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The Side Effects of Safe Asset Creation

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Abstract

We present an incomplete markets model to understand the costs and benefits of increasing government debt in a low interest rate environment. Higher risk increases the demand for safe assets, lowering the natural rate of interest below zero, constraining monetary policy at the zero lower bound, and raising unemployment. Higher government debt satiates the demand for safe assets, raising the natural rate and restoring full employment. While this permanently lowers investment, a policymaker committed to low inflation has no alternative. Higher inflation targets, instead, permit both full employment and high investment, but allow for harmful bubbles. Aggressive fiscal policy can prevent bubbles.

Key words: safe assets, negative natural rate, crowding out, risk premium, liquidity traps, bubbles

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1 Introduction

The most striking macroeconomic fact of the last three decades has been the dramatic decline in real interest rates in the United States and other advanced economies. Policy debates have been dominated by discussion of the causes, consequences, and remedies for this fact. A growing literature attributes this decline to a *shortage of safe assets*, which pushed the natural rate of interest well below zero. This negative natural rate pushed many advanced economies to the zero lower bound (ZLB), leaving conventional monetary policy unable to prevent a deep and lasting recession.[1] This diagnosis suggests that exiting such recessions may require raising natural rates above zero by increasing the *supply* of safe assets (Caballero et al., 2017b), such as U.S. government debt. Indeed, a recent literature documents that U.S. Treasuries enjoy a *convenience yield* reflecting their liquidity and safety attributes, which responds to the aggregate supply of Treasury debt (Krishnamurthy and Vissing-Jorgensen, 2012)(henceforth KVJ).[2] This suggests that by increasing debt, fiscal authorities can reduce the safety premium, satiate the demand for safe assets, and raise the natural rate of interest, allowing monetary policy to regain its potency. Even if governments *can* address the shortage of safe assets in this way, however, it remains unclear whether they *should* do so. We center our investigation around this question.

When Ricardian Equivalence holds, the supply of government debt does not affect equilibrium outcomes unless it provides non-pecuniary benefits; while such a modeling strategy is convenient, it does not provide satisfactory answers to the questions we are interested in. We instead present an analytically tractable incomplete markets model. The economy has a simple overlapping generations (OLG) structure. Young households supply labor inelastically, pay lump sum taxes, and invest in both risky capital and safe government debt. Risky capital earns a risk premium over safe government debt, which depends endogenously on the supply of safe assets.[3] By increasing the supply of government debt, a policymaker *can* reduce the risk premium and raise the safe rate of interest. We ask whether a policymaker *should* do so.

To understand the forces at play, we first study a model without nominal rigidities. We refer to allocations in this benchmark as "*natural allocations*", with the understanding that there is a continuum of natural allocations corresponding to different levels of government debt.[4] An increase in the riskiness of the return on capital reduces real interest rates, as households attempt to substitute away from risky capital towards safe debt; a large enough increase in risk pushes interest rates below zero. Higher government debt can offset this decline in the *natural rate of interest*[5] by satiating the demand for safe assets. But while higher debt insures old households against increased risk, it also crowds out investment in physical capital. Absent nominal rigidities, this cost is so strong that it is never optimal to prevent real interest rates from falling below zero. If risk is low enough, the *optimal natural allocation* features no safe asset creation and positive interest rates. For intermediate risk, it remains optimal to refrain from safe asset creation even when this entails negative real rates, because the costs outweigh the benefits. For a high level of risk, the insurance benefits overwhelm the costs associated with crowding out, and the optimal natural allocation

[1] See for example Caballero and Farhi (2016), Del Negro et al. (2017).

[2] Del Negro et al. (2017) find that liquidity and safety premia are the main drivers of the secular decline in the natural rate.

[3] In most of the paper, government debt is the only safe asset. We therefore use the terms government debt and safe assets interchangeably. In Section 5, we also allow for the possibility of privately provided pseudo safe assets such as rational bubbles.

[4] This feature arises due to the lack of of Ricardian Equivalence. Fiscal policy can choose between these natural allocations.

[5] By the *natural rate of interest*, we mean the real interest rate arising in the natural allocation. Fiscal policy can determine the natural rate via the choice of government debt.

features positive debt - but not enough to make interest rates positive. Importantly, even when some debt is optimal, this is not because the economy is dynamically inefficient: it is *always* dynamically efficient.

We use these insights to study how an increase in risk interacts with monetary policy in an economy with nominal rigidities. As in the natural allocation, higher risk induces households to substitute away from risky capital towards safe government debt. With monetary policy constrained by the ZLB, the interest rate cannot fall to clear the bond market. Higher demand for safe assets reduces demand for capital and consumption, causing prices to fall. With downward nominal wage rigidity, deflation raises real wages, lowering labor demand and employment. Worse still, the fall in employment is expected to persist, reducing the expected marginal product of capital and further reducing investment, leading to a permanent slump.

An increase in government debt satiates the demand for safe assets without requiring negative interest rates, allowing conventional monetary policy to restore full employment. This short-circuits the adverse feedback loop between unemployment and low investment, resulting in higher steady state capital than would occur without an increase in the supply of safe assets. But this level of capital is lower than the *optimal natural allocation*, which featured no safe asset creation and negative real rates. In this sense, the costs of a risk-induced recession may persist even after the economy has returned to full employment, manifesting as sluggish investment and low labor productivity.

The fundamental problem is that the optimal natural allocation in a risky economy requires negative real rates to sustain high investment. When the ZLB binds, monetary policy cannot replicate this allocation. Safe asset creation shifts the goalposts, presenting monetary policy with the easier task of implementing a *different, suboptimal* natural allocation with positive real rates. Policies such as higher target inflation which permit negative real rates would instead implement the optimal natural allocation with high investment and full employment. While these policies have their own trade-offs,[6] they are worth considering, since safe asset creation is no panacea. In this regard, our analysis forces us to reassess the question of whether low safe rates indicate a *shortage of safe assets*, as is sometimes argued.[7] We formalize the notion of a safe asset shortage as a situation in which issuing more safe assets increases welfare. Whether low rates indicate a shortage in this sense depends critically on whether negative real rates are implementable.

Besides pushing an economy to the ZLB, an increase in risk can also generate bubbles - assets with no intrinsic value which trade at a positive price. As in Samuelson (1958), in an environment with non-positive real interest rates, such assets can be held in equilibrium even when they have a stable price and pay no dividend. At zero interest rates, pseudo-safe bubbles with a zero probability of bursting may emerge in equilibrium. Pseudo-safe bubbles are a perfect substitute for government debt, and crowd out capital - which reduces welfare since our economy is dynamically efficient. This contrasts with classic models of rational bubbles (Tirole, 1985), in which bubbles can arise only in dynamically inefficient economies, and thus raise welfare if they emerge.[8] Worse still, risky bubbles which burst with some probability may arise. Risky bubbles reduce welfare both because they crowd out capital, and when they burst. It is often suggested that monetary policy should lean against the wind to prevent bubbles; our model suggests that fiscal policy should do so, by committing to aggressively increase the public supply of safe assets to crowd

[6] See for example Rogoff (2017).

[7] See for example Gourinchas and Jeanne (2012), Caballero et al. (2017b), Gourinchas and Rey (2016).

[8] A newer literature discusses conditions under which bubbles can arise in dynamically efficient economies owing to financial frictions. In such environments, bubbles may or may not raise welfare. See Martin and Ventura (2018) for a review.

out privately provided *safe-ish* assets. This resonates with the argument of Greenwood et al. (2016) that public creation of safe assets should crowd out inefficient private creation of money-like assets.

The adverse consequences of bubbles are even worse when monetary policy faces constraints, since the bursting of a bubble pushes the natural rate of interest below zero, potentially constraining monetary policy at the ZLB and increasing unemployment. Some commentators have argued that, prior to 2008, advanced economies 'needed' bubbles to maintain full employment; our model clarifies the sense in which this is true. When risk is sufficiently high, full employment requires one of three things: negative real interest rates, public safe assets, or private pseudo-safe assets. A bubble can sustain full employment with positive interest rates even when public debt is insufficient to meet safe asset demand - for a while. When the bubble bursts, however, it can cause a deep recession. Substituting public safe assets for private pseudo-safe bubbles maintains full employment, but fails to raise investment below the inefficiently low levels prevailing even before the recession.

Our framework provides a new perspective on the austerity debate during the Great Recession, when many advanced economies pursued fiscal austerity in the midst of a recession with near-zero interest rates. Some economists objected that governments who *can* borrow at negative real rates in a recession obviously *should* do so: deficit spending reduces unemployment in the short run, while negative real rates make it a perfect time to borrow, since the private sector is paying governments to take its money. In our model, this argument is correct as far as it goes. When the ZLB binds, increasing government debt raises the natural rate, allowing monetary policy to restore full employment. If instead a government fails to increase (or attempts to decrease) debt when the ZLB binds, this increases unemployment and reduces GDP. Paradoxically, such policies result in a higher debt to GDP ratio than would occur with an increase in debt. But even if increasing debt is better than not doing so, it comes with side effects: low investment, output, and labor productivity. Safe asset creation allows monetary policy to implement a natural allocation, but not the *optimal* natural allocation; it fails to address the fundamental problem that risky economies require negative real rates to sustain high investment. Policies such as higher inflation targets, which permit negative real rates, may dominate both fiscal austerity, and an expansion of government debt.

In our analysis, we treat government debt as a "safe asset" in the literal sense that its return does not covary with a household's marginal utility, unlike the return on capital, the other asset in our economy. In this regard, we differ from other definitions of safe assets used in the literature which emphasize liquidity, default risk and so forth.[9] KVJ document empirically that U.S. Treasuries earn a both a liquidity and safety premium relative to comparable private assets, and both premia depend on the supply of Treasury debt. While our baseline model emphasizes the safety premium, in Appendix J we show that our results are qualitatively unchanged if government debt also earns a liquidity premium.

Related Literature A large literature studies the macroeconomic consequences of the secular decline in safe rates of interest and the supply of safe assets. Caballero and Farhi (2016) study an endowment economy in which safe asset shortages generate a persistent recession. Relative to their work, we study the interaction between safe asset shortages and investment. Whereas in Caballero and Farhi (2016) higher

[9] For example, Gorton and Ordonez (2013) define safe assets as *information-insensitive* assets, which can be traded without fear of adverse selection and thus circulate widely. Azzimonti and Yared (2017), He et al. (2016) and Farhi and Maggiori (Forthcoming) define a safe asset as one which has no default risk. In Barro et al. (2014), safe assets are the riskless bonds issued by less risk averse agents to more risk averse agents.

government debt prevents a safety trap without adverse consequences, in our environment it comes at the cost of crowding out investment. Farhi and Maggiori (Forthcoming) and Gourinchas and Rey (2016) explain another tradeoff arising in an international setting. Safe asset provision is intrinsically good because it can avert a liquidity trap, but may be restricted because issuing safe assets exposes a sovereign to self-fulling "confidence crises" or real appreciations. In our closed economy setting with lump sum taxes, these concerns are not present. Instead, we focus on a different trade-off: while issuing safe assets prevents liquidity traps, it crowds out investment, reducing the full-employment level of GDP.

Our paper contributes to a large literature which studies the optimal supply of public debt, such as Woodford (1990) and Aiyagari and McGrattan (1998). In these papers, the potential benefit of public debt is that it relaxes private constraints. Instead, we consider an environment where a binding ZLB introduces a new reason to increase public debt - namely, to raise the natural rate of interest. Also, relative to this literature, our setup features capital income risk. More recently, Angeletos et al. (2016) study optimal debt policy in a flexible price economy when debt provides liquidity services. Their Ramsey planner trades off the liquidity benefits of higher debt against the cost of raising interest rates, requiring higher distortionary taxes to satisfy the government budget constraint. Our government has access to lump sum taxes, so relaxing the government budget constraint is irrelevant. The cost of issuing more debt is instead that it reduces investment relative to the optimal natural allocation;[10] the benefits are that it avoids liquidity traps and bubbles. This trade-off is absent in the papers just discussed which study flexible-price models.

Our paper also relates to a recent literature which explains how a contraction in private borrowing constraints can push economies with nominal rigidities into a liquidity trap (Eggertsson and Krugman, 2012; Guerrieri and Lorenzoni, 2015). Guerrieri and Lorenzoni (2011) noted that government debt can completely offset such a shock.[11] Since these models abstract from capital, there are no trade-offs associated with increasing the supply of government debt, since it offsets private borrowing constraints without any cost in terms of crowding out capital. In our environment, instead, while government debt can prevent the ZLB from binding, this comes at the cost of crowding out capital.

We are not the first to study the interaction of public debt and the ZLB in economies with capital. Auclert and Rognlie (2016) study how labor income inequality affects aggregate demand in an incomplete markets model with nominal rigidities. Similarly, Eggertsson and Mehrotra (2014) discuss how shocks such as a tightening of borrowing constraints can lead to "secular stagnation" in such environments. Like us, these authors find that public debt issuance can restore full employment when monetary policy is constrained.[12] In their models, capital is riskless; public debt can accommodate higher desired savings but does not act on the risk premium since capital and bonds earn the same return in equilibrium. Our setup features a risky return on capital allowing us to study how public debt can *optimally* moderate increases in *risk premia*.[13]

Our results relate to the literature on dynamic efficiency and rational bubbles. Absent risk, real interest

[10]In this regard, our result is reminiscent of Yared (2013) who shows that while increasing government debt can in principle substitute for limited private credit, it is not optimal to do so since this distorts investment decisions.

[11]Bilbiie et al. (2013) demonstrated a similar result in a Eggertsson and Krugman (2012)-type model.

[12] Bacchetta et al. (2016) also study the interaction between government debt and capital in a liquidity trap, albeit in a flexible price economy. Like them, we show that safe assets crowd out capital even in a liquidity trap. Unlike them, we study an economy with nominal rigidities, giving policymakers a reason to increase the natural rate which is absent in their flexible price economy.

[13]Like these papers, our model permits permanently negative real rates. This is not essential: an earlier version of this paper considered scenarios with temporarily negative real rates and found similar results.

rates can only be negative if the economy is dynamically efficient, in which case it is desirable to issue more public debt and crowd out capital (Diamond, 1965). Abel et al. (1989) argued that in an economy with aggregate risk, the safe rate of interest can be negative even when the economy is dynamically efficient in the sense that capital income is larger than investment. They also conjectured that rational bubbles can never arise in dynamically efficient economies. We provide a counterexample.

Gali (2014) argued that monetary policy should not necessarily 'lean against the wind', since in equilibrium, a systematic response of interest rates to the size of a bubble may increase bubble growth. Allen et al. (2017) argued that, on the contrary, policymakers may be able to raise interest rates and crowd out bubbles, for example by issuing more government debt. Our results are consistent with both authors: government debt policy can crowd out bubbles by raising the *natural* rate of interest, and this may be more effective than a monetary policy rule which adjusts the policy rate in response to bubbles. This prescription relates to a emerging literature which focuses on the financial stability consequences of low real interest rates, and the role of public debt management in regulating these. For example, Greenwood et al. (2016) and Woodford (2016) study whether a central bank should increase its supply of short term claims to promote financial stability. Whereas these papers interpret financial instability as socially excessive private sector maturity transformation, we interpret this as the risk of bubbles bursting.

Finally, Asriyan et al. (2016) also study the interaction between monetary policy and bubbles; like us, they find that the bursting of a bubble can can lead to a liquidity trap characterized by low investment. In their flexible price economy, this occurs because bubbles provide collateral, and the bursting of a bubble deprives the economy of collateral. Instead, in our economy with nominal rigidities, the bursting of a bubble reduces the natural rate below zero, making monetary policy unable to preserve full employment. In this regard, our analysis is similar to Boullot (2016) who shows that rational bubbles can ameliorate liquidity traps in an environment with nominal rigidities.

Duarte and Rosa (2015) present evidence from a variety of asset pricing models that the equity risk premium increased significantly between 2000 and 2013. While recent work has emphasized a number of factors potentially driving the decline in safe rates of interest – a slowdown in technological progress, demographic forces, a savings glut, and so forth (Eichengreen, 2015) – these need not predict an increase in risk premia. The shock we consider, an increase in idiosyncratic capital income risk, instead explains both a decline in safe rates and an increase in risk premia. This is consistent with Del Negro et al. (2017), who find that liquidity and safety premia are the main factors explaining the secular decline in the natural rate, and with Caballero et al. (2017a), who find using the methodology of Gomme et al. (2011) that the real return on productive capital remained flat or even increased over the past three decades, while the return on U.S. Treasuries declined dramatically.[14]

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 characterizes the natural allocations in our economy. Section 4 describes the effects of risk and safe asset creation. Section 5 studies the interaction between safe asset creation and rational bubbles. Section 6 concludes.

[14]Note that while Gomme et al. (2015) interpret the recent increase in the marginal product of capital as evidence against versions of the secular stagnation hypothesis which emphasize a shortage of investment opportunities, it is entirely consistent with out risk-based view of stagnation.

2 Model

Households Time is discrete. At each date t, a cohort of ex-ante identical individuals with measure 1 is born and lives for two periods. Each individual $j \in [0,1]$ has identical preferences given by:

$$\mathbb{U}(c_t^Y, c_{t+1}^O) = (1-\beta)\ln c_t^Y + \beta \mathbb{E}_t \ln c_{t+1}^O$$

where $\beta \in (0, 1/2)$. When young, each household is endowed with one unit of labor which it is willing to supply inelastically and earns a nominal wage W_t per unit. The household also receives a lump-sum transfer T_t from the government. Young households can invest in two assets: *risky* capital and *safe* government debt. The budget constraints of a household can be written as:

$$P_t c_t^Y + P_t k_{t+1} + \frac{1}{1+i_t} B_{t+1} = W_t l_t + P_t T_t \tag{1}$$

$$P_{t+1} c_{t+1}^O(z) = P_{t+1} R_{t+1}^k(z) k_{t+1} + B_{t+1} \tag{2}$$

where i_t is the nominal interest rate on government debt and $R_{t+1}^k(z)$ is the real return on capital earned by old household i at date $t+1$, which depends on a random variable z described below. A young household must decide how much to invest in capital without knowing the realization of z in the next period. Importantly, we assume this risk is uninsurable: households cannot trade Arrow securities contingent on the realization of z.[15] Appendix A shows that the households' optimal decisions are described by

$$c_t^Y = (1-\beta)(\omega_t l_t + T_t) \tag{3}$$

$$k_{t+1} = \beta \eta_t (\omega_t l_t + T_t) \tag{4}$$

$$\frac{b_{t+1}}{R_t} = \beta(1-\eta_t)(\omega_t l_t + T_t) \tag{5}$$

where $b_t = \dfrac{B_t}{P_t}$ denotes real debt, $R_t = \dfrac{(1+i_t)P_t}{P_{t+1}}$ is the real return on government debt and η_t, the portfolio share of risky capital, is defined by

$$\eta_t \equiv \frac{k_{t+1}}{k_{t+1} + b_{t+1}/R_t} = \mathbb{E}_z \left[\frac{R_{t+1}^k(z) k_{t+1}}{R_{t+1}^k(z) k_{t+1} + b_{t+1}} \right] \tag{6}$$

Young households consume a fraction $1-\beta$ of labor income net of transfers when young and save the rest. Out of the β fraction saved, households invest a fraction η_t in risky capital and $1-\eta_t$ in safe bonds. Appendix A shows that the optimal η_t solves a portfolio choice problem maximizing risk-adjusted returns:

$$\eta = \underset{\eta_t \in [0,1]}{\text{argmax}}\, \mathbb{E}_z \ln \left[\eta_t R_{t+1}^k(z) + (1-\eta_t) R_t \right]$$

Agents demand more safe assets (lower η) if bonds are relatively cheap (low $1/R_t$) or risk is high:

Lemma 1 (Portfolio Choice)**.** *The optimal portfolio choice η_t depends negatively on R_t. Compare two distributions of $R^k(z)$, F and G where G is a mean-preserving spread of F. Then $\eta_F < \eta_G$.*

[15] Appendix I provides microfoundations for this market incompleteness.

Proof. See Hadar and Seo (1990). □

Firms At each date t, each old household operates a firm with a Cobb-Douglas production technology:

$$Y_t(z) = (z_t k_t)^\alpha (\ell_t(z))^{1-\alpha}$$

where k_t is the amount of capital that household i invested when young. z is the firm-specific productivity and is i.i.d across all firms with distribution $\ln z \sim N\left(-\frac{\sigma_t^2}{2}, \sigma_t^2\right)$. Importantly, there is no market for capital among old households so households with low z cannot sell their capital to those with high z.[16]

Given its productivity and capital, the firm hires labor in order to maximize profits:

$$R_t^k(z)k_t = \max_\ell (zk_t)^\alpha \ell_t^{1-\alpha} - \omega_t \ell_t$$

where ω_t denotes the real wage. Labor demand is given by:

$$\ell_t(z) = \left(\frac{1-\alpha}{\omega_t}\right)^{\frac{1}{\alpha}} zk_t \tag{7}$$

and we can write the return to capital as:[17]

$$R_t^k(z) = \alpha \left(\frac{1-\alpha}{\omega_t}\right)^{\frac{1-\alpha}{\alpha}} z \tag{8}$$

Government At date t, the government issues non-defaultable nominally safe one period debt B_{t+1} at price $1/(1+i_t)$, using the proceeds to repay outstanding debt B_t and disburse transfers P_tT_t to the young, and purchase G_t units of the output good:

$$\frac{1}{1+i_t} B_{t+1} = B_t + P_tT_t + P_tG_t \tag{9}$$

We set $G_t = 0$ unless otherwise specified. The monetary authority sets nominal interest rates i_t according to some rule which we specify later.

2.1 Natural Allocations

Our ultimate goal is to consider how risk and the supply of safe assets interact with monetary policy in the presence of nominal rigidities. To this end, in Section 4 we will introduce nominal rigidities by assuming that nominal wages are sticky downwards but flexible upwards. However, in order to understand outcomes in the economy with nominal rigidities, it will be instructive to compare these to the outcomes arising in an economy with flexible prices and wages. To this end, we spend the remainder of Sections 2 and 3 characterizing allocations in such a benchmark economy, which we call *natural allocations*.

[16] Again, see Appendix I for a microfoundation of this market incompleteness.

