Monetary Policy Frameworks and the Effective Lower Bound on Interest Rates

Thomas M. Mertens
John C. Williams

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Abstract

This paper applies a standard New Keynesian model to analyze the effects of monetary policy in the presence of a low natural rate of interest and a lower bound on interest rates. Under a standard inflation-targeting approach, inflation expectations will be anchored at a level below the inflation target, which in turn exacerbates the deleterious effects of the lower bound on the economy. Two key themes emerge from our analysis. First, the central bank can mitigate this problem of a downward bias in inflation expectations by following an average-inflation targeting framework that aims for above-target inflation during periods when policy is unconstrained. Second, a dynamic strategy such as price-level targeting that raises inflation expectations when inflation is low can both anchor expectations at the target level and potentially further reduce the effects of the lower bound on the economy.

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The adoption of inflation targeting by many central banks succeeded in bringing high and variable inflation rates of the 1970s and early 1980s under control and thereby anchoring inflation expectations at the targeted rate. The significant decline in the natural rate of interest observed in many countries over the past quarter-century implies that central banks are now likely to be constrained by the lower bound on nominal interest rates relatively frequently, interfering with their ability to offset negative shocks to the economy (see, for example, Laubach and Williams (2016) and Holston, Laubach and Williams (2017)). The experiences of many advanced economies over the past decade are a testament to this new reality. As a result, central banks now face the challenge of inflation expectations potentially being anchored at too *low* a level, rather than too high. In this paper, we investigate alternative monetary policy frameworks designed to anchor inflation expectations at the desired level even if the lower bound frequently constrains monetary policy actions.

We use a simple New Keynesian model as a laboratory for our analysis. The economy is governed by a Phillips curve that links inflation to a supply shock, the output gap, and expected future inflation and an IS-curve that links the output gap to a demand shock, the ex ante real interest rate, and expectations of the future output gap. The central bank sets the nominal interest rate to minimize the variability of the inflation rate and the output gap around their target values. We assume that the interest rate is the central bank's sole policy tool and abstract from unconventional policies such as asset purchases and quantitative easing. We start by assuming that the central bank follows optimal policy under discretion and then explore alternative policies that incorporate features designed to mitigate the deleterious effects of the lower bound.

Absent a lower bound on interest rates, the optimal monetary policy under discretion fully offsets demand shocks, partially offsets supply shocks, and anchors inflation expectations at the target level. This policy behaves like a standard textbook inflation-targeting policy. However, when a lower bound on interest rates constrains policy, interest rates will not be able to respond optimally to negative shocks to the economy and the output gap and inflation will be lower than would otherwise occur. Owing to the inherent asymmetry of the lower bound, the average inflation rate will be below the target rate, and inflation expectations will likewise be anchored at too low a level. This reduction in expected inflation further exacerbates the effects of the lower bound on the economy.

We contrast outcomes from the optimal policy under discretion with three main alternative approaches that seek to raise inflation expectations. Each of these requires some degree of commitment to future actions that a discretionary policymaker would not take. The first "dovish policy" alternative reduces the monetary policy responses to shocks in order to limit the asymmetry implied by the lower bound. The second "average-

inflation targeting" policy implicitly aims for above-target inflation when policy is unconstrained, thereby offsetting the effects of the lower bound on expected inflation. The third "price-level targeting" strategy, along with its offshoots, targets the price level rather than the inflation rate.

The main conclusion of this analysis is that all three of these approaches work through the same mechanism of raising the inflation rate above the target rate when policy is not constrained. But, some do so at a greater cost in terms of stabilizing inflation and the economy. In particular, we find that average-inflation targeting dominates the dovish policy strategies, which allow excessive pass-through of shocks to the economy. In addition, we find that dynamic strategies like price-level targeting dominate average-inflation targeting because the former create expectations of relatively high inflation and output gaps following periods when the lower bound is binding. Importantly, the success of all of these three approaches depends crucially on affecting private-sector expectations and therefore on both the credibility and the public's clear understanding of the policy.

