a dZ_{at} + \lambda_k dt - \left(\sigma_{ka,t} - \sigma_a\right) \sigma_a dt.$$

Thus, equating coefficients, we obtain

$$\begin{aligned} 0 &= 2 p_{kt} \left(\theta_{kt} \omega_t p_{kt} + \beta \left(1 + \mathcal{B} - \omega_t\right) \right) \left(\mu_{Rk,t} - p_{kt}^{-1} - \bar{a} - \frac{\sigma_a^2}{2} + \lambda_k - \left(\sigma_{ka,t} - \sigma_a\right) \sigma_a \right) \\ &+ p_{kt}^2 \theta_{kt} \omega_t \left[\left(\sigma_{ka,t} - \sigma_a\right)^2 + \sigma_{k\xi,t}^2 \right] + \left(p_{kt}^2 - \frac{4}{\phi_0^2 \phi_1^2} \right) \theta_{kt} \omega_t \left(1 - \theta_{kt} \omega_t\right) \Phi\left(i_t\right) - 2\beta \omega_t p_{kt} \mu_{\omega t} \\ &- 2\beta \omega_t p_{kt} \left[\sigma_{\omega a,t} \left(\sigma_{ka,t} - \sigma_a\right) + \sigma_{\omega \xi,t} \sigma_{k\xi,t} \right] \\ 0 &= \left(\theta_{kt} \omega_t p_{kt} + \beta \left(1 + \mathcal{B} - \omega_t\right) \right) \left(\sigma_{ka,t} - \sigma_a\right) - \beta \omega_t \sigma_{\omega a,t} \\ 0 &= \left(\theta_{kt} \omega_t p_{kt} + \beta \left(1 + \mathcal{B} - \omega_t\right) \right) \sigma_{k\xi,t} - \beta \omega_t \sigma_{\omega \xi,t}. \end{aligned}$$

Using

$$\begin{aligned} \sigma_{\omega a,t} &= \left(\theta_{kt} - 1\right) \sigma_{ka,t} - \theta_{bt} \sigma_{ba,t} \\ \sigma_{\omega \xi,t} &= \left(\theta_{kt} - 1\right) \sigma_{k\xi,t} - \theta_{bt} \sigma_{b\xi,t}, \end{aligned}$$

we can rewrite the above as

$$\begin{aligned}
\beta\omega_t\theta_{bt}\sigma_{ba,t} &= -\left(\theta_{kt}\omega_t p_{kt} + \beta\left(1+\mathcal{B}-\omega_t\right)\right)\left(\sigma_{ka,t}-\sigma_a\right) + \beta\omega_t\left(\theta_{kt}-1\right)\sigma_{ka,t} \\
\beta\omega_t\theta_{bt}\sigma_{b\xi,t} &= -\left(\theta_{kt}\omega_t p_{kt} + \beta\left(1+\mathcal{B}-\omega_t\right)\right)\sigma_{k\xi,t} + \beta\omega_t\left(\theta_{kt}-1\right)\sigma_{k\xi,t}.
\end{aligned}$$

Simplifying, we obtain

$$\begin{aligned}
\sigma_{ba,t} &= -\frac{\beta\left(1+\mathcal{B}-\theta_{kt}\omega_t\right)+\theta_{kt}\omega_t p_{kt}}{\beta\omega_t\theta_{bt}}\left(\sigma_{ka,t}-\sigma_a\right) + \frac{\theta_{kt}-1}{\theta_{bt}}\sigma_a \\
\sigma_{b\xi,t} &= -\frac{\beta\left(1+\mathcal{B}-\theta_{kt}\omega_t\right)+\theta_{kt}\omega_t p_{kt}}{\beta\omega_t\theta_{bt}}\sigma_{k\xi,t},
\end{aligned}$$

and

$$\begin{aligned}
\beta\omega_t\sigma_{\omega a,t} &= \left(\theta_{kt}\omega_t p_{kt} + \beta\left(1+\mathcal{B}-\omega_t\right)\right)\left(\sigma_{ka,t}-\sigma_a\right) \\
\beta\omega_t\sigma_{\omega\xi,t} &= \left(\theta_{kt}\omega_t p_{kt} + \beta\left(1+\mathcal{B}-\omega_t\right)\right)\sigma_{k\xi,t}.
\end{aligned}$$

B.5 Risk-free rate

Consider finally the risk-free rate in the economy. From the households' Euler equation, we have

$$r_{ft} = \left(\rho_h - \frac{\sigma_\xi^2}{2}\right) + \frac{1}{dt}\mathbb{E}_t\left[\frac{dc_t}{c_t}\right] - \frac{1}{dt}\mathbb{E}_t\left[\left\langle\frac{dc_t}{c_t}\right\rangle^2 + \left\langle\frac{dc_t}{c_t}, d\xi_t\right\rangle\right].$$

From the goods' market clearing condition, we have

$$c_t = A_t K_t\left(1 - i_t\theta_{kt}\omega_t\right) = A_t K_t\left[1 - \frac{1}{\phi_1}\left(\frac{\phi_0^2\phi_1^2}{4}p_{kt}^2 - 1\right)\theta_{kt}\omega_t\right].$$