[17] We assume that there is full depreciation of capital. This is without loss of generality. Without complete depreciation, the return to capital can be written as $R^k(z) = \alpha\left(\frac{1-\alpha}{\omega_t}\right)^{\frac{1-\alpha}{\alpha}} z + 1 - \delta$

Labor Market In the benchmark economy, wages adjust to achieve full employment:

$$l_t = 1 \text{ and } \omega_t = (1-\alpha)k_t^\alpha \tag{10}$$

Return on capital Given equilibrium wages (10), the return to investing in capital can be written as:

$$R_t^k(z) \quad = \quad \alpha z k_t^{\alpha-1} \tag{11}$$

Throughout, we will refer to increases in σ as *increases in risk*. Note that since $\ln z \sim N(-\sigma^2/2, \sigma^2)$, an increase in σ is a mean-preserving spread to the distribution of idiosyncratic productivity, leaving the average return on capital (11) unchanged.

Goods Market Clearing The aggregate resource constraint of this economy can be written as:

$$c_t^Y + \int_z c_t^O(z)dF_t(z) + k_{t+1} = \int_z (zk_t)^\alpha \ell_t(z)^{1-\alpha} dF_t(z) = k_t^\alpha \tag{12}$$

where $F_t(z)$ is the cdf of the log-normal distribution defined above. The LHS of the equation above is the sum of total consumption and investment in capital in period t while the RHS is GDP.

Definition 1 (Equilibrium in the economy without nominal rigidities)**.** *Given a sequence* $\{B_{t+1}, i_t, T_t\}_{t=0}^\infty$ *and initial conditions* $\{B_0, k_0\}$*, an equilibrium is a sequence* $\{c_t^Y, c_t^O(z), k_{t+1}, l_t, \ell_t(z), R_t^k(z), P_t, W_t\}_{t=0}^\infty$ *such that*

1. $\{c_t^Y, c_t^O(z), k_{t+1}, B_{t+1}\}$ *solves the household's problem for each cohort* t*, given prices* $\{i_t, R_t^k(z), P_t, W_t\}$ *and transfers* $\{T_t\}$
2. $\{\ell_t(z), R_t^k(z)\}$ *solve the firm's problem at each date* t
3. *the government budget constraint* (9)*, labor market clearing* (10) *and goods market clearing* (12) *are satisfied.*

In this economy without nominal rigidities, the classical dichotomy holds and we can discuss real prices and allocations without reference to nominal variables. We refer to equilibrium allocations in such an economy as *natural allocations*, and call the prevailing real interest rate R_t the *natural rate of interest*. Importantly, there are many natural allocations in our economy, and they depend on fiscal policy, in particular on the path of government debt. There are two key equations that help us describe the dynamics of the economy in any natural allocation:

Aggregate supply of savings The first of these equations is the *aggregate supply of savings*:[18]

$$k_{t+1} + \frac{b_{t+1}}{R_t} = \beta\left[(1-\alpha)k_t^\alpha + \frac{b_{t+1}}{R_t} - b_t\right] \tag{13}$$

where the LHS of (13) denotes the total savings in the economy at date t.

[18]To derive (13), use the equilibrium expression for labor income (10) and government budget constraint (9) to substitute out for transfers from equation (4).

Demand for capital The other equation of interest concerns the demand for capital which is described by the optimal portfolio choice of young households:[19]

$$\eta_t \quad = \quad \mathbb{E}_z\left[\frac{\alpha z k_{t+1}^{\alpha}}{\alpha z k_{t+1}^{\alpha} + b_{t+1}}\right] = \mathbb{E}_z\left[\frac{\alpha z}{\alpha z + \tilde{b}_{t+1}}\right] \tag{14}$$

where $\tilde{b}_t = \frac{b_t}{k_t^\alpha}$ denotes the debt-to-GDP ratio. (14) shows that the equilibrium portfolio share of capital only depends on capital and bonds only via debt-to-GDP. In what follows, it will be convenient to work with $\tilde{b}$ instead of b as our measure of fiscal policy. It is also straightforward to see that η_t is decreasing in σ.[20] Finally, using the expression for η_t, the *demand for capital* can be expressed as:[21]

$$\alpha k_{t+1}^{\alpha-1} \quad = \quad g(\tilde{b}_{t+1}, \sigma) R_t \qquad \text{where } g(\tilde{b}, \sigma) = \frac{\mathbb{E}_z\left[(\alpha z + \tilde{b}_{t+1})^{-1}\right]}{\mathbb{E}_z\left[z(\alpha z + \tilde{b}_{t+1})^{-1}\right]} > 1 \tag{15}$$

The demand for capital is decreasing in the safe real interest rate, as is standard. However, it also depends on the supply of safe assets and the level of idiosyncratic risk. Since the LHS of equation (15) is the expected return on capital $\mathbb{E}_z R_{t+1}^k(z)$, $g(\tilde{b}_{t+1}, \sigma)$ can be interpreted as the premium earned by capital relative to bonds owing to the inherent risk in holding capital. $g(\tilde{b}_{t+1}, \sigma)$ is increasing in σ. An increase in the riskiness of capital, σ, decreases the demand for capital and widens the spread between the expected return on capital and the safe rate. As capital becomes more risky, investors would like to substitute away from capital towards government debt; if no increase in the supply of debt is forthcoming, either the price of debt must rise or investment in capital must fall. $g(\tilde{b}_{t+1}, \sigma)$ is also decreasing in $\tilde{b}_{t+1}$: increasing $\tilde{b}_{t+1}$ reduces the safety premium by satiating the demand for safe assets. In this sense, our model provides a micro-founded channel through which the supply of public safe assets affects the risk premium as found empirically by KVJ, although here the premium reflects safety rather than liquidity.[22]

The intersection of the aggregate supply of savings (13) and the demand for capital (15) determines the equilibrium level of investment k_{t+1} and real interest rates R_t given today's capital stock and government debt policy. Capital accumulation in any natural allocation, given a sequence $\{\tilde{b}_{t+1}\}_{t=0}^{\infty}$, is described by:

$$k_{t+1} = s(\tilde{b}_t, \tilde{b}_{t+1}, \sigma) k_t^\alpha \qquad \text{where } s(\tilde{b}_t, \tilde{b}_{t+1}, \sigma) = \frac{\beta(1-\alpha-\tilde{b}_t)}{\beta + (1-\beta)\mathbb{E}_t\left[\frac{\alpha z}{\alpha z + \tilde{b}_{t+1}}\right]^{-1}} \tag{16}$$

This equation is pleasingly reminiscent of the Solow model, with the aggregate savings rate given by $s(\tilde{b}_t, \tilde{b}_{t+1}, \sigma)$.[23] The savings rate is decreasing in both the current period and next period's debt-to-GDP

[19](14) can be derived by plugging in (11) into (6).

[20]Note that $1 - \eta_t = \mathbb{E}_z\left[\frac{\tilde{b}_{t+1}}{\alpha z + \tilde{b}_{t+1}}\right]$ is increasing in σ by Jensen's inequality.

[21]See Appendix B for details and for the characterization of $g(\tilde{b}, \sigma)$.

[22]KVJ and many others have used models in which bonds earn a convenience yield arises because (by assumption) they provide direct utility to the holder; as in models with money in the utility function, these utility benefits are a reduced form for the transaction services provided by this asset. In our model, bonds earn a premium relative to capital despite not being in the utility function and this premium responds to the public supply of safe assets. This premium is a *safety* premium rather than a *liquidity* premium: it arises endogenously due to incompleteness of markets. In section J we augment the model to include a liquidity premium following KVJ and show that our results are qualitatively unchanged.

[23]Note that the aggregate savings rate is different from the private savings rate of the young which is given by β.

ratio. Higher $\tilde{b}_t$ requires higher taxes on young savers, reducing their disposable income and thus the amount they save. High $\tilde{b}_{t+1}$ tomorrow, in equilibrium, requires that young households hold more bonds in their portfolio, reducing the amount they invest in capital. Finally, higher risk induces young savers to shift their portfolio away from riskier capital towards safe government debt, reducing aggregate savings.

Steady State In steady state, the aggregate supply of savings (13) becomes:[24]

$$k^{1-\alpha} = \beta(1-\alpha) - \left[\frac{1-\beta}{R} + \beta\right]\tilde{b} \tag{17}$$

Equation (17) shows that government debt crowds out capital, diverting savings away from physical investment, and (if $R > 1$) increasing taxes on young savers. Issuing zero debt maximizes steady state capital. Given debt to GDP, (17) defines an increasing relation between capital and the interest rate, depicted by the upward sloping curves in Figure 1a: higher interest rates make the same amount of debt cheaper for young savers, leaving ample funds available for investment. The downward sloping curves depict the demand for capital (15). The intersection of the two curves determines capital and interest rates in the steady state of the natural allocation with steady state debt-to-GDP $\tilde{b}$:

$$k\left(\tilde{b},\sigma\right) = \left[\frac{\beta(1-\alpha-\tilde{b})}{\beta + (1-\beta)\mathbb{E}\left[\frac{\alpha z}{\alpha z+\tilde{b}}\right]^{-1}}\right]^{\frac{1}{1-\alpha}} \tag{18}$$

$$R\left(\tilde{b},\sigma\right) = \frac{1}{1-\alpha-\tilde{b}}\left[\beta^{-1}\mathbb{E}\left[\frac{1}{\alpha z+\tilde{b}}\right]^{-1} - \tilde{b}\right] \tag{19}$$

3 Inspecting the Mechanism

We now show that an increase in risk can reduce the natural rate of interest, while an increase in the supply of safe assets can increase the natural rate (which is the same as the *prevailing* real interest rate, without nominal rigidities). Prior to date 0, capital and the natural rate are at their steady state levels, $k(\tilde{b}_L, \sigma_L)$, $R(\tilde{b}_L, \sigma_L)$. At date 0, σ increases unexpectedly and permanently from σ_L to $\sigma_H > \sigma_L$.

Can the natural rate be negative in the absence of risk? Even before we consider *changes* in σ or $\tilde{b}$, equation (19) reveals that even in the riskless case with $\sigma = 0$, the steady state natural rate can be negative for small enough $\tilde{b}$. For instance, with $\tilde{b} = 0$, $R(0,0) = \frac{\alpha}{\beta(1-\alpha)}$, which could be less than 1 even in the absence of risk. In this case, the economy would be *dynamically inefficient* in the sense of Diamond (1965). For the rest of the paper, we rule this out.[25]

Assumption 1. *The riskless economy is dynamically efficient:* $\frac{\alpha}{\beta(1-\alpha)} > 1$

[24] Here and elsewhere, quantities and prices without time sub-scripts denote steady state values.

[25] We refer the reader to end of Section 3.3 for a definition and discussion of dynamic efficiency.

3.1 The effects of an increase in risk

We now show that an increase in risk can reduce the natural rate of interest. To build intuition, maintain the assumption that bonds are in zero net supply, $\tilde{b} = 0$, but suppose $\sigma > 0$. In this case, the steady state natural rate (19) becomes $R = \frac{\alpha}{\beta(1-\alpha)} e^{-\sigma^2}$.[26] Ceteris paribus, an increase in the riskiness of capital causes young households to demand more safe assets; with no increase in the supply of safe assets forthcoming, their price must rise. In particular, if risk σ^2 exceeds $\underline{\sigma}^2 := \ln\left[\frac{\alpha}{\beta(1-\alpha)}\right] > 0$, the steady state natural rate is negative. This decreasing relationship between risk and natural rates holds more generally for $\tilde{b} > 0$.

Lemma 2. *For a given level of $\tilde{b}$, the steady state level of capital k is weakly decreasing in σ while the steady state natural rate of interest R is strictly decreasing in σ.*

To see this, apply Jensen's inequality to (18) and (19). Higher risk makes households substitute away from risky capital towards safe bonds. Given a fixed supply of bonds, their price $1/R$ must rise to clear the market. Facing higher prices of safe assets, young savers, who save a fixed fraction β of their total income, have less left over to invest in capital. Thus the aggregate saving rate and capital stock fall. Figure 1a depicts this graphically. An increase in σ shifts the capital demand schedule leftwards while leaving the aggregate supply of savings unchanged, reducing steady state capital and real interest rates. Importantly, high enough σ can result in a negative natural rate in steady state, $R < 1$.

3.2 The effects of an increase in safe assets

While risk can depress the natural rate of interest, an increase in the supply of safe assets always increases the natural rate. However, this crowds out investment, reducing steady state capital.

Lemma 3. *The steady state levels of capital k is strictly decreasing in $\tilde{b}$ while the steady state real interest rates R is strictly increasing in $\tilde{b}$.*

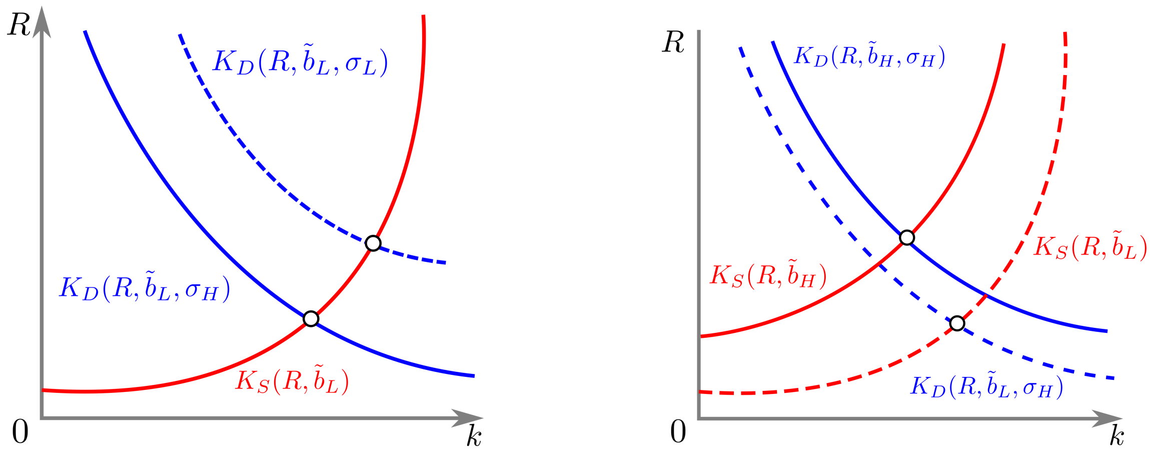
Figure 1b depicts this graphically. An increase in government debt satiates the demand for safe assets and reduces the safety premium in equation (15). Consequently, young households are willing to hold more capital for a given real rate, shifting the capital demand schedule rightwards. However, higher government debt diverts savings away from capital crowding out investment, shifting the aggregate supply of savings to the left. Overall, the steady state capital stock is unambiguously lower, and real interest rates higher, with a higher supply of safe assets. Thus in response to any increase in risk, a sufficiently large increase in the supply of safe assets can always keep the natural rate positive – at the cost of crowding out investment.

3.3 The optimal natural allocation

By increasing the supply of debt, the fiscal authority can always implement a natural allocation with a positive natural rate. Just because policy *can* do this does not mean that it *should*. As we now show, a planner who maximizes steady state welfare would *not* create enough safe assets to prevent negative interest rates. We consider a social planner who seeks to maximize steady state welfare subject to the implementability constraint (18):

$$\max_{k,\tilde{b}} \quad (1-\beta)\ln\left[(1-\alpha-\tilde{b})k^\alpha - k\right] + \beta\mathbb{E}_z \ln\left[(\alpha z + \tilde{b})k^\alpha\right] \text{ s.t. } k = s(\tilde{b}, \tilde{b}, \sigma)k^\alpha \tag{20}$$

[26] Since $\ln z \sim N(-\sigma^2/2, \sigma^2)$, $\mathbb{E}\left[z^{-1}\right] = e^{\sigma^2}$.



(a) A permanent increase in σ

(b) A permanent increase in $\tilde{b}$

Figure 1. Steady States

The planner faces the following trade-off. In equilibrium, an increase in government debt is essentially a forced transfer from the young to the old.[27] Since the planner cannot directly insure old people against low realizations of z, she can only raise their consumption in low z states via an unconditional transfer. This provides an *insurance motive* for creating safe assets. But if the old consume a greater share of GDP, the young must consume a smaller share. The more risk old households face, the higher the expected marginal utility of the average old individual, and thus the stronger the insurance motive, i.e. the gains from redistribution from young to old. However, safe asset production also crowds out physical capital investment. This harms both the young, who earn lower wages, and the old, who earn less capital income. Absent crowding out, the planner would create just enough safe assets that the real interest rate is zero.

Lemma 4. *Consider the unconstrained problem in which the planner maximizes* (20)*, ignoring the constraint. The solution to this problem is unique, with either* $R \geq 1$ *and* $\tilde{b} = 0$*, or* $R = 1$ *and* $\tilde{b} > 0$*.*

Proof. See Appendix C. □

Intuitively, the real rate R measures how much an individual values a unit of consumption when young relative to when old. If risk is relatively low, impatience outweighs the desire to insure against consumption risk when old, and a unit of consumption is worth more when young than when old, i.e. $R > 1$. Conversely, when risk is high, a young individual would willingly forgo one unit of consumption when young to receive one unit when old, i.e. $R < 1$. While the planner shares the individuals' preferences, unlike them, she has a technology which transfers one unit of consumption from young to old, namely government debt. Thus the planner would never permit $R < 1$; that would signal an unmet desire for transfers from young to old, which could easily be satiated with more government debt.[28]

However, safe asset creation does crowd out investment. Thus, it is in fact *not constrained optimal* to produce enough safe assets to keep real interest rates positive, as we now show.

[27]Here debt is financed by lump sum taxes on the old. If in addition there were lump sum taxes on the old, our results would be unchanged, if we redefine b_t as government debt net of taxes on the old. In this sense, $\tilde{b}$ can be broadly interpreted as private holdings of safe assets plus public transfers to the old.

[28]In contrast when $R > 1$, households would like to transfer resources from tomorrow to today but the planner has no technology to facilitate such a transfer.

Proposition 1 (Constrained optimal natural allocation)**.** *If the riskless steady state is dynamically efficient, there exist $\overline{\sigma} \in (\underline{\sigma}, \infty)$ such that the solution to (20) has the following properties:*[29]

i. If risk is low enough, i.e. $\sigma \leq \underline{\sigma}$, then the planner chooses $\widetilde{b} = 0$, $R \geq 1$.

ii. If risk is in the intermediate range $\sigma \in (\underline{\sigma}, \overline{\sigma}]$, the planner still does not choose to create safe assets and her optimal choices satisfy $\widetilde{b} = 0$, $R < 1$.

iii. If risk is high, $\sigma > \overline{\sigma}$, then the planner chooses to create some safe assets, but not enough to make real interest rates positive. The optimal choices satisfy $\widetilde{b} > 0$, $R < 1$.

Proof. See Appendix D. □

Intuitively, consider the net social benefit from a marginal increase in b starting from $b = 0$ when $\sigma = \underline{\sigma}$, so $R = 1$ absent any safe asset creation. This net benefit is

$$\underbrace{\beta \mathbb{E}_z \frac{1}{c^O(z)} - (1-\beta)\frac{1}{c^Y}}_{\text{net benefit of transferring \$1 from young to old}} \quad + \quad \underbrace{\frac{\partial \mathbb{W}}{\partial k}}_{\substack{\text{social benefit}\\ \text{of more capital}}} \underbrace{\left(\frac{dk^{ss}(b)}{db}\right)}_{\text{crowding out}} \tag{21}$$

This net benefit is negative when $b = 0$ and $\sigma = \underline{\sigma}$. In the private economy $\beta R \mathbb{E}_z \frac{1}{c^O(z)} - (1-\beta)\frac{1}{c^Y} = 0$. When $\sigma = \underline{\sigma}$, $R = 1$, and the first term in (21) is zero. That is, creating safe assets does not directly increase welfare, to first order, when $\sigma = \underline{\sigma}$. Issuing debt transfers resources from young to old, but the average marginal utility of young and old is already equalized, since $R = 1$.

While creating safe assets has no direct effect on welfare, to first order, it does indirectly reduce welfare by crowding out capital. There are two steps in this argument: safe assets crowd out capital (so the last term $\frac{dk^{ss}(b)}{db} < 0$), and this is costly because capital is socially valuable ($\frac{\partial \mathbb{W}}{\partial k} > 0$). While we have discussed the first point at length, the second point deserves more discussion. The first order effect of capital on social welfare is

$$\frac{\partial \mathbb{W}}{\partial k} = (1-\beta)\frac{1}{c^Y}\frac{d\omega}{dk} + \beta \mathbb{E}_z \left[\frac{1}{c^O(z)}\frac{dR^k(z)}{dk} k\right] > (1-\beta)u'(c^Y)\left[\frac{d\omega}{dk} + \mathbb{E}_z \frac{dR^k(z)}{dk} k\right] = 0$$

By the Envelope Theorem, a reduction in the capital stock only affects welfare via its effect on factor prices. Lower investment in capital by the young involves a *pecuniary externality*, reducing wages but increasing the rate of return to capital. Since all income is earned by either labor or capital, the *average* increase in capital income for the old is exactly offset by the fall in labor income for the young. Since $R = 1$, the expected marginal utility of old and young agents is equal, so this redistribution from young to old would not change welfare if the gain in capital income was enjoyed by all old agents equally. However, $\frac{d^2 R^k(z)}{dkdz} < 0$: capital income increases more for those with a high z and therefore a lower marginal utility.[30] Thus, the gain in expected utility of the old is outweighed by the loss in utility of the young. In other words, the pecuniary externality associated with a lower capital stock increases the share of risky income and reduces

[29] Recall that $\underline{\sigma}^2 = \log\left[\frac{\alpha}{\beta(1-\alpha)}\right]$ is defined in Section 3.1.

[30] To see this, note that $R^k(z) = z\alpha k^{\alpha-1}$ and so $\frac{d^2 R^k(z)}{dkdz} = \alpha(\alpha-1)k^{\alpha-2} < 0$.

the share of safe income, lowering welfare in this incomplete markets economy.[31] Appendix D.1 shows that the main result in Proposition 1 extends to an economy with homothetic time-separable utility and less than full depreciation. More generally, the appendix shows that with arbitrary concave preferences and neoclassical production technology, debt crowds out capital in the neighborhood of any stable steady state, and this reduces welfare as long as $\frac{d^2 R^k(z)}{dkdz} < 0$.

To be clear, our results do not imply that crowding out is undesirable in all circumstances. Economies which feature negative real interest rates even in the absence of risk can exhibit dynamic inefficiency (Diamond, 1965); in such cases government debt crowds out capital but this is desirable. The critical assumption in our analysis is that, instead, the riskless steady state is dynamically efficient in the sense that capital is not inefficiently high.[32] In our setting, this is guaranteed by Assumption 1. This assumption also guarantees that the economy would feature positive real interest rates absent risk, and it is the presence of risk which is responsible for negative real interest rates. More generally, though, our results would go through if the steady state capital stock was inefficiently low for other reasons, e.g. due to distortionary taxes or markups.