1 Model economy and monetary policy frameworks

We augment the standard New Keynesian model as described, for example, in Clarida, Gali and Gertler (1999) to include a lower bound on interest rates. The model consists of three equations describing the evolution of three endogenous variables: the rate of inflation, π_t, the output gap, x_t, and the short-term nominal interest rate, i_t. Inflation is determined by a forward-looking Phillips curve

$$\pi_t = \mu_t + \kappa x_t + \beta \mathbb{E}_t \pi_{t+1}, \tag{1}$$

where $\mathbb{E}_t$ denotes mathematical expectations based on information at time t, μ_t is a supply shock, $\beta \in (0,1)$ is the discount factor, $\kappa > 0$, and $\mu_t \sim iid\ U(-\hat{\mu}, \hat{\mu})$. We assume that all shocks are uniformly distributed i.i.d over time and independent from each other. An IS-curve relationship describes the determination of the output gap

$$x_t = \epsilon_t - \alpha(i_t - \mathbb{E}_t \pi_{t+1} - r^*) + \mathbb{E}_t x_{t+1}, \tag{2}$$

where $\alpha > 0$, r^* is the long-run natural real rate of interest, ϵ_t is a demand shock, and $\epsilon_t \sim iid\ U(-\hat{\epsilon}, \hat{\epsilon})$. Calculations for the model are provided in the appendix.

The central bank's objective is to minimize the expected weighted sum of the squared values of the output gap and inflation rate. We assume a long-run inflation target of zero, but it is straightforward to extend the

analysis to alternative values of the inflation target. Specifically, the central bank sets the nominal interest rate, i_t, to minimize the expected quadratic loss:

$$\mathcal{L} = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \lambda x_t^2) \right],$$ (3)

where $\lambda \geq 0$ is the relative weight the central bank places on the stabilization of the output gap. The central bank decision for i_t is assumed to occur after the realizations of the shocks in the current period.

1.1 Optimal policy under discretion

Assuming that the lower bound does not constrain policy, optimal policy under discretion can be implemented by setting the nominal interest rate according to the following policy rule:

$$i_t^u = \theta_0 + \theta_\mu \mu_t + \theta_\epsilon \epsilon_t + \theta_E \mathbb{E}_t \pi_{t+1}.$$ (4)

The coefficient values describing the optimal policy under discretion are given by: $\theta_0 = r^*$, $\theta_\epsilon = \frac{1}{\alpha}$, $\theta_\mu = \frac{\kappa}{\alpha(\kappa^2+\lambda)}$, and $\theta_E = 1 + \frac{1}{\alpha\kappa} - \frac{\lambda\beta}{\alpha\kappa(\kappa^2+\lambda)}$. This policy fully offsets demand shocks and partially offsets supply shocks depending on the degree of concern for output stabilization in the central bank objective.

In this model, the lower bound alters the optimal policy under discretion only in that the interest rate is set to the lower bound when the unconstrained interest rate is below the lower bound. The optimal values of the coefficients of the policy rule are unaffected. That is, the realized interest rate is given by: $i_t = \max\{i^{\text{LB}}, i_t^u\}$. Under the model assumptions, we can combine the two equations to make the expression for inflation independent of the output gap. Plugging in the rule for interest rates results in two equations, one for when monetary policy is constrained

$$\pi_t = \mu_t + \kappa \epsilon_t - \alpha\kappa(i^{\text{LB}} - r^*) + (1 + \alpha\kappa)\mathbb{E}_t \pi_{t+1}$$

and one when it is unconstrained

$$\pi_t = \alpha\kappa(r^* - \theta_0) + (1 + \alpha\kappa(1 - \theta_E))\mathbb{E}_t \pi_{t+1}$$
$$+ (1 - \alpha\kappa\theta_\mu)\mu_t + \kappa(1 - \alpha\theta_\epsilon)\epsilon_t.$$

The constraint binds when the realization of the two shocks satisfies $\theta_\epsilon \epsilon_t + \theta_\mu \mu_t \leq i^{\text{LB}} - \theta_0 - \theta_E \mathbb{E}_t \pi_{t+1}$. With

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a policy rule of the form (4), we can solve the model analytically.