Applying Ito's lemma, we obtain

$$\begin{aligned}
dc_t = {} & c_t\frac{dA_t}{A_t} + c_t\frac{dK_t}{K_t} - c_t\frac{i_t\theta_{kt}\omega_t}{1-i_t\theta_{kt}\omega_t}\frac{d\left(\theta_{kt}\omega_t\right)}{\theta_{kt}\omega_t} - 2A_tK_t\theta_{kt}\omega_t\left(\frac{\phi_0^2\phi_1}{4}\right)p_{kt}^2\frac{dp_{kt}}{p_{kt}} \\
& - A_tK_t\theta_{kt}\omega_t\left(\frac{\phi_0^2\phi_1}{4}\right)p_{kt}^2\left(\left\langle\frac{dp_{kt}}{p_{kt}}\right\rangle^2 + 2\left\langle\frac{dp_{kt}}{p_{kt}}, \frac{dA_t}{A_t}\right\rangle\right),
\end{aligned}$$

so that

$$\frac{dc_t}{c_t} = \frac{dA_t}{A_t} + \frac{dK_t}{K_t} - \frac{i_t\theta_{kt}\omega_t}{1-i_t\theta_{kt}\omega_t}\frac{d\left(\theta_{kt}\omega_t\right)}{\theta_{kt}\omega_t} - 2\frac{\left(i_t+\phi_1^{-1}\right)\theta_{kt}\omega_t}{1-i_t\theta_{kt}\omega_t}\left(\frac{dp_{kt}}{p_{kt}} + \frac{1}{2}\left\langle\frac{dp_{kt}}{p_{kt}}\right\rangle^2 + \left\langle\frac{dp_{kt}}{p_{kt}}, \frac{dA_t}{A_t}\right\rangle\right).$$

Thus

$$\frac{1}{dt}\mathbb{E}_t\left[\frac{dc_t}{c_t}\right] = \bar{a} + \frac{\sigma_a^2}{2} - \lambda_k + \frac{\theta_{kt}\omega_t}{1 - i_t\theta_{kt}\omega_t}\Phi\left(i_t\right)\left(1 - i_t\right) - \frac{\left(i_t + \phi_1^{-1}\right)\theta_{kt}\omega_t}{1 - i_t\theta_{kt}\omega_t}\left(\left(\sigma_{ka,t} - \sigma_a\right)^2 + \sigma_{k\xi,t}^2\right)$$
$$- 2\frac{\left(i_t + \phi_1^{-1}\right)\theta_{kt}\omega_t}{1 - i_t\theta_{kt}\omega_t}\left(\mu_{Rk,t} - p_{kt}^{-1} - \bar{a} - \frac{\sigma_a^2}{2} + \lambda_k\right)$$

$$\frac{1}{dt}\mathbb{E}_t\left[\left\langle\frac{dc_t}{c_t}\right\rangle^2\right] = \left(\sigma_a - 2\frac{\left(i_t + \phi_1^{-1}\right)\theta_{kt}\omega_t}{1 - i_t\theta_{kt}\omega_t}\left(\sigma_{ka,t} - \sigma_a\right)\right)^2 + \left(2\frac{\left(i_t + \phi_1^{-1}\right)\theta_{kt}\omega_t}{1 - i_t\theta_{kt}\omega_t}\right)^2\sigma_{k\xi,t}^2$$

$$\frac{1}{dt}\mathbb{E}_t\left[\left\langle\frac{dc_t}{c_t}, d\xi_t\right\rangle\right] = -2\frac{\left(i_t + \phi_1^{-1}\right)\theta_{kt}\omega_t}{1 - i_t\theta_{kt}\omega_t}\sigma_{k\xi,t}\sigma_\xi,$$

and the risk-free rate is given by

$$r_{ft}\left(1 + 2\frac{\left(i_t + \phi_1^{-1}\right)\theta_{kt}\omega_t}{1 - i_t\theta_{kt}\omega_t}\right) = \rho_h - \frac{\sigma_\xi^2}{2} + \bar{a} + \frac{\sigma_a^2}{2} - \lambda_k + \frac{\theta_{kt}\omega_t}{1 - i_t\theta_{kt}\omega_t}\Phi\left(i_t\right)\left(1 - i_t\right)$$
$$- \frac{\left(i_t + \phi_1^{-1}\right)\theta_{kt}\omega_t}{1 - i_t\theta_{kt}\omega_t}\left(\left(\sigma_{ka,t} - \sigma_a\right)^2 + \sigma_{k\xi,t}^2\right)$$
$$- 2\frac{\left(i_t + \phi_1^{-1}\right)\theta_{kt}\omega_t}{1 - i_t\theta_{kt}\omega_t}\left(\mu_{Rk,t} - r_{ft} - p_{kt}^{-1} - \bar{a} - \frac{\sigma_a^2}{2} + \lambda_k\right)$$
$$- \left(\sigma_a - 2\frac{\left(i_t + \phi_1^{-1}\right)\theta_{kt}\omega_t}{1 - i_t\theta_{kt}\omega_t}\left(\sigma_{ka,t} - \sigma_a\right)\right)^2 - \left(2\frac{\left(i_t + \phi_1^{-1}\right)\theta_{kt}\omega_t}{1 - i_t\theta_{kt}\omega_t}\right)^2\sigma_{k\xi,t}^2$$
$$+ 2\frac{\left(i_t + \phi_1^{-1}\right)\theta_{kt}\omega_t}{1 - i_t\theta_{kt}\omega_t}\sigma_{k\xi,t}\sigma_\xi.$$