Figure 2 illustrates $\tilde{b}$ in the optimal natural allocation as a function of σ (blue line) and the corresponding natural rate (red line). When risk is low ($\sigma \leq \underline{\sigma}$), natural rates are positive even with zero debt, and there is no insurance benefit from safe asset creation; even an unconstrained planner would not issue debt. For intermediate risk ($\sigma \in (\underline{\sigma}, \overline{\sigma}]$), natural rates are negative with zero debt, but zero debt remains optimal: the insurance benefits are outweighed by the costs associated with crowding out. In this sense, negative natural rates need not indicate a *shortage of safe assets*. Of course, when risk is high enough ($\sigma > \overline{\sigma}$), the optimal natural allocation features some debt – but not so much that the natural rate becomes positive. Thus, the dashed red line, indicating the natural rate under optimal policy, lies above the solid red line indicating the natural rate with zero debt, but below the horizontal line indicating $R = 1$. As R approaches 1, the insurance benefit vanishes, and the costs outweigh the benefits.

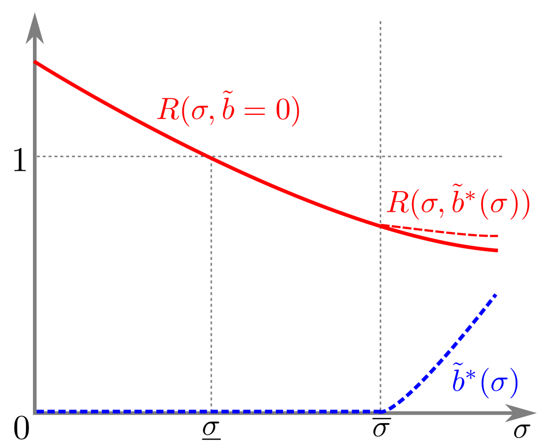


Figure 2. Optimal $\tilde{b}$ as a function of σ

This result is not driven by the fact that we have chosen to characterize optimal policy by maximizing steady state welfare. Appendix E characterizes the solution to the problem of a Ramsey planner who

[31] This result is similar to Davila et al. (2012) who find that an appropriately calibrated Aiyagari (1994) economy features under-accumulation of capital from the perspective of a utilitarian planner: higher capital would raise wages and depress returns on capital, benefiting poor individuals who hold less capital.

[32] We discuss dynamic efficiency in more detail on page 15.

maximizes intertemporal welfare taking into account costs and benefits associated with transitions. More precisely, we consider the Ramsey problem of a planner who puts arbitrary Pareto weights on the welfare of different cohorts, and define a *constrained efficient* allocation as one which is not Pareto dominated by any other allocation. Appendix E shows that constrained efficient allocations are very similar to the characterization in Proposition 1. The natural allocation with zero safe assets remains constrained efficient as long as risk remains below a certain level $\sigma^{\diamond} > \underline{\sigma}$ - even if the natural rate is negative ($\sigma \in (\underline{\sigma}, \sigma^{\diamond}]$). However, if risk is large enough, $\sigma > \sigma^{\diamond}$, the welfare gains from increased insurance outweigh the cost of crowding out, and the zero-debt policy is Pareto dominated.

Liquidity as a driver of safe asset demand Thus far, we have considered an economy in which capital earns a risk premium over safe government debt because capital bears idiosyncratic risk. Assets such as government debt may be valued not just for their safety but also for their liquidity or "moneyness". Empirically, KVJ document that the premium between US Treasury yields and comparable private assets contains both a liquidity and a safety component. Further, both liquidity and safety premia are affected by changes in the supply of public safe assets. Thus, one might wonder whether Proposition 1 would still apply if government debt is also valued for its liquidity in addition to its safety. Appendix J introduces liquidity services into our framework by positing that government debt provides direct utility to the holder, following Angeletos et al. (2016) and KVJ.[33] We show that Proposition 1 does indeed still hold. To be clear, liquidity services do make it more socially desirable to produce government debt. However, liquidity services also reduce real rates for a given level of risk. Thus our previous characterization remains accurate: negative real rates are a necessary but not sufficient condition for safe asset creation to be desirable.

Dynamic Efficiency It is natural to ask whether outcomes in our economy are dynamically efficient. Answering this question is not straightforward, however, since there is no single definition of dynamic efficiency which is obviously the correct one for our economy with idiosyncratic risk. Phelps (1965) and Diamond (1965) study economies without risk, and label an equilibrium *dynamically inefficient* if there exists another sequence for the aggregate capital stock that produces more aggregate consumption in some periods and never produces less aggregate consumption. Under this criterion, our economy is never dynamically inefficient given Assumption 1. However, this definition is not very relevant in our heterogeneous agent economy with risk. Abel et al. (1989) study economies with aggregate risk, and call an allocation dynamically efficient if no generation's ex-ante welfare can be increased without reducing the ex-ante welfare of another. This definition makes sense in their setting with only aggregate risk, but is not suitable to evaluate outcomes in our economy with uninsured idiosyncratic risk, because such Pareto improvements exist in any equilibrium. Trivially, starting from any equilibrium allocation, an unconstrained planner could equalize consumption across all old agents in a given cohort, increasing each cohort's welfare. Thus, existing definitions of dynamic efficiency are not appropriate in our economy.

The key to understanding our results is that our economy is not dynamically inefficient even when safe rates are negative - in fact, given Assumption 1, our economy features *under-accumulation* of capital, in the following precise sense. Consider a planner who can choose a path for the aggregate capital stock, but cannot redistribute between old agents with different realizations of productivity. Appendix F shows that,

[33] Angeletos et al. (2016) forcefully argue that modeling the liquidity property as a preference for safe assets affords great tractability while still retaining features which are present in richer environments.

starting from a steady state with zero safe assets (which features the highest level of capital attainable in equilibrium), this planner can deviate to a path which increases ex ante welfare for each cohort and involves higher capital in every period. While this deviation cannot be supported as an equilibrium in our setting, it highlights that this economy is not characterized by an inefficiently high level of capital - quite the reverse. The reason, as described above, is that a higher capital stock raises wages (the safe component of income) and reduces capital income (the risky component of income), increasing welfare on net.[34] If Assumption 1 were violated, then in the absence of risk, our economy would feature capital over-accumulation as in Diamond (1965): real interest rates and the return to capital would be negative, absent safe assets. In this case, producing safe assets would still have the side-effect of reducing investment, but this side-effect would be a desirable one - producing safe assets would increase the welfare of all cohorts precisely by crowding out capital.

Following Abel et al. (1989), it should not be surprising that a dynamically efficient economy can feature negative safe interest rates. In their OLG economy with aggregate risk (but no idiosyncratic risk), an allocation is dynamically efficient (according to their definition, described above) if capital income is larger than investment, or equivalently if the expected net return on capital is positive. While their definition of efficiency is not appropriate in our economy, as described above, our economy satisfies this criterion: even when the safe real interest rate is negative, the expected return on capital is positive because capital earns a risk premium. Thus, whether or not safe asset creation is desirable is not driven by over-accumulation of capital.

4 Nominal rigidities

The analysis above revealed that optimal natural allocations feature zero safe assets even if risk pushes the natural rate below zero. However, in the presence of nominal rigidities and a ZLB, a negative natural rate may prevent monetary policy from implementing the optimal natural allocation, as we now discuss. We introduce nominal rigidities by assuming nominal wages are sticky downwards but flexible upwards. Following Eggertsson and Mehrotra (2014) and Schmitt-Grohé and Uribe (2016), workers are unwilling to work for wages below a wage norm $\tilde{W}_t$; the prevailing wage is given by:

$$W_t = \max\left\{\widetilde{W}_t, P_t\omega_t^*\right\} \text{ where } \ln\widetilde{W}_t = (1-\gamma)\ln\left(\Pi^\star W_{t-1}\right) + \gamma\ln\left(P_t\omega_t^*\right) \tag{22}$$

$\Pi^\star \geq 1$ denotes the monetary authority's inflation target (described below) and $\omega_t^* = (1-\alpha)k_t^\alpha$ is the real wage that delivers full employment given capital k_t. $\gamma \in [0,1)$ is a measure of wage flexibility. With $\gamma = 0$, nominal wages are rigid downwards; with $\gamma = 1$, wages are fully flexible. When nominal wages exceed the market clearing nominal wage $P_t\omega_t^*$, labor demand is less than supply, resulting in unemployment: $\int_0^1 \ell_{i,t}di < 1$.[35] Firms are always on their labor demand curve and the prevailing nominal wage satisfies

[34] Again, this result is similar to Davila et al. (2012) who find that an appropriately calibrated Aiyagari (1994) economy features under-accumulation of capital.

[35] When there is unemployment, we assume that households are proportionally rationed, so each young household supplies the same amount of labor $l_t = \int_z \ell_t(z)dF_t(z)$.

$W_t = (1-\alpha)k_t^\alpha l_t^{-\alpha} P_t$. This yields a relationship between employment and inflation:

$$l_t = \min\left\{ l_{t-1}^{1-\gamma}\left(\frac{k_t}{k_{t-1}}\right)^{1-\gamma}\left(\frac{\Pi_t}{\Pi^\star}\right)^{\frac{1-\gamma}{\alpha}}, 1\right\} \tag{23}$$

The labor market can be in one of two regimes. When last period's nominal wage lies below the wage that would clear markets today, and full employment requires nominal wages today to rise, wages jump to their market clearing level and and there is full employment, $l_t = 1$. However, when last period's wage lies above today's market clearing wage, and full employment requires wages to fall, the wage norm binds, and wages only partially fall towards their market clearing level, resulting in unemployment. In this unemployment regime, employment will be higher, all else equal, if it was higher last period (which signals that wages were not too high and don't have far to fall); if capital is higher today than last period (which means the market clearing wage is higher today than last period); or if current inflation is higher. Temporarily higher inflation greases the wheels of the labor market by reducing $\tilde{W}/P$, lowering labor costs and increasing labor demand. Note however that there is no money illusion in the long-run, since we include $\Pi^\star$ in (22): higher target inflation does not relax downward nominal wage rigidity. In sections 4.1 and 4.2 we normalize $\Pi^\star$ to 1 without loss of generality. In Section 4.3 we consider the effects of an increase in $\Pi^\star$.[36]

Monetary Policy Monetary policy sets nominal interest rates according to the following flexible inflation targeting rule subject to the ZLB:

$$\left(\frac{\Pi_t}{\Pi^\star}\right)\left(\frac{Y_t}{Y_t^\star}\right)^\psi \leq 1, \qquad i_t \geq 0, \qquad \left\{\left(\frac{\Pi_t}{\Pi^\star}\right)\left(\frac{Y_t}{Y_t^\star}\right)^\psi - 1\right\} i_t = 0 \tag{24}$$

where $Y_t^\star = k_t^\alpha$ is the level of output consistent with full employment, given capital, and $\Pi^\star$ is the monetary authority's inflation target. Intuitively, the monetary authority aims to implement the target of $\Pi^\star$ and full employment whenever the ZLB does not prevent this. The monetary authority is willing to tolerate above target inflation if employment is below target; ψ denotes the weight placed on output stabilization, relative to price stability (e.g. $\psi = 0$ implies a strict inflation targeting regime).[37] When the ZLB constrains policy, however, both inflation and output may be below target.

Equilibrium with nominal rigidities The remaining equations characterizing equilibrium are similar to those characterizing natural allocations, except that the economy might not be at full employment. The aggregate supply of saving is given by:

$$k_{t+1} + \frac{b_{t+1}}{R_t} = \beta\left[(1-\alpha)k_t^\alpha l_t^{1-\alpha} + \frac{b_{t+1}}{R_t} - b_t\right] \tag{25}$$

Notice that if there is full employment ($l_t = 1$), (25) is identical to (13) in the natural allocation. More generally, unemployment today ($l_t < 1$) reduces the income of the young, reducing their savings and

[36]For technical reasons, we assume $\Pi^\star < \alpha^{-1}$. This is not a demanding restriction. The standard value for the capital share, α, is 1/3. Thus, our assumption states that the inflation target is less than 200 percent ($\Pi^\star < 3$). This assumption is sufficient to prove the uniqueness of steady states. See Proposition 2 for details.

[37]This can be thought of as the limit of a rule $1 + i_t = \max\left\{1, R^\star\Pi^\star\left(\frac{\Pi_t}{\Pi^\star}\right)^{\phi_\pi}\left(\frac{Y_t}{Y_t^\star}\right)^{\phi_y}\right\}$ as $\phi_\pi \to \infty$ and $\phi_y/\phi_\pi \to \psi$.

therefore demand for both capital and bonds. Similarly, savers' optimal portfolio decision (14) becomes:

$$\eta_t \quad = \quad \mathbb{E}_z\left[\frac{\alpha z k_{t+1}^{\alpha} l_{t+1}^{1-\alpha}}{\alpha z k_{t+1}^{\alpha} l_{t+1}^{1-\alpha} + b_{t+1}}\right] \tag{26}$$

As before, the equilibrium portfolio share of capital depends on the expected ratio of capital income to total income of the old. Unemployment reduces the marginal product of capital (MPK) and increases the portfolio share of safe assets (reduces η_t), given k_{t+1} and b_{t+1}. The demand for capital becomes:

$$\alpha\left(\frac{k_{t+1}}{l_{t+1}}\right)^{\alpha-1} \quad = \quad g(\tilde{b}_{t+1} l_{t+1}^{\alpha-1}, \sigma) R_t \tag{27}$$

where the average MPK is now $\mathbb{E}_z R_{t+1}^k(z) = \alpha\left(\frac{k_{t+1}}{l_{t+1}}\right)^{\alpha-1}$. The demand for capital in the natural allocation (15) is simply equation (27) with $l_{t+1} = 1$. Unemployment tomorrow ($l_{t+1} < 1$) affects the demand for capital in two ways. First, it lowers the average MPK, reducing capital demand for a given R_t. However, lower l_{t+1} also increases the portfolio share of safe assets, narrowing the spread between the safe rate on bonds and the risky return on capital. Intuitively, the consumption of the old contains a risky (capital) and a safe component (bonds). The higher the risky share of income, the higher the covariance of consumption and the return to capital and the higher the risk premium demanded by the young. Higher future unemployment lowers the risky share, leaving old households less exposed to risk, and reducing the risk premium.

Overall, the dynamics of the economy with nominal rigidities are described by equations (23)-(27).

4.1 The possibility of risk-induced stagnation

An increase in risk in the presence of nominal rigidities can result in persistent or even permanent unemployment, as we now show. As in Section 3, there is a permanent unanticipated increase in σ at date 0 from σ_L to $\sigma_H > \sigma_L$, where the corresponding steady state natural rates of interest satisfy $R(\tilde{b}_L, \sigma_L) > 1 > R(\tilde{b}_L, \sigma_H)$. For now, fiscal policy keeps $\tilde{b}_t$ constant at the same level $\tilde{b}_L$ as before date 0.[38] The following proposition describes equilibrium behavior of the economy from date 0 onwards.

Proposition 2 (Stagnation)**.** *Suppose $\tilde{b}_t = \tilde{b}_L$ for all $t \geq 0$ and for $t < 0$ the economy is in steady state with $R(\tilde{b}_L, \sigma_L) > 1$. At date 0, σ_t unexpectedly and permanently increases to σ_H with $R(\tilde{b}_L, \sigma_H) < 1$. Then:*

1. *There is no bounded equilibrium in which the economy returns to a steady state with full-employment.*
2. *For ψ sufficiently high and γ sufficiently low, there exists a unique equilibrium in which $i_t = 0$ for all $t \geq 0$ and the economy converges to a steady state with unemployment.*

Proof. See Appendix G. □

At date 0, young savers want to reallocate their portfolios away from increasingly risky capital towards safe government debt. With $\tilde{b}$ fixed, the excess demand for bonds necessitates a fall in the real return on bonds to equilibrate the market. Absent inflation, this requires a large cut in nominal interest rates, but the ZLB prevents this. Thus the real rate is *too high*, lowering the demand for capital, and thus the price

[38] Recall that $\tilde{b}_t = b_t / k_t^{\alpha}$ is the ratio of debt to the the level of GDP in the natural allocation. $\tilde{b}$ might be smaller than the ratio of debt to actual GDP because of unemployment.

of output (i.e. consumption and capital). With sticky nominal wages, the fall in price is only partially met by a fall in nominal wages, causing higher real wages, lower labor demand, and unemployment. The fall in young households' income reduces their demand for both bonds and capital – clearing the bond market, but reducing investment. This is only the beginning of a *risk-induced recession*.

It gets worse. Next period, the capital stock is lower, reducing the marginal product of labor and hence labor demand. Since nominal wages are slow to adjust to their market clearing level, unemployment persists and is expected to persist in the future. Anticipating a lower MPK, young households have even less reason to invest in capital – which is now permanently more risky – rather than safe government debt. With $\tilde{b}$ fixed, an excess demand for bonds persists, the ZLB prevents interest rates from falling to clear markets, and investment slumps further. Unemployment remains permanently high, since there is a permanent excess demand for safe assets (even with $i_t = 0$ forever), and so it takes permanently lower income to equate demand and supply.

Figure 3 depicts this graphically.[39] In the figure, k_{-1}, y_{-1}, w_{-1} and $\tilde{b}_{-1}$ are each normalized to unity. For comparison, the dashed gray lines illustrate the natural allocation. With $\tilde{b}$ constant, higher risk pushes the natural rate of interest (shown in panel (2,2)) permanently below zero . While this causes a very slight decline in capital, output, and real wages (panels (1,1), (1,2) and (2,1) respectively), the economy naturally remains at full employment throughout (panel (1,3)).

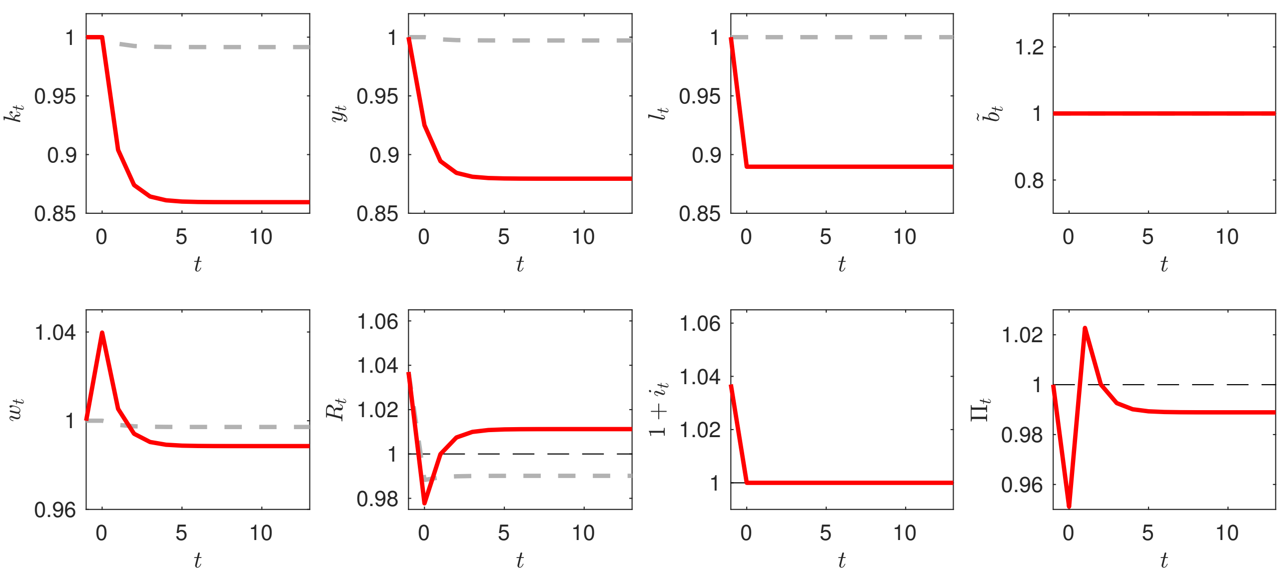


Figure 3. Risk-induced stagnation. Dashed lines denote the natural allocation following the permanent increase in σ while the solid lines denote the equilibrium trajectory of the economy with nominal rigidities.

In contrast, the solid red lines illustrate dynamics in the economy with nominal rigidities. The increase in risk, and the associated fall in employment, permanently reduce the aggregate saving rate, causing capital to decline to a lower steady state level (panel (1,1)). Real interest rates (panel (2,2)) fall on impact, as the spread between the safe rate and the expected MPK increases. As the capital stock declines, expected MPK rises while the spread remains wider, leading the real rate to increase to its new steady state level.

[39] In this and the subsequent figures, we set $\alpha = 1/3$, $\beta = 0.495$, $\sigma_L = 0.49$, $\sigma_H = 0.55$, $\gamma = 0.22$, $\tilde{b}_L = 0.065$. Since this is not intended as a quantitative exercise, we choose these particular values of $\sigma_L, \sigma_H, \gamma, \tilde{b}_L$ purely to make the qualitative features of equilibrium described in Proposition 2 easy to see. These properties of equilibrium do not depend qualitatively on the choice of parameters.

Employment (panel (1,3)) falls to its new lower steady state level. The fall in capital and employment combine to create a sustained decline in output (panel (1,2)). Finally, panel (2,4) depicts inflation. The collapse in demand at date 0 causes a large fall in prices, pushing up real wages (panel (2,1)) and creating unemployment. Inflation then recovers somewhat before declining to its new steady state level. Intuitively, this economy requires lower interest rates early on in the transition to a new steady state, as the capital stock remains high and the MPK remains low. With the nominal rate stuck at zero (panel (2,3)), real rates can only be temporarily low if inflation is temporarily high. As the capital stock declines, the real interest rate rises somewhat, and inflation falls further.

Notice that the shock we consider - an increase in risk - does not increase the *overall* level of desired savings relative to consumption as would, for example, an increase in the discount factor.[40] Instead, it shifts the desired *composition* of savings by increasing the demand for safe assets relative to capital. Thus, even though a (risky) physical storage technology is available, an increase in the desire to save in the form of *safe* assets can cause a recession, and in fact a permanent investment slump.