Under these assumptions, expected inflation in all future periods is constant and is below target if the lower bound ever constrains policy.[1] This downward bias in inflation expectations relative to the target stems from expectations taking into account future inflation rates under both constrained and unconstrained policy. The resulting reduction in inflation expectations in turn implies that the lower bound constrains policy more often and that monetary policy provides less stimulus when policy is constrained due to the higher resulting real interest rate when at the lower bound.

We now consider three alternative policy approaches that assume some form of commitment.

1.2 Dovish policy

As shown above, the lower bound on interest rates leads to below-target inflation expectations that, in turn, put downward pressure on inflation in the current period through the forward-looking Phillips curve. One way to mitigate this problem is for the central bank to follow a more "dovish" policy with smaller policy responses to shocks. In this way, the central bank can limit the asymmetry implied by the lower bound and the resulting reduction in inflation expectations.

A simple way to reduce the variance of interest rates would be for the central bank to impose an upper bound on interest rates. The central bank sets the nominal interest rate as it would under discretion but imposes an additional constraint preventing the interest rate from exceeding the upper bound.[2] For example, an upper bound symmetric to the lower bound around r^* fully eliminates the downward bias to inflation expectations. However, it achieves this by responding suboptimally to large positive shocks, which increases their pass-through to the economy. A similar, more nuanced approach is to reduce the overall response to shocks (see, e.g., Nakata and Schmidt (2016)). This works through the same mechanism as an upper bound on interest rates and increases expected inflation, but also at the cost of greater pass-through of shocks to the economy.

Either of these dovish policy approaches — the imposition of an upper bound or more muted overall responses to shocks — may change the central bank loss relative to the optimal discretionary policy in this model by reducing the downward bias to expected inflation relative to target. However, there are more direct

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[1]There are two steady-state equilibria, a target equilibrium and a liquidity trap equilibrium, as in Benhabib, Schmitt-Grohé and Uribe (2001) and Mertens and Williams (2018). As is standard in the literature and supported by the empirical analysis of Mertens and Williams (2018), we focus on the target equilibrium.

[2]In addition to the two steady-state equilibria under discretion, a third equilibrium associated with the interest rate at the upper bound emerges. We again restrict our analysis here to the target equilibrium.
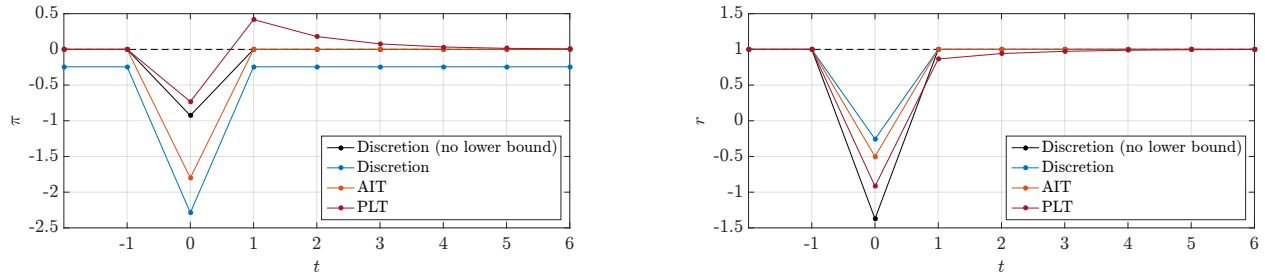
Figure 1: The above figures show the path of inflation (left panel) and real interest rate (right panel) in response to a negative supply shock $\mu_0 = -\mu$ for various monetary policy frameworks. "Discretion" refers to optimal monetary policy under discretion, "AIT" to average-inflation targeting such that inflation expectations are at target, and "PLT" to price-level targeting.

ways to achieve inflation expectations anchored at the target level, to which we now turn.

1.3 Average-inflation targeting

The second approach is to aim for above-target inflation whenever policy is unconstrained to offset the below-target inflation outcomes when policy is constrained. This average-inflation targeting framework can achieve the desired level of inflation expectations through an adjustment of the intercept of the policy rule, θ_0. In particular, a downward adjustment of the intercept raises inflation expectations, with the size of the adjustment needed to achieve zero expected inflation depending on the probability of a binding lower bound and the potential magnitude of negative shocks to the economy.

In fact, in this model it can be optimal to set the intercept below this level, thus pushing expected inflation above zero. This higher-than-target mean inflation rate further helps reduce the effects of the lower bound on the economy. Given this value of θ_0, it is optimal to respond to the shocks exactly as one would under the optimal discretionary policy. In this sense, the decisions on level and variance are separable. Taken together, the optimal average-inflation targeting policy dominates the dovish policy strategies. Note that the calibration of the optimal value of θ_0 requires detailed knowledge of the probabilities and associated costs of hitting the lower bound implied by the model.

1.4 Price-level targeting

The two alternative policy approaches described above illustrate how policies that raise inflation when the central bank is unconstrained can lift inflation expectations, with the beneficial side-effect of mitigating the effect of adverse shocks when the lower bound binds. The same logic suggests that a more refined approach

can yield even better outcomes by aiming to raise inflation expectations when inflation is running below the target rate, say due to the effects of the lower bound. These dynamic strategies are the focus of price-level targeting policies and their variants such as temporary price-level targeting and nominal GDP targeting.

An example of such a dynamic strategy is a policy rule that responds to the price level,

$$i_t^{u\text{PL}} = \theta_0 + \theta_p p_t + \theta_\mu \mu_t + \theta_\epsilon \epsilon_t + \theta_E \mathbb{E}_t^p \pi_{t+1},$$

where $\theta_p > 0$ and the log of the price level p_t evolves according to $p_t = p_{t-1} + \pi_t$. As a result, inflation expectations become a function of the price level $\mathbb{E}_t^p \pi_{t+1} = \mathbb{E}_t[\pi_{t+1}|p_t]$.

To solve the model with this policy rule, we substitute the interest rate rule into the two equations for the economy. We then take conditional expectations of these two model equations (conditional on the price level). We approximate expectations for future inflation and output gaps as a function of the price level and iterate backwards until the process converges and the fixed points for conditional expectations functions emerge.

Unlike the optimal discretionary policy, the price-level targeting policy delivers mean inflation equal to the target rate. Inflation expectations are state-dependent with their mean at the target rate, reflecting the fact that the policy delivers above-target inflation when policy is unconstrained by the lower bound. This result does not require detailed knowledge of model parameters, as was the case for average-inflation targeting, but relies on the nature of a price-level target that does not treat bygones as bygones in terms of past misses of the inflation target. In addition, because this policy raises inflation expectations during periods when the lower bound constrains policy, it lowers the real interest rate during those periods, thereby reducing the effects of the lower bound.

One variant of price-level targeting, called temporary price-level targeting, imposes a price-level target only following an episode when the lower bound constrains policy (see Bernanke (2017)). The main benefit of this policy is that it aims to raise inflation and output gap expectations when the lower bound constrains policy in the same way that standard price-level targeting does. Unlike standard price-level targeting, it does not automatically deliver mean inflation at the target rate. This is because of the asymmetric nature of the policy rule, which introduces a second source of asymmetry into the model. To achieve a mean inflation rate at the target rate, the intercept of the rule, θ_0, must be calibrated to take into account the effects of the lower bound and the asymmetric nature of the temporary price-level target.

1.5 Comparison of policies

The various aspects of the analysis are best illustrated with a concrete numerical example of the model. Therefore, we set $\beta = 0.99$, $\lambda = 0.25$, $\alpha = 1.25$, $\kappa = 0.8$, $r^* = 1\%$, $i^{LB} = -0.5\%$, and $\hat{\mu} = 3.3\%$. The standard deviation of the demand shock is set to zero.[3] Under the optimal discretionary policy, the probability of hitting the lower bound is about 27%. Inflation expectations are at -0.24% and thus below target.