Stagnant steady states To understand why nominal rigidities permit permanently high unemployment, it is useful to revisit our analysis of steady states. While the flexible wage economy was always at full employment, nominal wage rigidities allow the labor market to be in one of two regimes. In steady state, (23) becomes the long run Phillips curve:

$$l = \min\left\{\Pi^{\frac{1-\gamma}{\alpha\gamma}}, 1\right\} \tag{28}$$

Intuitively, since real wages are constant in steady state, positive steady state inflation ($\Pi \geq 1$) implies that nominal wages must be rising, effectively making wages flexible and ensuring full employment, $l = 1$. In contrast, with negative inflation ($\Pi < 1$), nominal wages must be falling. Since nominal wages are slow to adjust downwards, they cannot catch up with declining prices. Thus real wages exceed their market clearing level and there is unemployment in steady state. (28) defines an increasing relationship between inflation and employment in this regime whose slope depends on the degree of wage flexibility γ. When $\gamma = 1$ (perfect flexibility), the Phillips curve is vertical at full employment. When $\gamma = 0$ (perfect downward rigidity), the Phillips curve is inverse-L shaped and is horizontal at zero inflation ($\Pi = 1$). Thus, with $\gamma < 1$, in the deflation regime, inflation affects real allocations in the long run.

Our monetary policy rule (24) also implies two steady state regimes. Either the ZLB is slack ($i > 0$) and monetary policy achieves its target $\Pi l^{\psi(1-\alpha)} = 1$, or the ZLB binds ($i = 0$) and $\Pi l^{\psi(1-\alpha)} \leq 1$. In fact, the two regimes of the Phillips curve and monetary policy coincide in steady state: we either have $l = 1$ or $i = 0$ and $R = \Pi^{-1}$. Figure 4 illustrates. The red upward sloping line describes the Phillips curve. The blue solid line depicts monetary policy when the ZLB does not bind. A higher relative weight on output stabilization, ψ, makes the blue curves steeper.[41] When the ZLB is slack, the curves intersect at full employment and zero inflation, $(1, 1)$. When the ZLB binds, inflation and employment are both below target (depicted by the intersection of the dashed blue curve and the red curve).

In the ZLB regime, since $i = 0$ and $\Pi < 1$, the real interest rate $R > 1$. In fact, higher steady

[40]In standard New Keynesian models with no physical storage technology, a large enough increase in the discount factor raises desired savings, pushing the economy to the ZLB causing a recession. However, if a physical storage technology such as capital were available, higher desired savings could be accommodated by an increase in investment, leading to a boom.

[41]With a strict inflation target, $\psi = 0$, the blue curve is horizontal.

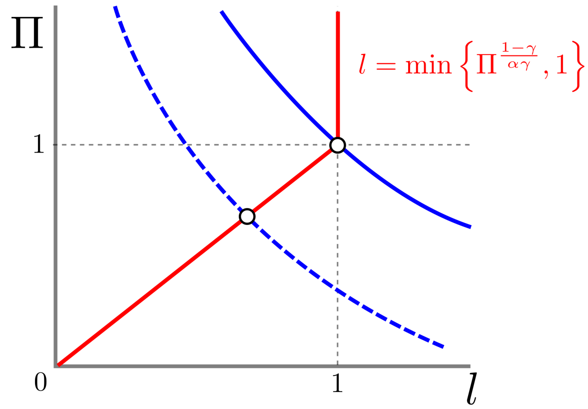


Figure 4. Determination of steady state inflation and employment

state unemployment generates more deflation and a higher steady state real interest rate. Combining the monetary policy rule and the Phillips curve yields the following set-valued map which we refer to as the '*LM*' curve (Labor markets and Monetary policy), depicted by the red curve in the left panel of Figure 5:

$$R = \begin{cases} l^{-\frac{\alpha\gamma}{1-\gamma}} & \text{if } l < 1 \\ r & \text{for any } r \geq 1 \text{ if } l = 1 \end{cases} \tag{29}$$

The remaining ingredient to complete the characterization of steady state is the young households' investment and savings decisions. Evaluating (25) and (27) at steady state, we can solve explicitly for k and R as functions of steady state employment l. In fact, these are the same functions defined in (18)-(19):

$$k = k\left(\tilde{b}l^{\alpha-1}, \sigma\right) l \tag{30}$$

$$R = R\left(\tilde{b}l^{\alpha-1}, \sigma\right) \tag{31}$$

(30) defines an increasing relationship between the capital stock and employment, depicted in the dashed black line on the right panel of Figure 5.[42] Intuitively, higher employment raises labor income, increasing savings and steady state capital. Similarly, (31) defines a decreasing relationship between interest rates and employment, depicted in the dashed blue curve on the left panel of Figure 5.[43] Higher employment implies higher steady state capital and investment; for households to invest more in capital, rather than safe government debt, real interest rates must fall. Thus, the blue curves show the relation between l and R required to equate saving and investment in steady state. We refer to them as IS curves (Hicks, 1937).

The intersection of the IS and LM curves determines steady state R and l. The dashed blue line in the left panel of Figure 5, which denotes the IS curve when risk is low (σ_L), intersects the full employment portion of the LM curve at $R > 1$. The right panel shows that full employment generates a high steady state capital stock. The green curves on the right panel indicate isoquants of the aggregate production function, $Y = k^\alpha l^{1-\alpha}$. With full employment and a high capital stock, output is relatively high in this low risk steady state, shown by the dashed-green higher isoquant.

Higher risk (σ_H) shifts the IS curve left (solid blue curve in the left panel), as savers substitute from

[42] Recall that $k(\cdot, \sigma)$ is decreasing in its first argument.
[43] Recall that $R(\cdot, \sigma)$ is increasing in its first argument.

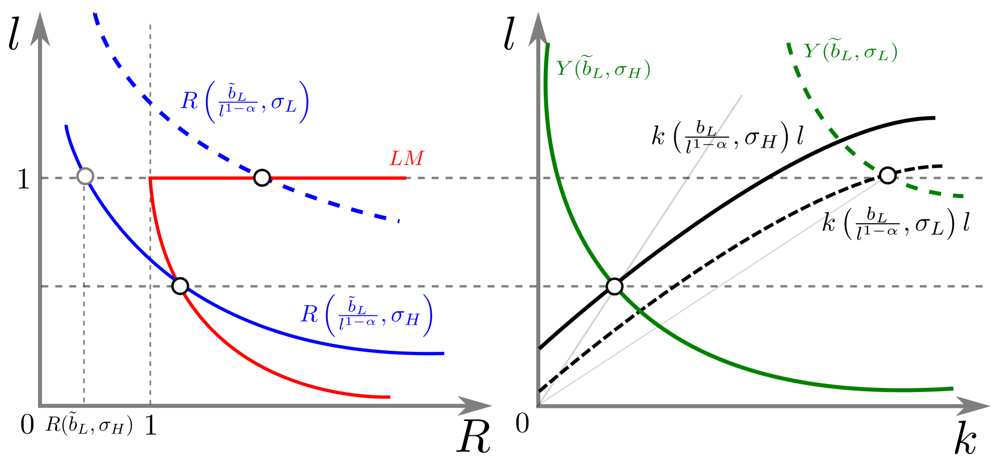


Figure 5. A permanent increase in σ keeping $\tilde{b}$ fixed

riskier capital towards safe debt, so that a lower real interest rate is required for them to hold capital. Indeed, this IS curve intersects the dashed horizontal full employment line at $R < 1$, indicating that negative rates are required to sustain full employment. That is, the steady state natural rate is negative. Given the ZLB, the LM curve does not permit $R < 1$. Instead, the ZLB binds, and the IS and LM curves intersect at $l < 1$. Unemployment in turn generates persistent deflation, raising real rates further above the natural rate with the nominal rate stuck at zero. The economy enters a stagnant steady state. Permanent unemployment implies lower income for young savers, less investment, and lower steady state capital (solid black line in the right panel). With a decline in both capital and employment, output falls dramatically (lower solid green isoquant in the right panel). In particular, higher risk reduces the capital-labor ratio (gray lines passing through the the origin in the right panel).

Given the lower capital-labor ratio, stagnation is accompanied by a *higher* expected MPK. Gomme et al. (2015) argue that while the return on government debt has remained low following the financial crisis, the real return on productive capital has rebounded, with the after-tax return on business capital at its highest level over the past three decades. They interpret this as evidence against versions of the secular stagnation hypothesis which emphasize a scarcity of investment opportunities. It is, however, entirely consistent with our risk-based view of stagnation. Higher risk may deter investment even though the *average* return on capital remains high. Indeed, in our model, an increase in σ implies a decline in the safe rate and a larger risk premium, as documented empirically by Duarte and Rosa (2015) and Caballero et al. (2017a).

4.2 How safe asset creation can restore full employment

An increase in government debt can offset an increase in risk and keep the natural rate of interest positive by satiating the demand for safe assets. Absent nominal rigidities, there was no reason to do so; but in the presence of nominal rigidities, a negative natural rate can cause a permanent recession, as shown above. It is therefore natural to ask whether issuing more debt can prevent a recession by raising the natural rate.

The answer is a qualified yes. The blue solid lines in Figure 6 depict the equilibrium when the fiscal authority raises $\tilde{b}_{t+1}$ permanently from $\tilde{b}_L$ to a higher level $\tilde{b}_H$ starting at date 0.[44] This increase in

[44] In Figure 6 we set $\tilde{b}_H = 0.077$. The precise value of $\tilde{b}_H$ is not important as long as it corresponds to a positive steady state natural rate of interest with high risk, i.e. $R(\tilde{b}_H, \sigma_H) > 1$. We set $\psi = 0.1$, which implies a relative weight on output

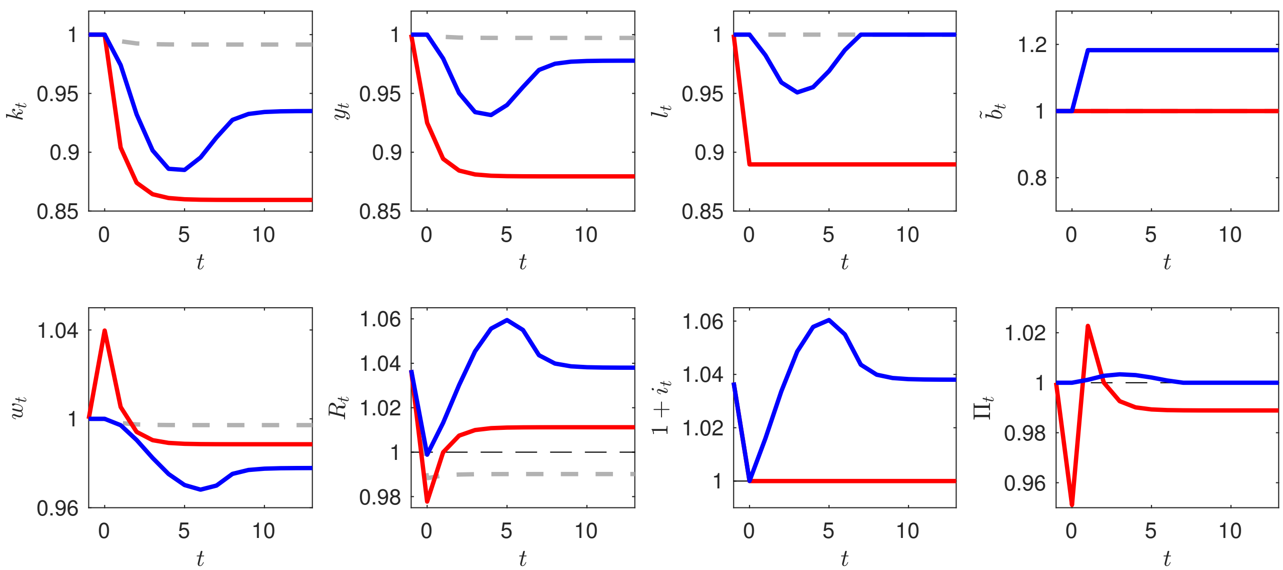


Figure 6. An increase in the supply of safe assets Dashed lines denote the natural allocation. Solid red lines denote equilibrium with nominal rigidities and no increase in safe assets. Solid blue lines denote equilibrium with nominal rigidities and an increase in safe assets.

the supply of safe assets (panel (1,4)) accommodates the higher demand induced by the increase in risk, equilibrating asset markets without requiring the monetary authority to cut nominal rates below zero. With more safe assets in their portfolio, households are less averse to investing in capital;[45] at the same time the government rebates the proceeds from debt issuance to households, allowing them to spend more on consumption and capital. This mitigates the fall in aggregate spending, preventing prices from falling. With no fall in prices,the nominal wage rigidity does not bind and there is no unemployment on impact.

While higher $\tilde{b}$ prevents an increase in unemployment on impact, it still crowds out capital - in the precise sense that investment is lower than in the *natural allocation* corresponding to no increase in safe assets. The blue line in panel (1,1) shows that the trajectory of capital lies below its trajectory in the 'low-debt' natural allocation. However, increasing safe assets results in *higher* investment than would obtain if there were no increase in safe assets, *given that nominal wages are not fully flexible.* Thus while the blue line lies *below* the dashed line describing the low-debt natural allocation, it lies *above* the red line describing equilibrium with nominal rigidities and no increase in safe assets. While producing safe assets reduces the capital-labor ratio, real wages, and labor productivity, this is more than compensated by an increase in employment, and so producing safe assets increases long-run output relative to the low debt economy with nominal rigidities which becomes stagnant (panel (1,2)).

Lower investment due to the increase in safe assets reduces labor demand. Since wages are sticky downwards, this causes unemployment in the short run. Since monetary policy puts some weight on output gap stabilization and is not constrained by the ZLB, it permits moderate inflation shortly after the initial shock (panel (2,4)). This inflation helps real wages fall to their new steady state level (panel (2,1)). In the long run an increase in safe assets results in full employment and zero inflation, albeit at a lower level of capital and output than before the initial shock. The increase in safe assets ultimately short-circuits

stabilization of 10 percent which is relatively standard. Again, the precise value of ψ does not affect the outcomes qualitatively.

[45] Recall that the households demand for capital is increasing in $\tilde{b}$: As the consumption when old has a lower covariance with the return on capital, capital becomes a more attractive option.

the adverse feedback loop between unemployment and low investment, resulting in higher steady state capital than would occur without an increase in the supply of safe assets. However, the temporary increase in unemployment does depress investment somewhat, which accounts for the undershooting of capital in panel (1,1). Output (panel (1,2)) and wages (panel (2,1)) inherit the behavior of capital. Note that while real interest rates fall on impact, as the economy suddenly finds itself with a high level of capital relative to the new steady state, they rise again to their new steady state as capital declines (panel (2,2)).

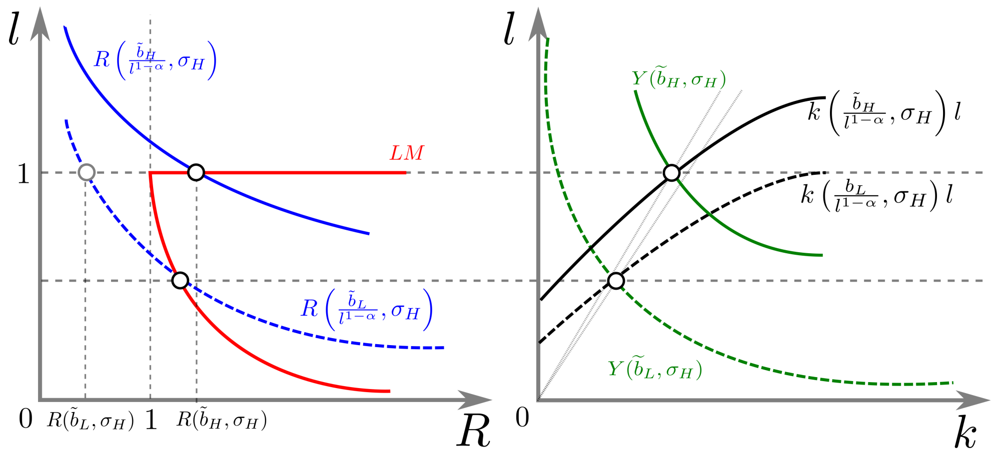


Figure 7. A permanent increase in $\tilde{b}$ in a high σ environment

New steady state with a higher supply of safe assets A permanent increase in $\tilde{b}_H$ does more than just smooth transitions; it also ensures full employment in the long run. The dashed lines in Figure 7 depict the steady state without the increase in debt (same as the dashed curves in Figure 5); the solid lines depict steady state with higher debt. Higher debt satiates the demand for safe assets, reducing the risk premium, shifting the IS curve rightwards (solid blue curve), and raising the natural rate of interest (intersection of the IS curve and $l = 1$). A large enough increase in $\tilde{b}$ pushes the natural rate above zero, allowing monetary policy to equate the real rate and the natural rate and achieve full employment. As the right panel shows, higher $\tilde{b}$ increases steady state capital relative to the stagnant steady state: higher employment raises the MPK, encouraging investment.

This is not costless. As the right panel of Figure 7 shows, higher $\tilde{b}$ reduces steady state capital for any level of employment (dashed black line), reducing investment relative to the natural allocation with no additional safe assets. Higher $\tilde{b}$ ultimately leaves the capital stock lower than before the increase in risk. With lower capital, output falls below its pre-crisis level even though full employment has been restored, as the isoquants (solid green curve) show. Indeed the new steady state has a lower capital-labor ratio, not just relative to the low risk steady state but also the stagnant steady state (see the gray lines in the right panel). This *capital shallowing* in turn reduces real wages and labor productivity (Y/l). In this sense, a risk-induced recession can continue to depress output, wages and labor productivity even when fiscal policy has restored full employment.

Should we create safe assets to increase the natural rate? Despite these costs, if safe asset creation is the only tool available to a policy-maker, it should always be used to restore full employment. Consider the problem of a constrained planner who chooses $\tilde{b}$ to maximize steady state welfare, given the constraints imposed by nominal rigidities and the monetary policy regime:

$$\max_{k,\tilde{b},l,R,\Pi,i} (1-\beta)\ln\left[(1-\alpha)k^\alpha l^{1-\alpha} - \tilde{b}k^\alpha - k\right] + \beta \mathbb{E}_z \ln\left[\alpha z k^\alpha l^{1-\alpha} + \tilde{b}k^\alpha\right] \tag{32}$$

subject to equations (30) and (31) which describe steady state capital labor ratio and real interest rates, the steady state Phillips curve (28) and the monetary policy rule which, in steady state, reduces to:

$$\left(\frac{\Pi}{\Pi^\star}\right) l^{(1-\alpha)\psi} \leq 1, \qquad 1+i = R\Pi \geq 1, \qquad i\left[\left(\frac{\Pi}{\Pi^\star}\right) l^{(1-\alpha)\psi} - 1\right] = 0$$

In the presence of nominal rigidities, it is constrained optimal to create enough safe assets to keep the natural rate nonnegative, as the Proposition below shows.

Proposition 3. *Let $\tilde{b}_{real}(\sigma)$ be the choice of $\tilde{b}$, given σ, which maximizes steady state welfare in Proposition 1, and define $\tilde{b}_{zlb}(\sigma,\Pi^\star)$ as the smallest level of $\tilde{b}$ such that the steady state features $i \geq 0$ and $l = 1$. Then the level of $\tilde{b}$ which solves (32) is:*

$$\tilde{b} = \max\{\tilde{b}_{zlb}(\sigma,\Pi^\star), \tilde{b}_{real}(\sigma)\}$$

In particular if $\Pi^\star = 1$, then $\tilde{b} = \tilde{b}_{zlb}(\sigma,1)$ whenever $\sigma > \underline{\sigma}$ (i.e. if the ZLB binds with $\tilde{b} = 0$)

Proof. See Appendix H. □

In response to higher risk which pushes the economy to the ZLB, an increase in $\tilde{b}$ increases both employment and capital relative to keeping $\tilde{b}$ unchanged – even though this level of capital is lower than in the optimal natural allocation in which $\tilde{b} = 0$. Absent nominal rigidities, $\tilde{b} = 0$ was optimal because increasing $\tilde{b}$ would reduce steady state capital. In an economy with nominal rigidities, since increasing $\tilde{b}$ up to $\tilde{b}_{\text{zlb}}(\sigma,\Pi^\star)$ actually *increases* capital relative to the stagnant steady state, this reason for abstaining from safe asset production no longer applies. Of course, once the economy has reached full employment, a further increase in $\tilde{b}$ would only crowd out capital, reducing welfare. Figure 8 illustrates the trade-off between steady state k and $\tilde{b}$ when $\sigma > \underline{\sigma}$, i.e. we would have $R(0,\sigma_H) < 1$ absent safe assets. The blue line illustrates this relation in the natural allocation, which is always decreasing. Absent nominal rigidities, increasing safe assets from $\tilde{b}_L$ to $\tilde{b}_H$ always decreases steady state capital. Thus such an increase is generally undesirable. In contrast the red curve depicts the same relationship, but with nominal rigidities. Now refraining from additional safe asset production results in unemployment, lowering steady state capital. Increasing $\tilde{b}$ up to $\tilde{b}_{\text{zlb}}$ increases steady state capital. Beyond this point though, the ZLB no longer binds and the economy behaves as in the natural allocation. Thus, it is generally optimal to increase $\tilde{b}$ to $\tilde{b}_{\text{zlb}}$ but no more.

A high-risk economy needs negative real rates to sustain high investment, as in the optimal natural allocation. In the presence of nominal rigidities and a zero long run inflation target, negative rates are simply not possible. Thus an economy with a negative natural rate experiences a recession, as monetary policy loses its potency at the ZLB. Issuing public debt satiates the demand for safe assets and raises the natural rate of interest, relaxing the ZLB and rendering monetary policy potent again. But this does not change the fact that a risky economy *requires negative real rates* to sustain high *investment*. The same increase in debt which restores full employment crowds out investment in physical capital, selecting a different, sub-optimal natural allocation.

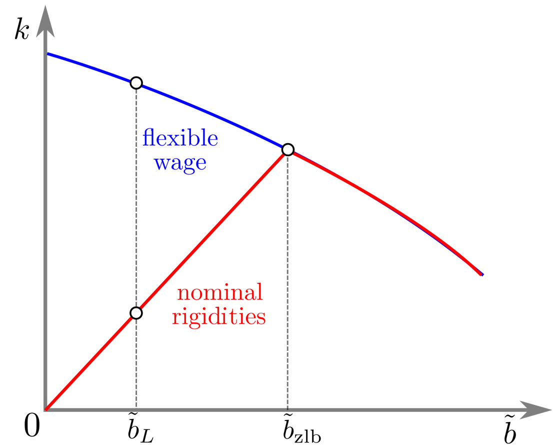


Figure 8. An environment with $\sigma > \underline{\sigma}$. The negative sloped blue curve represents the steady state trade-off between k and $\tilde{b}$ in the flexible wage economy. The non-monotonic red curve represents the steady state trade-off between k and $\tilde{b}$.