Figure 1 shows the mechanism by which the various monetary policy frameworks affect inflation expectations. The left panel plots the paths of inflation in response to a negative supply shock at time 0, $\mu_0 = -\hat{\mu}$, taking the unconditional expectation of all shocks for future periods. The right panel shows the same calculations for the path of the real interest rate, defined as the nominal interest rate net of expected inflation. The black lines show the responses under the optimal discretionary policy absent a lower bound on interest rates.

Under the optimal discretionary policy with a lower bound (blue line), the shock causes inflation to drop significantly further below target. In future periods, inflation equals its unconditional mean. The average-inflation targeting policy is assumed to have an intercept of 0.90, which achieves an unconditional mean inflation of zero. Under this policy, the decline in inflation is somewhat smaller than under the optimal discretionary policy supported by a sharper decline in the real interest rate. Inflation again rebounds to its mean immediately. For this calibration, the optimal mean inflation rate is 0.10%, which implies an intercept of 0.85. Except for the slightly higher mean inflation rate, the resulting simulation is very similar to the one shown.

The price-level targeting policy differs more significantly from the other two policies. For this exercise, we used a value for θ_p of 0.36, the value that minimizes the central bank loss for this calibration of the model. Inflation exceeds the target rate in all future periods due to the real interest rate staying lower than the natural rate. This "lower-for-longer" policy boosts expectations of future output gaps and inflation (see Reifschneider and Williams (2000)).

Figure 2 shows the social loss for the various policies associated with this model calibration. As seen in this chart, getting the average inflation right delivers benefits in terms of the central bank loss. Adding a moderate response to the price level produces additional benefits owing to the state-dependence of expectations.
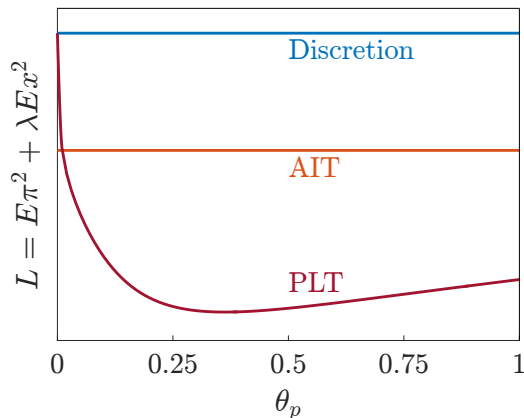
Figure 2: The above graph shows the social loss for different responses to the price-level target. Note that the price-level target does not appear in the interest rate rule for policy under discretion and average-inflation targeting.

2 Conclusion

This paper applies a standard New Keynesian model to analyze the effects of monetary policy in the presence of a low natural rate of interest and a lower bound on interest rates. Under a standard inflation-targeting approach, inflation expectations will be anchored at a level below the inflation target, which in turn exacerbates the deleterious effects of the lower bound on the economy. Two key themes emerge from our analysis. First, the central bank can correct for the downward bias in inflation expectations by following an average-inflation targeting framework that aims for above-target inflation during periods when policy is unconstrained. The resulting policy rule is equivalent to the one under discretion with a lower natural rate of interest than its true value. Second, a dynamic strategy such as price-level targeting that raises inflation expectations when inflation is low can both anchor expectations at the target level and potentially further reduce the effects of the lower bound on the economy.

Each of these alternative policy strategies works through its effects on expectations of future interest rates, the output gap, and inflation. In addition, each requires a commitment to take future policy actions that a future policymaker would prefer not to follow. Moreover, for inflation-targeting and temporary price-level targeting policies to be successful in anchoring inflation expectations at the desired level requires knowledge of the effects of the lower bound on the economy. Therefore, for any of these frameworks to work as well in practice as they do in theory requires clear communication and consistent execution of the policy and a belief by the public that the policy is credible.

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[3]The results for supply shocks presented here carry over to case of demand shocks.

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A Appendix

A.1 Derivation of inflation equations

Combining equations (1) and (2) from the New Keynesian model yields the equation for inflation

$$\pi_t - \mathbb{E}_t\pi_{t+1} = \mu_t + \kappa(\epsilon_t - \alpha(i_t - \mathbb{E}_t\pi_{t+1} - r^*))) + \beta\mathbb{E}_t(\pi_{t+1} - \pi_{t+2}).$$

With the interest rate rules of the form (4), the final term is zero and inflation is determined by

$$\pi_t = (1 + \alpha\kappa)\mathbb{E}_t\pi_{t+1} + \mu_t + \kappa\epsilon_t - \alpha\kappa(i_t - r^*).$$

Plugging in the interest rate rule from equation (4) for when the central bank is unconstrained, delivers inflation of the form

$$\pi_t = \alpha\kappa(r^* - \theta_0) + (1 + \alpha\kappa(1 - \theta_E))\mathbb{E}_t\pi_{t+1} + (1 - \alpha\kappa\theta_\mu)\mu_t + \kappa(1 - \alpha\theta_\epsilon)\epsilon_t. \tag{5}$$

In the case where the interest rate rule would ask for nominal rates below the lower bound, the central bank sets the policy rate as low as possible, to i^{LB}

$$\pi_t = \mu_t + \kappa\epsilon_t - \alpha\kappa(i^{LB} - r^*) + (1 + \alpha\kappa)\mathbb{E}_t\pi_{t+1}. \tag{6}$$

These two equations in the appendix are used in the main body of the paper. Note that they imply the existence of two steady-state equilibria in the deterministic version of the model. In both equation, inflation expectations and inflation rates appear linearly. Hence each equation may be associated with a steady-state equilibrium. We refer to the equilibrium associated with the first equation to the "target equilibrium" and the one associated with the second equation as a "liquidity trap".

A.2 Derivation of inflation expectations

With demand or supply shocks, the lower bound can become an occasionally binding constraint. In this case, both equations for inflation (5) and (6) have to be used to determine inflation expectations

$$\mathbb{E}\pi = \text{Prob}\left(i_t^{opt} < i^{LB}\right)\mathbb{E}\left[\pi_t | i_t^{opt} < i^{LB}\right] + \text{Prob}\left(i_t^{opt} \geq i^{LB}\right)\mathbb{E}\left[\pi_t | i_t^{opt} \geq i^{LB}\right]. \tag{7}$$

For this equation, we drop the period t subscript from the expectations operator. In this model with the specified monetary policy rule, there is no information at time t that predicts period $t + 1$ inflation and therefore conditional and unconditional expectations are identical.

For illustrative purposes, we drop the demand shock from the model, i.e., we set its variance to zero. Then the constraint on nominal interest rates binds when μ_t falls below a cutoff value $\bar{\mu} = \frac{1}{\theta_\mu}(i^{LB} - \theta_0 - \theta_E \mathbb{E}\pi_{t+1})$.

There are three different cases: The cutoff value $\bar{\mu}$ can fall below, in, or above the support of the distribution for the supply shock. The probability of being constrained by the lower bound can thus be expressed as

$$\text{Prob}\left(i_t^{opt} < i^{LB}\right) = \begin{cases} 1 & \text{if} \quad -\bar{\mu} \leq -\hat{\mu} \\ \frac{1}{2\hat{\mu}}(\hat{\mu} + \bar{\mu}) & \text{if} \quad -\hat{\mu} < -\bar{\mu} < \hat{\mu} \\ 0 & \text{if} \quad -\bar{\mu} \geq \hat{\mu}. \end{cases}$$

If we plug this expression in equation (7) and compute conditional expectations of inflation from equations (5) and (6) respectively, we get inflation expectations as

$$\mathbb{E}\pi_t = \begin{cases} -\alpha\kappa(i^{LB} - r^*) + (1 + \alpha\kappa)\mathbb{E}\pi_{t+1} & \text{if} \quad -\bar{\mu} \leq -\hat{\mu} \\ -\frac{\alpha\kappa}{4\hat{\mu}}\theta_\mu(\bar{\mu} + \hat{\mu})^2 + (1 + \alpha\kappa(1 - \theta_E))\mathbb{E}\pi_{t+1} - \alpha\kappa(\theta_0 - r^*) & \text{if} \quad -\hat{\mu} < -\bar{\mu} < \hat{\mu} \\ -\alpha\kappa(\theta_0 - r^*) + (1 + \alpha\kappa)\mathbb{E}\pi_{t+1} & \text{if} \quad -\bar{\mu} \geq \hat{\mu}. \end{cases}$$