Public debt vs. government spending We have just seen that while an increase in government debt can raise the natural rate and restore full employment, this comes at the cost of crowding out. While we have focused on government debt, the same trade-off applies to other aspects of fiscal policy. Eggertsson and Krugman (2012), among others, argue that government purchases are an especially effective way to stimulate output and consumption when monetary policy is constrained by the ZLB. Indeed, in our economy an increase in government purchases $G > 0$ raises the natural rate, as we show in Appendix K.[46] Higher G leaves less resources available to the young, reducing their savings for a given level of labor income. This raises the natural rate, but reduces the steady state capital stock. A large enough increase in G restores full employment after an increase in risk, substituting for the fall in private demand for capital, and preventing the deflationary spiral described earlier. However, higher $\tilde{b}$ can also restore full employment; in fact, it is strictly superior in welfare terms to an increase in G, as Appendix K shows. A marginal increase in debt elicits a larger increase in the natural rate than a marginal increase in government purchases; both policies reduce current resources of the young but only $\tilde{b}$ narrows the risk premium. Thus, a positive natural rate can be restored with a smaller increase in $\tilde{b}$ (and less crowding out) than would be required if one relied on an increase in G. Further, an increase in safe assets provides more insurance to the old, while an increase in G does not. Because government purchases are strictly inferior to an increase in safe assets, for the remainder of the paper we continue to focus on safe asset creation rather than government purchases.

Government purchases of risky assets Rather than increasing the supply of safe assets, another way for the fiscal authority to lower risk premia and circumvent the ZLB would be to directly purchase risky assets held by private agents. Indeed, during the recent financial crisis, the Federal Reserve directly purchased privately issued financial assets such as mortgage backed securities; Hancock and Passmore (2011) find that these purchases reduced risk premia in mortgage markets. In our economy if the government could directly purchase risky capital and rebate the returns equally to old households, this would reduce the risk exposure of households, reducing risk premia and raising equilibrium real interest rates. Indeed, in our economy with only idiosyncratic risk, such a policy is very powerful - it can completely undo the effects of an increase in risk - and comes at no cost, since there is no aggregate risk in our economy. More

[46] We consider a balanced budget increase in G. Deficit financed government spending can be thought of as a combination of higher G and higher $\tilde{b}$.

generally, in the presence of both idiosyncratic and aggregate risk, the policy would be less powerful, since the government (and therefore taxpayers) retains some exposure to aggregate risk after purchasing risky assets. Whether purchases of risky capital raise the natural interest rate in such a model would depend on the nature of market incompleteness.[47]

4.3 Raising the Inflation Target

While an increase in the supply of safe assets prevents a stagnant steady state, it has the undesirable side effect of lowering investment relative to the optimal natural allocation. An alternative policy is to raise the inflation target. Even in the presence of a ZLB on nominal rates, higher inflation permits negative real interest rates, allowing both full employment and higher investment, as in the optimal natural allocation.

Suppose that at date 0, in response to the increase in σ, the fiscal authority keeps $\tilde{b}$ unchanged at $\tilde{b}_L$ but the monetary authority raises its inflation target to $\Pi^\star > 1$. Turning first to steady state outcomes, with positive target inflation, the LM relation (23) becomes

$$R = \begin{cases} \frac{l^{-\frac{\alpha\gamma}{1-\gamma}}}{\Pi^\star} & \text{if } l < 1 \\ r & \text{for any } r \geq \frac{1}{\Pi^\star} \text{ if } l = 1 \end{cases} \tag{33}$$

With a higher $\Pi^\star$, monetary policy permits full employment and positive inflation without seeking to tighten policy. This allows for a steady state with full employment and modestly negative real interest rates, which would not be possible if monetary policy targeted zero inflation. In other words, as the left panel of Figure 9 shows, raising $\Pi^\star$ from 1 shifts the LM curve leftwards, moving from the dashed to the solid red line. This closes the gap between a lower natural rate and a higher prevailing rate of interest by reducing the prevailing rate, rather than by increasing the natural rate (as in Figure 7). A large enough increase in inflation target maintains full employment even after the increase in risk and a fall in the natural rate. Graphically, this allows the shifted LM curve to intersect the IS curve (solid blue curve) at full employment and negative real rates.

The right panel of Figure 9 shows that by attaining full employment, higher $\Pi^\star$ allows both higher output and a higher capital-labor ratio relative to zero target inflation and no increase in safe assets. Unlike an increase in the supply of safe assets (see Figure 7), higher $\Pi^\star$ does not crowd out investment. Graphically, the black curve in the right panel of Figure 9, depicting the relation between steady state capital and employment, does not shift leftwards as it did in Figure 7. Thus higher target inflation permits higher output, capital-labor ratio and labor productivity relative to an increase in safe assets.

Higher inflation targets also promote smoother transitions. As before, the blue and red lines in figure 10 depict transitional dynamics with and without an increase in safe assets, respectively, and with $\Pi^\star = 1$. The black lines depict transitional dynamics with no increase in safe assets but an increase in the inflation target from $\Pi^\star = 1$ to $\Pi^\star = 1.02$ starting from date $t = 0$. With higher target inflation, real rates can fall persistently below zero without the ZLB binding. Low real rates keep investment high, and the decline in capital is smaller than in the case with safe asset creation (blue line in panel (1,1)), not to mention the case without safe asset creation and $\Pi^\star = 1$ (red line in panel (1,1)). Indeed, the black line in panel (1,1)

[47] In section 5 we discuss another way to interpret "risky asset purchases". In the presence of asset bubbles, a fall in the value of bubbly assets can cause the ZLB to bind; by buying the bubbly assets at a high price, the government can prevent such an event from causing the ZLB to bind.

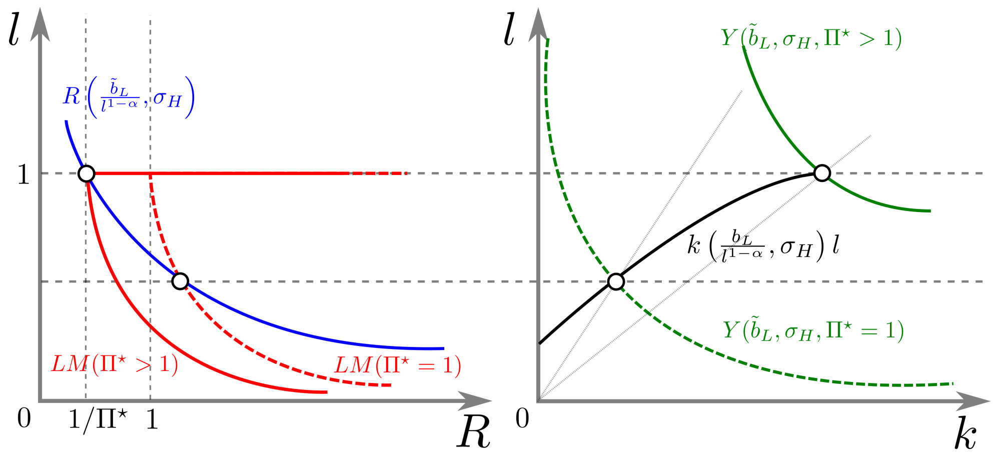


Figure 9. An increase in $\Pi^\star$ in a high σ environment.

closely mimics the trajectory of the capital stock in the natural allocation with no increase in safe assets. The only difference is that in the short run a reduction in capital requires a small decline in real wages, which causes temporary unemployment (panel (1,3)) and above-target inflation (panel (1,4)). However, this unemployment is temporary and long run outcomes coincide with those in the natural allocation.

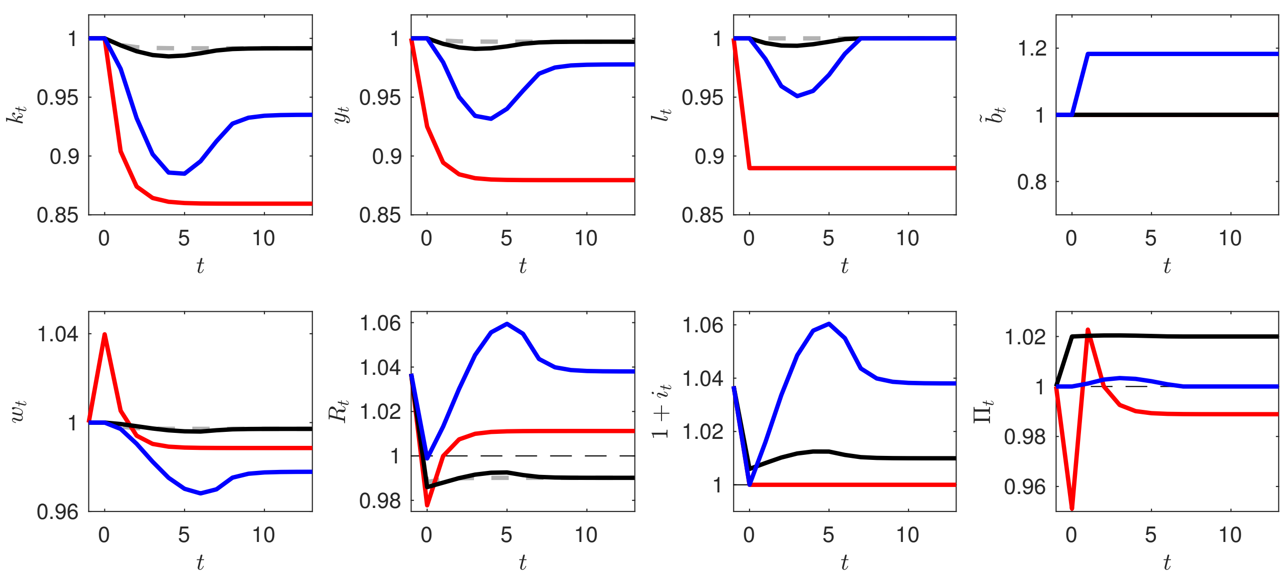


Figure 10. An increase in $\Pi^\star$. Dashed lines denote equilibrium in the absence of nominal rigidities. Solid red lines denote equilibrium with nominal rigidities and no increase in safe assets. Solid blue lines denote equilibrium with an increase in safe assets and no change in the inflation target. Solid black lines denote equilibrium with an increase in the inflation target and no increase in safe assets.

Traditionally, economists have argued that monetary policy should seek to replicate natural allocations (Goodfriend and King, 1997). Our economy has many natural allocations indexed by $\tilde{b}$, which is a *choice variable* of the fiscal authority. $\tilde{b} = 0$ selects the optimal natural allocation,[48] even if this involves negative real rates. Without higher target inflation, it is not possible to replicate this optimal natural allocation (or more generally the pre-recession level $\tilde{b}_L$). Safe asset creation shifts the goal posts, presenting monetary policy with the easier task of implementing a different, suboptimal natural allocation. But to replicate

[48]Provided that $\sigma < \bar{\sigma}$.

the *optimal* natural allocation requires higher inflation. A high-risk economy is crying out for negative real rates; a higher inflation target is the only way to deliver this given the ZLB.[49] To put this another way, even if it is desirable to close the gap between a negative natural rate and a prevailing real rate stuck above zero, there are two ways to do this. Safe asset creation raises the natural rate to meet the higher prevailing rate, which crowds out capital. A higher inflation target instead reduces the prevailing rate to meet the lower natural rate in the optimal natural allocation, sustaining high investment.

Our model does not permit a full cost-benefit analysis of higher inflation targets, since it abstracts from the costs associated with higher steady state inflation (Coibion et al., 2016). If these costs are large, risk-induced stagnation may present problems which cannot be solved by either monetary or fiscal policy. Nonetheless, such policies are worth considering, because safe asset creation is no panacea. Our analysis provides a new perspective on the idea that deficit spending is doubly desirable in ZLB episodes, because higher deficits reduce unemployment in the short run, while negative real rates make it an exceptionally good time for the government to borrow. In our model, this argument is correct as far as it goes: higher deficits prevent unemployment, and actually increase investment. This is preferable to the alternative of tight fiscal policy and a low inflation target. But while deficit spending implements *a* natural allocation, this is not the *optimal* natural allocation. A better alternative than either deficit spending or a tight fiscal policy combined with a low inflation target, is to raise the inflation target. This permits permanently negative real interest rates *in equilibrium*, and high investment, without the need for safe asset creation.

5 Low real interest rates and bubbles

We have seen that high risk can lead to a negative natural rate of interest. This creates problems for monetary policy; while an increase in safe assets cannot wholly solve these problems, a higher inflation target can, allowing monetary policy to deliver the negative real rates required for full employment and high investment. However, introducing negative real interest rates in equilibrium has its own side effect: it creates a breeding ground for bubbles. As is well known, negative real rates permit *rational bubbles* (Tirole, 1985): assets in finite supply which pay no dividend yet have a positive price. We now explore how rational bubbles interact with the supply of safe assets in our setting.[50]

Suppose there exists a stock of intrinsically useless assets in measure 1. At date 0, these are all owned by the date 0 old generation. Let Q_t denote the nominal price of this asset, and x_{t+1} the quantity of this asset purchased by a young household at date t. The budget constraints of cohort t become:

$$P_t c_t^Y + P_t k_{t+1} + \frac{1}{1+i_t} B_{t+1} + Q_t x_{t+1} = W_t l_t + P_t T_t \tag{34}$$

$$P_{t+1} c_{t+1}^O(z) = P_{t+1} R_{t+1}^k(z) k_{t+1} + B_{t+1} + Q_{t+1} x_{t+1} \tag{35}$$

where $Q_t x_{t+1}$ denotes expenditure on bubbles by the young household and $Q_{t+1} x_{t+1}$ in (35) denotes the payoff from holding x_{t+1} bubbles when old. Equilibrium in the market for bubbles requires $x_t = 1$. All of our analysis above considered equilibria in which $Q_t = 0$ for all t.

To isolate the problems introduced by bubbles, as distinct from those due to a binding ZLB, we begin

[49] Of course, negative nominal rates would also deliver this outcome, if possible.

[50] Aoki et al. (2014) also find that in a flexible price AK model with idiosyncratic risk, negative real interest rates can allow bubbles to exist. Unlike us they do not explore the interaction of such bubbles with fiscal policy and nominal rigidities.

by studying bubbles in a full employment steady state where the ZLB does not bind. One interpretation is that the inflation target $\Pi^{\star}$ is high enough that the monetary authority can deliver full employment even if the natural rate of interest is negative. We later describe how bubbles interact with a binding ZLB.

Pseudo-Safe bubbles Following Weil (1987), we consider equilibria in which the bubble bursts with a constant probability $1-\rho$ for $\rho \in (0,1]$ in each period. The simplest case is a *pseudo-safe bubble* which does not burst ($\rho = 1$). Since pseudo-safe bubbles and government debt are perfect substitutes (if bubbles have positive value), they must offer the same return in equilibrium: $R_t = q_{t+1}/q_t$ where $q_t = Q_t/P_t$ is the real price of a bubble at date t. In particular, if such a bubble has a positive price in steady state, we must have $R = 1 = q_{t+1}/q_t$. Then if σ is low enough such that $R(\tilde{b}, \sigma) \geq 1$ absent bubbles, no bubbly equilibrium exists.[51] However, if $R < 1$ in the absence of pseudo-safe bubbles, there exists a steady state in which $R = 1$ and pseudo-safe bubbles have a constant price $q > 0$ which solves

$$R\left(\tilde{b} + \tilde{q}, \sigma\right) = 1 \tag{36}$$

where $R(\cdot)$ is defined in (19) and $\tilde{q}_t = q_t/k_t^{\alpha}$ is the bubble's size relative to output. Take two steady states with the same σ and $\tilde{b}$, one with bubbles and one without: the bubbly steady state has a higher real interest rate and lower capital stock. Bubbles provide insurance and crowd out investment, like government debt. For this reason, bubbles reduce welfare, as we now show.

Lemma 5 (Welfare and Pseudo-safe Bubbles)**.** *For any σ, the solution to the planner's problem in* (20) *strictly dominates any steady state featuring pseudo-safe bubbles (if such steady states exist) .*

Proof. (36) implies that any steady state with $q > 0$ has the same capital and consumption as one with $q = 0$ and $\tilde{b}$ such that $R = 1$. From Proposition 1, the solution to the steady state planner's problem features either $\tilde{b} = 0$ or $R < 1$, and welfare-dominates any steady state with $\tilde{b} > 0$ and $R = 1$. □

Bubbles have the same effect as a level of government debt large enough to deliver $R = 1$. Proposition 1 tells us that this allocation is suboptimal: at $R = 1$, the marginal benefit of transferring resources from young to old via higher debt is zero, while the cost in terms of crowding out is positive.

Lemma 5 contrasts with the classic literature on rational bubbles. Generally, rational bubbles can only exist when the bubble-less equilibrium would feature over-accumulation of capital.[52] Bubbles increase welfare *by crowding out capital* in such economies, facilitating a transfer of wealth from the young to the old without inefficiently high capital. In our environment, bubbles arise even when the bubble-less equilibrium is dynamically efficient – they decrease welfare by crowding out capital, and emerge even when they are not desirable. This provides a counterexample to a conjecture of Abel et al. (1989) that rational speculative bubbles can only exist in economies with capital over-accumulation, even in the presence of risk.[53]

[51] If pseudo-safe bubbles had a positive price and $R > 1$, their price would have to grow forever, $q_{t+1}/q_t = R > 1$, and the value of bubbles would eventually exceed the income of young households who must purchase them, a contradiction.

[52] A newer literature discusses conditions under which bubbles can arise in dynamically efficient economies owing to financial frictions. See Martin and Ventura (2018) for a review and Asriyan et al. (2016) for an example of an economy in which bubbles arise even though there is under-investment.

[53] See paragraph 3 on page 15 of Abel et al. (1989).

Leaning against the wind We have just seen that low interest rates are a breeding ground for welfare-reducing bubbles. This is unfortunate, because when risk is high, implementing the optimal natural allocation requires negative real rates. One might worry that even if we would like to refrain from safe asset creation in order to prevent crowding out, this will not be possible, because the resulting low interest rate environment permit bubbles to arise, which crowd out investment anyway. In fact, this need not be the case if the fiscal authority can credibly commit to burst any bubble that arises.

Suppose that instead of committing to a fixed path of $\tilde{b}_{t+1} = \tilde{b}^*$, the fiscal authority commits to implement this path using the following policy rule: For any date t, if $q_t = 0$, choose $\tilde{b}_{t+1} = \tilde{b}^*$. If instead $q_t > 0$, set $\tilde{b}_s = \tilde{b}_{ss}$ for all $s > t$, where $R(\tilde{b}_{ss}, \sigma) > 1$. The off-equilibrium threat to crowd out rational bubbles with government debt prevents bubbles from ever emerging.[54] One interpretation of our results in Sections 2-4 is that such a rule prevented bubbles from arising. A large literature attempts to formalize the notion that monetary policy should lean against the wind to prevent bubbles; however, it has proven challenging to construct models in which bubbles can exist, reduce welfare, and can be prevented with contractionary monetary policy (Gali, 2014; Allen et al., 2017). Our analysis suggests that it may be more appropriate for fiscal policy, rather than monetary policy, to lean against the wind.

Risky bubbles Consider instead risky bubbles which have a constant positive probability $1 - \rho$ of bursting each period.[55] In this case, risky bubbles are no longer a perfect substitute for safe government debt, so we must have $R_t < q_{t+1}/q_t$ (assuming the bubble does not burst at date $t + 1$). Bubbles still crowd out capital, but now introduce another risk: they can burst, leading to consumption losses for old households whose wealth vanishes. In principle this can be prevented via commitment to a fiscal rule as described above. However, if such commitments are not credible, a government wishing to eliminate bubbles must increase the supply of public safe assets on-equilibrium. This resonates with the argument of Greenwood et al. (2016) that the government should supply short-term safe assets to crowd out socially excessive private safe asset creation. While our model abstracts from the externalities associated with private transformation which are the focus of Greenwood et al. (2016), risky bubbles can be thought of as an example of excessive *private* safe asset creation - which public safe asset creation can prevent.

Another way to prevent risky bubbles in our economy would be to impose *macroprudential* taxes on holdings of the bubble asset (see for example Biswas et al. (2018)). Clearly though, such a policy requires a policy maker to know which assets are bubbles and to observe private holdings of such assets. This is challenging since the risks we have discussed are not confined to tightly regulated intermediaries which are usually the targets of macroprudential policy, but could arise in securities markets or in markets for nonfinancial assets.[56] While it comes at the cost of crowding out capital, increasing the supply of publicly issued safe assets is a less informationally demanding policy: it only requires the policymaker to check that safe real interest rates are positive. This bears some similarity to the argument in Stein (2013) that higher interest rates "get in all the cracks", affecting corners of financial markets which regulation and supervision may not be able to reach. However, unlike in Stein (2013)'s argument, here it is fiscal rather than monetary policy that has a role to play in generating persistently higher real interest rates.

[54] See also Biswas et al. (2018) for a discussion of how macro-prudential policy can prevent costly bubbles from emerging.

[55] We assume that once a bubble bursts, a new bubble never appears to replace it.

[56] Of course, the fiscal policy rule above, which commits to aggressively expand the issuance of public debt if a bubble should arise, also requires the policymaker to know when a bubble exists, although in this case it is not necessary to observe who holds these assets.

Bubbles can mask stagnation Some commentators have argued that even prior to 2008, several advanced economies required bubbles just to maintain full employment; on this account, the bursting of such a bubble caused the ensuing liquidity trap. Our model allows us to formalize this hypothesis. Suppose the economy is initially in a bubbly steady state with $R(\tilde{b} + \tilde{q}, \sigma) = 1$ and $\tilde{q} > 0$.[57] Absent bubbles, the steady state natural rate of interest would be negative, and the economy with nominal rigidities would experience unemployment. Bubbles prevent this from happening, by increasing the effective supply of safe assets and raising the natural rate. This frees monetary policy from the ZLB, allowing it to implement full employment. This economy does indeed require bubbles in order to sustain full employment.