The expression in the middle where the constraint is occasionally binding is of particular interest. The cutoff for the supply shock that depends linearly on inflation expectations appears quadratically. To find a steady-state equilibrium for inflation expectations, we need to solve a quadratic equation which results in two solutions for a parameter range.[4]

A.3   Inflation expectations in the presence of an upper bound

The derivation of inflation expectations for the case where both a lower and upper bound are present follows the same steps as in Appendix A.2. Due to the additional constraint, however, the list of distinct cases increases. With various conditions $\mathbb{C}^{LB}$ and $\mathbb{C}^{UB}$ on the lower and upper bounds, respectively, we distinguish

[4]For cases of a single equilibrium or non-existence, see Mertens and Williams (2018).

the cases:

$$
\mathbb{E}[\pi_t] = \begin{cases}
(1 + \alpha\kappa)\mathbb{E}[\pi_{t+1}] - \alpha\kappa(i^{\mathrm{LB}} - r^*) & \text{if } \mathbb{C}_c^{\mathrm{LB}} \\[3ex]
\begin{aligned}
&\frac{\alpha\kappa}{4\theta_\mu\hat{\mu}}\left(i^{\mathrm{UB}} + i^{\mathrm{LB}} - 2\theta_0 - 2\theta_E\mathbb{E}[\pi_{t+1}]\right)\left((i^{\mathrm{UB}} - i^{\mathrm{LB}}) - 2\theta_\mu\hat{\mu}\right) + \\[1ex]
&\quad + (1 + \alpha\kappa(1 - \theta_E))\mathbb{E}[\pi_{t+1}] + \alpha\kappa(r^* - \theta_0)
\end{aligned} & \text{if } \mathbb{C}_o^{\mathrm{LB}} \text{ and } \mathbb{C}_o^{\mathrm{UB}} \\[4ex]
\begin{aligned}
&-\frac{\alpha\kappa\theta_\mu}{4\hat{\mu}}\left(\hat{\mu} + \frac{1}{\theta_\mu}\left(i^{\mathrm{LB}} - \theta_0 - \theta_E\mathbb{E}[\pi_{t+1}]\right)\right)^2 + \\[1ex]
&\quad + (1 + \alpha\kappa(1 - \theta_E))\mathbb{E}[\pi_{t+1}] + \alpha\kappa(r^* - \theta_0)
\end{aligned} & \text{if } \mathbb{C}_o^{\mathrm{LB}} \text{ and } \mathbb{C}_u^{\mathrm{UB}} \\[4ex]
(1 + \alpha\kappa)\mathbb{E}[\pi_{t+1}] - \alpha\kappa(i^{\mathrm{UB}} - r^*) & \text{if } \mathbb{C}_c^{\mathrm{UB}} \\[3ex]
\begin{aligned}
&\frac{\alpha\kappa\theta_\mu}{4\hat{\mu}}\left(\hat{\mu} - \frac{1}{\theta_\mu}\left(i^{\mathrm{UB}} - \theta_0 - \theta_E\mathbb{E}[\pi_{t+1}]\right)\right)^2 + \\[1ex]
&\quad + (1 + \alpha\kappa(1 - \theta_E))\mathbb{E}[\pi_{t+1}] + \alpha\kappa(r^* - \theta_0)
\end{aligned} & \text{if } \mathbb{C}_u^{\mathrm{LB}} \text{ and } \mathbb{C}_o^{\mathrm{UB}} \\[4ex]
(1 + \alpha\kappa(1 - \theta_E))\mathbb{E}[\pi_{t+1}] + \alpha\kappa(r^* - \theta_0) & \text{if } \mathbb{C}_u^{\mathrm{LB}} \text{ and } \mathbb{C}_u^{\mathrm{UB}}
\end{cases}
$$