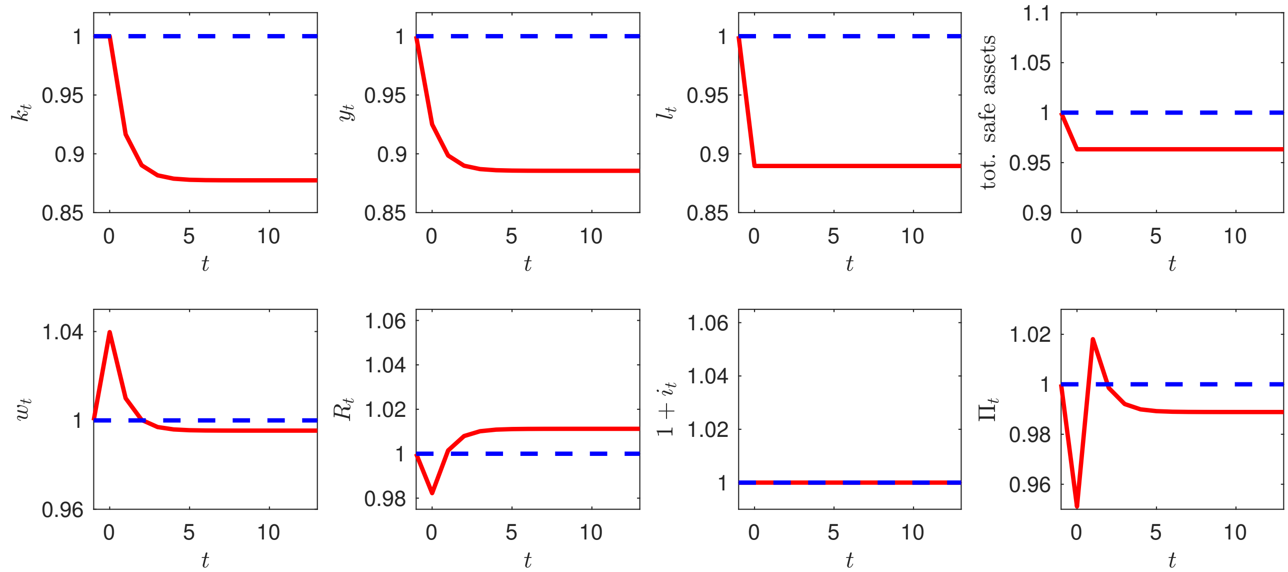


Figure 11. Solid red lines denote equilibrium without an increase in safe assets. Dashed blue lines denote denote equilibrium with an increase in safe assets.

Suppose this bubble bursts unexpectedly at some date 0, i.e. $\tilde{q}_t = 0$ for all $t \geq 0$. The solid red lines in Figure 11 depict the transition to a stagnant steady state after the bubble bursts. Here k_{-1}, y_{-1}, w_{-1} and the total stock of public plus private safe assets $\tilde{b}_{-1} + \tilde{q}_{-1}$ are all normalized to one. Note however that the initial capital stock in this scenario, which features a bubble, is lower than the capital stock which would obtain in the natural allocation without a bubble as in Figure 3.

As depicted in Figure 11, the dynamics of such an economy are broadly similar to those described in section 4, where we instead subjected the economy to an increase in risk starting from a bubble-free steady state. The bursting of the bubble reduces the available supply of pseudo-safe assets in the economy. This contraction in supply puts upward pressure on the price of safe assets, i.e. reduces the natural rate of interest. Since this economy features zero real interest rate even with the bubble, full employment requires negative real rates absent the bubble. The ZLB prevents this. Finding no bubbles to invest in, households attempt to re-balance their portfolios towards safe government debt, slashing spending on investment, resulting in a permanent decline in investment and economic activity. In this sense a bubble can mask risk-induced stagnation, and the bursting of such a bubble can reveal the rot within the economy.

An increase in the supply of publicly provided safe assets can counteract the reduced supply of privately

[57] For simplicity, consider the case of a bubble which is perceived to have zero probability of bursting. It is straightforward to extend this analysis to a case in which we have risky bubbles. Nothing would change qualitatively.

produced pseudo-safe bubbles, mitigating the fall in output.[58] Bear in mind, though, that the bubble, before it burst, was already crowding out capital investment relative to the optimal natural allocation. Replacing a private bubble with safe public debt, at best, only replicates this sublunary outcome. As discussed above, the cure the economy needs is negative real interest rates and not more safe assets.

6 Conclusion

We presented a model in which the natural rate of interest is affected both by idiosyncratic risk and by fiscal policy. By increasing the supply of safe assets, policymakers can prevent an increase in risk from driving the natural rate below zero, allowing monetary policy to operate effectively rather than being constrained by the ZLB. However, negative natural rates do not necessarily indicate a *shortage* of safe assets. While fiscal policy can keep the natural rate positive, the optimal natural allocation allows risk to drive the natural rate below zero, because increasing debt crowds out investment. With a binding ZLB, a policymaker committed to full employment and low inflation may be forced to deviate from the optimal natural allocation, producing safe assets in order to raise the natural rate and prevent a permanent economic slump. However, the reprieve comes at the cost of a permanent decline in investment. The return to full employment merely disguises the deeper problem – that the economy requires negative interest rates in order to operate at potential – which manifests as sluggish investment and labor productivity. In this sense, the cost of a risk-induced recession may linger even once the economy has returned to full employment. Rather than increasing government debt, it may be preferable to engineer negative real interest rates, for example through higher target inflation; this sustains high investment while preventing unemployment. However, negative real rates have their own side effect, providing a breeding ground for welfare reducing bubbles. Without aggressive fiscal policy to head off these bubbles, they may torment the economy, depressing investment while they persist, and creating deep recessions when they burst.

A full empirical evaluation of this theory lies beyond the scope of this paper. Nevertheless, the scenario we have described is in some respects disturbingly similar to the experience of the U.S. and other advanced economies during the recovery from the Great Recession. These economies experienced a large increase in publicly issued safe assets (government debt held by the public and central bank reserves). Even after returning to full employment, output, investment, labor productivity, and capacity utilization have remained persistently below their pre-crisis trends. Our analysis suggests that these outcomes might be the unavoidable consequence of an increase in safe asset creation.

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[58] One way to implement this would be for the fiscal authority to buy old households' worthless paper assets, financing the purchases with government debt which is rolled over forever. This replicates allocations in the old, bubbly steady state.

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Appendix

A Household's Optimal Choices

Using equations (1)-(2), the objective function of the households can be written as:

$$\max_{k_{t+1},b_{t+1}} (1-\beta)\ln\left[\omega_t l_t + T_t - \frac{1}{R_t}b_{t+1} - k_{t+1}\right] + \beta\mathbb{E}_z \ln\left[R^k_{t+1}(z)k_{t+1} + b_{t+1}\right]$$

where $\omega_t = \frac{W_t}{P_t}$ and $b_{t+1} = \frac{B_{t+1}}{P_{t+1}}$ and $R_t = \frac{\Pi_t}{1+i_t}$. The first order conditions w.r.t k_{t+1} and b_{t+1} can be written as:

$$\frac{1-\beta}{\omega_t l_t + T_t - \frac{b_{t+1}}{R_t} - k_{t+1}} = \beta\mathbb{E}_z\left[\frac{R^k_{t+1}}{R^k_{t+1}(z)k_{t+1} + b_{t+1}}\right] \tag{37}$$

$$\frac{1-\beta}{\omega_t l_t + T_t - \frac{b_{t+1}}{R_t} - k_{t+1}} = \beta\mathbb{E}_z\left[\frac{R_t}{R^k_{t+1}(z)k_{t+1} + b_{t+1}}\right] \tag{38}$$

Next multiply equation (37) by k_{t+1}, (38) by $\frac{b_{t+1}}{R_t}$ and add them up:

$$\frac{k_{t+1} + \frac{b_{t+1}}{R_t}}{\omega_t l_t + T_t - \frac{b_{t+1}}{R_t} - k_{t+1}} = \frac{\beta}{1-\beta} \tag{39}$$

which can be rearranged to yield:

$$k_{t+1} + \frac{b_{t+1}}{R_t} = \beta\left[\omega_t l_t + T_t\right] \tag{40}$$

i.e. the young household save a fraction β of its labor income net of transfers. Using the budget constraint, it is straightforward to see that

$$c^Y_{t+1} = (1-\beta)\left[\omega_t l_t + T_t\right]$$

Using these equations, we can re-write the objective as:

$$\max_{\eta_t}(1-\beta)\ln\Big[(1-\beta)(\omega_t l_t + T_t)\Big] + \beta\ln\Big[(\omega_t l_t + T_t)\Big] + \beta\mathbb{E}_z\ln\Big[\eta_t R^k_{t+1}(z) + (1-\eta_t)R_t\Big]$$

where we define the portfolio share of capital as η_t as $\frac{k_{t+1}}{k_{t+1}+\frac{b_{t+1}}{R_t}}$. Notice that only the last term of the expression depends on η_t. Thus, the choice of η_t can be seen as choosing portfolio weights to maximize risk-adjusted returns:

$$\eta = \arg\max_{\eta}\mathbb{E}_z\ln\Big[\eta_t R^k_{t+1}(z) + (1-\eta_t)R_t\Big]$$

The optimal choice of η_t can then be written as:

$$\mathbb{E}_z\Big[\frac{R^k_{t+1}(z) - R_t}{\eta_t R^k_{t+1}(z) + (1-\eta_t)R_t}\Big] = 0$$

Notice that the numerator of the expression above is the return earned by capital in excess of bonds and the denominator is just the return on a portfolio with share of capital being η_t. To derive equation 6, use the capital Euler equation of a household

$$\frac{1-\beta}{c^Y_t} = \beta\mathbb{E}_z\frac{R^k_{t+1}(z)}{c^O_{t+1}(z)}$$

where $c^O_{t+1}(z) = R^k_{t+1}(z)k_{t+1} + b_{t+1}$. Using the fact that $c^Y_t = \frac{1-\beta}{\beta}(k_{t+1} + b_{t+1}/R_t)$ and multiplying both sides of the Euler equation by k_{t+1} yields the expression (6) in the main text.

B Deriving an Expression for the Real Interest Rate

Using equations (4)-(5) we know that:

$$\frac{1-\eta_t}{\eta_t} = \frac{b_{t+1}}{R_t k_{t+1}} = \frac{\tilde{b}_{t+1}}{R_t}k^{\alpha-1}_{t+1}$$

where we used the definition of $\tilde{b}$ to go from the first to the second equality. Substituting out η_t using (14) and rearranging:

$$R_t = \frac{\mathbb{E}_z\left[\frac{z}{\alpha z+\tilde{b}_{t+1}}\right]}{\mathbb{E}_z\left[\frac{1}{\alpha z+\tilde{b}_{t+1}}\right]}\alpha k^{\alpha-1}_{t+1} = \frac{\mathbb{E}_z\left[\frac{z}{\alpha z+\tilde{b}_{t+1}}\right]}{\mathbb{E}_z\left[\frac{1}{\alpha z+\tilde{b}_{t+1}}\right]}\mathbb{E}_z R^k_{t+1}(z)$$

Rearranging we have equation (15). Notice that we can also write the expression for R_t as:

$$R_t = \left(\mathbb{E}_t\left[\frac{1}{\alpha z + \tilde{b}_{t+1}}\right]^{-1} - \tilde{b}_{t+1}\right)k^{\alpha-1}_{t+1}$$

Then since $\mathbb{E}_t\left[\frac{1}{\alpha z+\tilde{b}_{t+1}}\right]$ is increasing in σ (from Jensen's inequality), the whole expression is decreasing and thus, $\frac{\partial R_t}{\partial \sigma}<0$. Notice also that the inverse of the spread can be written as:

$$\frac{R_t}{\mathbb{E}_z R^k_{t+1}(z)} = \frac{1}{\alpha}\frac{1-\mathbb{E}_z\left[\frac{\tilde{b}_{t+1}}{\alpha z+\tilde{b}_{t+1}}\right]}{\mathbb{E}_z\left[\frac{1}{\alpha z+\tilde{b}_{t+1}}\right]} = \frac{1}{\alpha}\left[\mathbb{E}_z\left[\frac{1}{\alpha z+\tilde{b}_{t+1}}\right]^{-1}-\tilde{b}_{t+1}\right]$$

Next, from Jensen's inequality, we know that:

$$\frac{\partial\left(\frac{R_t}{\mathbb{R}^k_{t+1}(z)}\right)}{\partial \tilde{b}_{t+1}} = \frac{\mathbb{E}\left[\left(\frac{1}{\alpha z+\tilde{b}_{t+1}}\right)^2\right]-\left(\mathbb{E}\left[\frac{1}{\alpha z+\tilde{b}_{t+1}}\right]\right)^2}{\mathbb{E}\left[\frac{1}{\alpha z+\tilde{b}_{t+1}}\right]\mathbb{E}\left[\frac{1}{\alpha z+\tilde{b}_{t+1}}\right]}>0$$

C Proof of Lemma 4

The FOC for the choice of $\tilde{b}$ can be written as:

$$-\frac{1-\beta}{(1-\alpha-\tilde{b})k^\alpha-k}+\beta\mathbb{E}_z\left[\frac{1}{\left(\alpha z+\tilde{b}\right)k^\alpha}\right]\leq 0 \qquad \text{and} \qquad \tilde{b}\geq 0 \tag{41}$$

with at least one of the conditions holding with a strict equality. Also, note that the young household's bond Euler equation in equilibrium can be written as:

$$-\frac{1-\beta}{(1-\alpha-\tilde{b})k^\alpha-k}+\beta R\mathbb{E}_z\left[\frac{1}{\left(\alpha z+\tilde{b}\right)k^\alpha}\right]=0$$

Combining this equation with (41), we can write optimality as: $R\geq 1$, $\tilde{b}\geq 0$ and $(R-1)\tilde{b}=0$.

D Proof of Proposition 1

The problem of the steady state planner can be written as:

$$\mathcal{L}=\max_{k,\tilde{b}\geq 0}(1-\beta)\ln\left[(1-\alpha-\tilde{b})k^\alpha-k\right]+\beta\mathbb{E}_z\ln\left[(\alpha z+\tilde{b})k^\alpha\right]-\lambda^{ss}\left(k-s(\tilde{b},\tilde{b},\sigma)^{\frac{1}{1-\alpha}}\right)$$

The FOC for k can be written as:

$$\frac{\alpha}{k}-\frac{(1-\beta)(1-\alpha)}{(1-\alpha-\tilde{b})k^\alpha-k}-\lambda^{ss}=0 \tag{42}$$

The FOC for $\tilde{b}$ can be written as:

$$\frac{-(1-\beta)}{(1-\alpha-\tilde{b})k^\alpha-k}+\beta\mathbb{E}_z\left[\frac{1}{\left(\alpha z+\tilde{b}\right)k^\alpha}\right]+\frac{\lambda^{ss}s(\tilde{b},\tilde{b},\sigma)^{\frac{\alpha}{1-\alpha}}}{1-\alpha}\left[s_1(\tilde{b},\tilde{b},\sigma)+s_2(\tilde{b},\tilde{b},\sigma)\right] \leq 0 \tag{43}$$

$$\tilde{b} \geq 0 \tag{44}$$

To show that $\lambda^{ss} > 0$

Proof of (i) The objective function can also be written in terms of (k, b):

$$\mathbb{W} = \max_{\{k,b\}} (1-\beta) \ln \left[(1-\alpha) k^{\alpha} - b - k\right] + \beta \mathbb{E} \ln \left[\alpha z k^{\alpha} + b\right]$$

It is straightforward to see that $\mathbb{W}$ is concave in (k, b). Suppose $\sigma < \underline{\sigma}$ and evaluate the derivative of $\mathbb{W}$ at $(k_{\max}, 0)$ where $k_{\max} = s(0, 0, \sigma)^{\frac{1}{1-\alpha}}$:

$$\begin{aligned} \frac{\partial \mathbb{W}(k_{\max}, 0)}{\partial k} &= (1-\beta) \frac{\alpha(1-\alpha)k_{\max}^{\alpha-1} - 1}{(1-\alpha)k_{\max}^{\alpha} - k_{\max}} + \beta \frac{\alpha}{k_{\max}} \\ &= \frac{\alpha}{k_{\max}} \left[1 - \frac{\beta(1-\alpha)}{\alpha}\right] > 0 \end{aligned} \tag{45}$$

where the last inequality stems from Assumption 1. Similarly,

$$\begin{aligned} \frac{\partial \mathbb{W}(k_{\max}, 0)}{\partial b} &= -(1-\beta) \frac{1}{(1-\alpha)(1-\beta)k_{\max}^{\alpha}} + \beta \mathbb{E}_z \left[\frac{1}{\alpha z k_{\max}^{\alpha}}\right] \\ &= -\frac{1}{(1-\alpha)k_{\max}^{\alpha}} \left[1 - \frac{\beta(1-\alpha)}{\alpha} e^{\sigma^2}\right] < 0 \end{aligned} \tag{46}$$

where the last inequality holds since $\sigma < \underline{\sigma}$.

Next, take any feasible allocation where $b > 0$: it must feature $k < k_{\max}$. Since $\mathbb{W}(k, b)$ is concave,we have:

$$\begin{aligned} \mathbb{W}(k, b) &\leq \mathbb{W}\left(k_{\max}, 0\right) + \frac{\partial \mathbb{W}(k_{\max}, 0)}{\partial k}(k - k_{\max}) + \frac{\partial \mathbb{W}(k_{\max}, 0)}{\partial b} b \\ &< \mathbb{W}(k_{\max}, 0) \end{aligned}$$

Thus, $(k_{\max}, 0)$ must be optimal. Since $\sigma < \underline{\sigma}$ and $b = 0$, we know that $R(0, \sigma) > 1$.

Proof of (ii) Substituting the implementability constraint into $\mathbb{W}(k, b)$, we have

$$\mathbf{W}(\tilde{b}, \varepsilon) := \mathbb{W}\left(s(\tilde{b}, \tilde{b}, \underline{\sigma} + \varepsilon)^{\frac{1}{1-\alpha}}, \tilde{b} s(\tilde{b}, \tilde{b}, \underline{\sigma} + \varepsilon)^{\frac{\alpha}{1-\alpha}}\right)$$

for $\varepsilon > 0$. In order for $\tilde{b} > 0$ to be optimal given ε, we need $\mathbf{W}_b(\tilde{b}, \varepsilon) = 0$ and $\mathbf{W}(\tilde{b}, \varepsilon) \geq \mathbf{W}(0, \varepsilon)$. It is straightforward to show that there exists a function $\tilde{\mathbf{b}}(\varepsilon)$ such that for $\tilde{b} > \tilde{\mathbf{b}}(\varepsilon)$, $\mathbf{W}(\tilde{b}, \varepsilon) < \mathbf{W}(0, \varepsilon)$. Further, $\tilde{\mathbf{b}}(\varepsilon) \to 0$ as $\varepsilon \to 0$. Next, note that $\mathbf{W}_b(\tilde{b}, \varepsilon)$ is a continuous function and is strictly negative at $(0, 0)$. Thus, there exists (γ, δ) such that $\tilde{b} \in (0, \gamma)$, $\varepsilon \in (0, \delta)$ implies $\mathbf{W}_b(\tilde{b}, \varepsilon) < 0.5\mathbf{W}_b(0, 0) < 0$. Choose $\varepsilon_1 < \delta$ such that $\tilde{\mathbf{b}}(\varepsilon_1) < \gamma$. For all $\varepsilon \in (0, \varepsilon_1)$, we have $\mathbf{W}_b(\tilde{b}, \varepsilon) < 0$ for all $\tilde{b} \in (0, \tilde{\mathbf{b}}(\varepsilon))$. Thus, there are no interior optimum and $\tilde{b} = 0$ must be optimal in an open interval around $\underline{\sigma}$.

Proof of (iii) First, we show that for σ sufficiently large, the following expression is positive

$$\begin{aligned} d\mathbb{W}(k_{\max},0) &= \frac{\partial \mathbb{W}(k_{\max},0)}{\partial k}\frac{s(0,0,\sigma)^{\frac{\alpha}{1-\alpha}}}{1-\alpha}\Big[s_1(0,0,\sigma)+s_2(0,0,\sigma)\Big]+\frac{\partial \mathbb{W}(k_{\max},0)}{\partial \tilde{b}} \\ &= -\frac{1-\beta\,(1-\alpha)}{(1-\alpha)^2}+\left[\frac{\beta-\alpha}{\beta(1-\alpha)}+1-\beta\right]\frac{\beta}{\alpha}e^{\sigma^2} \end{aligned}$$

For large enough σ the second term overwhelms the first term making $d\mathbb{W}(k_{\max},0)>0$ if $\alpha<\beta$. In this case, there exists a finite $\bar{\sigma}$ such that as long as $\sigma>\bar{\sigma}$, it is optimal to create safe assets. If however α is large relative to β, and $\frac{\beta-\alpha}{\beta(1-\alpha)}+1-\beta<0$, then it may never be optimal to create safe assets for any level of σ since crowding out always dominates the benefits from insurance.

It remains to show that at the optimum whenever $\tilde{b}>0$, $R<1$. First, we show that we can never have an interior optimum with $\mathbb{W}_k\le 0$ and $\mathbb{W}_b<0$. Consider any point $(k_0,\tilde{b}_0)$ with $b_0>0$ s.t. $\mathbb{W}_k(k_0,\tilde{b}_0)\le 0$ and $\mathbb{W}_b(k_0,\tilde{b}_0)<0$. For any $\varepsilon>0$, define $k_\varepsilon=s(\tilde{b}_0-\varepsilon,\tilde{b}_0-\varepsilon,\sigma)<k_0$ as the steady state level of capital for $\tilde{b}_0-\varepsilon$. The gain in welfare from decreasing $\tilde{b}$ by ε is approximately $\mathbb{W}_k(k_0,\tilde{b}_0)(k_\varepsilon-k_0)+\mathbb{W}_b(k_0,\tilde{b}_0)\varepsilon$. For small ε, this gain is positive since $\mathbb{W}_k\le 0$, $k_\varepsilon<k_0$ and $\mathbb{W}_b>0$. So the initial point cannot be optimal. By a similar argument, we cannot have both $\mathbb{W}_b\le 0$ and $\mathbb{W}_k<0$ at an optimum. Finally, since $\mathbb{W}$ is concave and attains its maximum at $\tilde{b}=0$, $k>k_{max}$, we cannot have $\mathbb{W}_k=\mathbb{W}_b=0$ at any feasible point.

Take any interior optimal point. The first order necessary condition for optimality is

$$\mathbb{W}_b+\mathbb{W}_k\frac{\partial k}{\partial b}=0$$

If $\mathbb{W}_b\le 0$ at an optimum, then by the above arguments we must have $\mathbb{W}_k>0$, which contradicts the optimality condition. So at any interior optimum, we must have $\mathbb{W}_b>0$, which, again using the household's Euler equation for bonds, implies that $R<1$.

D.1 Relaxing the assumptions of log-utility and full depreciation

This section shows that the claims made in Proposition 1 generalize to a setting with homothetic time-separable utility functions and any depreciation rate $\delta\in[0,1]$. As is commonly known, for a single good, homothetic time-separable utility functions must take the form of CRRA utility functions, i.e. $u(c)=\frac{c^{1-\rho}}{1-\rho}$. Consider the young households first order condition for capital:

$$(1-\beta)\left[(1-\alpha)k^\alpha-k-b\right]^{-\rho}=\beta\mathbb{E}_z\left[\left(z\alpha k^{\alpha-1}+1-\delta\right)\left(zk^\alpha+(1-\delta)\,k+b\right)^{-\rho}\right]$$

Dividing by $k^{-\rho\alpha}$ and defining $\tilde{b}=b/k^\alpha$, we have

$$(1-\beta)\left[1-\alpha-k^{1-\alpha}-\tilde{b}\right]^{-\rho}=\beta\mathbb{E}_z\left[\left(z\alpha k^{\alpha-1}+1-\delta\right)\left(z\alpha+(1-\delta)k^{1-\alpha}+\tilde{b}\right)^{-\rho}\right]$$

Since the LHS is decreasing in k and $\tilde{b}$ while the RHS is increasing in both arguments, it is immediate that k is decreasing in $\tilde{b}$. Thus even with general CRRA preferences and less than full depreciation, increases in $\tilde{b}$ always crowd out capital, $dk/d\tilde{b}<0$. Next we show that this decreases welfare to first order when $R=1$. As in the main text, we assume that $R>1$ in the absence of risk. To find conditions under which this is the case, we use the young households' first order condition for the riskless bond:

$$(1-\beta)\left[(1-\alpha)k^{\alpha}-k-b\right]^{-\rho}=\beta R\mathbb{E}_z\left[(zk^{\alpha}+(1-\delta)\,k+b)^{-\rho}\right]$$

Evaluating both Euler equations at zero risk and imposing $R>1$, we have

$$\frac{\beta}{1-\beta}<\left[\frac{(1-\alpha)\,\delta-\alpha}{\alpha}\right]^{-\rho}$$

which is a generalization of Assumption 1 in the main text.[59] This assumption implies that if there is a sufficiently high level of risk in steady state, $R=1$ in the absence of government debt. If this is in fact the case, then subtracting the two Euler equations we have $\mathbb{E}_z\left\{\left[\alpha zk^{\alpha-1}-\delta\right]\left[\alpha zk^{\alpha}+(1-\delta)\,k\right]^{-\rho}\right\}=0$ which can be rewritten as:

$$\mathbb{E}_z\left[\alpha zk^{\alpha-1}-\delta\right]\mathbb{E}_z\left[\alpha zk^{\alpha}+(1-\delta)\,k\right]^{-\rho}+cov[\alpha zk^{\alpha-1}-\delta,\alpha zk^{\alpha}+(1-\delta)\quad=\quad 0$$

Since the covariance term is negative, it must be that $\alpha k^{\alpha-1}-\delta>0$ in steady state: even when safe rates are zero, risky capital earns a positive expected return. Next, the welfare of the representative cohort is:

$$\mathbb{W}=\frac{1-\beta}{1-\rho}\left[(1-\alpha)\,k^{\alpha}-k-\tilde{b}k^{\alpha}\right]^{1-\rho}+\frac{\beta}{1-\rho}\mathbb{E}_z\left[z\alpha k^{\alpha}+(1-\delta)\,k+\tilde{b}k^{\alpha}\right]^{1-\rho}$$

Taking derivatives with respect to k, evaluating at $\tilde{b}=0$ and using the capital Euler equation:

$$\left.\frac{d\mathbb{W}}{dk}\right|_{\tilde{b}=0}=(1-\alpha)\left\{(1-\beta)\left[(1-\alpha)\,k^{\alpha}-k\right]^{-\rho}\alpha k^{\alpha-1}-\beta\mathbb{E}_z\left[z\alpha k^{\alpha}+(1-\delta)\,k\right]^{-\rho}z\alpha k^{\alpha-1}\right\}$$

Using the Euler equation for bonds evaluated at $R=1$, we can rewrite this as

$$\left.\frac{d\mathbb{W}}{dk}\right|_{\tilde{b}=0}=(1-\alpha)\,(1-\beta)\left[(1-\alpha)\,k^{\alpha}-k\right]^{-\rho}\left[\alpha k^{\alpha-1}-\delta\right]>0$$

Thus the overall effect of a small increase in bonds on welfare is negative:

$$\begin{aligned}\left.\frac{d\mathbb{W}}{d\tilde{b}}\right|_{\tilde{b}=0}&=\left\{-(1-\beta)\left[(1-\alpha)\,k^{\alpha}-k-\tilde{b}k^{\alpha}\right]^{-\rho}+\beta\mathbb{E}_z\left[z\alpha k^{\alpha}+(1-\delta)\,k+\tilde{b}k^{\alpha}\right]^{-\rho}\right\}k^{\alpha}+\frac{d\mathbb{W}}{dk}\frac{dk}{d\tilde{b}}\\&=0+\frac{d\mathbb{W}}{dk}\frac{dk}{d\tilde{b}}<0\end{aligned}$$

where we again use the bond Euler equation with $R=1$. Thus, even with more general preferences and technology than in our benchmark model with log utility and full depreciation, the main result in Proposition 1 goes through: it is not optimal to create government debt even when risk is high enough to make real interest rates slightly negative.

[59] When $\rho=\delta=1$, this reduces to Assumption 1.

D.2 More general conditions under which safe assets crowd out capital and reduce welfare

The content of Proposition 1 holds even more generally. In this section, we prove Proposition 1 for general preferences and technology. We assume that a household maximizes some objective $E_z U(c_t^Y, c_t^O)$ subject to their budget constraints (1) and (2). U is a strictly concave function which is strictly increasing in both arguments. We assume that the production technology is neoclassical, represented by $f(k)$ where k denotes the capital labor ratio. Consequently, the real wage $w(k) = f(k) - f'(k)k$ is increasing in k and the average return on capital $\mathbb{E}_z R^k(z, k) = f'(k) + 1 - \delta$. Note that the average return of capital is decreasing in the quantity of capital: $\mathbb{E}_z \frac{\partial R^k(z,k)}{\partial k} = f''(k) < 0$. We also assume that $\frac{\partial R^k(z,k)}{\partial z} > 0$ and $\frac{\partial^2 R^k(z,k)}{\partial z \partial k} < 0$. These assumptions imply that the realized return on capital is increasing in realized productivity z and that a higher capital stock reduces the realized returns proportionally more for those those who draw high z. These assumptions are clearly satisfied in the case of purely multiplicative risk as in the paper. More generally, it holds more generally with less than full depreciation as in this case $R^k(z) = zf'(k) + 1 - \delta$.

Consider the problem of households who cannot trade the riskless bond; instead, they pay a transfer $b \geq 0$ when young and receive the same transfer when old. It is immediate that the allocations in this economy are the same as those in an economy where they can trade bonds and the government fixes the supply at b. For any preferences and technology, the date t solution to the young households choice of capital k_{t+1} can be expressed as some function:

$$k_{t+1} = \mathcal{S}(K_t, K_{t+1}, b)$$

where K_t denotes the aggregate capital at date t. Strict concavity of households preferences over consumption in both periods implies that $\partial \mathcal{S}/\partial b < 0$. Intuitively, higher b reduces marginal utility when old (and raises it when young), inducing households to invest less in capital holding aggregate variables constant. Similarly, provided that the production technology is such that higher capital stock increases wages, $\partial \mathcal{S}/\partial K_t > 0$, i.e, households respond to higher capital today by saving more. Given initial condition K_0, an equilibrium satisfies:

$$K_{t+1} = \mathcal{S}(K_t, K_{t+1}, b) \tag{47}$$

for all t and the steady state capital stock satisfies:

$$K^* = \mathcal{S}(K^*, K^*, b)$$

We call this steady state *stable* if there exists $\varepsilon > 0$ such that for all $K_0 \in (K^* - \varepsilon, K^* + \varepsilon)$, $K_t - K^*$ has the same sign as $K_0 - K^*$ and $\lim_{t\to\infty} K_t = K^*$, i.e. the economy returns monotonically to the original steady state K^* after a small perturbation.

Claim: In the neighborhood of any stable steady state, there is crowding out: $dK^*/db < 0$.

Proof: Notice that (47) implies that close to steady state, dynamics can be written as:

$$\frac{dK_{t+1}}{dK_t} = \frac{\mathcal{S}_1(K^*, K^*, b)}{1 - \mathcal{S}_2(K^*, K^*, b)}$$

where $\mathcal{S}_1$ and $\mathcal{S}_2$ denote the partial derivatives of the $\mathcal{S}$ function w.r.t the first and second arguments respectively. This steady state is stable if $\frac{dK_{t+1}}{dK_t} \in (0,1)$, i.e. $1 - \mathcal{S}_1(K^*, K^*, b) - \mathcal{S}_2(K^*, K^*, b) > 0$. Applying the implicit function theorem to the steady state, it is immediate that

$$\frac{dK^*}{db} = \frac{\mathcal{S}_b(K^*, K^*, b)}{1 - \mathcal{S}_1(K^*, K^*, b) - \mathcal{S}_2(K^*, K^*, b)} < 0$$

i.e., there is crowding out.

Next, we show that when risk is just high enough to make $R = 1$, increasing government debt strictly reduces the welfare of the representative cohort, which can be written as

$$\mathbb{W}(k, b) = \mathbb{E}_z \left[U\left(\omega(k) - k - b, R(k, z)k + b\right)\right]$$

The effect of an increase in debt is

$$\frac{d\mathbb{W}}{dk}\frac{dk}{db} = \underbrace{\mathbb{E}_z \left[-\frac{\partial U}{\partial c^Y} + \frac{\partial U}{\partial c^O} \right]}_{=0 \text{ since } R=1} + \frac{d\mathbb{W}}{dk} \underbrace{\frac{dk}{db}}_{<0}$$

Thus it suffices to show that the direct effect of a higher capital stock on welfare is positive. By the Envelope Theorem, the capital stock only affects welfare via factor prices:

$$\frac{d\mathbb{W}}{dk} = \mathbb{E}_z \left[\frac{\partial U}{\partial c^Y}\frac{\partial w}{\partial k} + \frac{\partial U}{\partial c^O}\frac{\partial R(z,k)}{\partial k}k \right]$$

Note that since the production function is neoclassical,

$$\frac{\partial w}{\partial k} = \frac{d}{dK}[f(k) - kf'(k)] = -f''(k)k = -\mathbb{E}_z \frac{\partial R(z,k)}{\partial k}k$$

Intuitively, since all income goes to capital or labor, an increase in the capital stock must raise labor income by exactly the same amount as it reduces average capital income. Thus, we have

$$\begin{aligned} \frac{d\mathbb{W}}{dk} &= -\mathbb{E}_z \left[\frac{\partial U}{\partial c^Y}\right] \mathbb{E}_z \left[\frac{\partial R(z,k)}{\partial k}k\right] + \mathbb{E}_z \left[\frac{\partial U}{\partial c^O}\frac{\partial R(z,k)}{\partial k}\right] \\ &> -\mathbb{E}_z \left[\frac{\partial U}{\partial c^O}\right] \mathbb{E}_z \left[\frac{\partial R(z,k)}{\partial k}k\right] + \mathbb{E}_z \left[\frac{\partial U}{\partial c^O}\right] \mathbb{E}_z \left[\frac{\partial R(z,k)}{\partial k}k\right] = 0 \end{aligned}$$

where we use the fact that $c^O(z)$ is increasing in z and $\frac{\partial U}{\partial c^O}$ is decreasing in $c^O(z)$, while $\frac{\partial R(z,k)}{\partial k}$ is decreasing in z, implying that the covariance between $\frac{\partial U}{\partial c^O}$ and $\frac{\partial R(z,k)}{\partial k}$ is positive. On average, higher capital stock decreases the income of old agents who own capital, and increase the income of young agents by the same amount. Since $R = 1$, the expected marginal utility of old and young agents is equal, so this redistribution from old to young would not change welfare if the loss in capital income was borne by all old agents equally. However, since in fact capital income falls relatively more for those with a low marginal utility, the loss in expected utility of the old is smaller than the gain in utility of the young. In other words, a higher capital stock increases the share of safe income and reduces the share of risky income, increasing welfare in this incomplete markets economy.

E Constrained efficiency of zero debt

The ex-ante welfare of cohort t, given an allocation $\{k_t, b_t\}_{t=0}^{\infty}$, is

$$U_t = (1-\beta)\ln((1-\alpha)k_t^{\alpha} - k_{t+1} - b_t) + \beta\mathbb{E}_z \ln(\alpha z k_{t+1}^{\alpha} + b_{t+1}).$$

We consider a Ramsey planner who solves

$$\mathbb{U}(\phi) = \max_{\{k_{t+1}, b_{t+1}\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty} \phi_t U_t + \phi_{-1}\mathbb{E}_z \ln c_0^O(z) \tag{48}$$

subject to:

$$k_{t+1} = s(\tilde{b}_t, \tilde{b}_{t+1}, \sigma)k_t^{\alpha} \quad , \quad \tilde{b}_t = \frac{b_t}{k_t^{\alpha}} \quad \text{and } k_0, b_0 \text{ given}$$

In the spirit of Negishi (1960), we call an allocation $\{k_t, b_t\}_{t=0}^{\infty}$ *constrained efficient* if it solves (48) for some sequence of Pareto weights $\{\phi_t\}$ with $\sum_{t=0}^{\infty}\phi_t < \infty$ with each $\phi_t \geq 0$ and at least one $\phi_t > 0$. The following Lemma characterizes conditions under which zero debt issuance is constrained efficient in this sense.

Lemma 6. *There exists $\sigma^{\diamond} > \underline{\sigma}$ such that, if $\sigma < \sigma^{\diamond}$, it is constrained efficient to choose $\tilde{b}_t = 0$ for all t.*

Proof. Define $\sigma^{\diamond} = \sqrt{\ln\left[\frac{\alpha}{(\beta-\alpha)(1-\alpha)}\right]}$ and $k_{\max}^{1-\alpha} = s(0,0,\sigma)$ We begin by showing that for all $\sigma \in [0, \sigma^{\diamond}]$, there exists at least one sequence of non-negative Pareto weights $\{\phi_i\}_{i=0}^{\infty}$ which satisfies absolute summability for which $k_t = k_{\max}$ and $\tilde{b}_t = 0$ for all $t \geq 0$ solve (48), while for $\sigma > \sigma^{\diamond}$, there is no sequence which of Pareto weights for which $(k_{\max}, 0)$ solves this problem.

Plugging the constraints into the objective function and rearranging yields:

$$\mathbb{U}(\phi) = \max_{\{\tilde{b}_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \phi_t \left\{(1-\beta)\ln\left[\left(1-\alpha-\tilde{b}_t\right) - s\left(\tilde{b}_t, \tilde{b}_{t+1}\right)\right] + \beta\mathbb{E}_z \ln\left[\alpha z + \tilde{b}_{t+1}\right]\right\}$$

$$+\phi_{-1}\beta\mathbb{E}_z \ln\left[\alpha z + \tilde{b}_0\right] + \sum_{t=0}^{\infty} \ln s\left(\tilde{b}_t, \tilde{b}_{t+1}\right)\left(\phi_t\beta + \sum_{j=t+1}^{\infty}\phi_j\alpha^{j-t}\right) + \text{constants independent of } \tilde{b}$$

The FOC is given by:

$$\phi_{t-1}\left\{(1-\beta)\frac{-s_2\left(\tilde{b}_{t-1}, \tilde{b}_t\right)}{\left(1-\alpha-\tilde{b}_{t-1}\right) - s\left(\tilde{b}_{t-1}, \tilde{b}_t\right)} + \beta\mathbb{E}_z\left[\frac{1}{\alpha z + \tilde{b}_t}\right]\right\} + \frac{s_2\left(\tilde{b}_{t-1}, \tilde{b}_t\right)}{s\left(\tilde{b}_{t-1}, \tilde{b}_t\right)}\left(\phi_{t-1}\beta + \sum_{j=t}^{\infty}\phi_j\alpha^{j-t+1}\right)$$

$$+\phi_t\left\{(1-\beta)\frac{-1-s_1\left(\tilde{b}_t, \tilde{b}_{t+1}\right)}{\left(1-\alpha-\tilde{b}_t\right) - s\left(\tilde{b}_t, \tilde{b}_{t+1}\right)}\right\} + \frac{s_1\left(\tilde{b}_t, \tilde{b}_{t+1}\right)}{s\left(\tilde{b}_t, \tilde{b}_{t+1}\right)}\left(\phi_t\beta + \sum_{j=t+1}^{\infty}\phi_j\alpha^{j-t}\right) \leq 0 \tag{49}$$

where

$$s_1\left(\tilde{b}_t, \tilde{b}_{t+1}\right) = \frac{\partial s\left(\tilde{b}_t, \tilde{b}_{t+1}\right)}{\partial \tilde{b}_t} = \frac{-\beta}{\beta + (1-\beta)\left(\mathbb{E}_z\left[\frac{\alpha z}{\alpha z + \tilde{b}_{t+1}}\right]\right)^{-1}}$$

and

$$s_2\left(\tilde{b}_t,\tilde{b}_{t+1}\right)=\frac{\partial s\left(\tilde{b}_t,\tilde{b}_{t+1}\right)}{\partial\tilde{b}_{t+1}}=-\frac{\beta\left(1-\alpha-\tilde{b}_t\right)}{\beta+(1-\beta)\left(\mathbb{E}_z\left[\frac{\alpha z}{\alpha z+\tilde{b}_{t+1}}\right]\right)^{-1}}\frac{(1-\beta)\left(\mathbb{E}_z\left[\frac{\alpha z}{\alpha z+\tilde{b}_{t+1}}\right]\right)^{-2}\mathbb{E}_z\left[\frac{\alpha z}{\left(\alpha z+\tilde{b}_{t+1}\right)^2}\right]}{\beta+(1-\beta)\left(\mathbb{E}_z\left[\frac{\alpha z}{\alpha z+\tilde{b}_{t+1}}\right]\right)^{-1}}$$

Evaluating (49) at $\tilde{b}_t=0$ for all $t\geq 0$ and rearranging yields:

$$\phi_{t-1}\frac{\beta(1-\alpha)e^{\sigma^2}}{1+(1-\alpha)(1-\beta)e^{\sigma^2}}\leq\alpha\sum_{s=0}^{\infty}\alpha^s\phi_{t+s}\tag{50}$$

Define $y_t=\sum_{s=0}^{\infty}\alpha^s\phi_{t+s}\in[0,\infty)$. So, $\phi_{t-1}=\alpha\left(\frac{1}{\alpha}y_{t-1}-y_t\right)$. Using these definitions, (50) can be written as:

$$\frac{\beta}{\alpha}\left[\frac{(1-\alpha)e^{\sigma^2}}{1+(1-\alpha)e^{\sigma^2}}\right]y_{t-1}\leq y_t$$

Since $y_t<\infty$ for any $\{\phi_s\}$ which satisfies absolute-summability[60], such a sequence $\{y_t\}$ exists iff

$$\frac{\beta}{\alpha}\left[\frac{(1-\alpha)e^{\sigma^2}}{1+(1-\alpha)e^{\sigma^2}}\right]<1$$

which holds as long as $\sigma<\sigma^{\diamond}$. Conversely, if $\sigma>\sigma^{\diamond}$, the above expression is strictly greater than one and no absolutely-summable positive sequence $\{\phi_t\}$ exists which satisfies (50). Finally, as is standard following Negishi (1960), an allocation is constrained efficient iff there exists Pareto weights $\{\phi_t\}$ such that the allocation solves the problem in (48) . So we are done. □

F Inefficiently low capital accumulation

Here we show that the allocation $(k_t,b_t)=(k_{\max},0)$ for all t, where $k_{\max}^{1-\alpha}=s(0,0,\sigma)$, features an inefficiently low level of capital, from the perspective of a social planner who can choose k_t and b_t without respecting individual savings decisions (but cannot redistribute within a generation). Starting from this allocation, consider a deviation which increases k_t by ε and increases b_t by δ for every $t>0$. We want to find (ε,δ) such that this deviation makes each cohort weakly better off. The effect of such a perturbation on the welfare of cohorts 0 and $t>0$ can be written as:

$$\begin{aligned}d\mathbb{W}_0&=-\left(\frac{1-\beta}{c^Y}\right)\varepsilon+\beta\mathbb{E}_z\left(\frac{\alpha^2zk^{\alpha-1}\varepsilon+\delta}{\alpha zk^{\alpha}}\right)\\ d\mathbb{W}_t&=\left(\frac{1-\beta}{c^Y}\right)\left[-\delta-\varepsilon+\alpha(1-\alpha)k^{\alpha-1}\varepsilon\right]+\beta\mathbb{E}_z\left(\frac{\alpha^2zk^{\alpha-1}\varepsilon+\delta}{\alpha zk^{\alpha}}\right)\end{aligned}$$

[60] y_t is the discounted sum of a absolutely-summable sequence and hence must be finite.

It is straightforward to show that $d\mathbb{W}_0 > 0$ if $\delta e^{\sigma^2} > \frac{\alpha}{\beta}\varepsilon$. Similarly, $d\mathbb{W}_t - d\mathbb{W}_0 > 0$ if $\frac{\alpha}{\beta}\varepsilon > \delta$, which implies that $d\mathbb{W}_t > 0$ if $d\mathbb{W}_0 > 0$. Thus, any sufficiently small (ε, δ) which satisfy:

$$\frac{\alpha}{\beta}\varepsilon = \left(\frac{1+e^{\sigma^2}}{2}\right)\delta$$

strictly increases welfare for all cohorts. Thus, the original allocation featured underaccumulation of capital. The deviation described here is not attainable in equilibrium for any debt policy: the original allocation already featured the highest possible level of capital attainable in equilibrium, namely $k_{\max}$.