The various conditions determine whether a constraint never binds, $\mathbb{C}_u^{\cdot}$, always binds, $\mathbb{C}_c^{\cdot}$, or occasionally binds, $\mathbb{C}_o^{\cdot}$. The specific conditions on the lower bound are

$$
\mathbb{C}_u^{\mathrm{LB}} = \left\{\frac{1}{\theta_\mu}(i^{LB} - \theta_0 - \theta_E E[\pi_{t+1}]) < -\hat{\mu}\right\}
$$

for the lower bound to never bind,

$$
\mathbb{C}_o^{\mathrm{LB}} = \left\{-\hat{\mu} \leq \frac{1}{\theta_\mu}(i^{LB} - \theta_0 - \theta_E E[\pi_{t+1}]) \leq \hat{\mu}\right\}
$$

for the lower bound to occasionally bind, and

$$
\mathbb{C}_c^{\mathrm{LB}} = \left\{\frac{1}{\theta_\mu}(i^{LB} - \theta_0 - \theta_E E[\pi_{t+1}]) > \hat{\mu}\right\}
$$

for the lower bound to always bind.

For the upper bound, the conditions are

$$\mathbb{C}_u^{\text{UB}} = \left\{ \frac{1}{\theta_\mu}(i^{UB} - \theta_0 - \theta_E E[\pi_{t+1}]) > \hat{\mu} \right\}$$

for the upper to never bind,

$$\mathbb{C}_o^{\text{UB}} = \left\{ -\hat{\mu} \leq \frac{1}{\theta_\mu}(i^{UB} - \theta_0 - \theta_E E[\pi_{t+1}]) \leq \hat{\mu} \right\}$$

for the upper bound to occasionally bind, and

$$\mathbb{C}_c^{\text{UB}} = \left\{ \frac{1}{\theta_\mu}(i^{UB} - \theta_0 - \theta_E E[\pi_{t+1}]) < -\hat{\mu} \right\}$$

for the upper bound to always bind.

When solving for inflation expectations, a third equilibrium besides the target equilibrium and the liquidity trap emerges. This equilibrium is associated with the upper bound on nominal interest rates. As in the case with only a lower bound, we restrict our analysis to the target equilibrium.

A.4   Comparison of policies

Table 1 shows a comparison of various statistics from the simulations using the different policies.

	Discretion		Dovish Policies		AIT		PLT
	No lower bound		Symmetric upper bound	Optimal θ_μ	$E\pi = 0$	Optimal θ_0	Optimal θ_p
$\mathbb{E}(\pi_t)$	0.000	-0.244	0.000	-0.085	0.000	0.104	0.000
$\mathbb{V}(\pi_t)$	0.287	0.675	0.642	0.668	0.501	0.446	0.190
$\mathbb{E}(x_t)$	0.000	-0.003	0.000	-0.001	0.000	0.001	0.002
$\mathbb{V}(x_t)$	2.934	2.053	2.035	1.912	2.381	2.501	2.788
$\mathbb{E}(\pi_t^2) + \lambda\mathbb{E}(x_t^2)$	1.020	1.248	1.150	1.153	1.096	1.082	0.887
$\mathbb{P}(i_t = i^{LB})$	——	0.273	0.184	0.172	0.205	0.178	0.076
$\mathbb{E}(\pi_t \| i_t = i^{LB})$	——	-1.389	-1.193	-1.402	-1.124	-1.005	-0.652
$\mathbb{E}(\pi_t \| i_t > i^{LB})$	0.000	0.185	0.269	0.189	0.290	0.344	0.054
θ_0	1.000	1.000	1.000	1.000	0.900	0.850	1.000
θ_μ	0.719	0.719	0.719	0.626	0.719	0.719	0.719
θ_E	1.722	1.722	1.722	1.722	1.722	1.722	1.722
θ_p	0.000	0.000	0.000	0.000	0.000	0.000	0.360

Table 1: The above table shows various statistics for the simulations discussed in the main body of the paper.