G Proof of Proposition 2

The first claim follows from our analysis in section 4.1 which shows that when $R(\tilde{b}_L, \sigma_H) < 1/\Pi^\star$, then no full employment steady state can exist. To see why the second claim is true, first, we establish that the unemployment steady state is unique for γ sufficiently small. After that we construct an equilibrium in which $i_t = 0$ for all $t \geq 0$ and show that it is unique.

Steady states are characterized by:

$$\Pi^\star R\left(\tilde{b}_L l^{\alpha-1}, \sigma_H\right) = l^{-\frac{\alpha\gamma}{1-\gamma}}$$

When $\gamma = 0$, this equation has a unique solution. It follows immediately that for γ sufficiently close to zero, the steady state remains unique. Equilibrium with $i_t = 0$ for all $t \geq 0$ must satisfy the following conditions:

$$\begin{aligned}
k_{t+1} + (1-\beta)\frac{\tilde{b}_L}{R_t}k_{t+1}^\alpha &= \beta\left[(1-\alpha)k_t^\alpha l_t^{1-\alpha} - \tilde{b}_L k_t^\alpha\right] && (51)\\
\Pi_{t+1}^{-1} = R_t &= \frac{1}{g\left(\tilde{b}_L l_{t+1}^{\alpha-1}, \sigma_H\right)}\alpha\left(\frac{k_{t+1}}{l_{t+1}}\right)^{\alpha-1} && (52)\\
l_t &= \min\left\{\left(\frac{k_t}{k_{t-1}}\right)^{1-\gamma} l_{t-1}^{1-\gamma}\left(\frac{\Pi_t}{\Pi^\star}\right)^{\frac{1-\gamma}{\alpha}}, 1\right\} && (53)\\
\left(\frac{\Pi_t}{\Pi^\star}\right) l_t^{(1-\alpha)\psi} &\leq 1 && (54)
\end{aligned}$$

We proceed by assuming that (54) is satisfied with a strict inequality and that (53) holds with $l_t < 1$ for all t. Plug in (52) into (51):

$$\left[1 + \left(\frac{1-\beta}{\alpha}\right)\tilde{b}_L l_{t+1}^{\alpha-1} g(\tilde{b}_L l_{t+1}^{\alpha-1}, \sigma_H)\right]\frac{k_{t+1}}{k_t^\alpha} = \beta\left[(1-\alpha)l_t^{1-\alpha} - \tilde{b}_L\right]$$

Similarly, using (53) and (52):

$$\frac{k_{t+1}}{k_t^\alpha} = \alpha\frac{l_{t+1}^{\frac{1-\gamma(1-\alpha)}{1-\gamma}}}{l_t^\alpha}\frac{\Pi^\star}{g\left(\tilde{b}_L l_{t+1}^{\alpha-1}, \sigma_H\right)} \quad (55)$$

Substitute the second equation into the first to get:

$$\begin{aligned}\Pi^\star\left[\frac{\alpha}{g\left(\tilde{b}_L l_{t+1}^{\alpha-1},\sigma_H\right)}+(1-\beta)\,\tilde{b}_L l_{t+1}^{\alpha-1}\right] l_{t+1}^{\frac{1-\gamma(1-\alpha)}{1-\gamma}} &= \beta\left[(1-\alpha)l_t-\tilde{b}_L l_t^{\alpha}\right] &(56)\\ LHS(l_{t+1}) &= RHS(l_t) &(57)\end{aligned}$$

It is easy to see that $LHS(l)$ is increasing and nonnegative, while $RHS(l)$ is negative for $l_t < l_{min} = \left(\frac{1-\alpha}{\tilde{b}_L}\right)^{\frac{1}{1-\alpha}}$, and positive and increasing after that. Furthermore, for γ sufficiently close to 0, the two curves have a unique intersection in $(0,1)$, as we now show. First let $\gamma = 0$. Then after some algebra one can show that intersections of the two curves satisfy:

$$\frac{\Pi^\star}{\beta} = \mathbb{E}\left[\frac{1-\alpha+(\Pi^\star-1)\,\tilde{b}_L l^{\alpha-1}}{\alpha z+\tilde{b}_L l^{\alpha-1}}\right]$$

A sufficient condition for the derivative of the RHS with respect to l to be positive is that $\Pi^\star \leq \frac{1}{\alpha}$:

$$\begin{aligned}\frac{\partial}{\partial l^{\alpha-1}}\left\{\mathbb{E}\left[\frac{1-\alpha+(\Pi^\star-1)\,\tilde{b}_L l^{\alpha-1}}{\alpha z+\tilde{b}_L l^{\alpha-1}}\right]\right\} &= \mathbb{E}\left[\frac{\alpha z\,(\Pi^\star-1)-(1-\alpha)}{\left[\alpha z+\tilde{b}_L l^{\alpha-1}\right]^2}\right]\tilde{b}_L\\ &\leq (\alpha\Pi^\star-1)\,\mathbb{E}\left[\frac{1}{\left[\alpha z+\tilde{b}_L l^{\alpha-1}\right]^2}\right]\tilde{b}_L\end{aligned}$$

Thus, the solution for l is unique. Again, by continuity it follows that the solution is also unique for γ sufficiently close to 0.

It follows that at the unique intersection l^*, RHS cuts LHS from above, i.e. $RHS'(l^*) < LHS'(l^*)$. Thus if $l_0 < l^*$, $LHS(l_1) = RHS(l_0)$ implies $l_1 < l_0$, and so forth: $\{l_t\}$ is monotonically decreasing. The sequence cannot converge to any positive number: if it did converge, that limit would be another steady state, a contradiction. So eventually we must have $l_t < l_{min}$, which cannot be an equilibrium. By a similar argument, if $l_1 > l_0$, we must eventually have $l_t > 1$, which contradicts our assumption that the ZLB binds in every period. Thus the unique equilibrium with $i_t = 0$ features $l_t = l^*$ in every period. It is straightforward to construct the rest of the equilibrium setting $l = l^*$. Iterating forwards on equation (55) delivers the dynamics of capital. Imposing $l_t = l^*$ in (55) for all $t \geq 0$ reveals that the path for capital is monotonically declining towards the new steady state. Plugging these into equation (52) we obtain a sequence of inflation rates. Finally since $l_t = l^* < 1$ for all $t \geq 0$, (54) is always satisfied for high enough ψ.

Why do we need to impose that ψ is large enough?. If the economy is hit with a large enough shock, the real return on bonds may actually be negative at date zero, as the economy's capital stock is far above its new steady state level. This in turn requires positive inflation in the short run, even though the economy will eventually arrive at a deflationary steady state. If ψ is too small, the monetary authority might be unwilling to keep rates at zero early on in the transition if the economy experiences positive inflation. In this case, no equilibrium exists, given our specification of fiscal and monetary policy. The economy desperately requires at least a few periods of negative real rates to smooth the transition to the new steady state, since capital is high in the short run, depressing interest rates even beyond the effect of the increase in risk. A monetary rule which will not accommodate temporarily negative real interest rates cannot even engineer a transition to a steady state with deflation and unemployment. Instead, employment spirals towards zero, eventually leaving the government unable to meet its fiscal obligations and such an equilibrium cannot exist. If ψ is sufficiently large, monetary policy is willing to tolerate short run inflation since output is below potential. In this case the equilibrium is described in the Proposition.

H Proof of Proposition 3

The problem of the steady state planner can be written as:

$$\mathbb{W} = \max_{k,\tilde{b},l,R,\Pi} (1-\beta)\ln\left[(1-\alpha)k^{\alpha}l^{1-\alpha} - \tilde{b}k^{\alpha} - k\right] + \beta\mathbb{E}_z \ln\left[\alpha z k^{\alpha}l^{1-\alpha} + \tilde{b}k^{\alpha}\right] \tag{58}$$

s.t.

$$\begin{aligned} \frac{k}{l} &= k\left(\tilde{b}l^{\alpha-1},\sigma\right) && (59)\\ R &= R\left(\tilde{b}l^{\alpha-1},\sigma\right) && (60)\\ \Pi &= \Pi^{\star}l^{\frac{\alpha\gamma}{1-\gamma}} && (61)\\ \left(\frac{\Pi}{\Pi^{\star}}\right)l^{(1-\alpha)\psi} &\leq 1 && (62)\\ R\Pi &\geq 1 && (63) \end{aligned}$$

$$(R\Pi-1)\left[\left(\frac{\Pi}{\Pi^{\star}}\right)l^{(1-\alpha)\psi}-1\right] = 0 \tag{64}$$

We begin by showing that the optimal choice always features full employment, $l=1$. Take any putative solution $(k_*, l_*, \tilde{b}_*, R_*, \Pi_*)$ which features $l_* < 1$. Now consider a deviation in which $k' = \frac{k_*}{l_*}$, $l' = 1$ and $\tilde{b}' = \frac{\tilde{b}_*}{l_*^{1-\alpha}}$. Note that $(k', l', \tilde{b}')$ still satisfy (59)-(60) with the same $R\left(\tilde{b}',\sigma\right) = R\left(\tilde{b}_* l_*^{\alpha-1},\sigma\right) = R_*$ and generate a higher level of inflation from (61). Since $l' = 1$, (62) is satisfied with equality and $\Pi' = \Pi^{\star}$. Since $\Pi' > \Pi_*$ and $R' = R_*$, (63) and (64) is satisfied. Thus, $(k', l', \tilde{b}', R', \Pi')$ is feasible if $(k_*, l_*, \tilde{b}_*, R_*, \Pi_*)$ is feasible. It is straightforward to check that this deviation increases social welfare by $-\ln l_* > 0$. Thus, in any solution to this problem we must have $l=1$. As a result we can rewrite the problem as:

$$\mathbb{W} = \max_{k,\tilde{b}}(1-\beta)\ln\left[(1-\alpha)k^{\alpha} - \tilde{b}k^{\alpha} - k\right] + \beta\mathbb{E}_z \ln\left[\alpha z k^{\alpha} + \tilde{b}k^{\alpha}\right]$$

s.t.

$$\begin{aligned} k &= k\left(\tilde{b},\sigma\right)\\ R\left(\tilde{b},\sigma\right)\Pi^{\star} &\geq 1 \end{aligned}$$

This problem is identical to the problem in Proposition 1 except for the ZLB constraint which essentially puts a lower bound on $\tilde{b}$. This lower bound can be written as:

$$\tilde{b} > \tilde{b}_{\text{zlb}}\left(\sigma, \Pi^{\star}\right)$$

where $\tilde{b}_{\text{zlb}}\left(\sigma, \Pi^{\star}\right)$ is defined as the level of $\tilde{b}$ such that $R\left(\tilde{b}_{\text{zlb}}\left(\sigma, \Pi^{\star}\right), \sigma\right)\Pi^{\star} = 1$. Since the problem has a strictly concave objective, the result follows that the optimal $\tilde{b}$ satisfies:

$$\tilde{b} = \max\{\tilde{b}_{\text{zlb}}(\sigma, \Pi^{\star}), \tilde{b}_{\text{real}}(\sigma)\}$$

where $\tilde{b}_{\text{real}}(\sigma)$ denotes the optimal $\tilde{b}$ which solves the problem in Proposition 1 given σ. In particular if $\Pi^{\star} = 1$, then the level of $\tilde{b}$ required to ensure full employment is such that $R\left(\tilde{b}, \sigma\right) = 1$. From Proposition 1, we know that this level of $\tilde{b}$ is strictly higher than the optimal level absent nominal rigidities, i.e. $\tilde{b}_{\text{zlb}}(\sigma, \Pi^{\star}) > \tilde{b}_{\text{real}}(\sigma)$ whenever $\tilde{b}_{\text{zlb}}(\sigma, \Pi^{\star}) > 0$.

I Foundations for incomplete markets

In the paper we assumed that capital income risk faced by each household is non-diversifiable. Here, we show that unobservable capital quality can micro-found the incompleteness of markets assumed in the main text. Suppose a household i invests k_{t+1} when young in physical capital and draws productivity shock z_i when old. This productivity is embodied in the units of capital that household i possesses when old. That is, even if another household j with a different productivity z_j were to operate the capital produced by i, that capital would continue to have productivity $z_i$even in the hands of households j. We assume that household i cannot directly observe the realization of z_j for $j \neq i$.

No trade in spot markets for capital Suppose old household i perceives that it can buy or sell capital after the realization of z_i at a price q^k. Then the problem of the firm operated by old household i's can be written as:

$$R^k(z)\,k = \max\left[z\left(k - k_s(z)\right) + \tilde{z}k_b(z)\right]^{\alpha} \ell(z)^{1-\alpha} - \omega\ell(z) - q^k\left(k_b - k_s\right)$$

subject to:

$$\begin{aligned} k_s(z) &\in [0, k] \\ k_b(z) &\geq 0 \end{aligned}$$

where k denotes the amount of capital household i had invested in when young, $k_s(z) \in [0, k]$ is the amount of capital that household i chooses to sell and $k_b(z)$ is the amount of capital chooses to buy. $\widetilde{z}$ denotes the average quality of capital being sold in equilibrium and is given by:

$$\widetilde{z} = \frac{\int_0^{\infty} z k_s(z) dF(z)}{\int_0^{\infty} k_s(z) dF(z)}$$

as long as the denominator is positive (i.e. there is some capital being sold) and zero otherwise (if no capital is sold). It is straightforward to see that given the a firm's optimal labor demand, this problem can be re-written as:[61]

$$R^{k}\left(z\right)k=\max_{k_{s}(z)\leq k,k_{b}(z)\geq 0}\alpha\left(\frac{1-\alpha}{\omega}\right)^{\frac{1-\alpha}{\alpha}}\left[z\left(k-k_{s}\right)+\tilde{z}k_{b}\right]-q^{k}\left(k_{b}-k_{s}\right)$$

which is linear in k_s and k_b implying that all firms who sell their capital are those with productivity:

$$z\leq\left(\frac{\omega}{1-\alpha}\right)^{\frac{1-\alpha}{\alpha}}\frac{q^{k}}{\alpha}\tag{65}$$

In addition, firms are willing to buy any amount of capital as long as

$$\alpha\left(\frac{1-\alpha}{\omega}\right)^{\frac{1-\alpha}{\alpha}}\tilde{z}\geq q^{k}\tag{66}$$

and demand an infinite amount of capital if the inequality is strict. Thus, if there is any trade in this spot market, equation (66) must hold with a strict equality. Plugging in the expression for q^k into (65) yields $z_i \leq \tilde{z}$ for all sellers. However, this cannot be the case given the definition of $\widetilde{z}$. Thus, there is no trade is such a spot market and hence old households cannot insure themselves against low realizations of z through such a spot market.

No trade in Arrow securities contingent on productivity realizations In the main text, we had also made the assumption that households could not insure themselves against low realizations of z by buying Arrow securities (when young) which pay off after such realizations. Since the actual realization of z_i is not publicly observable, the Arrow securities must payoff based on the profile of reports which we denote by $\hat{\mathbf{z}} := (\hat{z}_i, \hat{\mathbf{z}}_{-i})$ where $\hat{z}_i$ denotes the report by household i and $\hat{\mathbf{z}}_{-i}$ denotes the profile of all other household's reports.

Given each household's purchases of Arrow securities when young, and given everyones realization of productivity, all old households of a given generation play the following message game: each household announces $\hat{z}_i$ in order to maximize:

$$c^{O}(z_i)=R^{k}(z_i)k_i+b_i+a_i(\hat{z}_i,\hat{\mathbf{z}}_{-i})$$

Observe that household i's best response correspondence does not depend on the actual realization of z_i: i always reports whichever state maximizes the net transfers from the rest of the households to her. So do all other households. Thus, the Nash equilibrium of the message game does not depend on the true state of the world, and each household merely receives a constant transfer. Finally, it is easy to see that these transfers must be zero. Since the transfers must sum to zero, positive transfers for some households must be balanced by negative transfers from others. The households receiving negative transfers would prefer to deviate by not participating in these markets at all. Thus, Arrow securities cannot provide households insurance against low realizations of z.

[61] The labor demand conditional on k_s and k_b can be derived as:

$$\ell(z)=\left(\frac{1-\alpha}{\omega}\right)^{\frac{1}{\alpha}}\left\{z\left(k-k_{s}(z)\right)+\tilde{z}k_{b}(z)\right\}$$

J Liquidity as a driver of safe asset demand

Thus far, we have considered an economy in which capital earns a risk premium over safe government debt because capital bears idiosyncratic risk. Assets such as government debt may be valued not just for their safety but also for their liquidity or 'moneyness'. Empirically, KVJ document that the premium between US Treasury yields and comparable private assets contains both a liquidity and a safety component. Further, both liquidity and safety premia are affected by changes in the supply of public safe assets. One might wonder how our conclusions change when government debt provides liquidity as well as safety. Following much of the recent literature (KVJ, Angeletos et al. (2016)) we augment our model by assuming that households derive utility directly from holdings of government debt.Preferences are now:

$$(1-\beta)\ln c_t^Y + \beta\mathbb{E}_z \ln\left[c_{t+1}^O(z) + v\left(\frac{b_{t+1}}{\bar{k}_{t+1}^{\alpha}}\right)\bar{k}_{t+1}^{\alpha}\right]$$

where $v' \geq 0$, $v'' \leq 0$, $v(0) = 0$, and $\bar{k}_{t+1}$ denotes aggregate capital, taken as given by the household. Here $v\left(\frac{b}{k^{\alpha}}\right)k^{\alpha}$ represents the 'convenience' benefits provided by government debt. Following KVJ,the convenience function is increasing in the debt-to-GDP ratio, with the marginal benefit decreasing in debt.

Now that government debt provides liquidity services, the premium earned by capital relative to bonds reflects both a liquidity and a safety component. The demand for capital is now

$$\alpha k_{t+1}^{\alpha-1} = \underbrace{g\left(\tilde{b}_{t+1} + v\left(\tilde{b}_{t+1}\right), \sigma\right)}_{\text{safety premium}} \underbrace{\left(1 + v'\left(\tilde{b}_t\right)\right)}_{\text{liquidity premium}} R_t$$

with g defined as above. If $v(\cdot) = 0$, there is no liquidity premium, and the model collapses to the one studied above. Both the liquidity and safety premia are decreasing in the supply of safe assets $\tilde{b}$. Consequently, steady state comparative statics are qualitatively the same as those in an economy where government debt does not provide liquidity services. In particular, when bonds are in zero net supply, the steady state natural rate is

$$R = \frac{\alpha e^{-\sigma^2}}{\beta(1-\alpha)(1+v'(0))}$$

The natural rate can be depressed both by higher risk σ, and by a higher demand for liquidity - measured as the marginal convenience benefit of safe assets when safe assets are in zero supply. If risk increases, eventually the natural rate becomes negative, absent an increase in the supply of safe assets.[62] The only difference is that now that debt provides liquidity services, it takes less risk to generate a negative natural rate. Safe asset creation can push real rates back into positive territory but crowds out capital. Thus, as in Proposition 1, absent nominal rigidities, it is not necessarily optimal to produce safe assets even if real rates are negative - and it is never desirable to produce enough safe assets that real rates become positive.

To be clear, if government debt provides liquidity services, this does make it more socially desirable to produce government debt. However, liquidity services also reduce real rates for a given level of risk. Thus our previous characterization remains accurate: negative real rates are a necessary but *not* sufficient condition for safe asset creation to be desirable. In the presence of nominal rigidities and a binding ZLB,

[62] We have implicitly assumed that absent risk and with $\tilde{b} = 0$, the natural rate is positive, i.e. $\frac{\beta(1-\alpha)}{\alpha}(1+v'(0)) < 1$.

however, if the demand for safety and liquidity pushes an economy into a deep recession, safe asset creation may be necessary to restore full employment. Overall, introducing a liquidity motive for holding safe assets does not qualitatively change the positive or normative conclusions arising from our analysis.

K Proof of the optimality of $G = 0$

It is straightforward to show that with $G_t > 0$, the economy's evolution is described by:

$$k_{t+1} = s(\tilde{b}_t + \tilde{\mathcal{G}}_t, \tilde{b}_{t+1}, \sigma)k_t^\alpha \text{ where } \tilde{\mathcal{G}}_t := \frac{G_t}{k_t^\alpha}$$

Thus in steady state

$$\alpha k^{\alpha-1} = \frac{\alpha}{s(\tilde{b} + \tilde{\mathcal{G}}, \tilde{b})} = g(\tilde{b}, \sigma)R$$

Since s is decreasing in its first argument, an increase in $\mathcal{G}$ raises the natural rate R, but reduces steady state capital.

The planner's problem can be written

$$\mathbb{W} = \max_{k,\tilde{b},l,R,\Pi} (1-\beta)\ln\left[(1-\alpha)k^\alpha l^{1-\alpha} - \tilde{b}k^\alpha - \tilde{g}k^\alpha - k\right] + \beta\mathbb{E}_z \ln\left[\alpha z k^\alpha l^{1-\alpha} + \tilde{b}k^\alpha\right] \tag{67}$$

s.t. (61), (62), (63), (64), and

$$k = s((\tilde{b} + \tilde{g})l^{\alpha-1}, \tilde{b}l^{\alpha-1}, \sigma)k^\alpha l^{1-\alpha} \tag{68}$$

$$\alpha k^{\alpha-1} = g(\tilde{b}, \sigma)R \tag{69}$$

The same argument as in Proposition 3 implies that it is always optimal to choose $l = 1$. Then suppose by contradiction that $k_0, \tilde{b}_0, \tilde{g}_0$ is optimal with $\tilde{g}_0 > 0$ an optimum. Consider a deviation $k_1, \tilde{b}_1, \tilde{g}_1$ where $\tilde{g}_1 = 0$ and

$$\begin{aligned} k_1^{1-\alpha} &= s(\tilde{b}_1, \tilde{b}_1, \sigma) \\ \frac{\alpha k_1^{\alpha-1}}{g(\tilde{b}_1, \sigma)} &= \frac{\alpha k_0^{\alpha-1}}{g(\tilde{b}_0, \sigma)} \end{aligned}$$

This deviation increases the capital stock, i.e. $k_1 > k_0$, since it increases debt $\tilde{b}_1 > \tilde{b}_0$, reduces the spread, and so increases the level of capital consistent with maintaining the same real interest rate. It also decreases the net transfers from young households: $\tilde{b}_1 < \tilde{b}_0 + \tilde{g}_0$. Finally, it increases transfers to old households, i.e. safe assets. Since welfare is increasing in capital, this deviation increases welfare over all, contradicting the assumption that the original allocation with $\tilde{g} > 0$ was optimal.